The Formation of Filament Threads in a Sheared Magnetic Field

Lawrence Sparks

Jet Propulsion Laboratory

Solar Activity During the Onset of Solar Cycle 24

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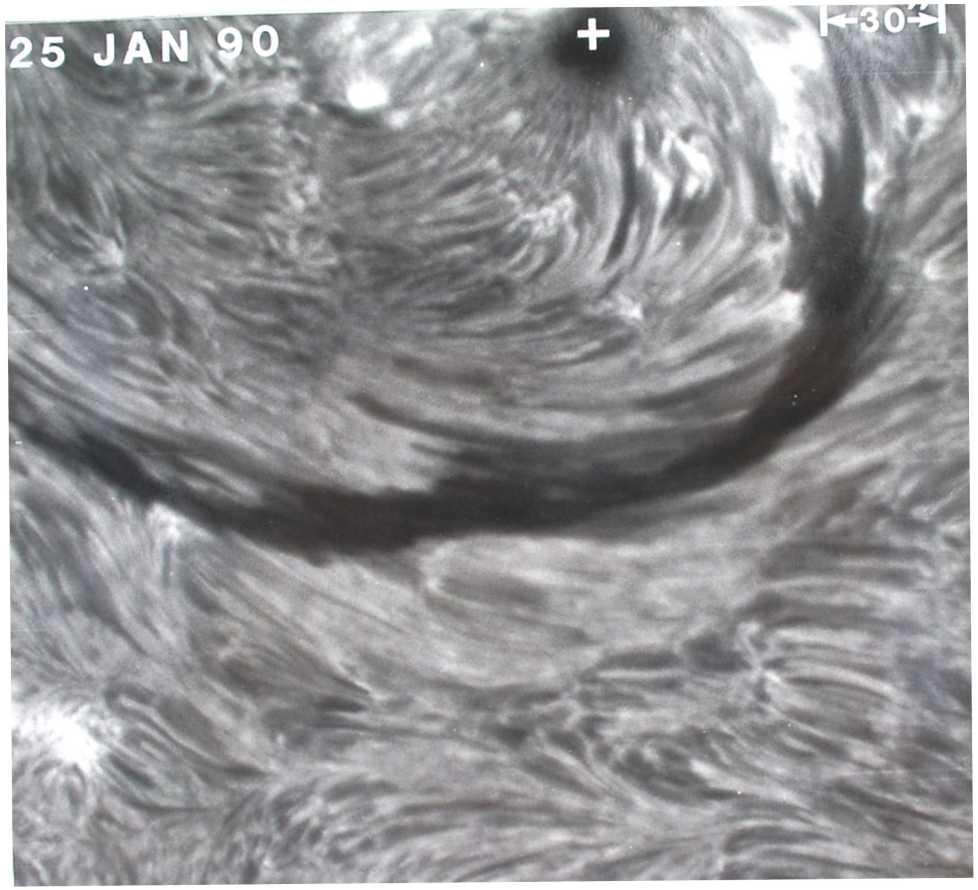
Outline

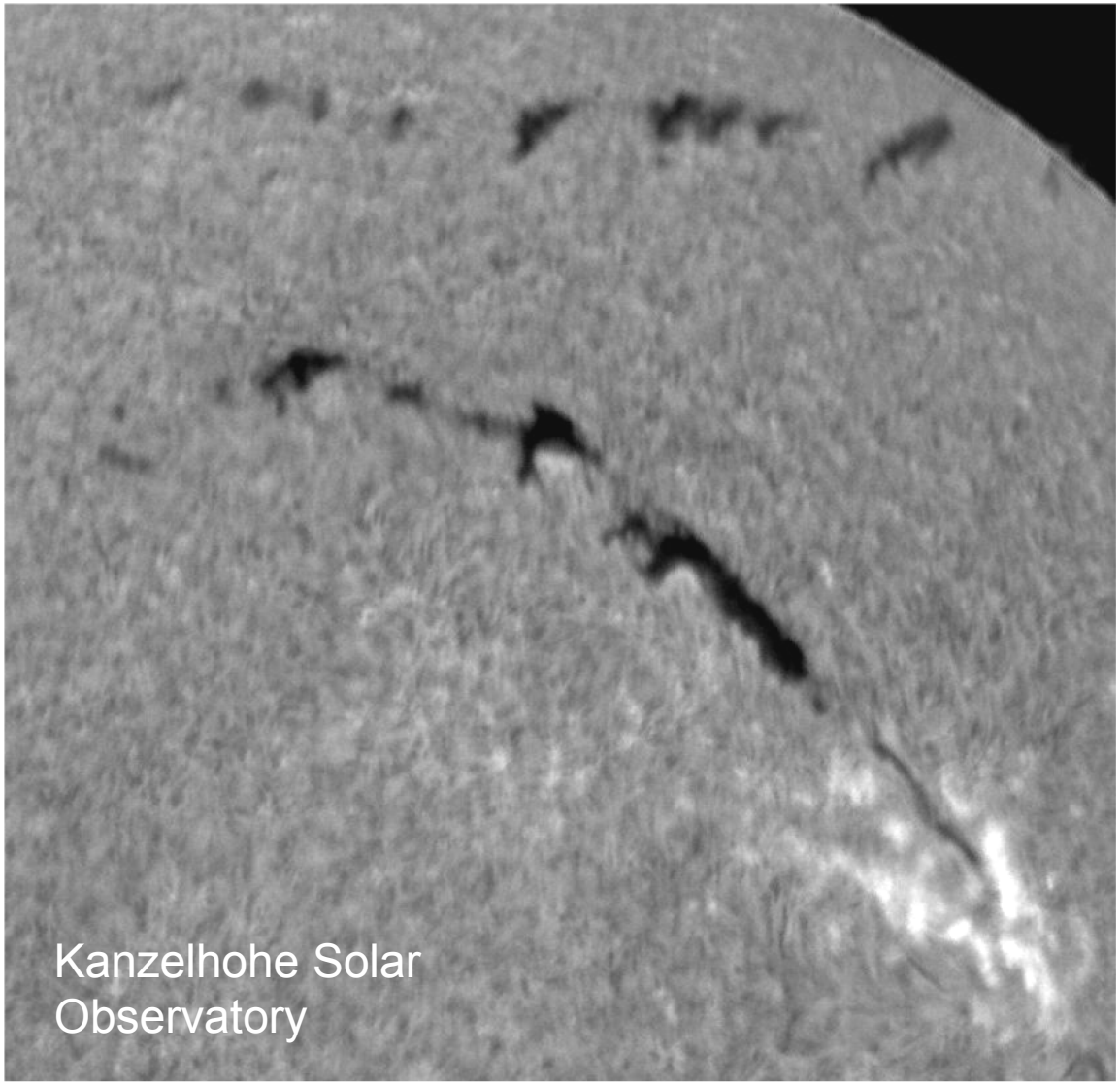
- Observations
- Thermal instability
- Condensation in a sheared magnetic field
- Linear analysis
- Nonlinear simulation

Observations



- Filaments are observed to separate regions of opposite, line-of-site magnetic polarity in the photosphere.

Spectrum of filaments



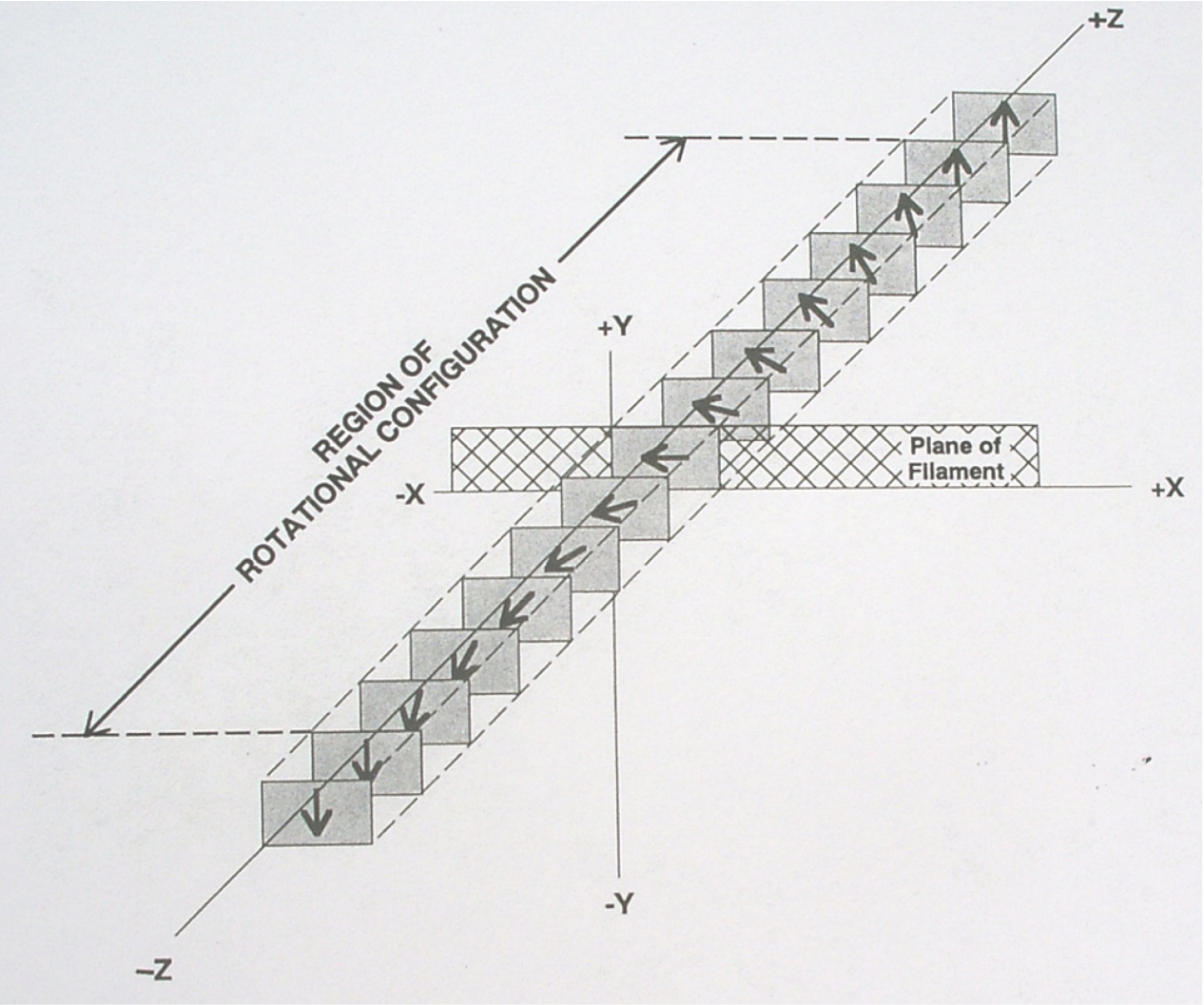
Quiescent polar crown filament

Quiescent filament

Intermediate filament

Active region filament

Magnetic field configuration for filaments (from Sara Martin)



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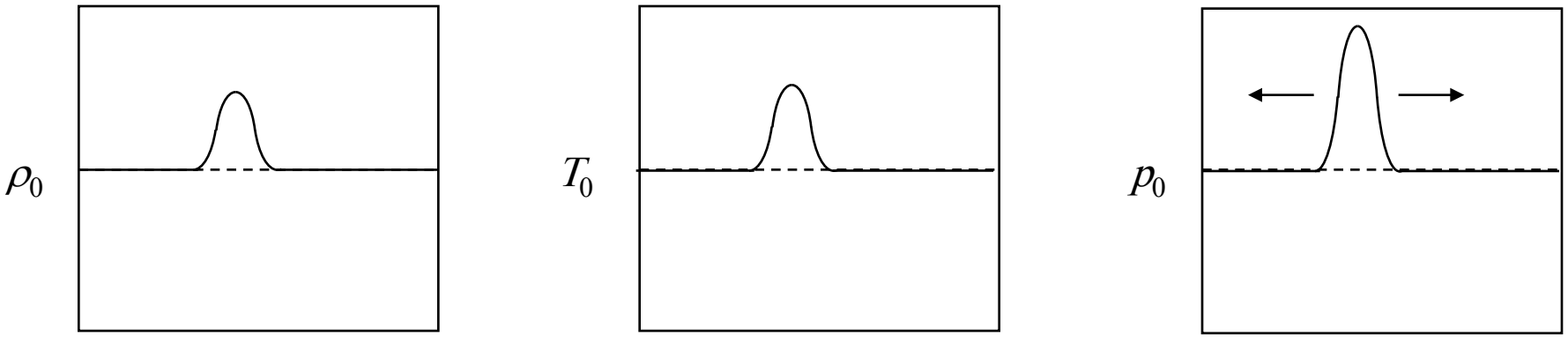
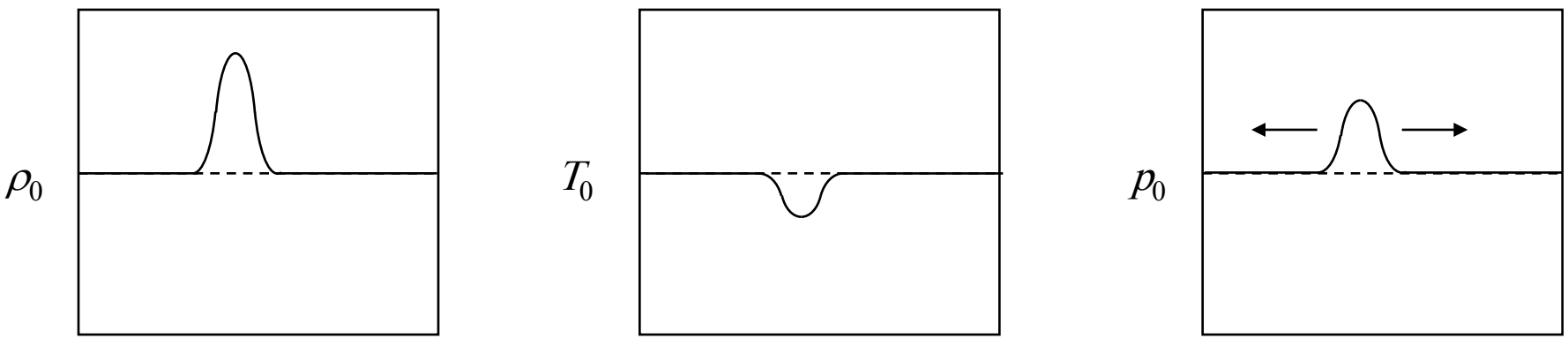
Thermal stability

density | temperature | pressure



Adiabatic compression: perturbed pressure provides a restoring force

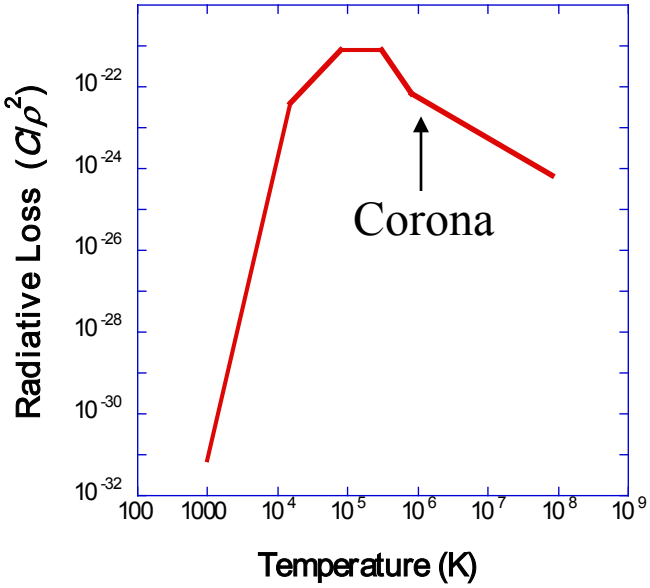


Thermal equilibrium with radiation: $H + C = 0$ $H = H(\rho)$ $C = R\rho^2 T^r$

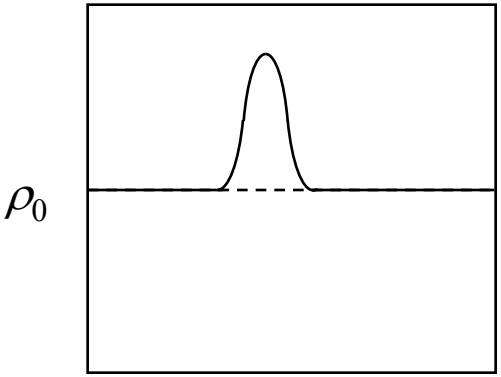
Radiative compression: here adiabatic and non-adiabatic heating dominates

Thermal instability

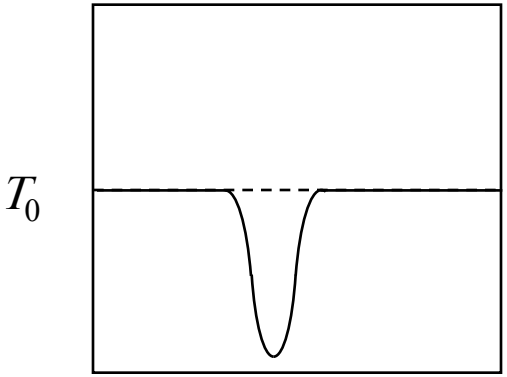
Condensation occurs when a density enhancement allows radiative losses to dominate.



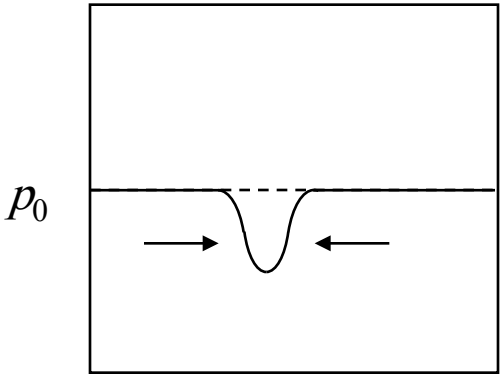
density



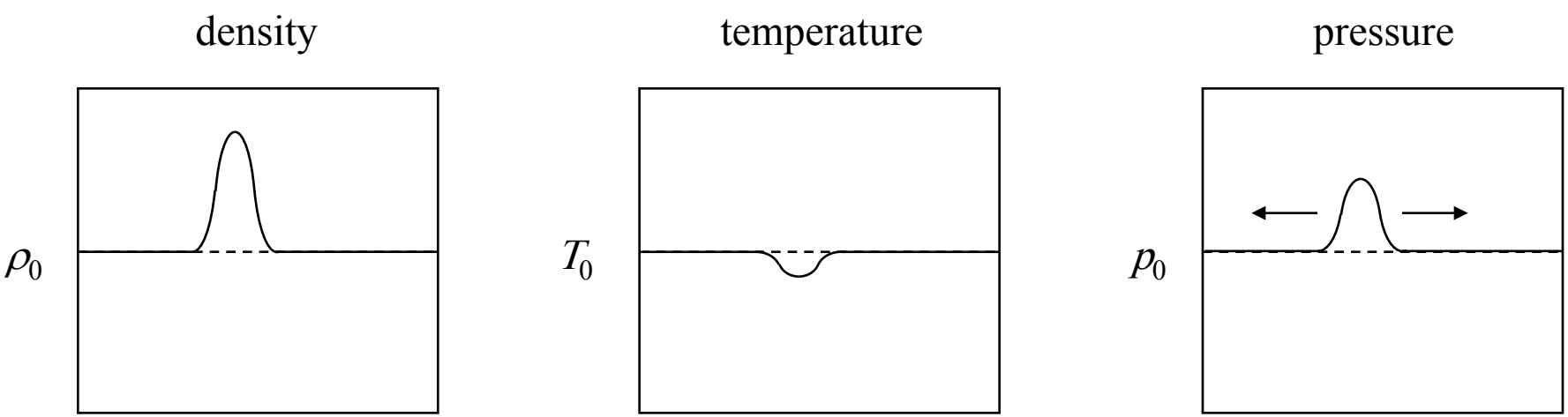
temperature



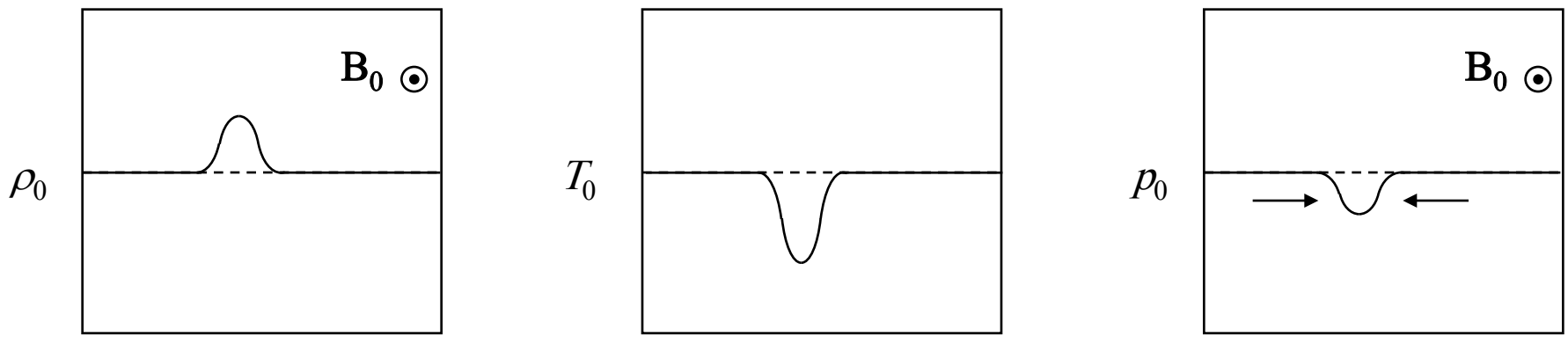
pressure



Thermal conduction



Compression with thermal conduction: heat flow stabilizes perturbation



Compression with magnetic field: field provides thermal insulation in transverse direction
Magnetic pressure opposes density enhancement

Filament thread formation by condensational instability?

- Under coronal conditions, thermal conduction inhibits the condensational instability.

- Filament thread formation from a condensational instability requires a magnetic field to inhibit thermal conduction.

- Coronal condensations cannot form in a uniform magnetic field:
  (1) the magnetic field prevents transverse mass flow from contributing to a local growth in density;
  (2) heat flow parallel to the field ensures that parallel mass flow will not contribute to a condensational instability.

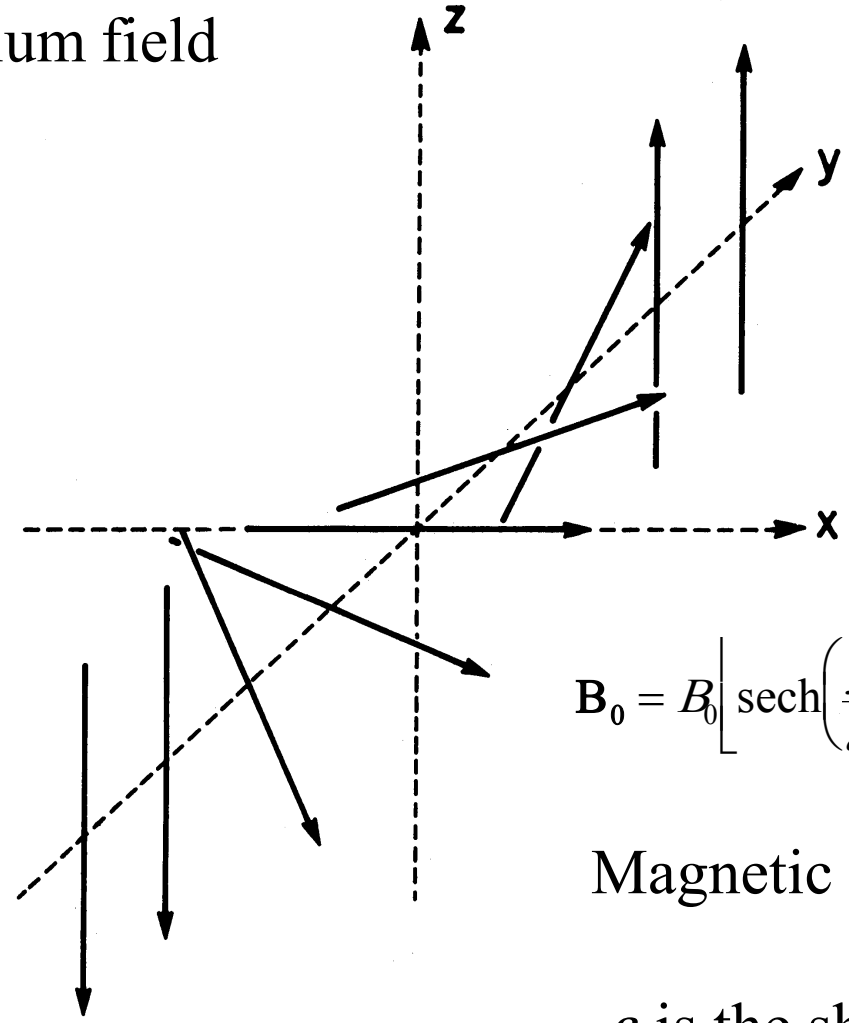
Key question:

How is it possible for a magnetic field to inhibit heat flow in a coronal plasma without simultaneously restricting the mass flow required for the growth of density condensations?

Condensations in a sheared magnetic field

Model equilibrium field

Perturbation
wave vector

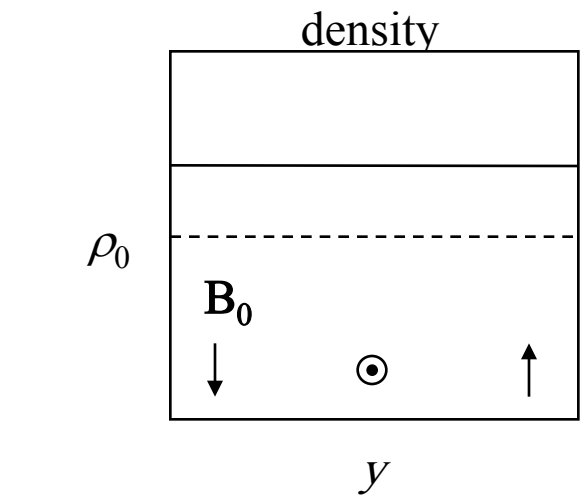


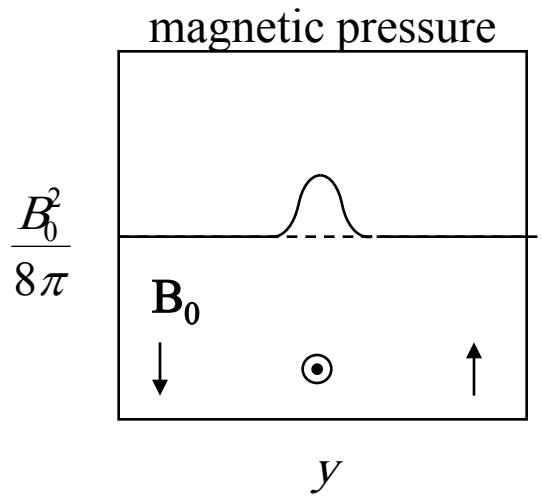
$$\mathbf{B}_0 = B_0\left[\operatorname{sech}\left(\frac{y}{a}\right)\hat{\mathbf{e}}_x + \tanh\left(\frac{y}{a}\right)\hat{\mathbf{e}}_z\right]$$

Magnetic field is force-free.

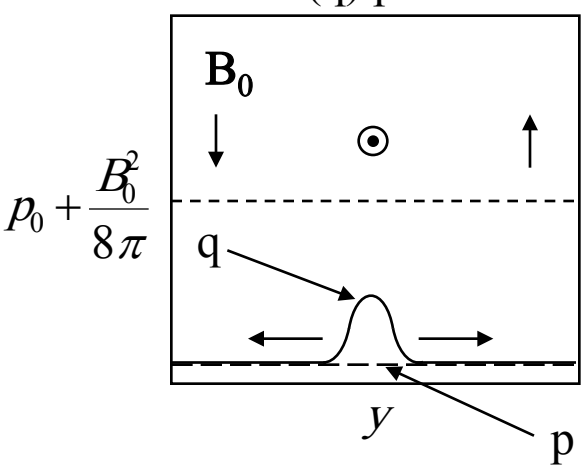
a is the shear scale

Localization of density in the shear layer



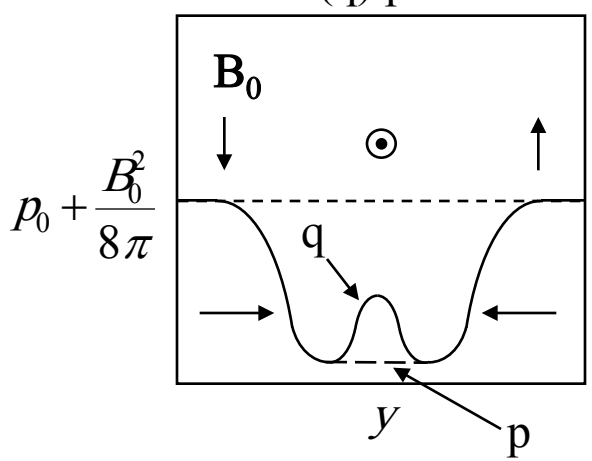


kinetic (p) and
total (q) pressure



No heat flow

kinetic (p) and
total (q) pressure



Parallel heat flow

Parallel heat flow forces perturbation to vanish at boundary.

Total pressure gradient pushes plasma into the shear layer.

Model equations

Continuity eq.

$$\frac{\partial \rho}{\partial t} + \boldsymbol{u} \cdot \nabla \rho + \rho \nabla \cdot \boldsymbol{u} = 0 \,,$$

Momentum eq.

$$\rho \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} \right) + \nabla p - \frac{1}{c} \boldsymbol{J} \times \boldsymbol{B} = 0 \,,$$

Maxwell's eqs.

$$\frac{1}{c} \frac{\partial \boldsymbol{B}}{\partial t} + \nabla \times \boldsymbol{E} = 0 \,, \qquad \nabla \times \boldsymbol{B} = \frac{4\pi}{c} \boldsymbol{J} \,,$$

$$\nabla \cdot \boldsymbol{B} = 0 \,, \qquad \boldsymbol{E} + \frac{1}{c} \boldsymbol{u} \times \boldsymbol{B} = \eta \boldsymbol{J} \,,$$

Assume classical (small) resistivity.
Note: no gravity.

Energetics

Ideal gas law

$$p = 2\rho k_{\mathrm{B}} T/m_i$$

Energy eq.

$$\frac{\partial p}{\partial t} + \boldsymbol{u} \cdot \nabla p = \frac{\gamma p}{\rho}\left(\frac{\partial \rho}{\partial t} + \boldsymbol{u} \cdot \nabla \rho\right) + (\gamma - 1)(\eta J^2 + H - C + \nabla \cdot \boldsymbol{\kappa} \cdot \nabla T)$$

Heating function H is assumed constant in time.

Classical thermal conductivity tensor $\boldsymbol{\kappa}$ has $\kappa_{\perp} << \kappa_{\parallel}$.

Characteristic time scales

TABLE 1

CHARACTERISTIC CORONAL FREQUENCIES

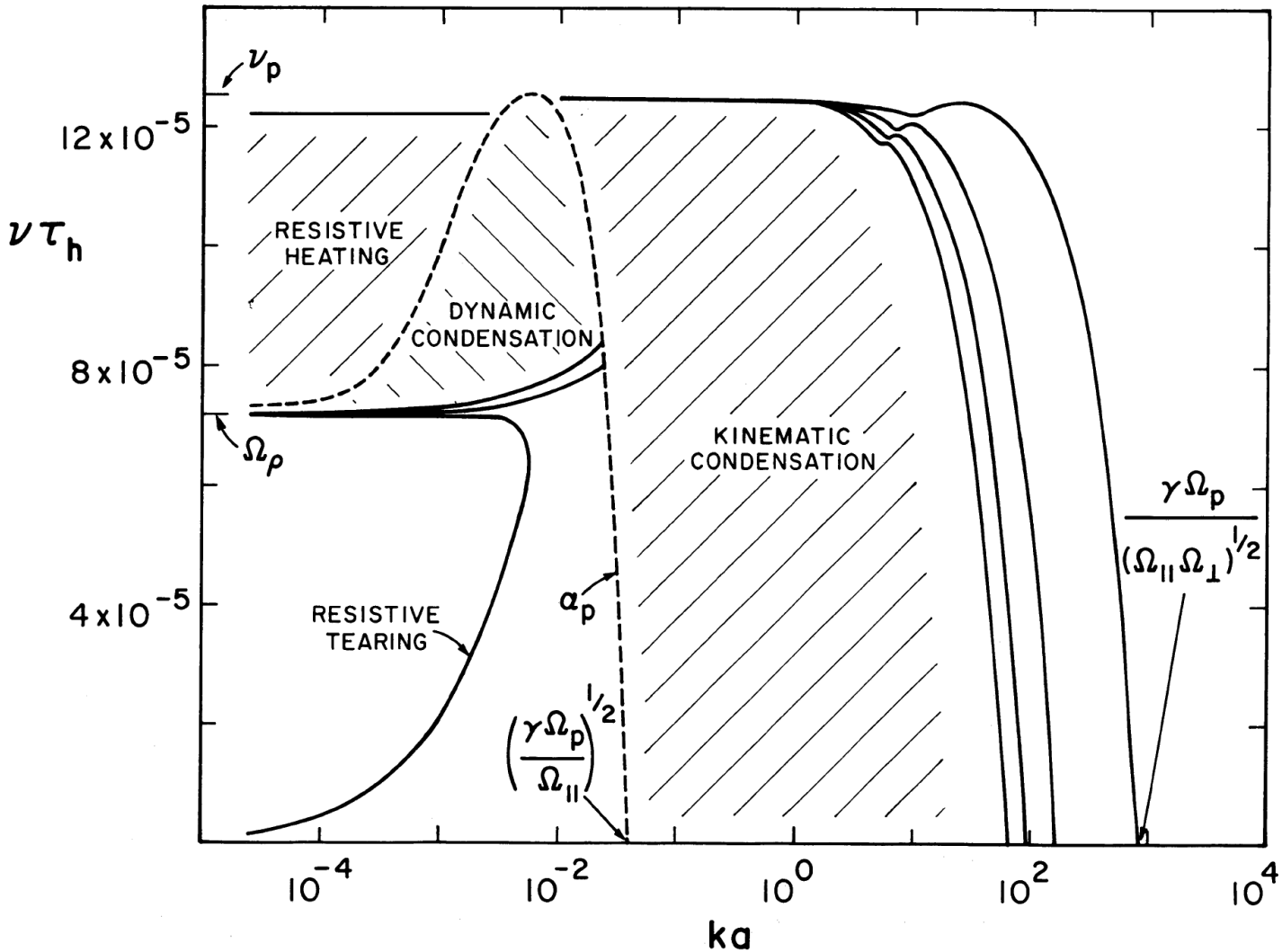
Frequency	Definition	Value ($\Omega\tau_h$)
Ω_ρ	$-(\gamma-1)\left(\frac{T_0}{p_0}\right)\left(\frac{\partial C}{\partial T}\bigg\vert_\rho\right)_0$	7.32×10^{-5}
Ω_T	$(\gamma-1)\left(\frac{\rho_0}{p_0}\right)\left(\frac{\partial C}{\partial \rho}\bigg\vert_T\right)_0$	1.46×10^{-4}
Ω_p	$(\gamma-1)\left(\frac{\rho_0}{\gamma p_0}\right)\left(\frac{\partial C}{\partial \rho}\bigg\vert_p\right)_0$	1.32×10^{-4}
$\Omega_\parallel$	$(\gamma-1)\left(\frac{\kappa_\parallel T_0}{p_0 a^2}\right)$	1.45×10^{-1}
$\Omega_\perp$	$(\gamma-1)\left(\frac{\kappa_\perp T_0}{p_0 a^2}\right)$	4.83×10^{-13}
Ω_r	$\frac{\eta_0 c^2}{4\pi a^2}$	9.57×10^{-12}
Ω_J	$-(\gamma-1)\frac{d\ln\eta_0}{d\ln T_0}\frac{\eta_0 J_0^2(0)}{p_0}$	1.91×10^{-9}

$n_0 = \rho_0/m_i = 10^{10}\ \mathrm{cm}^{-3}$, $\quad T_0 = 10^6\ K$,

$a = 100\ \mathrm{km}$, $\quad B_0 = 83.3\ \mathrm{G}$

$\tau_h = a\sqrt{4\pi\rho_0 / B_0^2}$

Linear modes for thermal instability in a sheared magnetic field



Growth rate vs. wavenumber

Dynamic condensations

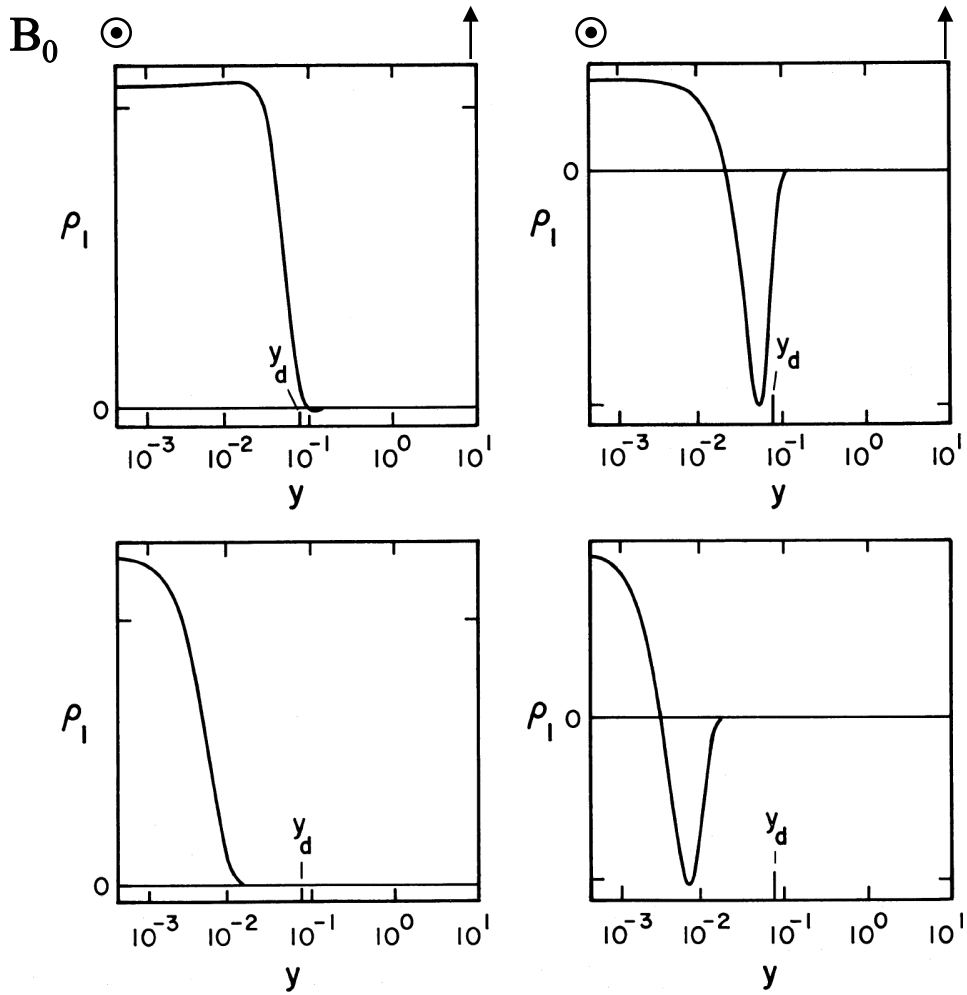
Characteristics of *dynamic* condensations:

- spatial scale is determined primarily by force balance
- plasma mass flow is directed perpendicular to the magnetic field
- less compressible, the magnetic field inhibits (transverse) plasma compression
- only a drop in temperature contributes to growth
- the growth rate increases with number of nodes in mode
- analog of classical resistive tearing mode is a spatial case

Density perturbations for principal dynamic condensation mode and 1st harmonic

$S = 10^8$

$S = 10^{10}$



$$k_{\parallel}(y_d)\, v_A = \Omega_p$$

$$ka = 2 \times 10^{-3}$$

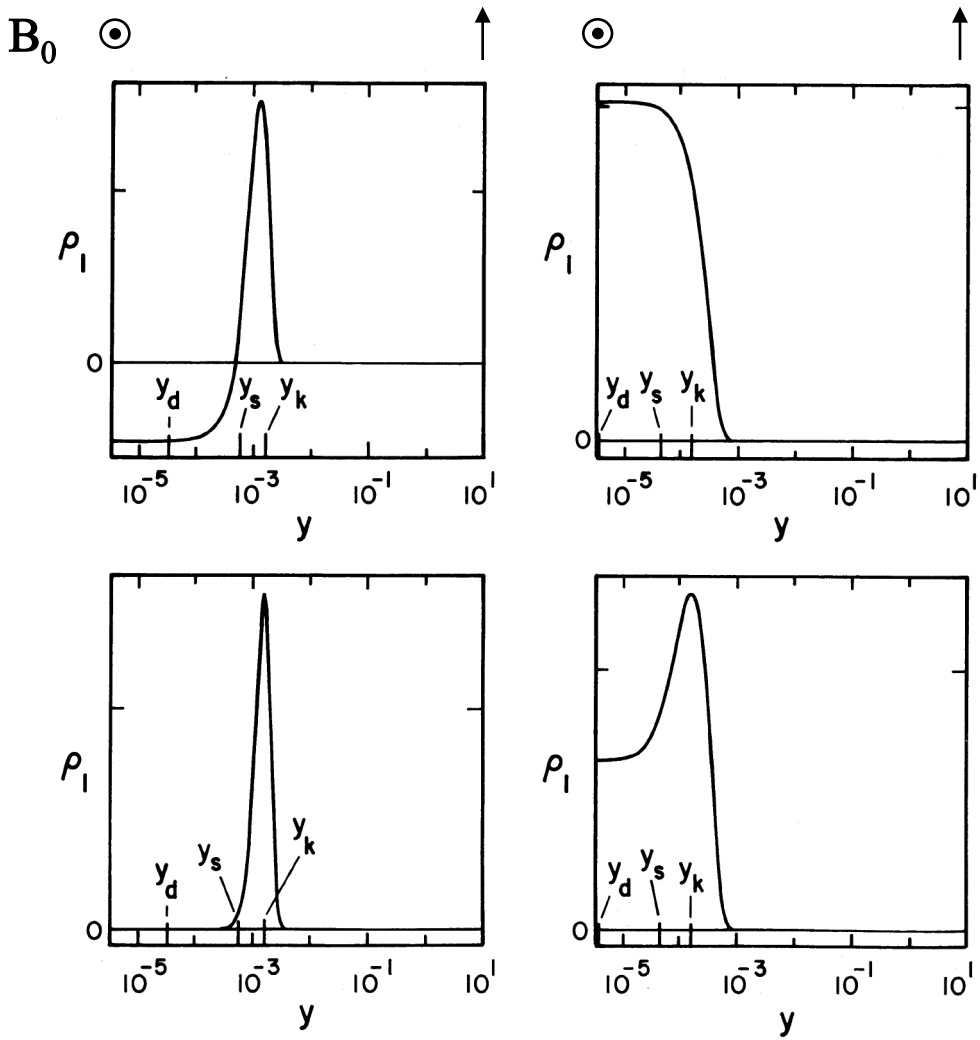
Kinematic condensations

kinematic - "of or relating to aspects of motion apart from considerations of mass and force"

Characteristics of *kinematic* condensations -

- exist only in the presence of anisotropic heat flow
- spatial scale is determined primarily by energy balance
- plasma mass flow is directed parallel to the magnetic field
- highly compressible; most compressible when sound waves traveling parallel to the magnetic field can maintain pressure balance
- both a drop in temperature and mass flow parallel to field lines contribute to growth
- exhibit higher growth rates than dynamic modes
- growth rate decreases with number of nodes in mode
- growth rate increases as number of nodes in mode decrease

Density perturbations for principal kinematic condensation mode



$ka = 4$ $\qquad$ $ka = 40$

$$S = 10^8$$

$$S = 10^{10}$$

$$k_{\parallel}(y_d)v_A = \Omega_p$$

$$k_{\parallel}(y_s)c_s/\gamma^{1/2} = \upsilon$$

$$\frac{k^2 B_{0z}^2(y_k)}{B_0^2} = \frac{\gamma p_0(\Omega_p - \upsilon) - \upsilon^2 \rho_0 a^2 \Omega_{\parallel}}{2 p_0 a^2 \Omega_{\parallel}}$$

Non-linear evolution of thermal instability in sheared field

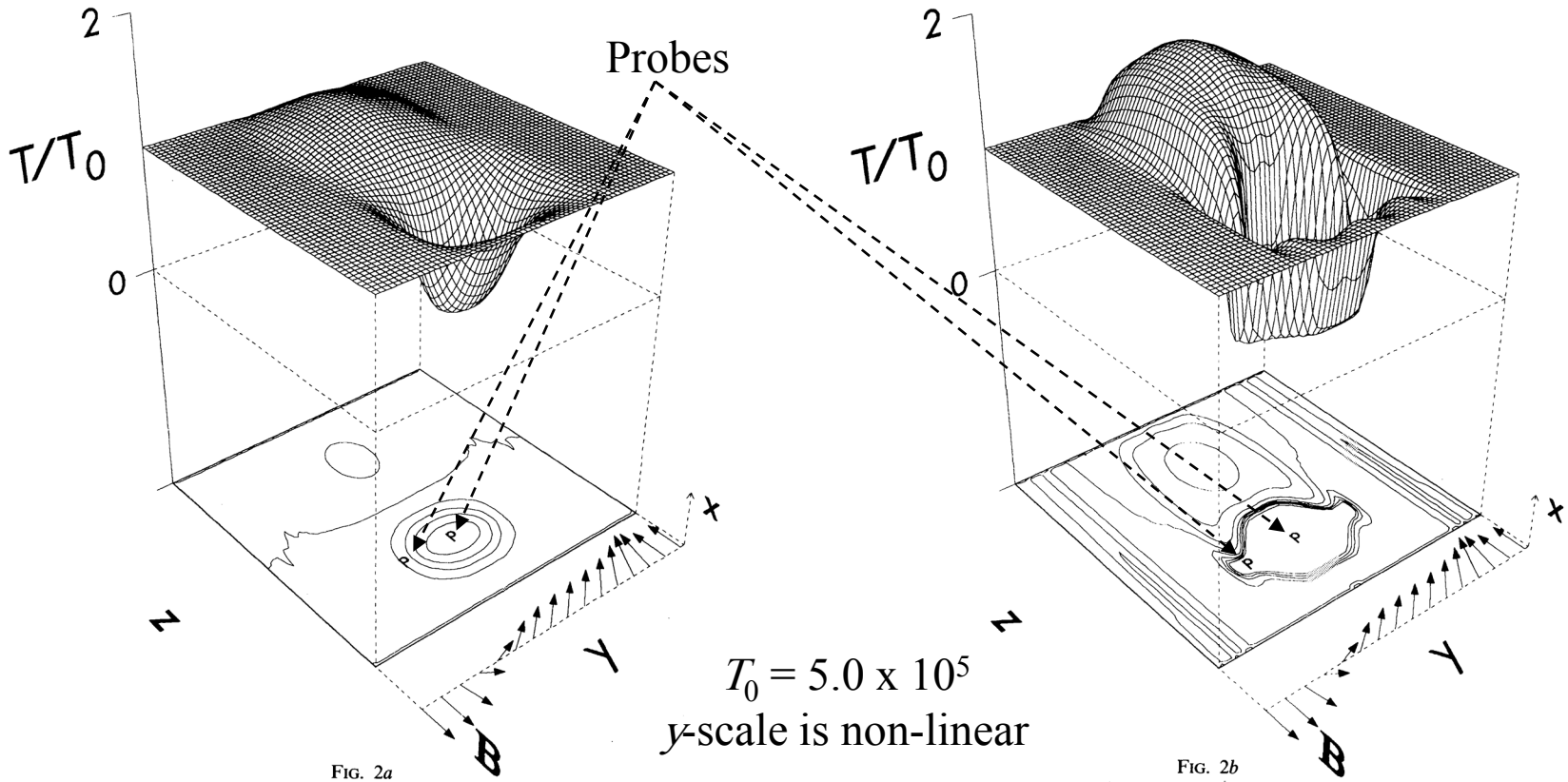
Types of perturbations studied:

(1) Generic perturbation with $T_0 < T_c$

(2) Generic perturbation with $T_0 = T_c$

(3) Random perturbation with $T_0 < T_c$

Temperature evolution for generic perturbation with $T_0 < T_c$



$T_0 = 5.0 \times 10^5$
y-scale is non-linear

T = 178 sec

T = 293 sec

Choose **k** so that only kinematic modes are thermally unstable.

Density evolution for generic perturbation with $T_0 < T_c$

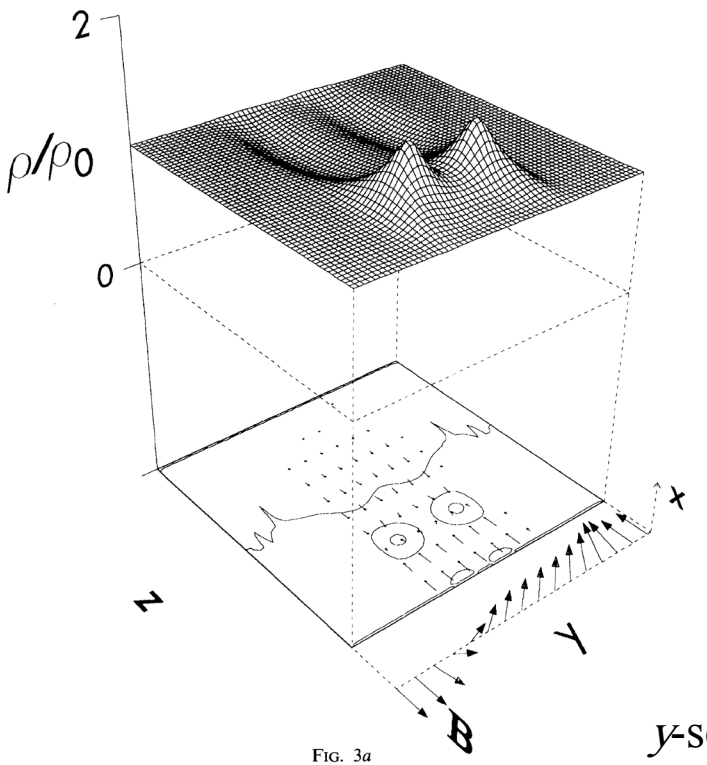


FIG. 3a

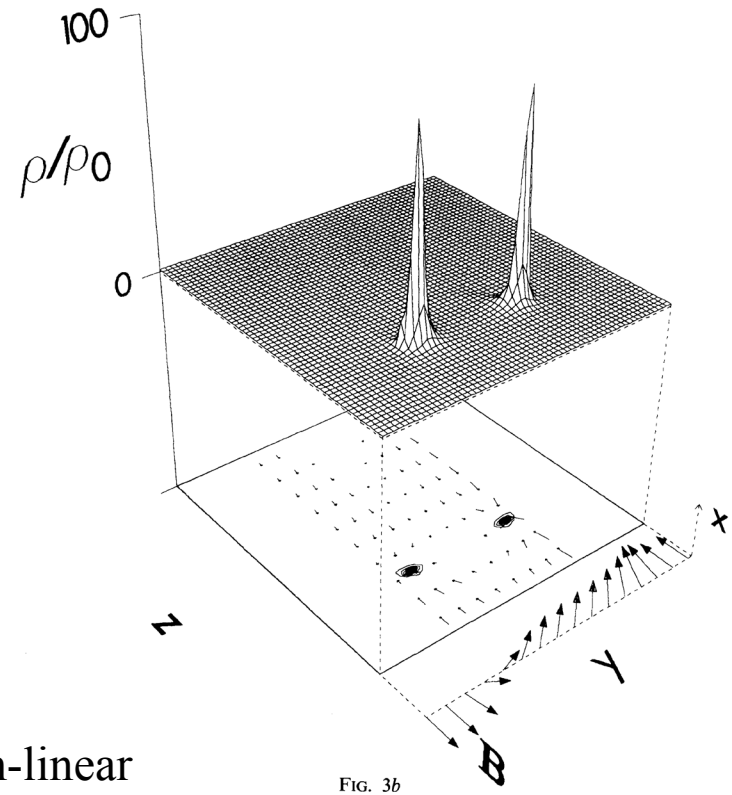


FIG. 3b

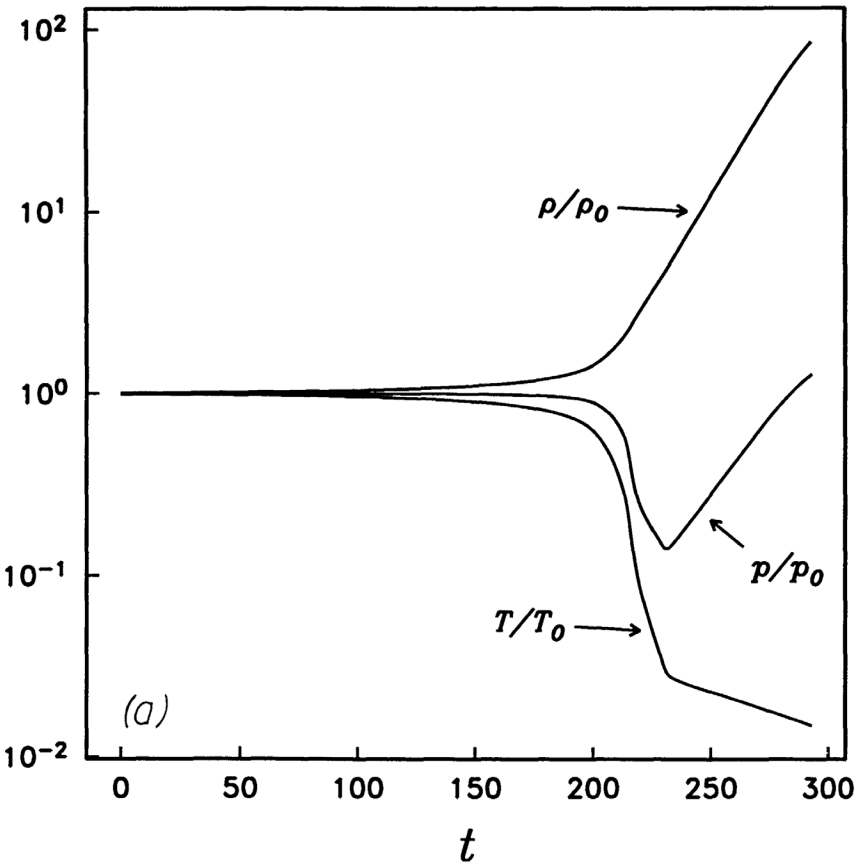
y-scale is non-linear

T = 178 sec

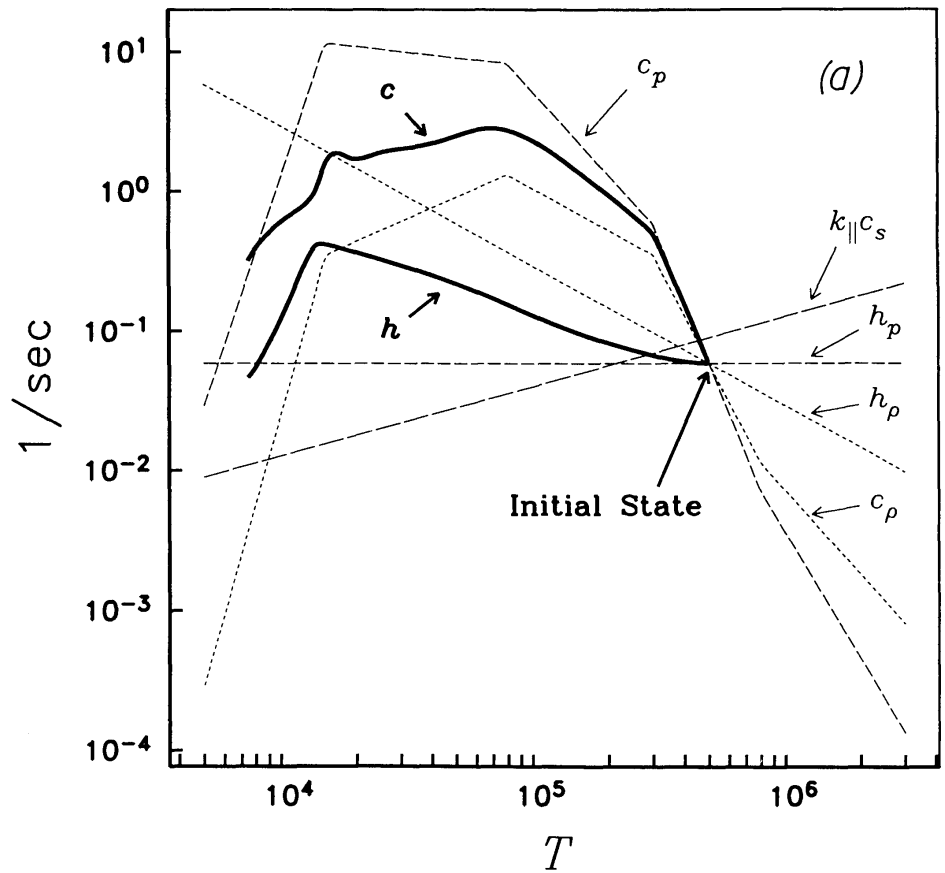
T = 293 sec

Density at peaks increases by nearly two orders of magnitude.

Time evolution at outer probe for generic perturbation with $T_0 < T_c$



Temperature, density, and pressure

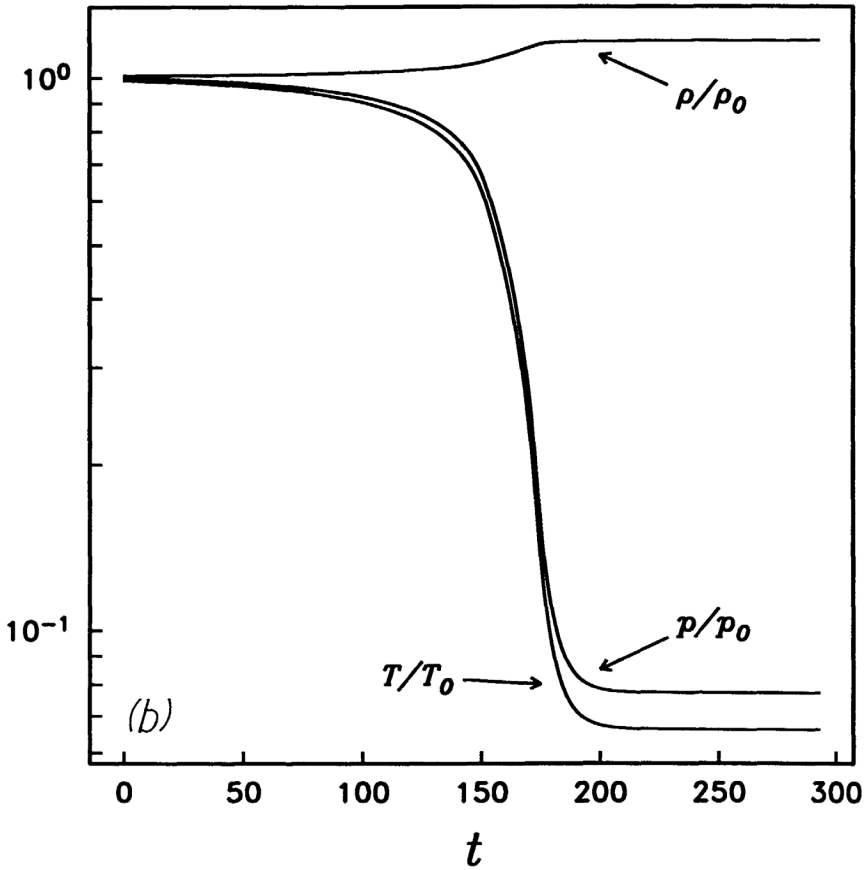


Heating rate: $h = H/p$

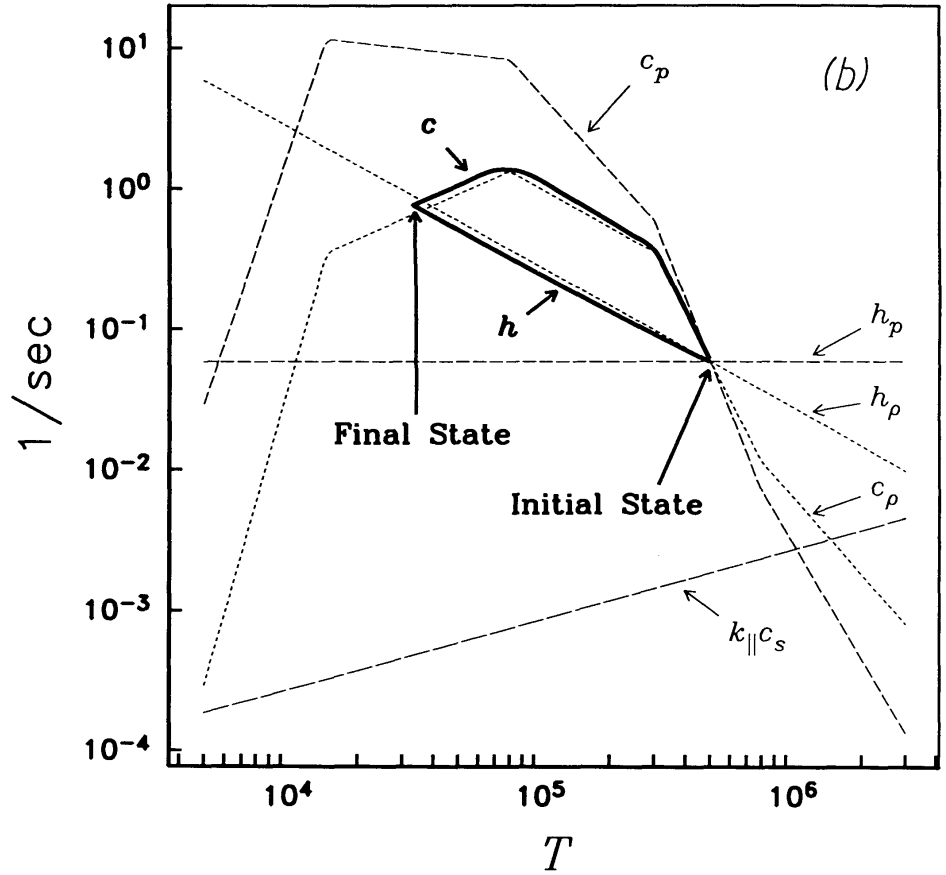
Cooling rate: $c = C(T)/p$

Simulation is run until gradients can no longer be resolved.

Time evolution at inner probe for generic perturbation with $T_0 < T_c$



Temperature, density, and pressure

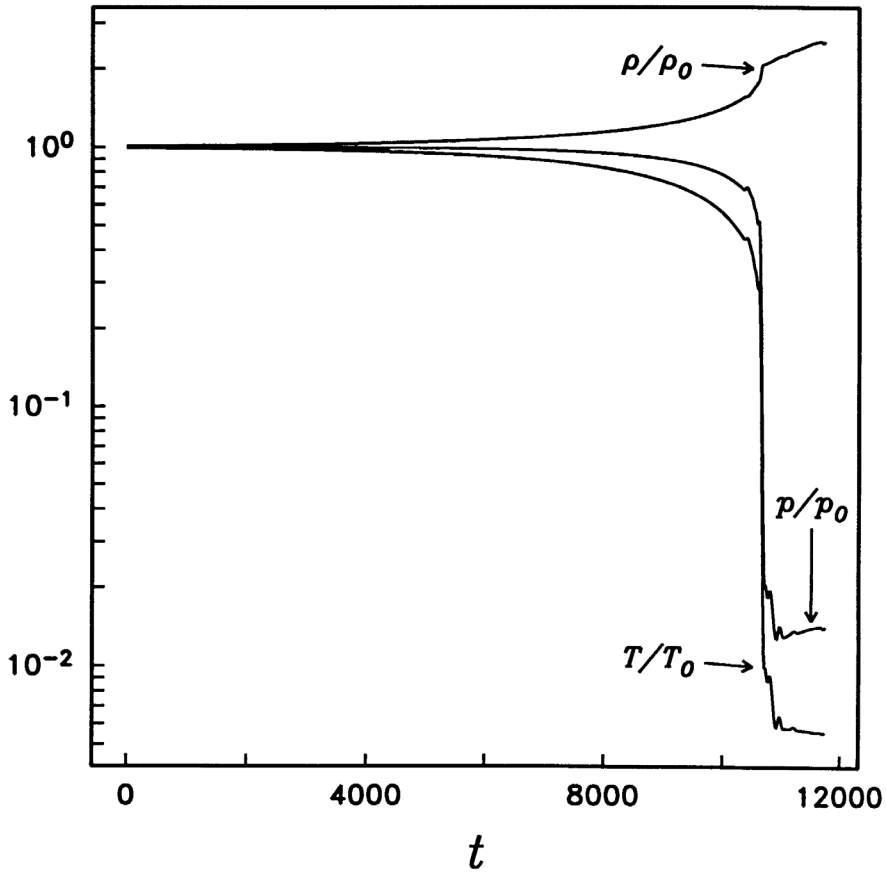


Heating rate: $h = H/p$

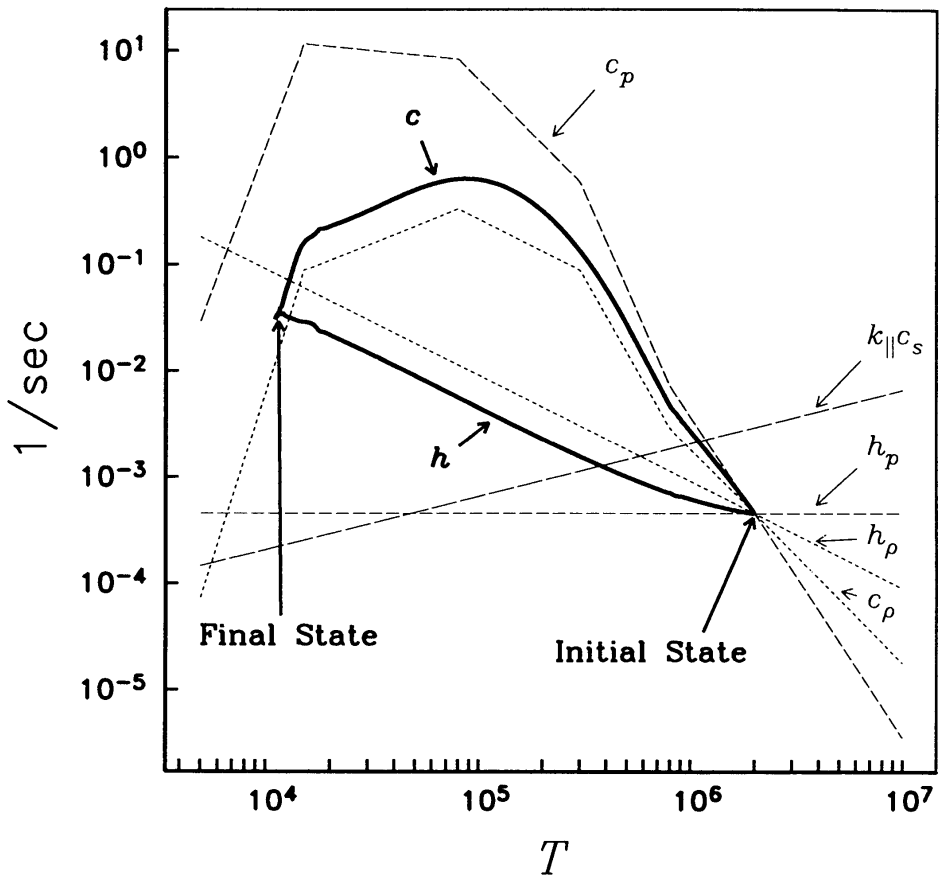
Cooling rate: $c = C(T)/p$

A new radiative equilibrium is established at the inner probe.

Time evolution at probe for generic perturbation with $T_0 = T_c$



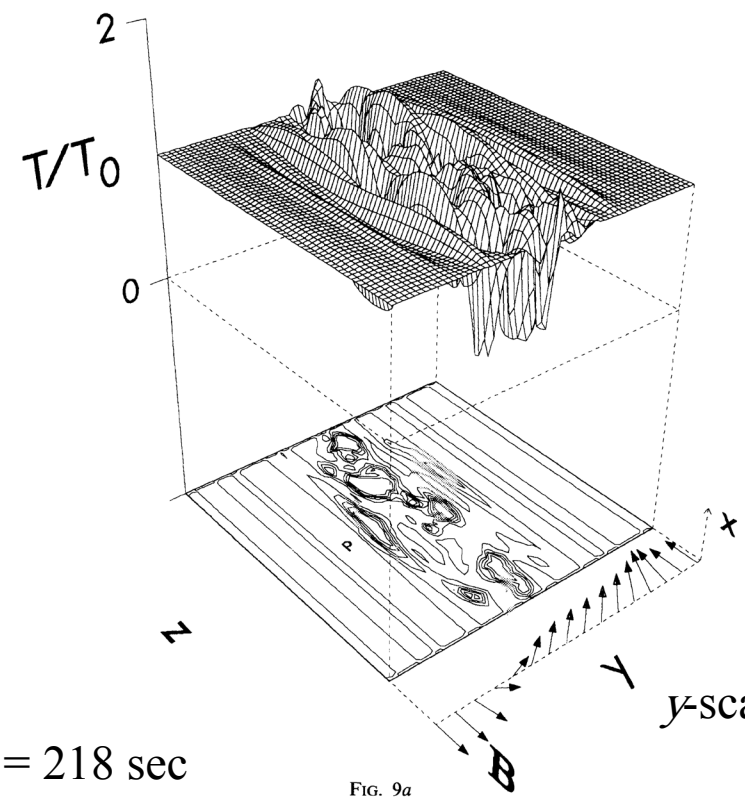
Temperature, density, and pressure



Heating rate: $h = H/p$
Cooling rate: $c = C(T)/p$

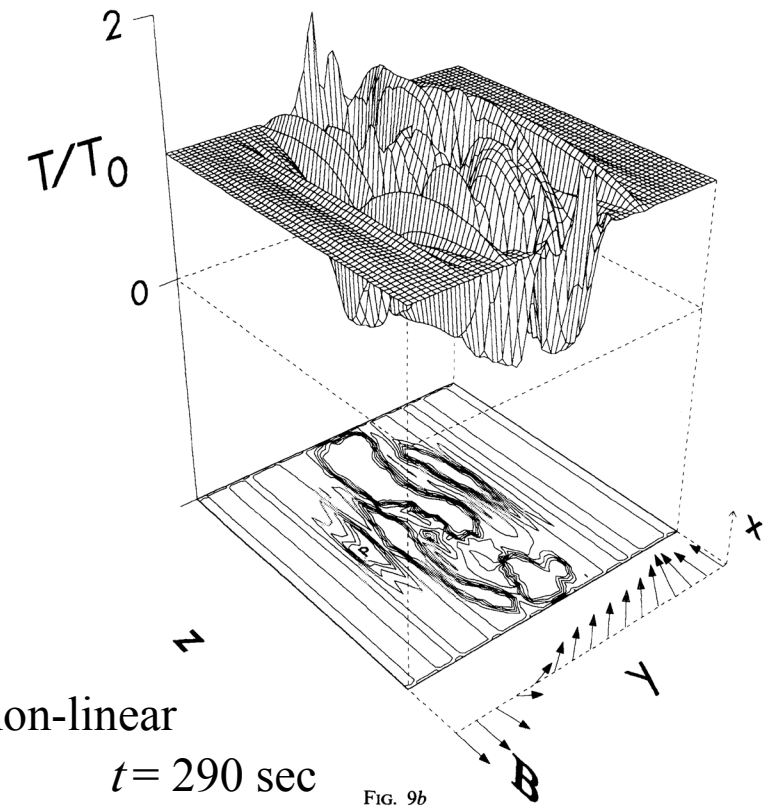
Similar results but slower to develop - a much longer computation.

Temperature evolution for random perturbation with $T_0 < T_c$



$t = 218$ sec

FIG. 9*a*



$t = 290$ sec

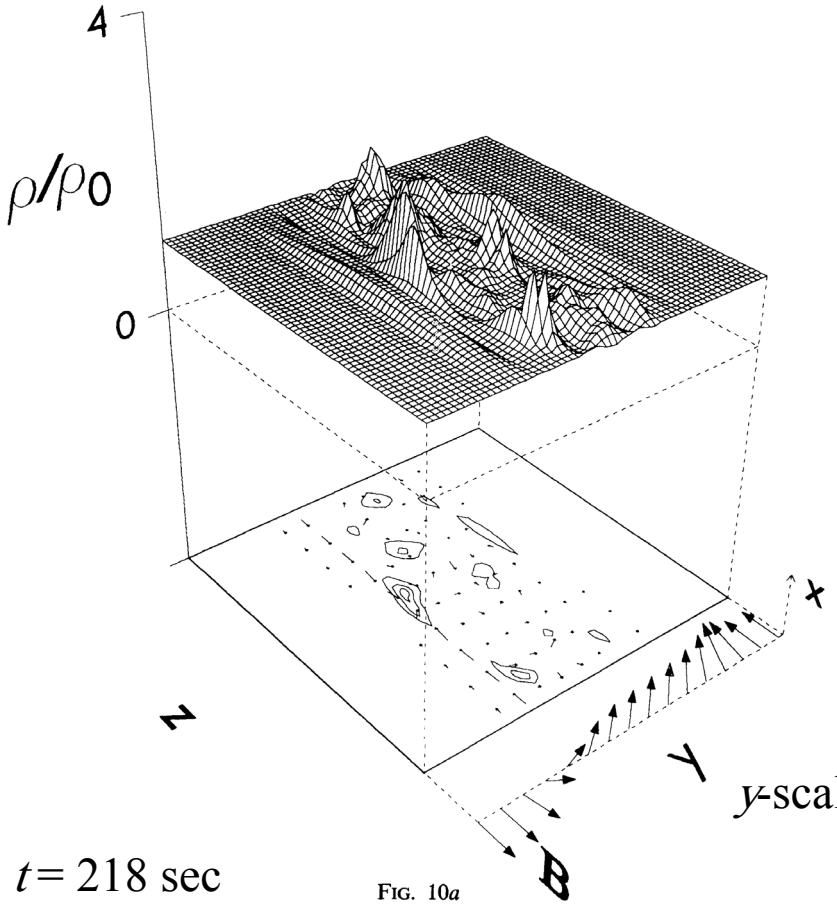
FIG. 9*b*

y-scale is non-linear

Initial temperature is $T_0 = 5 \times 10^5$ K

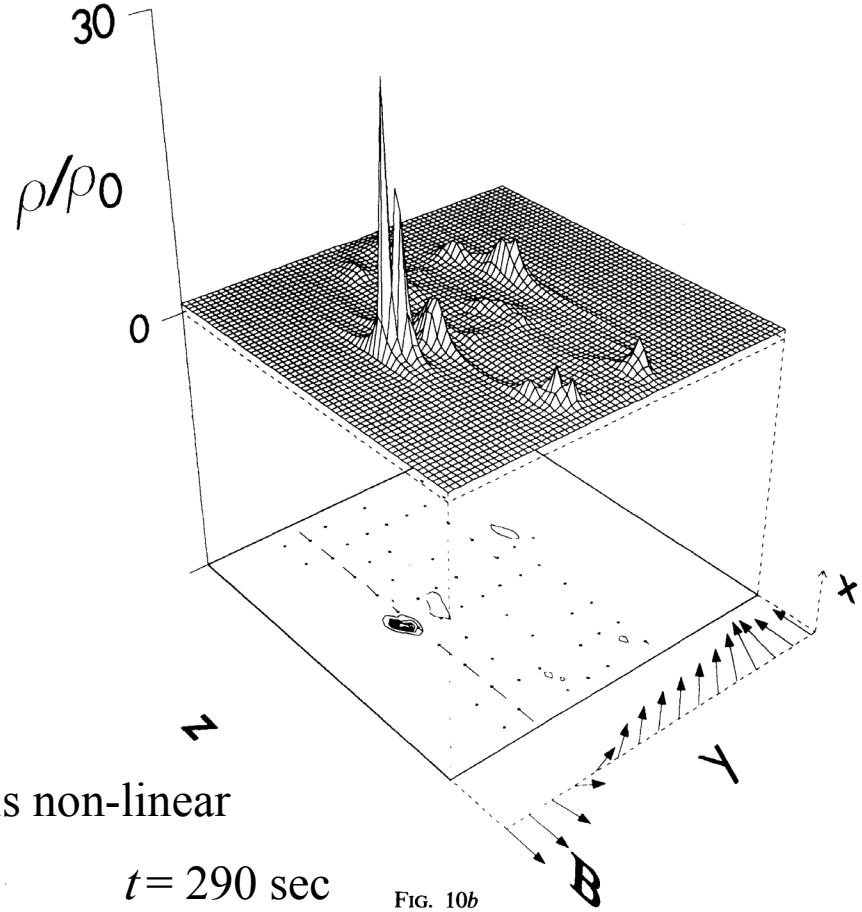
Final temperature in condensation is ~ $T_0 / 100$.

Density evolution for random perturbation with $T_0 < T_c$



$t = 218$ sec

FIG. 10*a*



$t = 290$ sec

FIG. 10*b*

y-scale is non-linear

Final peak density is ~ $100\rho_0$.

Summary for linear analysis

- A condensation will form preferentially in regions near where **k** is perpendicular to $\mathbf{B}_0$.

- *Dynamic* condensations:
  - spatial structure is determined primarily by force balance
  - have plasma mass flow perpendicular to the magnetic field
  - only a drop in temperature contributes to growth;

- *Kinematic* condensations:
  - exist only in the presence of anisotropic heat flow.
  - have plasma mass flow parallel to the magnetic field
  - are most compressible when sound waves traveling parallel to the magnetic field can maintain pressure balance.
  - exhibit the fastest growth
  - both a drop in temperature and mass flow parallel to field lines contribute to growth

Summary for non-linear simulations

Nonlinear two-dimensional MHD simulations (neglecting gravity) have traced the local genesis and growth of plasma filament threads in a force-free, sheared magnetic field until they attain both a minimum temperature and a maximum mass density characteristic of observed solar filaments.

A locally sheared magnetic field can thermally insulate regions of a coronal plasma without simultaneously impeding the mass flow required for the growth of condensations.

Linearized equations of motion

$$\frac{1}{\rho_0}\frac{\partial \rho_1}{\partial t} + \nabla \cdot \boldsymbol{u} = 0\,,$$

$$\rho_0 \frac{\partial \boldsymbol{u}}{\partial t} + \nabla p_1 + \boldsymbol{B}_0 \times (\nabla \times \boldsymbol{B}_1)/4\pi$$

$$+ \boldsymbol{B}_1 \times (\nabla \times \boldsymbol{B}_0)/4\pi = 0$$

$$\frac{1}{T_0}\frac{\partial T_1}{\partial t} + (\gamma - 1)\nabla \cdot \boldsymbol{u} + \frac{(\gamma - 1)}{p_0}\left(\frac{\partial C}{\partial \rho}\rho_1 + \frac{\partial C}{\partial T}T_1\right)$$

$$- \frac{(\gamma - 1)\eta_0 c^2}{8\pi^2 p_0}(\nabla \times \boldsymbol{B}_0) \cdot (\nabla \times \boldsymbol{B}_1)$$

$$- \frac{(\gamma - 1)\eta_1 c^2}{16\pi^2 p_0}|\nabla \times \boldsymbol{B}|^2 - \frac{(\gamma - 1)}{p_0}\nabla \cdot \underset{\sim}{\kappa}_0 \cdot \nabla T_1 = 0$$

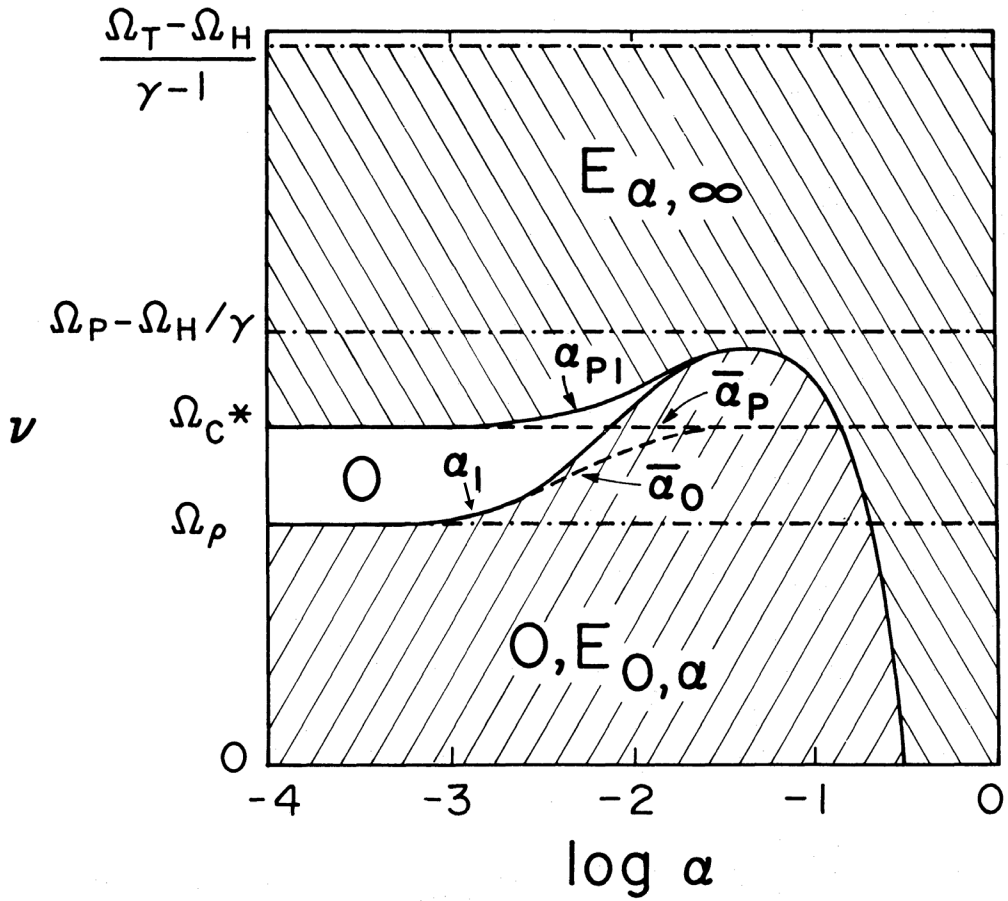
$$\frac{\partial \boldsymbol{B}_1}{\partial t} + \frac{c^2}{4\pi}\nabla \eta_1 \times (\nabla \times \boldsymbol{B}_0)$$

$$+ \frac{c^2 \eta_0}{4\pi}\nabla \times (\nabla \times \boldsymbol{B}_1) - \nabla \times \boldsymbol{u} \times \boldsymbol{B}_0 = 0$$

$$\nabla \cdot \boldsymbol{B}_1 = 0\,,$$

$$\frac{p_1}{p_0} = \frac{\rho_1}{\rho_0} + \frac{T_1}{T_0}\,.$$

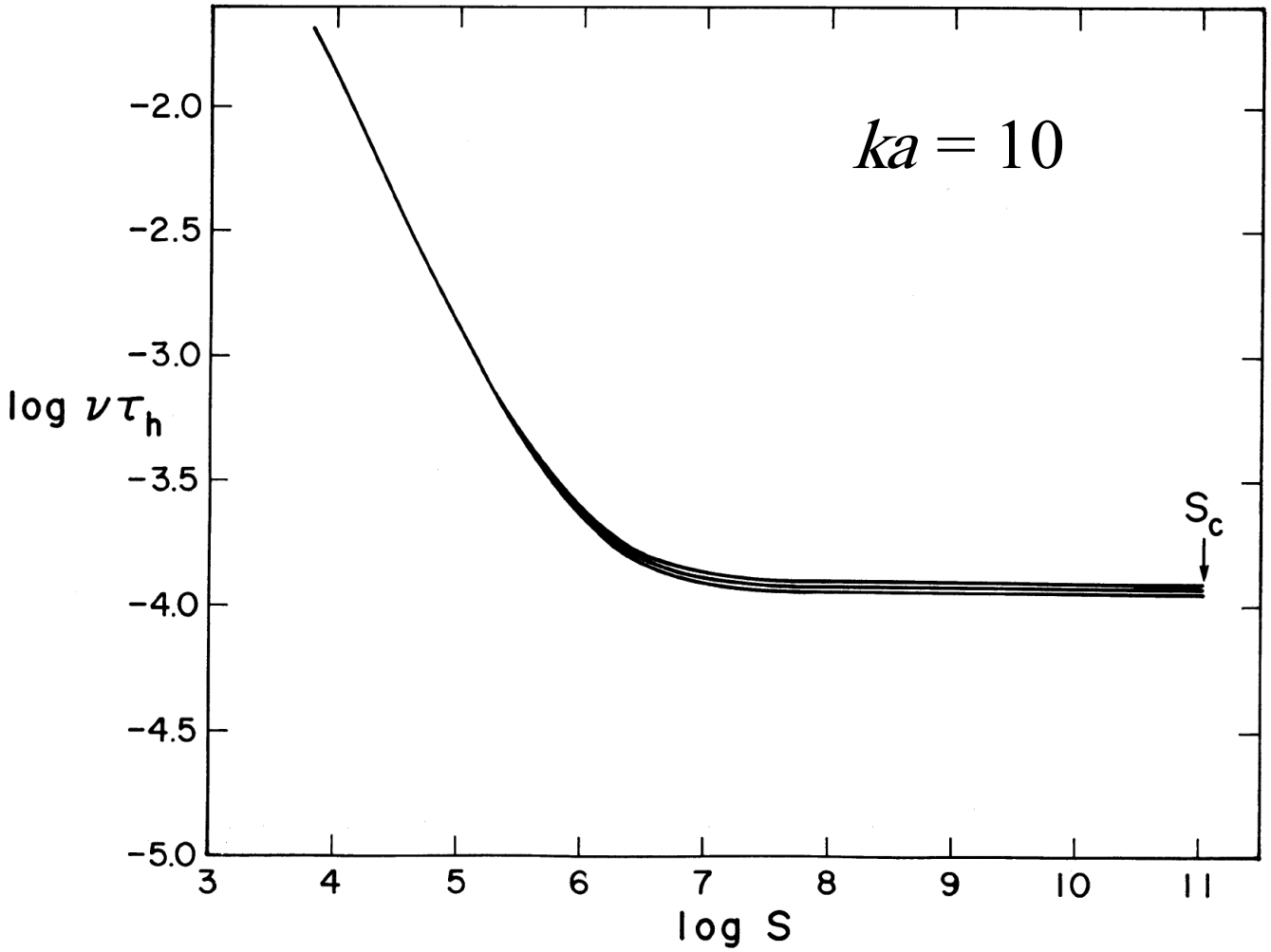
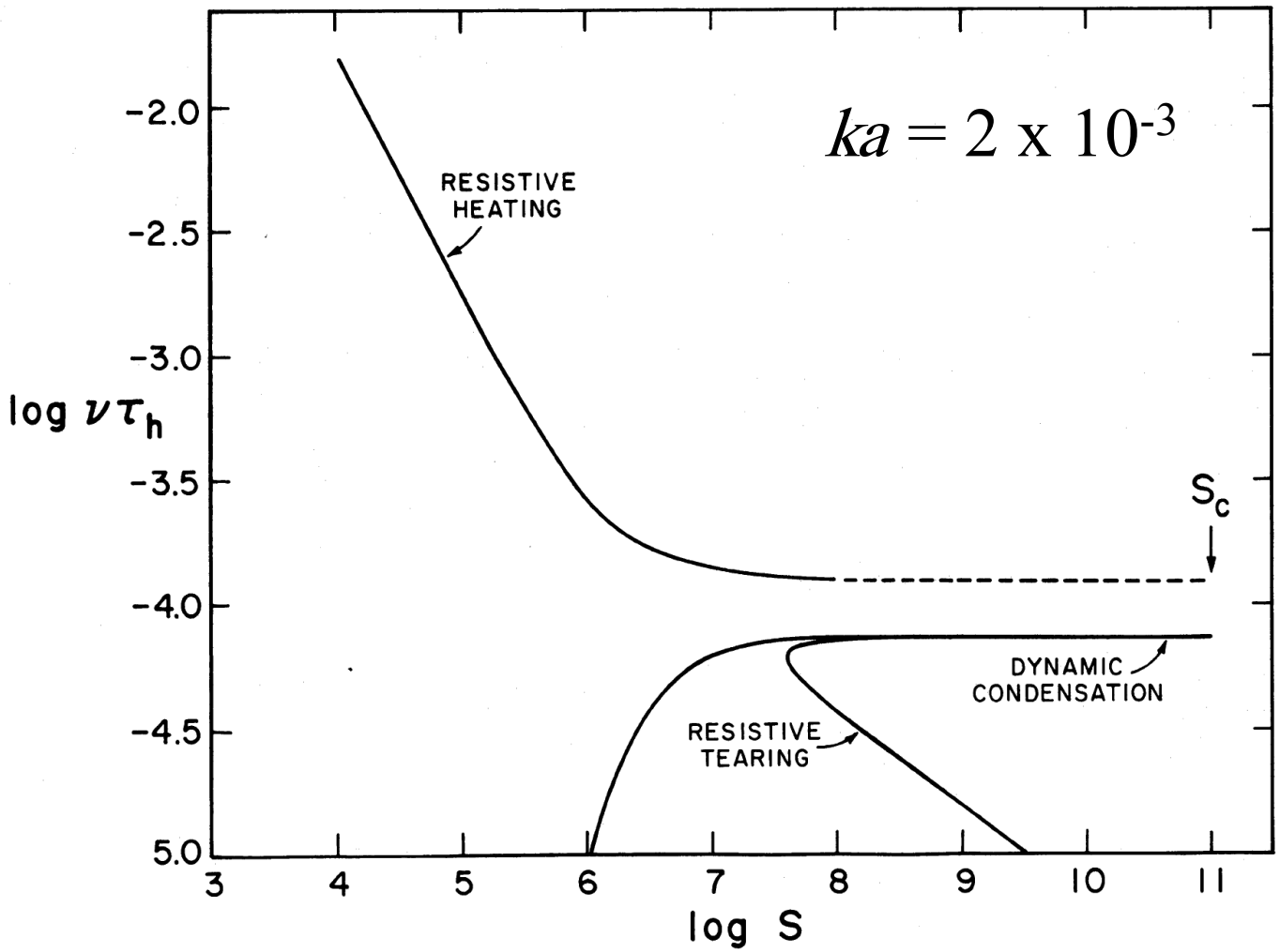
Linear modes in a *uniform* magnetic field



$\mathbf{k} \cdot \mathbf{B}_0 = 0$

Solutions of the form: $q(y) = (A\, e^{+sy} + B e^{-sy})\, e^{ikx}$

Growth rate vs. Lundquist number for kinematic condensation modes



Growth rate vs. S for dynamic condensation, resistive heating mode and resistive tearing mode