Risk Analysis for Resource Planning Optimization

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What this paper is NOT about

• NOT about anatomy of planning and optimization algorithms
  – But to formulate a risk analysis and planning framework that plugs in different planning and optimization schemes like FMINCON, ILOG, and GA

• NOT about generation of an “optimal” plan
  – But to provide a “near-optimal plan” of non-deterministic events whose probability of failure $P_F$ can be quantified analytically and by simulation

• NOT about tedious mathematical derivations
  – But to demonstrate that non-deterministic events and their relationships (constraints) can be mathematically modeled, and lend itself to mathematical optimization and empirical simulation
Main goals

• The main purpose of this paper is to introduce a risk management approach that allows planners to quantify the risk and efficiency tradeoff in the presence of uncertainties, and to make forward-looking choices in the development and execution of the plan.

• Demonstrate a planning and risk analysis framework that tightly integrates mathematical optimization, empirical simulation, and theoretical analysis techniques to solve complex problems.
Problem statement (1)

• Extending link analysis techniques to resource planning optimization in the presence of uncertainties
  – Standard link analysis is a proven statistical risk analysis technique for evaluating communication system performance and trade-off
  – Many of the gain/loss parameters (in dB’s) of the link are statistical
    • Parameter \( x \) with designed value \( x_d \), minimum value, \( x_{\text{min}} \), maximum value \( x_{\text{max}} \), and a probability function \( f(x) \), result in \( x_{\text{mean}} \) and \( x_{\text{var}} \)
    – With the ‘hand-waving’ assumption that the sum of all gain/loss link parameters has a Gaussian distribution with distribution \( N(m, \sigma^2) \), one can design a link and establish link margin policy based on statistical confidence level measured in terms of \( \sigma \) (i.e. n-sigma event)
  
  – Non-deterministic events has variable time durations
  – Extend the link performance analysis (in dB’s) to non-deterministic event planning (in time)
Problem statement (2)

- Some notations
  - Planning horizon \([T_s, T_e]\): given start time \(T_s\), given end time \(T_e\), all events must fit within \([T_s, T_e]\)
  - Event \(E_i\): start time \(t_o^i\), duration \(d_i\), where \(t_o^i\) is the state variables to optimize, and \(d_i\) is a random variable that has a unimodal probability distribution function \(p_i(d_i)\) with mean \(m_i\) and standard deviation \(\sigma_i\)
  - A plan consists of a number of events within the planning horizon, and events \(E_i\) and \(E_j\) might bear certain pair-wise relationship \(R_{ij}\)
  - There are one or more resource limits that cannot be exceeded

\[
\begin{align*}
E_1 & \quad t_o^1 \quad d_1 \quad \text{Triangular distribution} \\
E_2 & \quad t_o^2 \quad d_2 \quad \text{Uniform distribution} \\
E_n & \quad t_o^n \quad d_n \quad \text{Gaussian distribution}
\end{align*}
\]
Problem statement (3)

• Some definitions of terms
  – Planning is the process of a priori scheduling the events within the planning horizon
  – There are one or more objective functions that the plan is trying to optimize subjected to the given rules and constraints
  – A plan is said to be successfully executed if
    • All events in the plan can be accommodate within the planning horizon
    • There is no resource usage that exceeds the maximum allowable limit
    • There is no violation to the set of pre-defined rules and constraints
Applications (1)

- Space mission planning and sequencing
  - Mission planning/sequencing translates science intents and spacecraft health and safety requests from the users into activities in the mission plan
  - Non-deterministic spacecraft events: star-tracker to acquire a star, data volume per pass, slew, … etc.
  - Spacecraft resources: power/energy, data rate/data volume, thermal limits, onboard storage, CPU etc.
  - Event-driven spacecraft activities: an activity could be contingent upon the complete of other activities, upon the state of the spacecraft and/or estimated resources, or triggered by real-time events such as observation of a supernova explosion
Applications (2)

• Risk analysis for cost and schedule planning
  – Model budget (resource) and schedule (duration) and their uncertainties
  – Model tasks dependencies

• Risk analysis for communication network planning
  – Model link durations and their uncertainties
    • Time uncertainty to transmit a certain fix data volume in the presence of retransmission (e.g. Prox-1)
  – Model link availabilities as resources
    • Number of users in a multiple access scheme
    • Data rates
  – Model link dependencies
    • Store-and-forward relay link: forbidden synchronous
    • Bent-pipe relay link: inclusion
Risk analysis approach by iterative simulation and optimization

- Given an acceptable risk level $P_{th}$, find a plan with $P_F \leq P_{th}$ by iterative optimization and simulation
- Plan is intentionally sub-optimal to ensure a stable solution
  - Start time $t_o^i$ is not dependent upon the completion time of any prior events
  - Ensure successful execution of plan as long as $d_i \leq \Delta_i$
- Simulation always converge
- $P_F$ is always “well-behaved”, i.e. increasing the task duration $\Delta_i$ will always yield lesser events to be accommodated but higher probability or completion or vice versa
Mathematical representation of non-deterministic events and constraints (1)

- Examples of objective Functions
  - Given start times $t_o^1, t_o^2, \ldots, t_o^n$ (state variables to optimize)

\[
\begin{align*}
  f_1(t_o^1, \ldots, t_o^n) &= \max_i \{t_o^i + d_i\} & \text{f}_1 & : \text{Minimizing maximum end time} \\
  f_2(t_o^1, \ldots, t_o^n) &= \sum_{i=1}^{n} t_o^i & \text{f}_2 & : \text{Minimizing initial time occurrence of all events} \\
  f_3(t_o^1, \ldots, t_o^n) &= \sum_{i=1}^{n} t_o^i + d_i & \text{f}_3 & : \text{Minimizing end time of all events} \\
  f_n & : \text{Priority weighted versions of the above}
\end{align*}
\]
**Mathematical representation of non-deterministic events and constraints (2)**

- Example of linear constraints
  - Ranges of start time $t_o^i$

\[
\overline{x} = \begin{bmatrix} t_o^1 \\ \vdots \\ t_o^n \end{bmatrix}_{nx 1} \quad \overline{lb} = \begin{bmatrix} T_{\min}^1 \\ \vdots \\ T_{\min}^n \end{bmatrix}_{nx 1} \quad \overline{ub} = \begin{bmatrix} T_{\max}^1 \\ \vdots \\ T_{\max}^n \end{bmatrix}_{nx 1}
\]

\[
\overline{lb} \leq \overline{x} \leq \overline{ub} \Rightarrow \begin{bmatrix} T_{\min}^1 \\ \vdots \\ T_{\min}^n \end{bmatrix} \leq \begin{bmatrix} t_o^1 \\ \vdots \\ t_o^n \end{bmatrix} \leq \begin{bmatrix} T_{\max}^1 \\ \vdots \\ T_{\max}^n \end{bmatrix}
\]
Mathematical representation of non-deterministic events and constraints (3)

- An example of non-linear constraints (with explanation)
  - Forbidden synchronic: when two given events are both scheduled, they must not occur simultaneously at any point in time

Direct form: \[ \max(t_o^i + \Delta_i, t_o^j + \Delta_j) - \min(t_o^i, t_o^j) \geq \Delta_i + \Delta_j \]

Alternate form: \[ [\Delta_i + \Delta_j - |2(t_o^i - t_o^j) + \Delta_i - \Delta_j|] \leq 0 \]
Other examples of non-linear constraints (with no explanation)

- Inclusion: if event $i$ is scheduled, then event $j$ must be initiated in some chosen time interval $[w_o^j, w_f^j]$
  \[
  \left(2t_o^j - w_o^j - w_f^j \right) + w_o^j - w_f^j \leq 0
  \]

- Exclusion: if event $i$ is scheduled, then event $j$ must not be initiated in some chosen time interval $[w_o^j, w_f^j]$
  \[
  \left(w_f^j - w_o^j - \left|2t_o^j - w_o^j - w_f^j\right|\right) \leq 0
  \]

- Others: precedence relationships, resource constraints, etc.
Empirical results and theoretical results (1)

- Theoretical result: a simple upper bound of $P_F$
  - Denote $P_{F,i}$ the probability that event $i$ would end with a duration $d_i$ that exceeds the predetermined duration $\Delta_i$, and $P_{S,i} = 1 - P_{F,i}$
  - Denote $P_S$ the probability that the schedule succeeds, meaning it does not violate constraints nor exceeds the planning horizon; it is obvious that $P_S \geq P_{S,1} P_{S,2} \cdots P_{S,n}$, because $P_{S,1} P_{S,2} \cdots P_{S,n}$ does not take into account all the possible ways in which event may exceed the designated durations determined by $P_{S,i}$, and still have a successful schedule
  - Therefore
    $$P_F = 1 - P_S \leq 1 - P_{S,1} \times \cdots \times P_{S,n} \leq 1 - (1 - P_{F,1}) \cdots (1 - P_{F,n})$$
  - Which results in an upper bound of $P_F$ given by
    $$P_F \leq P_{F,1} + P_{F,2} + \cdots P_{F,n}$$
- The upper bound of $P_F$ can be used to guide the adjustment of $\lambda_i$ in the iterative optimization/simulation process
Empirical results and theoretical results (2)

• Theoretical result: Saddle-Point approximation of $P' F$ of an Ensemble of Tasks in Tandem
  – In task planning, a common situation is that there are a number of tasks that are required to execute in tandem, sometime with a constraint on overall duration
  – If no dependencies between these tasks with other tasks, one can treat them as a single task to simplify downstream analysis and optimization
  – The probability that the total duration of tasks exceed $\alpha$, $P' F(z > \alpha)$, can be approximated by

$$q_+(\alpha) \approx \frac{e^{\psi(s_0)}}{\sqrt{2\pi \psi''(s_0)}}$$

  – See next chart for outline of derivation
Empirical results and theoretical results (3)

- Some notations
  - \( x_1, x_2, \ldots x_n \) are \( n \) independent random variable with pdf \( f_{x_i}(x_i) \)
  - \( z \) is the sum of \( x_1, x_2, \ldots x_n \)
  - \( \Psi_{x_i}(s) \) is the characteristic function of \( x_i \), and \( \Psi_z(s) \) is the characteristic function of \( z \)
  - \( q_+(\alpha) \) is the tail probability of \( z \)

\[
\psi(s) = -s\alpha + \log \Psi_z(s) - \log s
\]

\[
z = \sum_{i=1}^{n} x_i \quad f_z(z) = f_{x_1}(x_1) \ast f_{x_2}(x_2) \ast \ldots \ast f_{x_n}(x_n)
\]

\[
\Psi_z(s) = \int_{-\infty}^{\infty} e^{sz} f_z(z) \, dz \quad \Psi_z(s) = \Psi_{x_1}(s) \Psi_{x_2}(s) \ldots \Psi_{x_n}(s)
\]

\[
q_+(\alpha) = \int_{\alpha}^{\infty} f_z(z) \, dz \quad q_+(\alpha) \approx \frac{e^{\psi(s_0)}}{\sqrt{2\pi\psi''(s_0)}}
\]

- Analysis challenges
  - Evaluation of pdf of sum of \( n \) variables requires \( n-1 \) nested integration
  - Inverse of \( \Psi_z(s) \) is usually extremely difficult, if not impossible
10-event case toy problem
   - Events 1 and 3 may not overlap
   - Event 1 must finish before event 4 begins
   - Each event consumes 1 unit of resource, limit 3 at any time
   - PDF and its parameters of each of the ten event durations

<table>
<thead>
<tr>
<th>Event ID</th>
<th>Type of Dist.</th>
<th>Parameters</th>
<th>Min. Value</th>
<th>Max Value</th>
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<tbody>
<tr>
<td>1</td>
<td>Uni.</td>
<td>NA</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>Beta</td>
<td>$\alpha=4, \beta=4$</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Norm</td>
<td>$\mu=10, \sigma=.5$</td>
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<td>NA</td>
</tr>
<tr>
<td>4</td>
<td>Tri.</td>
<td>Peak=4</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>LogN</td>
<td>$\mu=2, \sigma=.5$</td>
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<td>NA</td>
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<tr>
<td>6</td>
<td>Uni.</td>
<td>NA</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>Beta</td>
<td>$\alpha=5, \beta=5$</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>Uni.</td>
<td>NA</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>Tri.</td>
<td>Peak=3</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>Tri.</td>
<td>Peak=4</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>
10-event case optimization and simulation results

Set durations $\Delta_i$ such that each event has a 99% confidence of successful completion.

<table>
<thead>
<tr>
<th>Simulation ID</th>
<th>Probability of Schedule (10 Events) Failing (5000 runs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0424</td>
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<tr>
<td>2</td>
<td>0.0430</td>
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<td>3</td>
<td>0.0458</td>
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<tr>
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<td>0.0448</td>
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<tr>
<td>5</td>
<td>0.0382</td>
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<td>8</td>
<td>0.0434</td>
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<td>9</td>
<td>0.0400</td>
</tr>
<tr>
<td>10</td>
<td>0.0430</td>
</tr>
<tr>
<td>Ave. $P_F$</td>
<td>0.0414</td>
</tr>
</tbody>
</table>

Upper Bound of $P_F$ 0.10
Empirical results and theoretical results (6)

- 30-event case
  - 2 precedence relations, 1 exclusion relation, 1 resource limit of 3
Using Stochastic Optimization to Find a Good Initial Point (1)

- **Challenges of optimization**
  - Speed and optimization performance depends strongly on the initial guess of the state vector \([t_o^1, t_o^2, \ldots t_o^n]^T\)
  - A bad guess results in slow convergence and/or poor locally-optimal solution

- **Improved optimization using stochastic optimization algorithm**
  - Use stochastic optimization algorithm (e.g. genetic algorithm) to find a set of viable and promising state vectors to serve as initial guesses
  - Use the initial guesses as input to more sophisticated optimization schemes (e.g. Sequential Quadratic Programming in Matlab’s FMINCON) to generate a set of locally-optimal solutions
  - Obtain an “overall” optimal solution out of all the local optima by subjecting them to a probabilistic simulation to determine likelihood of failure and to compare objective values
Using Stochastic Optimization to Find a Good Initial Point (1)

- Genetic Algorithm
- Set of Initial Guesses
- Sequential Quadratic Programming: FMINCON
- Locally Optimal Solution Set
Using Stochastic Optimization to Find a Good Initial Point (2)

- 100 event case, 40 constraints, 2 resources limit of 4
Using Stochastic Optimization to Find a Good Initial Point (3)

- Resource#1 usage profile of 100 event case