

# Error modeling of multibaseline optical truss, part 1: Modeling of system level performance

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## ABSTRACT

Global astrometry is the measurement of stellar positions and motions. These are typically characterized by five parameters, including two position parameters, two proper motion parameters, and parallax. The Space Interferometry Mission (SIM) will derive these parameters for a grid of approximately 1300 stars covering the celestial sphere to an accuracy of approximately 4uas, representing a two orders of magnitude improvement over the most precise current star catalogues. Narrow angle astrometry will be performed to a 1uas accuracy. A wealth of scientific information will be obtained from these accurate measurements encompassing many aspects of both galactic and extragalactic science. SIM will be subject to a number of instrument errors that can potentially degrade performance. Many of these errors are systematic in that they are relatively static and repeatable with respect to the time frame and direction of the observation. This paper and its companion define the modeling of the contributing factors to these errors and the analysis of how they impact SIM's ability to perform astrometric science.

## 1. INTRODUCTION

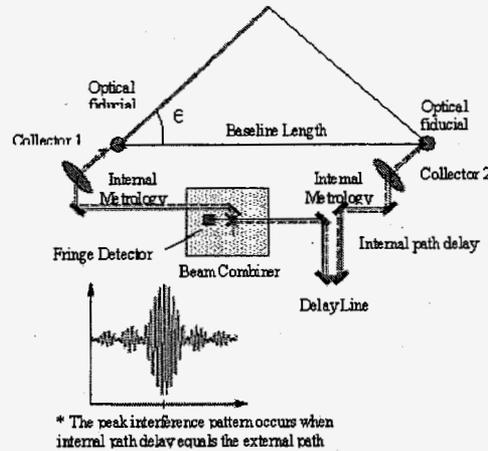
Global astrometry is the measurement of stellar positions and motions. These are typically characterized by five parameters, including two position parameters, two proper motion parameters, and parallax. The Space Interferometry Mission (SIM) will derive these parameters for a grid of approximately 1300 stars covering the celestial sphere to an accuracy of approximately 4uas [3]. This represents a two orders of magnitude improvement over the most precise current star catalogues. A wealth of scientific information is obtained by these accurate measurements, including the formation and dynamics of our galaxy, calibration of the cosmic distance scale, and a host of fundamental astrophysics. In addition to global astrometry, SIM will perform narrow angle astrometric measurements to 1uas accuracy. This will enable a deep survey of our local neighborhood of about 250 stars for planets of just a few ( $3\times$ ) Earth masses, and a more extensive survey of about 2000 stars for the detection of larger planets ( $15\times$  Earth mass).

Classical ground based global astrometry is performed by piecing together stellar images of a field of view of a few degrees. This stitching together is possible because objects belong to multiple fields of view, and from these a consistent set of position equations may be obtained. A number of auxiliary parameters are typically estimated in this stitching process to correct for field aberrations. Second and third order polynomial corrections are commonly used [4], [7]. The Hubble telescope also performs astrometric measurements and the coefficients of a fifth order polynomial in two variables are used for correction [1]. In contrast SIM enjoys a large field of regard (15deg) and makes one dimensional measurements. Within the process of solving for the astrometric parameters from the basic observable (the so-called regularized delay measurement) an analogous-polynomial is corrected. This polynomial is of the form  $p(x, y) = p_0 + p_x x + p_y y + p_r(x^2 + y^2)$ . In this sense SIM is a very pristine astrometric instrument that simultaneously enjoys a very large field of regard with very little aberration when compared with classical astrometry. However, SIM is not a perfect instrument, and it is subject to a number of errors that compromise its performance. Many of these errors are very repeatable and are a function of how the instrument is pointing with respect to the center of the

field of regard. Errors of this class are termed "field dependent". A representative sample of these include imperfect corner cubes, diffraction effects associated with moving delay lines, reflection phase shift changes as interrogating metrology beams change their angle of incidence, noncoincident vertices on multiple corner cubes, and others. This paper and its companion address the modeling and analysis issues of many errors of this type and how they affect SIM astrometric performance.

## 2. THE SIM REGULARIZED DELAY MEASUREMENT

A complete description of the process of the fundamental delay measurement made by the SIM instrument can be found in [5]. Here will give a very brief overview of this process to keep the paper relatively self-contained. The astrometric observations made by SIM require three white light Michelson interferometer measurements coupled with various metrology measurements to monitor changes in the distances between a set of fiducials that define the interferometer baseline vectors. The fiducial points correspond to the vertices of high precision corner cubes. Two of the interferometers lock onto bright guide stars to make precise measurements of changes in the attitude of the instrument. The third interferometer is the science interferometer that makes the delay measurement which is the projection of its baseline vector onto the unit direction vector of the science target. The external metrology system (there is also an internal metrology system) tracks changes in the three interferometer baseline vectors relative to each other as well as changes in the baseline lengths.



The delay measurement equation for the science interferometer is (see figure)

$$d = \langle s, B \rangle + C + \eta, \quad (1)$$

where  $d$  is the measured "external" delay (accomplished by white light fringe measurements and an internal metrology measurements),  $s$  is the star direction unit vector,  $B$  is the interferometer baseline vector,  $C$  is the interferometer constant term, and  $\eta$  is the noise in the measurement (encompassing both the white light noise and internal metrology measurement noise). The astrometric objective is to determine the unknown star direction vector. However, all of the other quantities on the right side of this equation are also unknown. Thus SIM cannot directly measure even the 1-D projection of a star vector. SIM circumvents this difficulty by observing multiple stars with the same baseline vector and constant term within its  $15^\circ$  field of regard:

$$d_i = \langle s_i, B \rangle + C + \eta, \quad i = 1, \dots, N. \quad (2)$$

The set of stars within the field of regard is referred to as a tile. SIM then revisits the tile with a new baseline vector and constant term:

$$d_{ij} = \langle s_i, B_j \rangle + C_j + \eta_j, \quad i = 1, \dots, N, \quad j = 1, \dots, M. \quad (3)$$

At this point we now have  $MN$  equations with  $2N + 4M$  unknowns (each star vector has two unknown parameters while the baseline vector and constant term require at most four parameters). Eventually  $MN \geq 2N + 4M$  so that a well (or over) determined system of equations results.

These equations were formed under the assumption that the baseline vector  $B$  is inertially fixed while the measurements are being made. While this is true at the arcsec level, it is definitely not true at the uas level, which is what is required of SIM, i.e.  $B(t)$  is in fact time-varying. The role of the external metrology subsystem and the guide interferometers is to compensate for this time-varying behavior. In [5] it is shown that so long as the guide interferometers are locked onto their guide stars and the external metrology subsystem is operating continuously, then an estimate of the science baseline vector,  $\hat{B}$ , can be constructed so that

$$B(t) - \hat{B}(t) = \delta B, \quad (4)$$

where  $\delta B$  is an unknown, but *constant* vector. Introducing the *a priori* estimates of the star position vectors,  $s_i^0$  and writing  $s_i = s_i^0 + \delta s_i$ , with the  $\delta s_i$  the unknown correction, the astrometric equations more correctly have the form

$$d_i - \langle s_i^0, \hat{B} \rangle = \langle \hat{B}, \delta s_i \rangle + \langle s_i^0, \delta B \rangle + C + \eta. \quad (5)$$

This expression is very close to (2), but the unknowns are now the correction terms  $\delta B$  and  $\delta s_i$ , and the measurement appearing on the left side above is the so-called "regularized delay" [2].

A number of different error sources enter into the right side of the equation above. Many of these are second order effects that are kept small by alignment and stability requirements. The focus of this paper is on the class of errors that are repeatable as a function of the position of the target star within the instrument's field of regard. The mechanisms that produce these errors are all connected with the instrument components that must move to acquire a new target in the field. These include, most prominently, the linear motion of the optical delay line and the rotation of the siderostats. The maximum delay line motion results in a change on the order of a meter of the internal metrology pathlength. This introduces a deviation, due to diffraction effects, of several nanometers between the geometric change in distance and the distance that is measured by metrology. The rotation of the siderostats introduce a number of effects. Firstly, because the corner cubes are imperfect (they have dihedral error and are subject to reflection phase shifts produced by polarization), the metrology gauges measure these effects as well as the sought after changes in geometric pathlength between the fiducials. Another effect is the offset between the corner cubes and the surface of the siderostats. Ideally the pathlength measured by the metrology beam should coincide (or at least have a constant difference) with the pathlength observed by the starlight. However, when there is an offset, the distances traversed by the two signals changes as the siderostat rotates.

To incorporate these field dependent errors into the basic astrometric system of equations we locally introduce a coordinate system  $(u, v, z)$  such that the science interferometer baseline vector is nominally aligned with the  $u$  coordinate and the center of the field corresponds to the unit vector  $(0, 0, 1)$ . In this coordinate system any star in the field of regard has coordinates  $(u, v, \sqrt{1 - u^2 - v^2})$  with  $\sqrt{u^2 + v^2} \leq \sin(7.5\pi/180)$ . The field dependent errors can then be introduced via a function  $C(u, v)$ . As a convenience the constant term contribution may also be absorbed into this function so that (5) takes the form

$$d_i - \langle s_i^0, \hat{B} \rangle = \langle \hat{B}, \delta s_i \rangle + \langle s_i^0, \delta B \rangle + C(u, v) + \eta. \quad (6)$$

A very important observation made in [6] is that since the unknown baseline compensation vector  $\delta B$  has the parameterization  $\delta B = \delta B_u u + v \delta B_v + \delta B_z \sqrt{1 - u^2 - v^2}$  with the constant terms  $\delta B_i$  unknown, these parameters are *indistinguishable* from the low order polynomial components

$$C(u, v) = c_0 + c_u u + c_v v + c_r (u^2 + v^2) + \text{HOT}. \quad (7)$$

(Here we have used the binomial expansion  $\sqrt{1 - u^2 - v^2} \approx 1 - (u^2 + v^2)/2$  to make the identification.)

### 3. FIELD DEPENDENT ERROR METRICS

In this section we will outline a general strategy for characterizing the field dependent error. There are two keys to this development. The first is the parameterization of the field dependent errors via Zernike polynomials and the recognition that the constant term, linear term, and radial quadratic term as discussed above are automatically compensated for in the grid solution, and thus *do not contribute an astrometric error*. This observation may be extended via a philosophy that the baseline vector is just an instrument parameter giving us license to introduce higher order Zernike terms as unknown parameters that are also solved for in the grid. The second key is the recognition that many of the errors that we do know have a parametric form  $C(u, v, p)$  where  $p$  is a parameter vector of errors (e.g., dihedral error, incorrect complex index, a misalignment term, etc.) This characterization is taken one step further via a linearization assumption

$$C(u, v, p) = C(u, v, 0) + \frac{\partial C(u, v, 0)}{\partial p} p + O(|p|^2) \quad (8)$$

with  $C(u, v, 0) = 0$  so that

$$C(u, v, p) = \frac{\partial C(u, v, 0)}{\partial p} p + O(|p|^2). \quad (9)$$

For now we will assume that the  $O(|p|^2)$  is sufficiently small so that it has negligible impact on the astrometric equation solution. And in a slight abuse of notation we will write the linearization as  $C(u, v, p) \approx C(u, v, p)$ .

If  $p$  is an  $r$ -vector, then  $C(u, v)$  is an  $r$ -vector valued function with components  $C_i(u, v)$ . Each component can be represented as a sum of Zernike polynomials over the field of regard. Thus we have

$$C_i(u, v) = \sum_j c_{ij} Z_{ij}(u, v), \quad (10)$$

where each  $Z_{ij}$  is a Zernike polynomial with coefficient  $c_{ij}$ . Hence the total error becomes

$$C(u, v, p) = \sum_i \sum_j c_{ij} Z_{ij} p_i. \quad (11)$$

Now that we have a convenient form for the field dependent error in terms of orthogonal polynomials and the error parameter vector, there are many metrics that can be developed to capture various aspects of instrument performance. Below we will give a representation that focuses on the "highest" level of performance — the astrometric error.

This analysis is facilitated by the observation that the astrometric equations derived from the set of regularized delay measurements (6) are weakly nonlinear in the astrometric parameters of position, proper motion and parallax. (This follows because the *a priori* values are quite good so that the linearized equation is in general an excellent approximation.) Thus as an analysis tool to characterize the effects of various field dependent instrument errors the linear model is more than adequate. Hence, each polynomial field dependent error  $Z_{ij}(u, v)$ , yields an astrometric error vector  $X_{ij}$  containing all of the astrometric parameter errors (position, proper motion, and parallax) for each star in the grid. By linearity we can form a basis set for the particular class of parameter error via

$$X_i = \sum_j c_{ij} Z_{ij}. \quad (12)$$

And the total error for a given parameter vector is just a linear combination of these basis errors, i.e. the total error vector  $E$  resulting from a given parameter vector  $p$  is simply

$$E = \sum_i p_i X_i, \quad p = (p_1, \dots, p_r). \quad (13)$$

A very reasonable metric is then  $|E|^2$  which is computed as

$$\begin{aligned}
 |E|^2 &= \left\langle \sum_i p_i X_i, \sum_j p_j X_j \right\rangle \\
 &= \sum_{i,j} p_i p_j \langle X_i, X_j \rangle \\
 &= \langle Xp, p \rangle
 \end{aligned} \tag{14}$$

where  $X$  is the  $r \times r$  matrix  $X_{ij} = \langle X_i, X_j \rangle$ . Thus the error for any parameter vector  $p$  is given by the quadratic form in (14). And because  $X$  is symmetric it follows that

$$\max_{|p|=1} |E|^2 = |X|. \tag{15}$$

This formula needs a slight modification, however, since the components of  $p$  need to be scaled when they represent different physical phenomena, e.g. dihedral error versus complex index error. To allow for this we introduce a scaling matrix  $D$  and compute

$$\begin{aligned}
 \max_{|Dp|=1} |E|^2 &= \max_{|Dp|=1} \langle Xp, p \rangle \\
 &= \max_{|p'|=1} \langle XD^{-1}p', D^{-1}p' \rangle \\
 &= |D^{-T}XD^{-1}|.
 \end{aligned} \tag{16}$$

This bound is sharp in that there is a parameter value  $p^*$  that attains the maximum value. (This vector is the eigenvector associated with the largest eigenvalue of  $D^{-T}XD^{-1}$ .) The spread of the eigenvalues of  $D^{-T}XD^{-1}$  indicates how conservative (likely) this bound is. The stochastic version of the metric treats  $p$  as a random vector. In this case the variance is the sum of the eigenvalues, or more simply the trace of  $D^{-T}XD^{-1}$ .

The metric just defined is very high level since it reaches to the top of the error budget. A metric that is perhaps more amenable to the design phase of the instrument (and not dependent on mission scenarios) is the mean square error within the field of regard. This error is

$$|E|^2 = \frac{1}{A} \iint |C(u, v, p)|^2 dudv, \tag{17}$$

where the integration is performed over the field of regard (a circle of radius  $7.5\pi/180$ ), and  $A$  is the area. This metric is investigated fully in Part 2 of the paper.

#### 4. AN EXAMPLE – MAPPING SINGLE GAUGE EXTERNAL METROLOGY ERRORS TO DELAY ERROR

We have previously described several mechanisms that introduce field dependent delay errors. Many of these involved single gauge external metrology errors (e.g., non-common vertex error, dihedral error, reflection phase shift errors). In this section we will derive the mapping that takes a single gauge error into a delay error. This mapping is the departure point for the more detailed investigations in Part 2 of the paper.

Given a set of  $N$  fiducials in 3-space with nominal positions  $X_i^0 \in R^3$ ,  $i=1, \dots, N$ , we define the  $3N$  vector  $X$  by  $X = (X_1^T, \dots, X_N^T)^T$ . The relative metrology measurement between fiducial  $i$  and fiducial  $j$  is denoted by  $l_{ij}$  where  $j > i$ . We associate with each measurement  $l_{ij}$  the function  $f_{ij} : R^{3N} \rightarrow R$ :

$$f_{ij}(X) = |X_j - X_i| - |X_j^0 - X_i^0|, \tag{18}$$

and the concatenated function  $F : R^{3N} \rightarrow R^M$  formed from stacking the  $f_{ij}$ 's. The rows of the differential of  $F$ , denoted  $F'(X^0)$  are formed from the gradients  $\nabla f_{ij}(X^0)$ :

$$\nabla f_{ij}(X^0) = [0_{1,3(i-1)}, (X_j^0 - X_i^0)^T / |X_j^0 - X_i^0|, 0_{1,3(j-i-1)}, -(X_j^0 - X_i^0)^T / |X_j^0 - X_i^0|, 0_{1,3(N-j)}]. \quad (19)$$

where  $0_{st}$  is the zero matrix with  $s$  rows and  $t$  columns. Thus  $F'(X^0)$  is an  $M \times 3N$  matrix. In general  $F'(X^0)$  will have a nontrivial null space that is spanned by the six rigid body modes of the system. Hence, a necessary condition for  $F'(X^0)$  to have full rank once these rigid body modes have been removed is that  $M \geq 3N - 6$ . The SIM optical truss is redundant so that  $M > 3N - 6$ . Now let  $U$  denote a matrix with orthonormal columns, each of which is orthogonal to the rigid body modes. We seek the solution to the least squares problem

$$\min_{\alpha} |F'(X^0)U\alpha - l|^2, \quad (20)$$

to form the fiducial displacement vector  $V \equiv U\alpha$ . The linearized solution is given simply as  $V = [F'(X^0)]^\dagger l$  where  $[F'(X^0)]^\dagger$  denotes the pseudoinverse of  $F'(X^0)$ . For small displacements ( $|l| < 10^{-7}$  m is easily good enough) the linearized solution yields subpicometer accuracy in determining the displacement vector.

The SIM truss consists of six fiducial positions so that  $N = 6$  in the analysis above. The active science baseline vector  $B_s$  is defined by the difference  $X_6 - X_1$  and the guide baseline vector  $B_g$  is the difference  $X_5 - X_2$ . Thus we may pick off the science and guide baseline vectors from the concatenated vector  $X$  via the mappings  $Z_s$  and  $Z_g$  defined

$$B_s = Z_s X, \quad Z_s = [-I_{3 \times 3} \quad 0_{3 \times 12} \quad I_{3 \times 3}], \quad (21)$$

and

$$B_g = Z_g X, \quad Z_g = [0_{3 \times 3} \quad -I_{3 \times 3} \quad 0_{3 \times 6} \quad I_{3 \times 3} \quad 0_{3 \times 3}] \quad (22)$$

where again  $0_{st}$  denotes the  $s \times t$  zeros matrix and  $I_{s \times s}$  is the  $s \times s$  identity matrix.

So for example, the effect of an external metrology error vector  $\delta l$  on the computed primary science baseline vector is

$$\delta B_s = Z_s [F'(X)]^\dagger \delta l. \quad (23)$$

Next recall that the instantaneous delay of the science interferometer is given by the basic astrometric equation (sans the constant term which does not contribute to the first order error analysis)

$$d = \langle s, B_s \rangle. \quad (24)$$

We will compute the error in  $d$  due to an error in the external metrology measurement. To compute this we must recapitulate some of the analysis in [5].

The purpose of the guide interferometers and external metrology are to provide the observables from which an estimate of  $B_s$  can be constructed. Call this estimate  $\hat{B}_s$ . The delay error  $\delta$  is defined as the difference

$$\delta = \langle s, B_s - \hat{B}_s \rangle. \quad (25)$$

To compute  $\delta$  we must determine  $B_s - \hat{B}_s$ , which in turn requires developing the estimate  $\hat{B}_s$ . To do this we assume that the spacecraft local frame is closely aligned with the inertial frame so that we may express the science baseline vector as

$$B_s = B_s^{loc} + \omega \times B_s^{loc}, \quad (26)$$

where the super "loc" refers to a vector written in the s/c local frame, and vectors without this superscript have their coordinates given in the inertial frame. The vector  $\omega$  has small magnitude (several  $\mu$ rad), and effects the transformation between the coordinate frames. The general scheme is that external metrology is used to develop an estimate of  $B_s^{loc}$ , while the guide interferometers are used to estimate  $\omega$ . Thus the estimate of  $B_s$  has the form

$$\hat{B}_s = \hat{B}_s^{loc} + \hat{\omega} \times \hat{B}_s^{loc}, \quad (27)$$

where  $\hat{\omega}$  is the estimate of  $\omega$ . Then to first order

$$B_s - \hat{B}_s = B_s^{loc} - \hat{B}_s^{loc} + (\omega - \hat{\omega}) \times B_s^{loc}. \quad (28)$$

Next let  $\delta l$  denote the vector of external metrology errors. In the previous section we saw how to compute  $B_s^{loc} - \hat{B}_s^{loc}$ , viz

$$B_s^{loc} - \hat{B}_s^{loc} = Z_s [F'(X^0)]^\dagger \delta l. \quad (29)$$

We will similarly need the error in computing  $B_g^{loc}$ :

$$B_g^{loc} - \hat{B}_g^{loc} = Z_g [F'(X^0)]^\dagger \delta l. \quad (30)$$

To compute  $\omega - \hat{\omega}$ , first observe that since (24) and (27) also apply to the guide interferometers [5], the basic astrometric equations for these are

$$d_{g_i} = \langle s_{g_i}, B_g^{loc} + \omega \times B_g^{loc} \rangle, \quad i = 1, 2, \quad (31)$$

where  $s_{g_1}$ ,  $s_{g_2}$  are the direction vectors to the two guide stars, and  $B_g^{loc}$  is the common baseline vector for the two guide interferometers. The fact that the *same*  $\omega$  vector works for the guide baseline vector and science baseline vector is an important consequence of the inversion of the external metrology measurements [5]. Now define the pair of  $2 \times 3$  matrices  $S$  and  $T$  by

$$S = \begin{bmatrix} s_{g_1}^T \\ s_{g_2}^T \end{bmatrix}, \quad T = \begin{bmatrix} (B_g \times s_{g_1})^T \\ (B_g \times s_{g_2})^T \end{bmatrix} \quad (32)$$

where  $B_g \equiv \hat{B}_g^{loc}(0)$  is a fixed vector. (How  $B_g$  is fixed is not critical to the linear analysis.) It follows from (31) and (32) that  $\omega$  is given by

$$\omega = T^\dagger [d_g - S B_g^{loc}], \quad d_g \equiv (d_{g_1}, d_{g_2})^T, \quad (33)$$

where  $T^\dagger$  denotes the pseudoinverse of  $T$ . The estimate of  $\omega$  is

$$\hat{\omega} = T^\dagger [d_g - S \hat{B}_g^{loc}]. \quad (34)$$

Thus,

$$\omega - \hat{\omega} = -T^\dagger [S(B_g^{loc} - \hat{B}_g^{loc})] \quad (35)$$

$$= -T^\dagger S Z_g [F'(X^0)]^\dagger \delta l. \quad (36)$$

Putting (25), (28–29), and (36) together, we obtain

$$\delta = \langle [F'(X^0)]^\dagger T \left\{ Z_s^T s - Z_g^T S^T T^\dagger T (B_s \times s) \right\}, \delta l \rangle, \quad (37)$$

which may be written more compactly as

$$\delta = \langle s, M \delta l \rangle \quad (38)$$

where

$$M = [Z_s + B T^\dagger S Z_g] [F'(X^0)]^\dagger T, \quad (39)$$

and  $B$  the skew symmetric matrix associated with  $B_s$  via  $Bx = B_s \times x$  for any 3-vector  $x$ .

Thus (38) is the sought after relationship between science interferometer regularized delay error and single gauge external metrology error.

## 5. CONCLUDING REMARKS

The machinery is now in place to insert physics based component errors and propagate these to higher level error metrics such as rms delay error or astrometric parameter error. The completion of this task is the subject of Part 2 of the paper. Specifically, single gauge errors will be derived for the components of the external metrology error vector  $\delta l$  as a function of the error parameter values (e.g., dihedral error) and the position of the target star in the field of regard. The error vector will then be propagated to a delay error via (38–39) as a function of these variables. Zernike decomposition, eigenanalysis and other techniques will then be applied to analyzing how parameter error affects the metric.

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