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AN INTRODUCTION TO JPL'S ORBIT DETERMINATION PROGRAM

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PREFACE

The work described in this report was performed by the Orbit Determination Group of JPL's Science and Engineering Computing Section (914).

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It is our pleasure to express to them our appreciation.

LIST OF ABBREVIATIONS

AFETR	Air Force Eastern Test Range
ATS	Applications Technology Satellites
CSC	Computer Sciences Corporation
DCS	Direct Coupled System
DPODP	Double Precision Orbit Determination Program
DRVID	Differenced Range Versus Integrated Doppler
DSIF	Deep Space Instrumentation Facility
DSN	Deep Space Network
DSS	Deep Space Stations
GCF	Ground Communications Facility
GPCF	General Purpose Computing Facility
HSDL	High Speed Data Lines
IPODP	Interplanetary Orbit Determination Program
JPL	Jet Propulsion Laboratory
LGFIO	Lawson's Generalized Format Input Output
MSFN	Manned Space Flight Network
MCCC	Mission Control Computing Center
MOPS	Maneuver Operations Planning System
NASA	National Aeronautics and Space Administration
NCS	Network Control System
OD	Orbit Determination
ODE	Orbit Data Editor
ODP	Orbit Determination Program
ONP	Optical Navigation Program
PET	Probe Ephemeris Tape
PTT	Project Tracking Tape
SATODP	Satellite Orbit Determination Program
SCF	Scientific Computing Facility
SFOF	Space Flight Operations Facility
SOP	Simulation Output Program
SPODP	Single Precision Orbit Determination Program
SSDPS	Solar System Data Processing System
TSAC	Tracking System Analytic Calibrations

CONTENTS

CONTENTS (contd)

I. INTRODUCTION

This document is designed to give a general introduction to the Orbit Determination Program (ODP) of the Jet Propulsion Laboratory (JPL). It is not to be an ODP user's guide, which will be produced as a separate document, and it is intended to supplement Moyer's TR 32-1527 and Spier's TM 33-451. It has two classes of readers in mind: those who have little or no background in spacecraft navigation; and those who are familiar with the basic concepts of orbit determination (OD) but wish to see how they are implemented at JPL.

Numerous experiences in educating newcomers have pointed to the need for a study tool which will supply the basic information in an organized manner to enable quicker assimilation of the fundamentals of OD, the structure of the ODP, and the relationship of the ODP to other programs. This last item is quite important. The ODP is a large software system which depends heavily on many other software systems for some of its inputs. The ODP also furnishes many other programs with basic data for specific navigation-related functions. There is a bewildering maze of acronyms for these programs and the organizations which design, implement and run them. This document attempts to place the major components in perspective to enable a better appreciation of the reasons for some of the capabilities found within the ODP and for the way the program has been implemented. Such an overview is expected to facilitate more thorough study of the program's theoretical basis, its implementation details, and its operation by the user.

This document describes in general terms the spacecraft navigation process at JPL and the institutional elements involved in this effort. It outlines the major computer and software systems used and indicates the basic functions of each of these components. It provides a thumbnail sketch of the OD portion of the navigation process by presenting the fundamentals of OD as performed at JPL. In discussing the more recent history of the ODP at JPL including its use in research and flight operations, it covers the implementation of the system on JPL's General Purpose Computing Facility (GPCF) computers, its maintenance, daily operation and continuing development.

It presents highlights of the basic mathematical model embodied in the ODP and gives an overall description of the system in terms of the constituent programs and their interaction with other major software systems.

The document is organized in such a fashion as to allow one to first obtain an overview of the basic concepts and then to delve in greater detail into the formulation and implementation if one so desires. The text is presented in large and small type, usually segregated into separate columns. The large type text contains the more elementary information, while the smaller type sections elaborate on the details of the mathematical model.

II. SPACECRAFT NAVIGATION

II.1 Spacecraft Navigation Overview

JPL is known primarily for design and operation of unmanned deep space missions to the moon and planets. Navigation of these spacecraft involves determining where they are and where they are going, and making such corrections to their paths (trajectories) that they will arrive at the intended place at the desired time. This process involves three conceptually different tasks:

1. Using some equipment to make observations of the spacecraft's location with respect to a given reference point, like a tracking station on Earth, or the target planet the spacecraft is approaching.

2. Using these observations to continually correct the prediction of where (with respect to the target) the spacecraft is going and what time it will arrive at the predicted place.

3. Using the predicted arrival place and time to determine whether the spacecraft will be sufficiently close to the desired aiming conditions to satisfy mission requirements, and if not, to compute the correction maneuver necessary.

The first task is currently performed by two basic techniques: measurements made by a station on the ground using radio signals sent to and from the spacecraft (radio-metric tracking), and TV pictures taken by the spacecraft of the target, usually against the background stars (on board optical guidance). There are other methods under development as well, but these will not be discussed here. Radio-metric observations are discussed in subsection II.1.1.1, while on-board optical guidance is covered in subsection II.1.1.2.

The second task, the actual orbit determination, is sometimes referred to as differential correction because the *difference* between what was actually observed, and what the formulas predict should have been observed based on the current assumption of where the spacecraft was, is used to *correct* the assumptions about the spacecraft's trajectory. Subsection II.1.3 presents the fundamental principles involved in this OD process.

The third task, trajectory correction maneuver determination, is becoming ever more important as the newer missions allow for multiple correction motor restarts in space. Aspects of this task are discussed in subsection II.1.4.

II.1.1 Observable Collection

II.1.1.1 Ground-Based Observations

II.1.1.1.1 The Deep Space Network

These observations are made by a network of radio telescopes owned by the National Aeronautics and Space Administration (NASA) and dedicated primarily for use with deep space missions. These stations, operated by JPL, include two sets of three

26-meter diameter antennas and one set of three 64m antennas. Each set (subnet) has a station at Goldstone, California, one near Madrid, Spain, and one in Australia. Thus, it is possible for at least one station from each subnet to have a given deep space probe in view at any time during a 24-hour day.

These stations are operated by local crews under control of a staff of personnel for the Network Control System (NCS) in the Space Flight Operations Facility (SFOF) at JPL. This staff determines which station tracks which spacecraft at which time and monitors each station's activity by voice and digital lines between SFOF and the stations. These communication lines, which also handle the spacecraft data, are part of the Ground Communications Facility (GCF) run by JPL. The radio telescopes, or Deep Space Stations (DSS) contain sophisticated radio equipment and support computers for communicating with the spacecraft and with computers in the SFOF via the GCF. The stations and their equipment are referred to collectively as the Deep Space Instrumentation Facility (DSIF). The Deep Space Network (DSN) is the JPL institution chartered by NASA to administer the DSIF, GCF and NCS.

The DSN is responsible for three types of communication with spacecraft: command, telemetry and radiometric tracking. The first involves transmission of commands to the spacecraft to control its activities. These commands are generated by those in charge of the spacecraft (the flight project) using software on the Mission Control Computing Center (MCCC) IBM 360/75 computers in SFOF, and are then transferred over the GCF lines to the stations. There, they are eventually sent to the spacecraft by high power radio transmitters operating at microwave frequencies. Telemetry containing engineering and science information is beamed from the spacecraft and collected at the station by the same antenna used for transmitting commands. The station forwards what it receives over the high speed data lines (HSDL) of the GCF to the NCS in the SFOF. Here, the data are transferred to the Project via the MCCC for additional processing. Radio-metric tracking data (which will be described below) are obtained by the stations simultaneously with the recording of telemetry or the transmission of commands. These data join the telemetry and certain monitor data describing the station's performance in the packets of information called GCF blocks that are sent from the station to the NCS over the HSDL.

II.1.1.1.2 The Observables and Their Physical Significance

The DSN stations make three fundamental types of measurements with these radio-metric tracking data. They record the pointing direction of their antennas in terms of two angles (typically, hour angle and declination or azimuth and elevation), thereby giving two components of the location of the spacecraft in the sky.

They measure the difference in frequency between the signals sent to the spacecraft and those received from it, which is caused in part by apparent motion of the spacecraft to or from the station. The portion of this difference due to relative motion between the spacecraft and the station is called the Doppler shift, and is used within the ODP to measure the relative velocity of the probe in a radial direction (along the line of sight) from the station. Finally, the stations measure a quantity related to the length of time it takes for a signal to be transmitted from the ground to the spacecraft and back, which can be used by the ODP to determine the distance (range) to the spacecraft.

II.1.1.1.2.1 Angles

The large radio telescopes at the stations are not pointed at the spacecraft with anything near the precision that optical telescopes can be pointed at the stars. The design of the equipment itself does not permit determination of the direction of the signal beam too much better than a thousandth of a degree. This is not a disadvantage because the other radio-metric observables (Doppler and range) can be used indirectly within the ODP to determine these same angles to greater precision than would even be possible with optical telescope quality pointing. It does mean, however, that angle data are very seldom used in OD at JPL. These data are helpful for the first few hours (4–6) after launch when the spacecraft's direction is changing quite rapidly with respect to the background stars, but their information content is soon superseded by that in the Doppler and range measurements. As a consequence, when the MCCC delivers their "Project Tracking Tape" (PTT) of radio-metric data to the project, project personnel in performing basic pre-processing of the data with the Orbit Data Editor (ODE) program will totally eliminate angle data from the file they pass to the ODP. The ODE is a program developed and maintained by Section 914 and operated by Navigation personnel from Section 391 on the GPCF Univac 1108's.

- **Angles**

Angle observations indicate the direction, relative to some coordinate system, in which the tracking station antenna was pointed while making spacecraft observations. Two angle pairs which specify the direction of the antenna are available. Figures 1 and 3 provide a description of these angles.

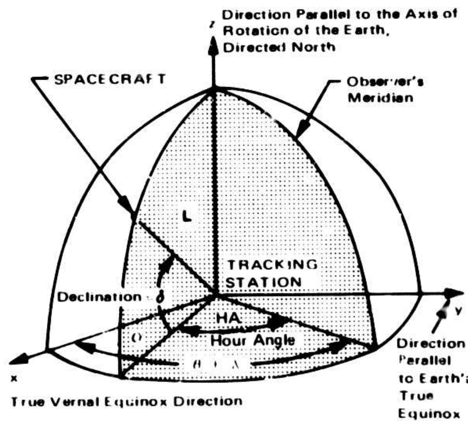


Fig. 1. Hour Angle (HA) and Declination (δ)

Hour Angle (HA) (Directly observed by most DSN-stations)

$$\boxed{HA = (\theta + \lambda) - \alpha, \qquad 0^\circ \leq HA < 360^\circ} \tag{1}$$

where

θ = True sidereal time at reception time t_3,

λ = East longitude of tracking station, relative to true pole,

α = Right ascension = $\tan^{-1}\left(\frac{L_y}{L_x}\right)$, $0^\circ \leq \alpha \leq 360^\circ$

Declination (δ) (Directly observed by most of DSN-stations)

$$\boxed{\delta = \sin^{-1} L_z, \qquad -90^\circ \leq \delta \leq 90^\circ} \tag{2}$$

$$\mathbf{L} = \text{unit vector} = \begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix}$$

11

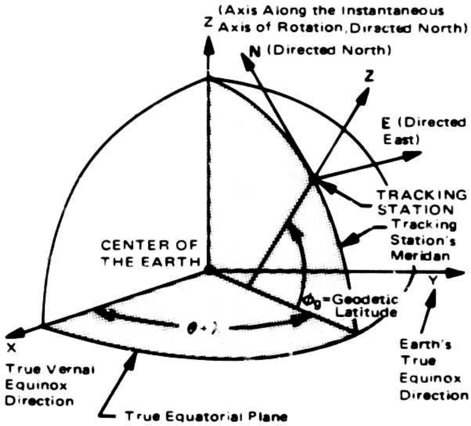


Fig. 2. The North-East-Zenith Coordinate System

where the unit vectors **N**, **E**, and **Z** are given by the following expressions:

$$\mathbf{N} = \begin{pmatrix} -\sin \phi_g \cos (\theta + \lambda) \\ -\sin \phi_g \sin (\theta + \lambda) \\ \cos \phi_g \end{pmatrix}$$

$$\mathbf{E} = \begin{pmatrix} -\sin (\theta + \lambda) \\ \cos (\theta + \lambda) \\ 0 \end{pmatrix}$$

$$\mathbf{Z} = \begin{pmatrix} \cos \phi_g \cos (\theta + \lambda) \\ \cos \phi_g \sin (\theta + \lambda) \\ \sin \phi_g \end{pmatrix}$$

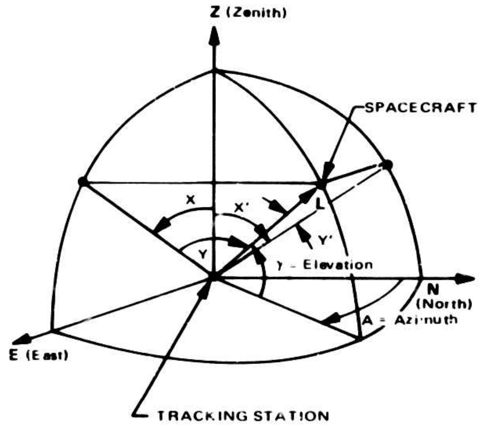


Fig. 3. Azimuth (A), Elevation (γ) and X–Y, X′–Y′

Elevation Angle (γ) (Observed directly by most AFETR stations and some DSN stations)

$$\gamma = \sin^{-1}(\mathbf{L}\cdot\mathbf{Z}), \qquad 0^\circ \le \gamma \le 90^\circ \tag{3}$$

Azimuth (A) (Observed directly by most AFETR stations and some DSN stations)

$$\mathbf{A} = \tan^{-1}\left(\frac{\mathbf{L}\cdot\mathbf{E}}{\mathbf{L}\cdot\mathbf{N}}\right), \qquad 0^\circ \le \mathbf{A} \le 360^\circ \tag{4}$$

X, Y (X30, Y30) angles (Observed by Manned Space Flight Network MSFN)

$$\begin{aligned} \mathbf{X} &= \sin^{-1}\left(\frac{\mathbf{L}\cdot\mathbf{E}}{\cos\mathbf{Y}}\right), & -90^\circ \le \mathbf{X} \le 90^\circ \\ \mathbf{Y} &= \sin^{-1}(\mathbf{L}\cdot\mathbf{N}), & -90^\circ \le \mathbf{Y} \le 90^\circ \end{aligned} \tag{5}$$

X′, Y′, (X85, Y85) angles (Observed by MSFN)

$$\begin{aligned} \mathbf{X'} &= \sin^{-1}\left(\frac{\mathbf{L}\cdot\mathbf{N}}{\cos\mathbf{Y'}}\right), & -90^\circ \le \mathbf{X'} \le 90^\circ \\ \mathbf{Y'} &= \sin^{-1}(\mathbf{L}\cdot\mathbf{E}), & -90^\circ \le \mathbf{Y'} \le 90^\circ \end{aligned} \tag{6}$$

Note that when $\left.\begin{matrix}\mathbf{Y} = \pm 90^\circ \\ \mathbf{Y'} = \pm 90^\circ\end{matrix}\right\}$ (spacecraft on the horizon) then **X**, **X′** are indeterminate.

II.1.1.1.2.2 Ranging

The ranging observables are a sophisticated cousin of the simple radar systems that send out a pulse, measure how long it takes for the pulse to return, and multiply the elapsed time by 300,000 km/sec to obtain the distance. Although there are a variety of ranging systems, they all work in basically the same way.

A coded signal of a very special waveform is sent repeatedly from the transmitter for a long period of time (continuously for the sake of an example). The length of this code, call it n seconds, is known very precisely, to within a few nanoseconds. The signal travels to the spacecraft, is amplified and retransmitted by a transponder, and returns to the station. Upon receipt at the station, a measurement is made to determine by how many nanoseconds this received code would have to be shifted to be in phase with the current outgoing coded signal. This amount of time may be converted to some arbitrary measurement units before release as the measured observable. It is important to note, however, that this observable is not the total round-trip signal travel time (light time) in range units. Instead, it is the round-trip light time, modulo n seconds, expressed in terms of some arbitrary range units. Given a reasonable estimate of the distance of the spacecraft in terms of integral numbers of n light seconds of round-trip light time, this observable allows one to complete the round-trip measurement to the limit of measurement precision of the system. M t of the systems allow use of a number of different length codes to help resolve the modulo number ambiguity.

The MARK1 and MARK1A systems are of Lunar Orbiter vintage. The 26-meter antennas currently all have MARK1A systems, which are useful out to a range of approximately 2×10^7 km for Mariner type spacecrafts, after which the signal-to-noise ratio becomes too low. The TAU system was implemented at DSS 14 in time for Mariner V in 1967 and until 1971 was the mainstay of deep space ranging. During 1971, the MU system was installed in an experimental basis at DSS 12 and DSS 14. It has since given rise to 4 new ranging systems: MU, MU2, and the Planetary Operational systems PLOP, and PLOP2.

Two-way ranging is the standard mode, preferred to three-way ranging because of the difficulty of synchronizing codes at the different stations to the nanosecond level.

Not all spacecraft have ranging transponders so ranging observables are not available for all missions.

II.1.1.1.2.3 Doppler

When there is relative motion (involving a change in distance) between a source emitting a signal of a given frequency and an observer, the observer detects a signal of a different frequency from that which the source is transmitting. The difference in frequency (the Doppler shift) is a function of the magnitude and sign of the relative speed of the source with respect to the observer.

The case of a spectator at a model airplane meet is a good example of this effect. The person flying the airplane in a horizontal circle using the usual control wires hears a constant pitch from the model's engine because he is always a fixed distance from the model. The spectator, on the other hand, hears the pitch alternate in a sinusoidal fashion, first higher than usual as the model comes in his direction on the circle, and then lower as it goes in the opposite direction. The amplitude of the variation in pitch is a function of the tangential velocity of the model, and in principle, once the normal pitch of the engine were known, the airspeed (tangential velocity) of the model could be determined by noting the variation in pitch over part or all of one complete variation.

The spectator would presumably also be able to imagine, while closing his eyes, the times at which the model was crossing the line of sight from him through the person flying the model, because at those times the pitch, halfway between its high and low extremes, would be equal to that of the airplane normally, when the model was held stationary.

Finally, the more mathematically inclined spectator would realize that he would hear a slightly different range of pitch variations if he sat at the top of the grandstand than if he sat at eye level to the plane in which the model was flying. This would occur because there would be a smaller component of relative velocity of the model in his direction the further out of its plane of flight he moved.

Since the Doppler effect is a function of the relative velocity between the principals in this scenario (the model airplane and the spectator), it should be obvious that if the spectator were blowing on a horn to give a continuous constant-pitched tone, the tone heard by a microphone on the airplane would also vary sinusoidally.

Spacecraft OD at JPL has long been based on just this principle. For the horn-blowing spectator, substitute a stationary spacecraft transmitting a radio signal. For the model airplane with a microphone, substitute a tracking station rotating with the Earth. The hour angle of the spacecraft can be determined by noting the variation in the rate at which the tone changes: when the spacecraft is directly over the observer's meridian, he should hear the frequency the spacecraft is transmitting. The declination of the spacecraft (the angular distance away from the plane described by the daily rotation of the tracking station), as with the spectator moving to higher rows in the grandstand, will affect the range of the variation in received frequency.

In practice, this concept quickly is complicated by the fact that the spacecraft is not stationary in space (the spectator is moving around in the grandstand) and the Earth has orbital motion in addition to the daily rotation (the person flying the model doesn't always stay in the same spot as he pirouettes). Moreover, there are not radio transmitters available for spacecraft that can reliably put out a constant frequency over a long period of time. This mode, called one-way Doppler because it involves only a signal from the spacecraft to the Earth, is, therefore, not often used.

A much more reliable means is to have the spacecraft listen for a signal from the tracking station and once it is received, adjust the frequency by a given factor to avoid interference and retransmit to the tracking station. As long as the stability of the oscillator at the station is sufficiently high that the frequency transmitted stays effectively constant over the full time it takes the signal (travelling at 300,000 km/sec) to get to the spacecraft and back, this "two-way" Doppler can be used in a similar manner to measure the spacecraft velocity relative to the station. Here, of course, the difference between the trasnmitted and received frequencies depends, among other things, on the Doppler shift heard at the spacecraft, the frequency multiplication factor at the spacecraft, and the Doppler shift of the signal re-broadcast by the spacecraft as it is finally received on the ground.

A third variation involves reception of a signal by a different station from the one that transmitted it. Although conceptually no different in principle, in practice this "three-way" Doppler is less reliable than "two-way" because the oscillators governing the transmitted and/or comparison frequency standards at the two stations do not have the same short-term stability.

The frequencies transmitted are in the microwave region of the electromagnetic spectrum. Up until the mid 1960's, frequencies in the 960 x 10^6 cycles per second (960 Megahertz (MHz)) region were used. These "L-band" systems gave way in time to the "S-band" systems at 2300 MHz and "X-band" systems at 6600 MHz are now making their entry. The higher frequency systems allow for greater telemetry transmission rates and are less susceptible to deleterious effects on the signal caused by charged particles in the medium through which it passes.

The tracking stations effectively mix the signal from their transmitter with that from their receiver to produce a beat frequency, to which is added a constant bias to keep the measured output positive. This output passes through a counter which increments by 1 every time another cycle passes. This counter is sampled periodically to give the current accumulated cycle count. A device called a resolver measures the fraction of a cycle that had passed since the last full cycle was counted. The counter is not reset except by an overflow. The period between samples is called the sample time, which can be as little as 0.1 second for entry and exit occultation measurements, and is usually either 10 or 60 seconds otherwise.

As part of the pre-processing mentioned earlier, the ODE computes average Doppler frequencies over intervals of one or more sample times. These frequencies are computed by dividing the difference in cycle counts at two times by the interval between them, and including the resolver effects. If the interval is longer than a single sample time, the Doppler data is said to be "compressed" to the length of that interval. For a sample rate of 1 minute, if the average frequency were computed by differencing samples taken on the 1st and 21st minutes, the data would be "compressed" to 20 minutes, or would have a 20-minute "count time." Compressing has the result of ameliorating the effect of quantization measurement errors in the resolvers and thus reducing "noise" on the data. It also decreases the number of data points to be processed while, under appropriate circumstances, preserving sufficient information content of the data.

The lower the rate of change of relative velocity, the less the information that can be lost by compressing. Conversely, the higher the rate of change, the shorter the count time must be to avoid washing out (by averaging) the information contained in the rapidly changing relative velocity.

II.1.1.1.3 Tracking and Acquisition Predictions

In order to obtain radiometric observables, the tracking station has to know in which general direction to point its antenna, and what are the optimum transmission and reception frequencies for its own and the spacecraft's benefit. The latter concern arises because the spacecraft receiver is designed to work over a given range of frequencies and the closer to the center of this range the incoming signal is, the better the reception. It is desirable, therefore, to transmit that frequency from the station which when it reaches the spacecraft after undergoing a Doppler shift will be within the acceptable range of the spacecraft receiver.

Part of the DSN's operation, therefore, involves generation of these pointing and acquisition predictions, or "predicts". These are generated from predicted trajectories produced by navigation personnel. The DSN, in concert with project personnel, determines what an acceptable spacecraft acquisition frequency will be for a given day, and combining that with the predicted Doppler shift factors for the path from the transmitting station to the spacecraft (the up-link), determines what frequency should actually be transmitted. This is done in the program PREDIX.

The predicts include among other things the rise and set time of the spacecraft as viewed by the station.

II.1.1.2 Spacecraft-Based Observables

These observations basically involve using a TV camera on board the spacecraft to view a target planet and/or its satellites against the background stars, or the edge (limb) of the planet itself. What results then is a measurement of the position of the spacecraft with respect to the target, rather than with respect to a tracking station on earth. The TV picture is transmitted from the spacecraft along with other science data over a telemetry link separate from that used for radio-metric tracking. Naturally, these observables can only be taken by spacecraft that carry imaging systems.

The most successful technique so far has been that used on the MM'71 approach to Mars, during which numerous images were taken of the tiny moons, Phobos and Deimos, against the background stars. By observing these bodies at different times, their positions with respect to the background stars would seem to change. This was a function both of their orbital motion and the gradual motion of the spacecraft nearer to Mars, which produced a slightly different viewing angle. These observations were used to determine the bodies' orbits more precisely, and thence to infer the position of Mars itself, around which the objects revolved.

This processing is currently performed by Optical Navigation Programs (ONP) like TGP, OOPG, ODAP and CERPLP, which interface with the ODP.

II.1.1.3 Tracking Data Flow

The following chart shows the radio-metric tracking data flow through the navigation system during the MVM'73 era.

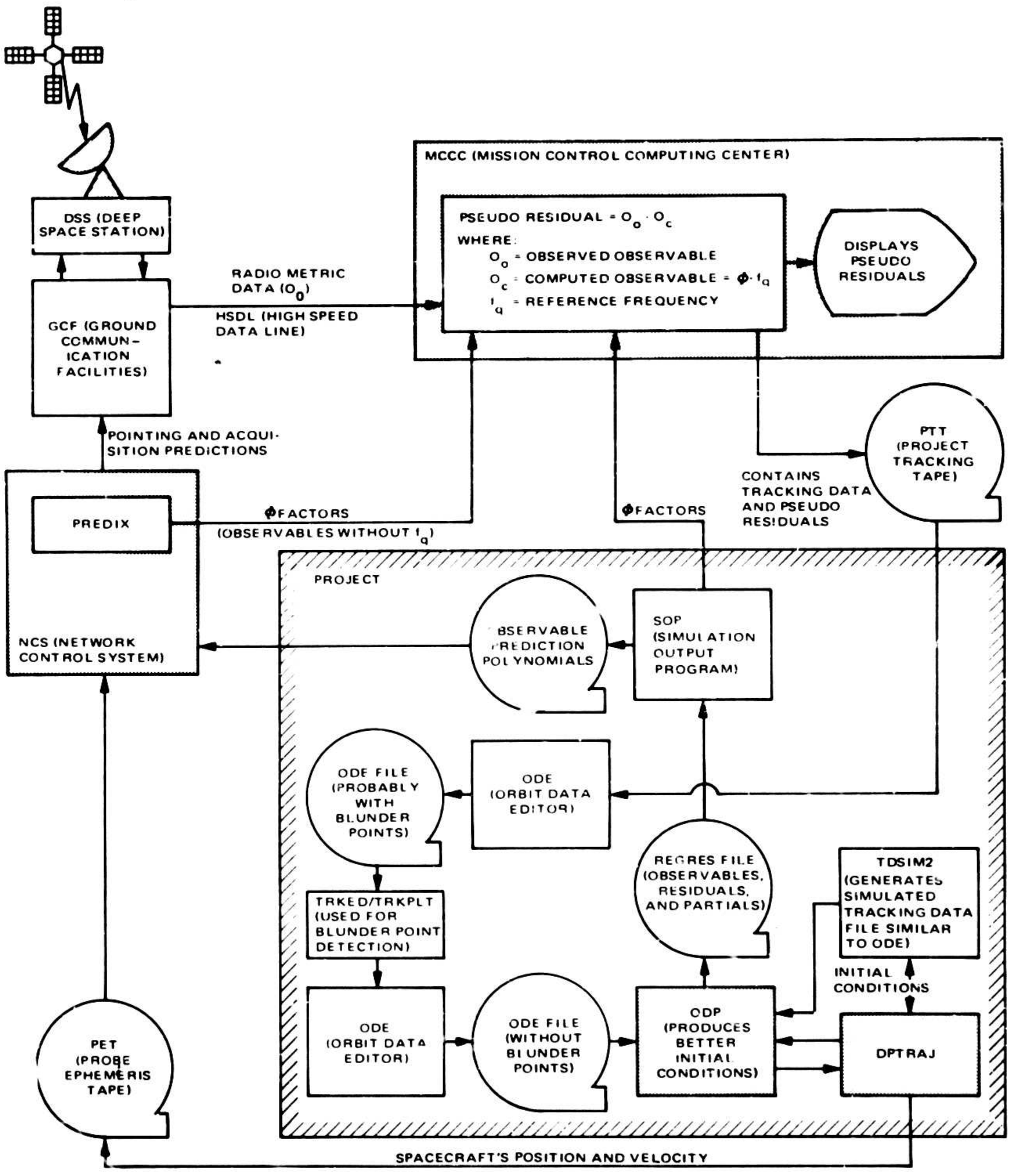


Fig. 4. Tracking Data Flow

II.1.2 Supporting Data

II.1.2.1 'lanetary and Lunar Ephemerides

One of the most important additional types of data used in OD are tables of positions of the Earth, Moon and planets. These ephemerides, as they are called, allow the ODP to compute the position of the spacecraft with respect to the planets, and thus the gravitational accelerations it feels. They enable the program to account for the motion of the Earth-Moon system around the Sun, and the Earth around the center of mass (barycenter) of the Earth-Moon system, all of which affect the observed Doppler shift, range and angles mentioned earlier.

These ephemerides are produced by the Solar System Data Processing System (SSDPS) which is maintained and operated by Section 391. The SSDPS performs many of the functions of an orbit determination program; by simultaneous numerical integration of the equations of motion of all the planets and the Moon, it computes dynamically consistent trajectories for all these bodies; by use of these trajectories, it computes what it would expect to see in terms of optical right ascensions and declinations and round-trip ranges for those times at which the real observations were taken; by use of partial derivatives of these computed observables with respect to quantities related to the initial positions and velocities of the Earth-Moon barycenter, Moon and planets, and the difference (the "residual") between the actual and computed observables, it differentially corrects the initial conditions in a fashion that minimizes the sum of squares of these residuals.

The SSDPS produces ephemerides in a specific form called the "type 66" format. This is the format used within the SSDPS for observable and partial derivative generation. In the past, a somewhat more compact but less accurate "type 50" format, had been used within the ODP. This form was generated by auxiliary programs of the SSDPS which used the "type 66" ephemerides as sources and combine them with nutation* data derived by analytical formulae. The "type 50" format, however, has been replaced by one which utilizes Chebyshev coefficients. This enables the ephemerides stored in the ODP to be of compatible accuracy with those in the "type 66" format.

II.1.2.2 Platform Parameters

Platform parameters define the position and motion of the tracking stations with respect to the instantaneous equator and rotation axis of the Earth.

The Earth rotates on its axis with a speed that varies somewhat predictably from season to season and less predictably from day to day. Over a long period of time, the rotation rate has been observed to be decreasing. This rate is measured by numerous institutions throughout the world. JPL has committed to using the measurements taken by the worldwide network of the Bureau Internationale de le Heure (BIH) which periodically sends JPL determinations of the current difference between time told by high precision atomic clocks and that told by the rotation of the Earth. These discrete measurements are fit by polynomials for use in the ODP by the programs PLATO and/or STOIC.

Another phenomenon quite noticeable in high accuracy orbit determination is the motion of the body-fixed axes with respect to the rotation axis of the Earth. This "polar motion" is significant enough that over the course of 13 months, the true rotation axis appears to wander with respect to the geographic pole, over an area the size of a tennis court. This pole wandering is also observed by BIH and subjected to curve fitting in PLATO and/or STOIC.

The "timing and polar motion" decks generated by PLATO and/or STOIC contain these polynomials and are an essential part of any attempt to represent real tracking data. The polar motion is expressed in terms of the X and Y coordinates, in seconds of arc, of the current instantaneous rotation axis with respect to the geographic north pole (Conventional International Origin, sometimes called the "mean pole of 1903.0").

The tracking station locations are given in terms of latitude and longitude with respect to the prime meridian and equator defined by this 1903.0 rotation axis and Greenwich England. Station locations can be expressed in a variety of coordinate

*Nutation is a periodic "wobbling" of the Earth's rotational axis which is caused by the rotation (with respect to the ecliptic) of the plane of the Moon's orbit about the Earth.

systems, but in the ODP they usually appear as cylindrical coordinates: longitude, the perpendicular distance from the rotation axis (the "spin-axis" distance) and the perpendicular distance from the equator (Z height). Station locations are derived by processing radiometric tracking data with the ODP itself.

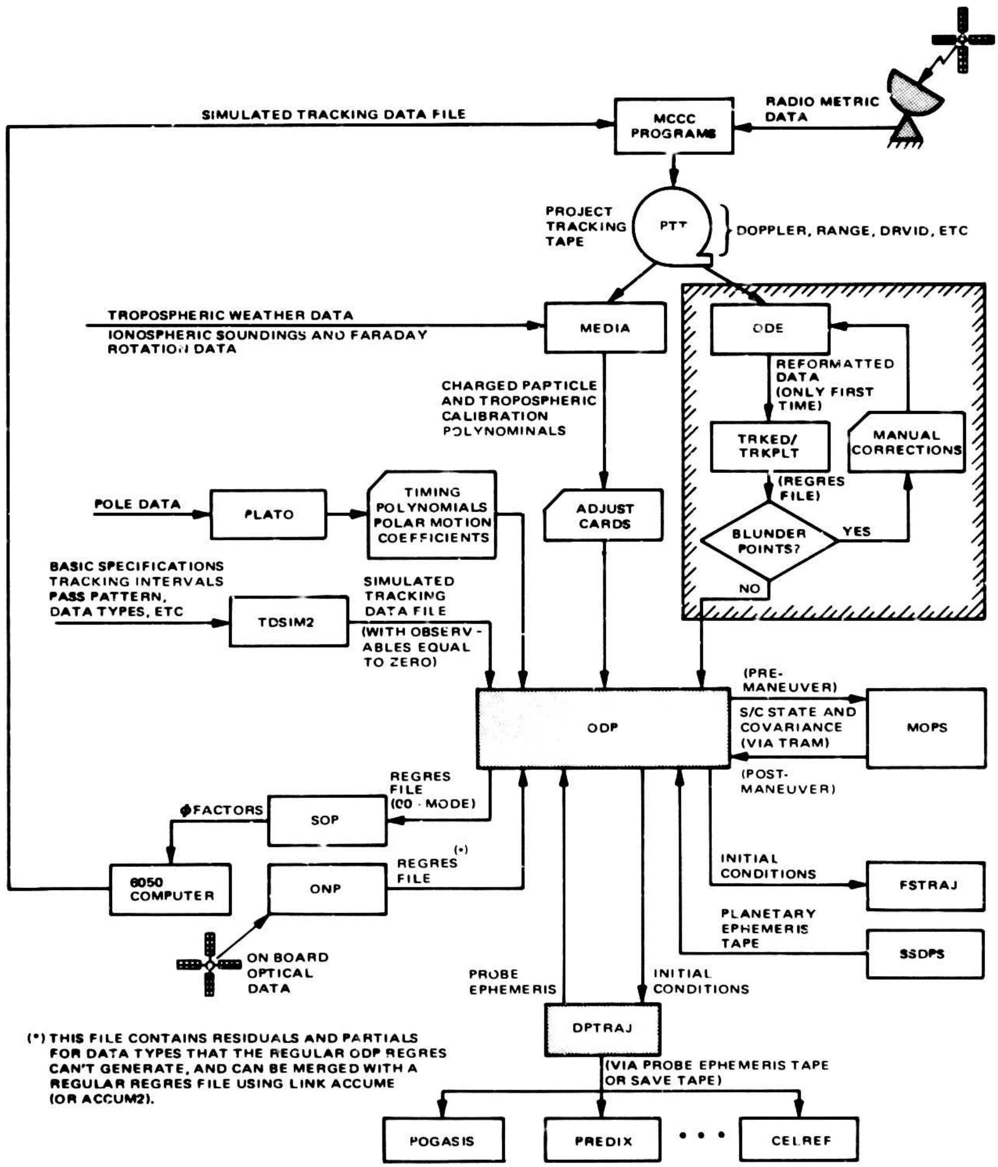
II.1.2.3 Tracking System Analytic Calibrations (TSAC)

The fact that the radio signals must pass through the Earth's atmosphere on the way to and from the tracking stations, combined with the presence of small but perceptible numbers of charged particles in the "void" of space causes the radio signals to behave differently from the way they would in a vacuum. Neutral particles in the Earth's troposphere slow the group velocity of electromagnetic waves passing through them, just as charged particles do, whether in the Earth's ionosphere or in the solar plasma in space. The phase velocity, which is what affects the Doppler observable, also decreases in the presence of neutral particles but increases when charged particles are traversed.

The program MEDIA maintained and operated by Section 391, generates polynomials to account for these effects in the ODP. Tropospheric calibrations are derived from temperature, pressure and relative humidity measurements averaged on a monthly basis. Charged particle calibrations come from a variety of sources. Faraday rotation measurements from Applications Technology Satellites (ATS) and other Earth orbiting satellites determine the amount a linearly polarized radio signal from the spacecraft is rotated as it passes through the ionosphere. The amount of rotation can be related to the charged particle content along the line of sight. The Differenced Range Versus Integrated Doppler (DRVID) technique makes use of the differing effects of charged particles on the group (range) and phase (Doppler) velocities to measure how the charged particle content varies in time. The dual frequency, or "S-X band", approach uses the difference in amounts of retardation due to charged particles in two signals simultaneously transmitted from the spacecraft at different carrier frequencies, to assess the number of charged particles traversed by the waves.

II.1.2.4 ODP Interfaces

The following chart shows the basic ODP interfaces with other navigation software during the MVM'73 era.



(*) THIS FILE CONTAINS RESIDUALS AND PARTIALS FOR DATA TYPES THAT THE REGULAR ODP REGRES CAN'T GENERATE, AND CAN BE MERGED WITH A REGULAR REGRES FILE USING LINK ACCUME (OR ACCUM2).

Fig. 5. ODP Interfaces

II.1.3 The Orbit Determination Process

II.1.3.1 General Information

The formulae used to describe the motion of the spacecraft, due to gravitational attractions by the various major bodies in the solar system, to solar radiation pressure, and to gas leaks on-board the probe, are all referred to collectively as the "model" of the spacecraft's motion. There are similar models describing the effect on the observable of the various motions the Earth undergoes, of particles in the Earth's atmosphere or in the interplanetary medium, etc.

These models are used to compute what the values of the observables should be at any given time, and when the computed values differ from those actually observed, and there is no reason to discredit the observed data, one can only conclude that the model is erroneous. The model can be viewed *conceptually* as a series of formulae, implemented on a given computer. Assuming the formulae are correctly *coded*, the model may produce incorrect results because of numerical imprecision, accumulated round-off or other computer related problems, or because the formulae themselves are deficient. The latter may be due to such things as incorrect coefficients and/or to incomplete structure of the basic expressions.

As an example of incorrect coefficients, consider the standard expression for the gravitational attraction of body **B** on body A. This attraction is proportional to the inverse square of the distance between the bodies, and one of the proportionality coefficients is the mass of body **B**. If the value used for that mass is incorrect, the derived acceleration is incorrect, and one could say the spacecraft motion is mismodeled. In fact, common usage tends to use "mismodelling" more to indicate formulational deficiencies, as for example those describing random gas leaks on the spacecraft. Although one might have a general idea of how to represent the forces generated by a sputtering attitude control jet, the fact that the acceleration as a function of time is not accurately derived from the engineering telemetry data makes it much more difficult to come up with a representative force function that need only be scaled up or down to make it agree with the observations.

The orbit determination process involves primarily the improvement of the values of coefficient-like quantities, and to a smaller degree, accounting for formulational deficiencies. ("Coefficient-like" quantities are meant to include true coefficients as mentioned earlier, arguments of trigonometric functions, initial conditions in differential equations, etc.). In general, the processing is done to sufficient precision and with such an eye to numerical difficulties that the computer-induced problems are insignificant.

The need for an accurate model of the spacecraft's motion should be obvious, since it is used to obtain the position of the probe at any time – past, present, or future. Such information helps the users know exactly where their data were taken and helps mission planners determine how and when to take new data, perform maneuvers, etc. The model that computes the observables based on given spacecraft coordinates is also important since errors in it might be misinterpreted as errors in the model of the spacecraft's motion.

The ODP is not the sort of program that would enable one to determine the orbit of an unknown object of which observations were available. It is a program for refining the coefficients (parameters) that describe an existing notion (estimate) of what the orbit is. By a series of steps, the initial parameters are iteratively corrected until there are no changes possible that further improve the agreement of the computed with the real observables for a given set of data. The nature of this "agreement" and the steps taken to achieve it will be described below.

The basic principle involved comes from an analysis of the Taylor series expansion of a function f(**x**) about the point **x=a**, in which

$$f(\mathbf{x}) = f(\mathbf{a}) + f'(\mathbf{a})(x-a) + \frac{f''(\mathbf{a}).(x-\mathbf{a})^2}{2!} + \frac{f'''(\mathbf{a}).(x-\mathbf{a})^3}{3!} + \ldots + \frac{f^{(n)}(a).(x-a)^n}{n!} + \ldots$$

Assume that the correct value of the parameter is **x** but that the initial estimate of it is **a**, and that the quantity f(**x**) is the actual observed data, which naturally depends on the true value **x**. Since one has an initial estimate **a** of that parameter (sometimes called a nominal value), it is possible to generate a computed value f(**a**) for that observable, and the derivative of the observable with respect to that parameter at **x**=**a**. Then, for **a** reasonably close to **x**, one can ignore the second and higher order derivatives and approximate the difference (called the residual) between the observed and computed value of the observable as

$$f(\mathbf{x}) - f(\mathbf{a}) \cong f'(\mathbf{a}) \cdot (\mathbf{x} - \mathbf{a})$$

and this can be used to solve for **x**. The more distant the initial estimate **a** is from the true value **x**, the poorer this approximation will be, but under proper circumstances, the procedure could be iterated as with Newton's method for approximating roots of equations until the new **a** becomes arbitrarily close to **x**. The problem in the ODP is somewhat more complicated because the observable is a function of many variables, and because the observed data often is slightly degraded due to equipment limitations.

The subsections that follow will outline how the computed observables are generated in the ODP, how the above derivatives are obtained, how new estimates of the parameters are determined, and what additional capabilities are provided to assess the quality of the new estimates and the amount they change existing ideas of where the spacecraft was, is currently, and will be located.

II.1.3.2 Trajectory Generation

This is the first step in observable generation because any datatype is basically either directly or indirectly trying to measure the spacecraft's position and/or velocity. The path (trajectory) that the spacecraft follows is computed by the ODP and represented as a series of coordinates (positions and velocities) at given times. The vector of position and velocity coordinates is often referred to as the probe's "state". This table of coordinates, called the probe's ephemeris, can be interpolated to give spacecraft states at intermediate points, if necessary.

The ephemeris is generated by numerically integrating the differential equations of the probe's motion. The probe is constantly experiencing a variety of forces in a number of directions which give it a time-varying acceleration. These forces include motor firings; gravitational attractions from the Sun, the planets and their satellites; radiation pressure from the Sun; reactions from gas expelled or leaking from the spacecraft; drag in tenuous upper atmospheres, etc. These forces are so numerous and the models describing them so complex that it is practically impossible with current technology to develop analytic expressions describing spacecraft motions over any significant period of time.

The approach, therefore, has been to start with a given set of initial conditions (the initial state) and by some means to approximate the state at equally-spaced intervals on either side of the initial starting time (epoch).

Using these approximate states, one computes the accelerations that would be predicted from the models if the spacecraft had those coordinates at those times. A polynomial of a given order is then fit to those accelerations, and evaluated to give better estimates of the states at the various equally spaced times. These new states are used to recompute the accelerations and the procedure is continued until successive polynomials differ by less than a user specified value, at which time the process can progress to a new time point.

For this progression, the integrated polynomial is extrapolated to produce a state at the new time point, which in turn is used to compute accelerations at that point. As before, the acceleration polynomial is refit to this new point and reintegrated to give a refined state estimate, and so on. The process is complicated somewhat by sudden quickly varying or even discontinuous accelerations, incremental velocity changes due to short motor firings, etc., and rather sophisticated algorithms have been developed for handling these types of problems.

Given a set of ephemerides for the planets, from which their positions can be interpolated, and an estimate of the forces expected to act on the spacecraft over a specified period of time, one could generate tracking station "predicts," target encounter time and location predictions, etc.

- **Differential Equations of Spacecraft Motion (PATH)**

This paragraph describes the differential equations of motion of spacecraft that are integrated numerically in a rectangular coordinate system to give the spacecraft ephemeris with ET as the independent variable. The X-axis is directed along the mean equinox of 1950.0; the Z-axis is normal to the mean earth equator of 1950.0, directed north; and the Y-axis completes the right-handed system. The center of integration is located at the center of mass of the Sun, the Moon, or one of the nine planets. It may be specified as one of these bodies or it may be allowed to change as the spacecraft passes through the sphere of influence of a planet (relative to the Sun) or the Moon (relative to the Earth). In this case, the center of integration will be the body within whose sphere of influence the spacecraft lies. At a change in the center of integration, the position and velocity of the spacecraft relative to the old center of integration are incremented by the position and velocity, respectively, of the old center relative to the new center. The injection position and velocity components may be referred to any body (not necessarily the center of integration).

The injection epoch may be specified in the UT1 (Universal time), A1 (Atomic time), or ET (Ephemeris time) time scales and must be transformed to ephemeris seconds past Jan. 1. 1950.0^h.

The acceleration of the spacecraft consists of (References 1, 5, 13, 15)

$$\begin{aligned}\ddot{r} = {} & \ddot{r}_{ip}\ \text{(Newtonian acceleration)} + \ddot{r}_{ip}\ \text{(Mass concentrations)} \\ & + \ddot{r}_{ip}\ \text{(Direct Oblateness)} + \ddot{r}_{ip}\ \text{(Indirect oblateness)} \\ & + \ddot{r}_{ip}\ \text{(Solar radiation pressure)} + \ddot{r}_{ip}\ \text{(Attitude control systems)} \\ & + \ddot{r}_{ip}\ \text{(Powered flight)} + \ddot{r}_{ip}\ \text{(Relativistic effects)} \\ & + \ddot{r}_{ip}\ \text{(Atmospheric forces)}\end{aligned} \tag{7}$$

where:

i is any planet, Sun, or Moon

p is the spacecraft

or, in functional form

$$\boxed{\ddot{r} = \ddot{r}(r, \dot{r}, q)} \tag{8}$$

where

$r, \dot{r}, \ddot{r}$ = position, velocity, and acceleration vectors of spacecraft relative to center of integration with rectangular components x, y, and z referred to the mean earth equator and equinox of 1950.0. The independent variable is ephemeris time.

q = Solve for parameter vector

• Numerical Integration

The computer algorithm used to numerically integrate the system of second order differential equation is a multi-step method in summed form. It is a modified version of the Cowell-Adams method and consists basically:

i. A starting procedure to produce the solution values at the first m + 1 time points.

ii. A stepping procedure of the predictor–corrector type to advance the solution one time step, making use of the solution at the first m immediately preceding points. A detailed description of the method is given in Reference 2.

The program can integrate forward or backward in time. The step size may be chosen automatically or obtained as a function of distance from perturbing planets (ranging tables). The automatic step size control is most commonly used and will be covered in slightly greater detail. The iteration is halted during the calculation of difference lines when at the jth difference line (Reference 15)

$$E_j(x) = \frac{\max\limits_{-2\leq i\leq 10} \left| \nabla^i \ddot{x}_o^j - \nabla^i \ddot{x}_o^{j-1} \right|}{\max\limits_{-2\leq i\leq 10} \left| \nabla^i \ddot{x}_o^{j-1} \right|} \qquad x \to y,z \tag{9}$$

is less than some specified tolerance ϵ where $\ddot{x}_o$ is the probe's acceleration at the time when the new difference lines are being iteratively constructed. These restarts will occur at the start of the trajectory, at any discontinuity and whenever the step size is reduced. The backward difference operator ∇ is defined as follows:

$$\nabla f_n = f_n - f_{n-1}$$

The integration scheme is a second-sum, 10th order multi-step method of the following form for each dimension:

$$\begin{aligned} x_{n-s} &= h^2 \sum_{i=-2}^{10} \alpha_i(s) \nabla^i \ddot{x}_n \\ \dot{x}_{n-s} &= h \sum_{i=-1}^{10} \beta_i(s) \nabla^i \ddot{x}_n \end{aligned} \tag{10}$$

Prediction is accomplished by s = -1 and correction by s = 0. After each step the local truncation (or discretization) error E is estimated by

$$E \approx \frac{h^2 \left|\alpha_{11}\right| \sum\limits_{i=1}^{3} \left|\nabla^{11} \ddot{x}_i\right|}{\sum\limits_{i=1}^{3} \left|x_i\right|} \tag{11}$$

where

h = current step size

α_{11} = the 11th coefficient in the expression for x_{n-s}

The user can keep this error within an interval set by two parameters E_{min} and E_{max}. If $E < E_{min}$ the step size is doubled and if $E > E_{max}$ the step size is reduced according to the formula

$$h_{new} = h_{old} \left[\frac{.25\, E_{min}}{E}\right]^{1/10} \tag{12}$$

The minimum and maximum step size allowable is governed by the variables h_{min} and h_{max} (in seconds)

PCOP is the integer prediction correction flag. Acceptable values are:

0 = Standard predictor-corrector
1 = Predict only
2 = Correct position and velocity only
3 = Predict partial-correct

The scheme used when PCOP is 0, 1 or 2 can be found in Reference 3. The predict partial-correct integration scheme (PCOP=3) is described in Reference 4.

The integration central body can be held fixed or allowed to vary with the physical control body under user control.

- **Variational Equations (VARY)**

Differentiating Eq. (8) with respect to **q** we get the variational equations

$$\boxed{\frac{\partial \ddot{\mathbf{r}}}{\partial \mathbf{q}} = \left(\frac{\partial \ddot{\mathbf{r}}}{\partial \mathbf{r}}\right)\frac{\partial \mathbf{r}}{\partial \mathbf{q}} + \left(\frac{\partial \ddot{\mathbf{r}}}{\partial \dot{\mathbf{r}}}\right)\frac{\partial \dot{\mathbf{r}}}{\partial \mathbf{q}} + \left.\frac{\partial \ddot{\mathbf{r}}}{\partial \mathbf{q}}\right|_{\mathbf{r},\,\dot{\mathbf{r}} = \text{constant}}} \tag{13}$$

II.1.3.3 Observable Generation

Once a spacecraft ephemeris has been generated, it is possible to simulate the data-types that were recorded at the actual observation times. For on-board TV data, one could have the computer generate a set of positions at which the various stars and satellites might be found in the picture. For radiometric data, the software would generate "computed" Doppler, range and angle data to compare with the actual observed values, using the approach described below.

It is important to realize that with radiometric data, one makes a measurement of some characteristic of a radio signal at a given instant at a receiver within a tracking station on the earth. That signal emanated from the spacecraft at some earlier time and will have been affected by the spacecraft's state at the time the signal left it; it is not at all affected by the spacecraft's state at the time the signal was actually received on the ground. By the same token, when the spacecraft detects a tracking signal that has been sent up from a transmitting station on the ground, the signal received will have been affected by the state of the transmitter when the signal emanated from it. The state of the transmitter at the time the spacecraft received the signal has no influence on the signal itself.

One identifies three separate instants, then, at which participants act to produce a two- or three-way radiometric observable: the transmission time, t_1; the spacecraft time, t_2, at which the signal is received and retransmitted; and the reception time, t_3, at which the signal is received and the observable is recorded. There are naturally delays within the electronics at the ground stations and in the spacecraft which cause a finite time to elapse between receipt of a signal and its retransmission from the spacecraft, or between receipt of a signal by an antenna on the ground and its recording by the station hardware. These are very small, however, with respect to the overall time taken by the signal for the round trip from station to spacecraft and back to the station, and are not considered further in this discussion.

The Doppler and range signals are transmitted continuously from the transmitting station and are continuously received at the receiving station. Observables are generated at distinct instants, however, and in order to compute the observable precisely, the program must know the state of each participant the instant it got into the act. The only time known accurately to start with is the instant t_3 at which the electronic hardware was sampled. The program must determine how much earlier the signal that impinged on the receiver at t_3 actually left the spacecraft. Given t_3, the program must determine t_2.

The determination of the time it took the signal to go from participant to participant (the light time) is the major part of the observable computation process. The process is really quite straightforward. One first obtains the heliocentric state of the tracking station at time t_3 using, among other things, the planetary ephemeris and the timing and polar motion data mentioned earlier. Then, unless

- **The Light Time Equation (REGRES)**

In order to compute Doppler, range and angular observables (Reference 5), the time for light to travel from the transmitting station on Earth to the spacecraft, and from there to the receiving station on Earth, must be computed. Thus, an equation is required which relates the position coordinates of two points to the coordinate time t for light to travel from one of the points to the other. This equation will be referred to as the light time equation and it is derived from the 1-body expression for the interval ds in the Brans-Dicke theory.

The following form of the light time equation was developed by D. Holdridge (Reference 6).

$$\boxed{t_j - t_i = \frac{r_{ij}}{C} + \underbrace{\frac{(1+\gamma)\mu_s}{C^3} \ln\left(\frac{r_i + r_j + r_{ij}}{r_i + r_j - r_{ij}}\right)}_{\text{relativistic perturbation term}}} \qquad (14)$$

where light travels from point i at coordinate time (ephemeris time) t_i to point j at coordinate time t_j.

and

$$r_{ij} = \left\| r_j^s(t_j) - r_i^s(t_j) \right\|$$

$$r_i = \left\| r_i^s(t_i) \right\|$$

$$r_j = \left\| r_j^s(t_j) \right\|$$

$r_i^s(t_i), r_j^s(t_j)$ = heliocentric position vectors of point i at transmission time t_j (ET) and point j at reception time t_j (ET), respectively, with rectangular components referred to a nonrotating frame of reference

μ_s = gravitational constant of Sun, km^3/sec^2

C = speed of light, km/sec

γ = relativity parameter

The subscripts i and j are equal to 1, 2 or 3 where

1 refers to the transmitting station on Earth at the transmission time t_1

2 refers to the spacecraft (free or landed) at the reflection time t_2

3 refers to the receiving station on Earth at the reception time t_2

The relativistic perturbation term is due to the fact that the coordinate speed of light (v_c) decreases slightly as the photon approaches the Sun ($v_c < c$) which means a longer light time. This term becomes very large (36 Km/s) when the spacecraft approaches superior conjunction and the minimum distance from the light path to the surface of the Sun becomes very small. This is the only really large effect of general relativity on Earth-based tracking data.

a better guess is available, one chooses t_2 to be equal to t_3 and computes the heliocentric spacecraft state from the probe ephemeris and n-body ephemerides. Given these two states, the scalar distance between spacecraft and receiver can be computed and divided by the known speed of light to give an initial estimate of the down-leg light time. This length of time is subtracted from t_3 to give a better estimate of t_2. The new value of t_2 is then used to reinterpolate the heliocentric state of the spacecraft and the process is iterated until there is no further significant change in the value of t_2. Once the value of t_2 has been determined, one can use a similar iterative procedure to derive t_1. This "light time solution" is rather time-consuming because of the number of interpolations involved and the complexity of the transformations and/or interpolations being performed for the "unknown" participant at each iteration.

As mentioned earlier, the ranging hardware measures the offset in time between a periodic signature on the signal received from the spacecraft and a similar periodic signature continuously being piped from the transmitter to the receiver. The station data does not determine the total round trip time, only the fractional interval beyond the nearest number of full signature cycles. It measures the round trip time, *modulo* some repetition interval.

It is important to note that the offset is measured by clocks at the tracking station in terms of Station Time (ST). Clocks at different stations may run at slightly different rates and clocks at the same station may run at different rates at different times due to relativistic effects and even electromechanical reasons. The computed observables must therefore include these effects as well as the actual "time of flight" described earlier. This latter component is measured in Ephemeris Time (ET) which a uniformly flowing time scale in the classic Newtonian sense.

The Doppler system does not record the instantaneous Doppler frequency but rather the average frequency during a given interval of time. Because of the ST effects described above, this average frequency cannot be related exactly to the average Earth-probe range rate during that interval, although the approximation is quite good. Instead the average Doppler frequency can be related *precisely* to the averaged difference of ranging observables made at the beginning and end of the count interval. This is the origin of the "differenced-range Doppler" formulation in the ODP.

Angle observables give the apparent incoming direction of the down leg signal in any of a variety of coordinate systems, but will not be discussed further here because of their limited usefulness for OD.

- **Computed Observable (O_c)**

Range (AFETR, MARK1, MARK1A, TAU, MU, PLOP, PLOP2, MU2):

$$\boxed{\rho = (t_3 - t_1)_{ST} F, \text{ modulo } M} \qquad (15)$$

where

$(t_3 - t_1)_{ST}$ = Round-trip time in seconds of station time (ST) of an electromagnetic signal transmitted from a tracking station on Earth at time t_1, received and retransmitted by the spacecraft at time t_2, and received by the same (or different) tracking station at time t_3.

F = Conversion factor from seconds of station time (ST) to the units of the range observable.

M = Modulo number

or

$$\boxed{\begin{aligned}\rho = \Big\{ & (t_3 - t_1)_{ET} \\ & - (ET - A1)_{t_3} + (ET - A1)_{t_1} \\ & - (A1 - UTC)_{t_3} + (A1 - UTC)_{t_1} \\ & - (UTC - ST)_{t_3} + (UTC - ST)_{t_1} \\ & + \text{Range bias} + \text{Range corrections} \Big\} F, \text{ modulo } M\end{aligned}} \qquad (16)$$

where

$$(t_3 - t_1)_{ET} = (t_3 - t_2)_{ET} + (t_2 - t_1)_{ET}$$

with

$$(t_2 - t_1)_{ET} = \frac{r_{12}}{C} + \frac{(1+\gamma)\mu_s}{C^3} \ln\left(\frac{r_1 + r_2 + r_{12}}{r_1 + r_2 - r_{12}}\right)$$

which is the up-leg of the light-path

and

$$(t_3 - t_2)_{ET} = \frac{r_{23}}{C} + \frac{(1+\gamma)\mu_s}{C^3} \ln\left(\frac{r_2 + r_3 + r_{23}}{r_2 + r_3 - r_{23}}\right)$$

which is the down-leg of the light path of the light-time solution.

The expressions for ET - A1, A1 - UTC and UTC - ST are given below (References 7, 8)

$$
\begin{aligned}
ET - A1 = {} & \Delta T_{1958} - (t - 252{,}460{,}800)\,\Delta f_{cesium}/f_{cesium} + 1.658 \times 10^{-3} \sin E + 0.317679 \times 10^{-9} u \sin (UT + \lambda) \\
& + 5.341 \times 10^{-12} u \sin (UT + \lambda - M) + 1.01 \times 10^{-13} u \sin (UT + \lambda - 2M) - 1.3640 \times 10^{-11} u \sin (UT + \lambda + 2L) \\
& - 2.27 \times 10^{-13} u \sin (UT + \lambda + 2L + M) + 1.672 \times 10^{-6} \sin D + 1.38 \times 10^{-13} u \sin (UT + \lambda - D) - 1.318 \times 10^{-10} v \cos L
\end{aligned} \tag{17}
$$

where

ΔT_{1958} = ET - UT2 on January 1, 1958, $0^h\,0^m\,0^s$ UT2 minus the periodic terms of Eq. (17) evaluated at this epoch using u and λ of the master A1 clock. The master A1 clock was set equal to UT2 on this date. The parameter ΔT_{1958} may be estimated by the ODP

f_{cesium} = 9,192,631,770 cycles of cesium atomic clock per second A1 time. This adopted length of the A1 second is the current experimentally determined average length of the ET second

$f_{cesium} + \Delta f_{cesium}$ = cycles of cesium atomic clock per ephemeris second. The parameter Δf_{cesium} may be estimated by the ODP; its current nominal value is zero

t = seconds past January 1, 1950, 0^h

252,460,800 = seconds from January 1, 1950, 0^h to January 1, 1958, 0^h

M = mean anomaly of heliocentric orbit of Earth-Moon barycenter

E = eccentric anomaly of heliocentric orbit of Earth-Moon barycenter

L = geometric mean longitude of the Sun, referred to mean equinox and ecliptic of date

D = ☾ - L = mean elongation of the Moon from the Sun, where

☾ = mean longitude of the Moon, measured in the ecliptic from the mean equinox of date to the mean ascending node of the lunar orbit, and then along the orbit

r = geocentric latitude

u = distance of atomic clock from Earth's spin axis, km

v = height of the tracking station above the equator in km

λ = east longitude of atomic clock

ϕ = geocentric latitude of tracking station

UT = universal time, hours past midnight, converted to radians. It is computed from

$$
UT = 2\pi \left[\frac{UT1}{86{,}400}\right]_{\text{decimal part}} \tag{18}
$$

where UT1 = seconds of UT1 time past January 1, 1950, 0^hUT1. The angles M, L, and D in radians are given by

$$
\left.
\begin{aligned}
M &= 6.248291 + 1.99096871 \times 10^{-7}\, t \\
L &= 4.888339 + 1.99106383 \times 10^{-7}\, t \\
D &= 2.518410 + 2.462600818 \times 10^{-6}\, t
\end{aligned}
\right\} \tag{19}
$$

$$
UTC - ST = a + (b + b_o)(t - t_o) + c(t - t_o)^2 \tag{20}
$$

where b_o is the following function of the station coordinates:

$$
\begin{aligned}
b_o = {} & 4.435{,}035 \times 10^{-6}\left(\frac{1}{r}\right) - 9.767{,}11 \times 10^{-2}\left(\frac{3\sin^2\phi - 1}{r^3}\right) \\
& + 2.958{,}254 \times 10^{-20} u^2 - 0.696{,}928{,}273 \times 10^{-9}
\end{aligned} \tag{21}
$$

The quantity b_o should be computed once for each tracking station

$$A1 - UTC = d + et \tag{22}$$

The coefficients a, b, c, d, e are obtained by curve-fitting techniques in the programs PLATO and/or STOIC mentioned earlier in Section II.1.2.2.

The time transformations in the above equation convert the precision round-trip light time from an interval of ephemeris time (ET) to an interval of station time (ST). The remaining terms account for the effects of the troposphere, ionosphere, constant range bias, and the tracking antenna w.r.t. the Earth-fixed 'Station location'. The troposphere and ionosphere effects are computed in ACCUME.

Range Units: 1 Range Unit $= \frac{C}{2F}$ km

Conversion factors and module numbers for ranging systems

Name	Conversion Factor F	Modulo Number M
AFETR	C/2	None
MARK I	$\frac{1440}{221} f_q(t_1)$	785,762,208
MARK IA	96 x 1,487,500	785,762,208
TAU	10^9	$\frac{1.00947}{1.0002} \times 10^9$
MU	10^9	$\frac{64 \times 2^n}{3 f_q(t_1)} \times 10^9$
PLOP	$48 f_q(t_1)$	2^{10+n}
PLOP2	$48 f_q(t_1)$	$2^{10} \times 1009470$
MU2	$6144 f_q(t_1)$	2^{17+n}

$f_q(t_1)$ = reference oscillator frequency at transmitting station, cycles per second of station time (ST) evaluated at transmission time $t_1 \approx 22 \times 10^6$ Hz

n = number of components of ranging code used with MU, PLOP, and MU2 ranging system

Doppler (differenced-range doppler):

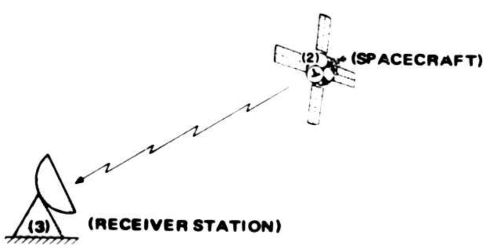


Fig. 6. One-Way Doppler

$$\boxed{FI = C_2 \left\{ f_{S/C} - \frac{\rho_{1e} - \rho_{1s}}{T_c} - \left[\Delta f_{T_0} + f_{T_1}(t_2 - t_0) + f_{T_2}(t_2 - t_0)^2 \right] \right\}} \tag{23}$$

Fig. 7. Two-Way Doppler

$$\boxed{F2 = C_3 f_q(t_1) \frac{\rho_{2e} - \rho_{2s}}{T_c}} \tag{24}$$

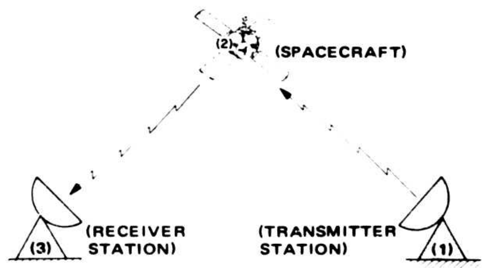


Fig. 8. Three-Way Doppler

$$\boxed{F3 = C_5 f_q(t_1) \frac{\rho_{3e} - \rho_{3s}}{T_c}} \tag{25}$$

where

$f_{s/c}$	=	$f_{T_0} + \Delta f_{T_0} + f_{T_1}(t_2 - t_0) + f_{T_2}(t_2 - t_0)^2$ is the spacecraft's transmitting frequency
f_{T_0}	=	Nominal probe transmitting frequency
$\Delta f_{T_0}, f_{T_1}, f_{T_2}$	=	Drift coefficients
C_2, C_3, C_5	=	Coefficients arising from the various frequency multipliers and adders in the DSIF instrumentation
t_0	=	Initial time for the drift coefficients
t_1	=	Station transmission time
t_2	=	Probe's transmission time
t_3	=	Station reception time
T_c	=	Count time
$\rho_{ie} - \rho_{is}$	=	The difference of the two pseudo range observables ρ_{ie} and ρ_{is} (F = 1, M = ∞) at times $t_3 + T_C/2$ and $t_3 - T_C/2$ correspondingly with i = 1,2,3

In case of F1, $\rho_{1e} - \rho_{1s}$ is computed from a one-way version of the range formulation. For details see Reference 5.

Doppler Units: Hertz (Hz) = cycles/second

At S-band 15.3 Hz = 1 m/sec

Angles (HA, δ, A, γ, X, Y, X′, Y′):

These are seldom used in ODP and are computed using expressions (1 – 6).

II.1.3.4 Residual

The algebraic difference between the computed observable described in the preceding subsection and the actual observable taken at the tracking station is called a residual, and is an indication of both the noise in the DSIF, and the deficiencies in the models of the observables and the spacecraft's motion. As was indicated earlier, part of these differences could be related to necessary changes in model parameters by expressions involving derivatives of the observables with respect to these parameters. Computation of such partial derivatives is another lengthy process, involving more complex algorithms than the observables themselves.

- **Residual**

$$\boxed{\delta Z = O_0 - O_c} \qquad (26)$$

O_0 = Observed observable

O_c = Computed observable

II.1.3.5 Partial Derivatives

Since Doppler observables can be precisely represented by a function of the differenced range observables at the beginning and end of a sample (or a count) interval, the partial derivatives of Doppler observables can be likewise related to differenced range partials. The range partials are quite simple in form and depending on the parameter are equally simple in fact. For all parameters, the partials reflect how the t_1 derived in the light time solution varies with the parameter in question.

If the parameter is one that has affected the numerical integration of the probe's trajectory, it will most likely have been computed by a similar numerical integration of the second order differential equations that represent the partials of the spacecraft equations of motion (variational equations). Other parameters that affect the observable computation have partials computed by simple analytic expressions involving the states of the various participants. Still others are independent of the state of any participant and affect merely the signal transmission process itself.

- **Partial Derivatives** $\partial O_c/\partial \mathbf{q}$ (REGRES)

$\mathbf{q}$ = vector of parameters for which partials are desired

Range:

$$\boxed{\frac{\partial \rho}{\partial \mathbf{q}} = -F\,\frac{\partial t_1\,(\mathrm{ST})}{\partial \mathbf{q}}} \tag{27}$$

Since it is only very seldom that partials are desired for the parameters that transform from ST to ET, this equation reduces in all other cases to

$$\frac{\partial \rho}{\partial \mathbf{q}} = -F\,\frac{\partial t_1\,(\mathrm{ET})}{\partial \mathbf{q}} \tag{27a}$$

where

$$\frac{\partial t_2(\mathrm{ET})}{\partial \mathbf{q}} = \frac{\dfrac{1}{c}\dfrac{\mathbf{r}_{23}^{T}}{r_{23}}\left[\dfrac{\partial \mathbf{r}_2^S(t_2)}{\partial \mathbf{q}} - \dfrac{\partial \mathbf{r}_3^S(t_3)}{\partial \mathbf{q}}\right]}{\left(1 - \dfrac{\dot{\rho}_{23}}{c}\right)} \tag{28}$$

$$\frac{\partial t_1(\mathrm{ET})}{\partial \mathbf{q}} = \frac{\dfrac{\partial t_2(\mathrm{ET})}{\partial \mathbf{q}}\left(1 - \dfrac{\dot{r}_{12} + \dot{\rho}_{12}}{c}\right) + \dfrac{1}{c}\dfrac{\mathbf{r}_{12}^{T}}{r_{12}}\left[\dfrac{\partial \mathbf{r}_1^S(t_1)}{\partial \mathbf{q}} - \dfrac{\partial \mathbf{r}_2^S(t_2)}{\partial \mathbf{q}}\right]}{\left(1 - \dfrac{\dot{\rho}_{12}}{c}\right)} \tag{28a}$$

with

$$\dot{\rho}_{12} = \frac{\mathbf{r}_{12}}{r_{12}} \cdot \dot{\mathbf{r}}_1, \quad \dot{\rho}_{23} = \frac{\mathbf{r}_{23}}{r_{23}} \cdot \dot{\mathbf{r}}_2, \quad \dot{r}_{12} = \frac{\mathbf{r}_{12}}{r_{12}} \cdot \dot{\mathbf{r}}_{12} \tag{29}$$

The partial derivatives of the heliocentric position vectors with respect to the solve-for-parameter vector $\mathbf{q}$ holding the epochs of participation constant are given by the following sums:

$$\left.\begin{aligned}
\frac{\partial \mathbf{r}_3^S(t_3)}{\partial \mathbf{q}} &= \frac{\partial \mathbf{r}_3^E(t_3)}{\partial \mathbf{q}} + \frac{\partial \mathbf{r}_E^S(t_3)}{\partial \mathbf{q}} \\
\frac{\partial \mathbf{r}_2^S(t_2)}{\partial \mathbf{q}} &= \frac{\partial \mathbf{r}_2^{B_2}(t_2)}{\partial \mathbf{q}} + \frac{\partial \mathbf{r}_{B_2}^S(t_2)}{\partial \mathbf{q}} \\
\frac{\partial \mathbf{r}_1^S(t_1)}{\partial \mathbf{q}} &= \frac{\partial \mathbf{r}_1^E(t_1)}{\partial \mathbf{q}} + \frac{\partial \mathbf{r}_E^S(t_1)}{\partial \mathbf{q}}
\end{aligned}\right\} \tag{30}$$

where

B_2 = Center of integration for free probe or body (other than Earth) on which a station is located

E = Earth

S = Sun

For a free probe $\partial r_2^{B_2}(t_2)/\partial q$ is obtained from the solution of the variational equations (13)

The partial derivatives $\partial r_E^S(t_3)/\partial q$, $\partial r_{B_2}^S(t_2)/\partial q$, and $\partial r_E^S(t_1)/\partial q$ are computed from the following (where B = Earth-Moon barycenter):

$$\frac{\partial r_E^S(t_3)}{\partial q} = \frac{\partial r_B^S(t_3)}{\partial q} - \frac{\partial r_B^E(t_3)}{\partial q}$$

$$\frac{\partial r_{B_2}^S(t_2)}{\partial q} = \frac{\partial r_B^S(t_2)}{\partial q} + \frac{\partial r_{B_2}^B(t_2)}{\partial q} \tag{31}$$

$$\frac{\partial r_E^S(t_1)}{\partial q} = \frac{\partial r_E^S(t_1)}{\partial q}$$

The columns of Equation (31) are non-zero only for the reference parameters A_E, R_E, μ_E, μ_M and osculating orbital elements **E** for the ephemeris of a planet, the Earth-Moon barycenter, or the Moon, where the right-hand terms are obtained from the following equations:

$$\frac{\partial r_P^S}{\partial A_E} = \frac{r_P^S}{A_E} \qquad r \rightarrow \dot{r} \qquad \text{(valid also for P = E)}$$

$$\frac{\partial r_M^E}{\partial R_E} = \frac{r_M^E}{R_E} \qquad r \rightarrow \dot{r} \tag{32}$$

The partial derivatives

$$\frac{\partial r_P^S}{\partial \mathbf{E}_P} \quad \text{and} \quad \frac{\partial r_M^E}{\partial \mathbf{E}_M} \qquad r \rightarrow \dot{r}$$

are computed from the formulation of Subsection IV B-3 and B-4 of Reference 5.

$$\frac{\partial r_M^B}{\partial R_E} = \frac{r_M^B}{R_E} \qquad r \rightarrow \dot{r}$$

$$\frac{\partial r_M^B}{\partial \mathbf{E}_M} = \frac{\mu}{1+\mu}\frac{\partial r_M^E}{\partial \mathbf{E}_M} \qquad r \rightarrow \dot{r}$$

$$\frac{\partial r_M^B}{\partial \mu_E} = \frac{r_M^E}{(1+\mu)^2 \mu_M} \qquad r \rightarrow \dot{r}$$

$$\frac{\partial r_M^B}{\partial \mu M} = -\frac{\mu r_M^E}{(1+\mu)^2 \mu_M} \qquad r \rightarrow \dot{r}$$

$$\frac{\partial r_B^E}{\partial R_E} = \frac{r_B^E}{R_E} \qquad r \to \dot{r}$$

$$\frac{\partial r_B^E}{\partial E_M} = \frac{1}{1+\mu}\frac{\partial r_M^E}{\partial E_M} \qquad r \to \dot{r}$$

$$\frac{\partial r_B^E}{\partial \mu_E} = -\frac{r_M^E}{(1+\mu)^2\,\mu_M} \qquad r \to \dot{r}$$

$$\frac{\partial r_B^E}{\partial \mu_M} = \frac{\mu r_M^E}{(1+\mu)^2\,\mu_M} \qquad r \to \dot{r}$$

where $\mu = \mu_E/\mu_M$ and B = Earth-Moon barycenter, M = Moon, P = Planet

Doppler:

$$\boxed{\begin{aligned}\frac{\partial F1}{\partial \mathbf{q}} &= C_2\frac{f_{s/c}}{T_c}\left(\frac{\partial \rho_{1e}}{\partial \mathbf{q}} - \frac{\partial \rho_{1s}}{\partial \mathbf{q}}\right)\\ \frac{\partial F2}{\partial \mathbf{q}} &= C_3\frac{f_q(t_1)}{T_c}\left(\frac{\partial \rho_{2e}}{\partial \mathbf{q}} - \frac{\partial \rho_{2s}}{\partial \mathbf{q}}\right)\\ \frac{\partial F3}{\partial \mathbf{q}} &= C_5\frac{f_q(t_1)}{T_c}\left(\frac{\partial \rho_{3e}}{\partial \mathbf{q}} - \frac{\partial \rho_{3s}}{\partial \mathbf{q}}\right)\end{aligned}} \tag{33}$$

where all the quantities in the right-hand side have been defined previously (II.1.3.3)

Angles:

$$\frac{\partial \mathbf{O}_c}{\partial \mathbf{q}} = \frac{\partial \mathbf{O}_c}{\partial r_3^s(t_3)}\left[\frac{\partial r_3^s(t_3)}{\partial \mathbf{q}}\right]_{t_3=\text{constant}} + \frac{\partial \mathbf{O}_c}{\partial r_2^s(t_2)}\left\{\left[\frac{\partial r_2^s(t_2)}{\partial \mathbf{q}}\right]_{t_2=\text{constant}} + \dot{r}_2^s(t_2)\left[\frac{\partial t_2(ET)}{\partial \mathbf{q}}\right]_{c,\text{time transformation}=\text{constant}}\right\}$$
$$+\left[\frac{\partial \mathbf{O}_c}{\partial \mathbf{q}}\right]_{c,\text{time transformations}=\text{variable}} + \left[\frac{\partial \mathbf{O}_c}{\partial c}\right]_{r_3^s(t_3),\ r_2^s(t_2),\ r_1^s(t_1)=\text{constant}} \tag{34}$$

where

$r_i^s(t_i)$, $\dot{r}_i^s(t_i)$ = Heliocentric position and velocity vector of participant i at t_i (i=1 for transmitter on Earth, i=2 for spacecraft, i=3 for receiver on Earth)

The expressions for $\frac{\partial t_2(ET)}{\partial \mathbf{q}}$, $\frac{\partial r_2^S(t_2)}{\partial \mathbf{q}}$ and $\frac{\partial r_3^S(t_3)}{\partial \mathbf{q}}$ have been defined previously in Equations (28 – 30).

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II.1.3.6 Least Squares Solution

Recalling the principles mentioned in subsection II.1.3.1, the idea is to determine those changes to a series of parameters, (a_1, a_2, $\cdots$, a_n) which eliminate the differences between observed and computed data (O - C). Thus, for each observation, i, there is a linearized "conditional" or "observational" equation in n unknowns of the form.

$$(O_i - C_i) = \frac{\partial C_i}{\partial a_1}\Delta a_1 + \frac{\partial C_i}{\partial a_2}\Delta a_2 + \cdots + \frac{\partial C_i}{\partial a_n}\Delta a_n, \quad (i = 1, \cdots, m)$$

On the assumption that the n parameters are distinct, there must be at least n independent equations to allow a solution. In fact, however, the residuals are assumed to have observational noise in them, and any individual sample could not be relied upon as accurate to more than some given tolerance. The approach therefore, is to combine the information in a large number of conditional equations in such a way that the effect of observational errors on the derived solutions can be minimized. The technique used is called the method of least squares because it chooses the parameter solutions which minimize the sum of the squares of the residuals that will remain after the solutions Δa_i are fed back into the conditional equations.

There are a variety of means available for combining m conditional equations in n equations in n unknowns. The classical method of least squares forms n "normal" equations by appropriate manipulation of the m conditional equations. The ODP uses a somewhat more sophisticated approach called square root filtering which minimizes potential numerical difficulties in the solution process. In this approach, the m "conditional equations" are compacted to a n x n matrix referred to as the "information matrix".

In the process of accumulating the information matrix, the individual conditional equations are multiplied by weighting factors which indicate how seriously the solution algorithm should take each equation. This weighting is used to account for two basic types of errors in the observation equation: those inherent in the measured data, like Doppler data noise which generally decreases with increasing count time; and deficiencies in the computed observable due, for example, to tropospheric corrections whose daily and seasonal fluctuations are very difficult to model. The ODP weights each equation individually and does not account for possible correlations between errors. The weighting serves a further purpose by eliminating the discrepancy between units of measure for the different data types.

The solution of the n equations in n unknowns can be obtained by a variety of means. Some of the techniques involve sophisticated analysis of the numerical stability and characteristics of the information matrix. Others account for the possibility that the nominal parameter values might be sufficiently far from the true values as to make the

- **Data Equation Formulation** (ACCUMF, ACCUM2)

The augmented data equation (Reference 10) including the given a priori information array $[\tilde{R}, \delta\tilde{\mathbf{Z}}]$ is

$$\begin{bmatrix} \tilde{R} \\ A \end{bmatrix} \delta\mathbf{q} = \begin{bmatrix} \delta\tilde{\mathbf{Z}} \\ \delta\mathbf{Z} \end{bmatrix} + \begin{bmatrix} \tilde{\epsilon} \\ \epsilon \end{bmatrix} \qquad (35)$$

Observe that the α priori information is interpreted as additional observations.

$\tilde{R}$ = is an (n x n) matrix of a priori information referenced to the start of the current data span.

A = is an (m x n) matrix of $\partial O_c/\partial \mathbf{q}$ corresponding to the linearized measurement (computed in REGRES)

$\delta\mathbf{q}$ = n-vector of incremental correction to be found and added to the initial estimate for $\mathbf{q}$ to yield to an "improved" estimate for $\mathbf{q}$

$\delta\tilde{\mathbf{Z}}$ = is an n-vector of normalized a priori estimate

$[\mathrm{Cov}\ (\delta\tilde{\mathbf{Z}}) = I]$

$\delta\mathbf{Z}$ = is an m-vector of pseudo residuals

$\epsilon, \tilde{\epsilon}$ = are n-vectors of random variables of zero mean and unity covariance

m = is the number of observations

n = is the number of modeled parameters

Since the observations cannot be treated in exactly the same manner, the matrices A and $\delta\mathbf{Z}$ are weighted so all measurements have uniform validity (see Data Weighting equations (46 – 50) on page 32).

The least-squares solution to the data equation (35) is equivalent to the least-squares solution to the

$$H \begin{bmatrix} \tilde{R} \\ A \end{bmatrix} \delta\mathbf{q} = H \begin{bmatrix} \delta\tilde{\mathbf{Z}} \\ \delta\mathbf{Z} \end{bmatrix} + H \begin{bmatrix} \tilde{\epsilon} \\ \epsilon \end{bmatrix} \qquad (36)$$

where H is a product of orthogonal **Householder** transformations (Reference 11).

approximation involved in truncating the Taylor series expansion mentioned earlier a poor one. Under this circumstance, the information matrix based on the conditional equations, which are linearized approximations of the full Taylor series expansion, can yield a set of corrections Δa which might push one further rather than closer to the true value. The so called "partial step algorithm" (Reference 9) limits the size of a correction Δa to be within predefined bounds so that the solutions can eventually converge to the correct one.

Almost as important as the solution is an estimate of what confidence should be placed in that solution. This is provided as a by-product of the solution process in the form of a covariance of estimate errors on the solved-for parameters. The covariance is determined by the contribution of the individual partial derivatives, which depends upon the probe-observer geometry when the observations were taken, on the weights applied to each conditional equation and on the number of parameters estimated. The covariance can further be modified by including the effects of uncertainties in the nominal values of other parameters that were not estimated but are known to affect the observables. These "considered" parameters are felt to be accurately modelable, but imprecisely obtainable from the data set at hand. The "consider" covariance increases the uncertainty in the standard solution to account for the fact that if the consider parameters were indeed allowed to change by as much as we felt their nominal values were uncertain, the standard solved-for parameters would vary by some additional amount over and above that from data noise alone. The uncertainty in the considered parameters is represented by an a priori "consider covariance."

The ODP also allows for a few additional types of information beyond the observations themselves to be included in the process of deriving a solution from a given set of data. The first is so called "a priori" information about the uncertainty in the parameters being solved for. One usually has some indication, either from physical considerations or previous experience, of the maximum size error expected in the nominal value of any given parameter. This can be viewed as additional information which can help to limit the size of the correction that will be obtained for the given parameter. This information is given to the solution program as an a priori covariance on the estimated parameters, and is included in the information matrix before the solution process starts.

Another type of information is less easy to conceptualize, but is equally important. On the assumption that the models for the spacecraft motion and/or the observable generation are in some way deficient, then both the observables and their partial derivatives with respect to different parameters can be expected to be inaccurate. Since these partial derivatives are what eventually become the information matrix, or coefficients of the n simultaneous equations in unknowns, errors in them can obviously prejudice the solution derived. There is a means within the ODP to degrade the strength of the coefficients in the information matrix that deal with the spacecraft state to allow for possible mismodelling of the observables and/or equations of motion.

With H properly chosen equation (36) is reduced to

$$\begin{bmatrix}\hat{R}\\ 0\end{bmatrix}\delta\hat{q} = \begin{bmatrix}\delta\hat{Z}\\ e\end{bmatrix} + \begin{bmatrix}\hat{\epsilon}\\ \epsilon_e\end{bmatrix} \tag{37}$$

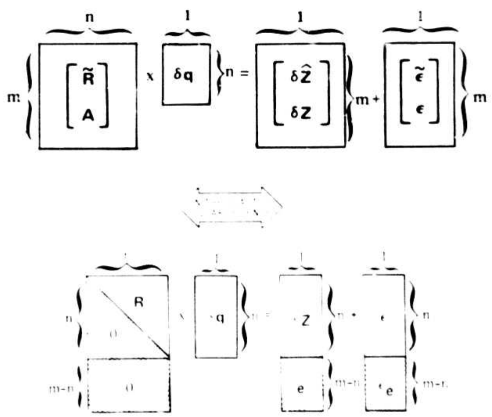
or

$$\left.\begin{aligned}\hat{R}\,\delta\hat{q} &= \delta\hat{Z} + \hat{\epsilon}\\ 0 &= e + \epsilon_e\end{aligned}\right\} \tag{38}$$

where $\hat{R}$ is an upper triangular (n x n) matrix and e is a projection of the residual vector after the fit:

$$\|e\|^2 = \|\tilde{R}\,\delta\hat{q} - \delta\tilde{Z}\|^2 + \|\delta Z - A\,\delta\hat{q}\|^2$$

The application of H to a matrix is referred as "packing" of the matrix since the resulting transformed matrix is more compact than the original matrix. The following diagram shows clearly the equivalence



Equation (38) is in the form of a data equation. This is important, because this data equation can now act as the a priori for the next set of observations and the procedure is repeated until no more data are available. In this way we have a recursive algorithm to sequentially process observations.

The weighting, the application of the Householder transformation, and the generation of the accumulation matrix (square root information array) are performed in link ACCUME or ACCUM2.

The initial least-squares problem is reduced to the solution of the following equation

$$\hat{R}\,\delta\hat{q} = \delta\hat{Z} \tag{39}$$

This is done by computing the effect of uncertainties in the so called "stochastic" parameters on the information content of all data for determination of the spacecraft state. These stochastic parameters could, for example, be sporadic gas leaks, the general behavior and effect of which are known, but the specific details of whose magnitude and direction at any time are indeterminate. One assumes that their variations are bounded and the values from time to time are correlated to a degree which changes as the length of time increases. One can then estimate over a particular period of time how much their variation can be expected to affect the spacecraft state, and decrease the strength of the information matrix for the state to account for this uncertainty. The result with this, as with a priori information, will be a different, and presumably better, solution and covariance than if the observed data alone had been used and treated in a straightforward fashion.

The **solution** is:

$$\delta\hat{\mathbf{q}} = \hat{R}^{-1}\delta\hat{\mathbf{z}} \tag{40}$$

When $\hat{R}$ is singular, the computation of the pseudo inverse of $\hat{R}$ is done by the singular value decomposition algorithm (Reference 10).

Remark: Navigation problems frequently involve large number of bias parameters (station location errors, ephemeris corrections, and planetary harmonics). Also, many random phenomena can be described or approximated by exponentially correlated process noise (solar pressure and attitude acceleration leaks). State vectors that are composed partly of biases and partly of exponentially correlated process noise are partitioned as follows:

$$\delta\mathbf{q} = \delta\begin{pmatrix} \mathbf{p} \\ \mathbf{x} \\ \mathbf{y} \end{pmatrix} \tag{41}$$

where

$\delta\mathbf{p}$ = exponentially correlated process noise (e.g., attitude accelerations, solar pressure, or measurement dependent errors that are exponentially correlated).

$\delta\mathbf{x}$ = states that are time varying but do not explicitly depend upon white process noise, (e.g., position and velocity errors).

$\delta\mathbf{y}$ = the bias parameters, (e.g., constant acceleration errors, planet harmonics, station location errors, or ephermeris corrections).

We partition $\delta\mathbf{q}$ in order to facilitate the analysis of the effects of individual parameters and to reduce computer storage.

From equation (39) we have

$$\begin{bmatrix} \hat{R}_p & \hat{R}_{px} & \hat{R}_{py} \\ 0 & \hat{R}_x & \hat{R}_{xy} \\ 0 & 0 & \hat{R}_y \end{bmatrix} \delta \begin{bmatrix} \hat{\mathbf{p}} \\ \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \end{bmatrix} = \delta \begin{bmatrix} \hat{\mathbf{z}}_p \\ \hat{\mathbf{z}}_x \\ \hat{\mathbf{z}}_y \end{bmatrix}$$

Then

$$\delta\hat{\mathbf{p}} = \hat{R}_p^{-1}\delta\hat{\mathbf{z}}_p - \hat{R}_p^{-1}\hat{R}_{px}\delta\hat{\mathbf{x}} - \hat{R}_p^{-1}\hat{R}_{py}\delta\hat{\mathbf{y}}$$

$$\delta\hat{\mathbf{x}} = \hat{R}_x^{-1}\delta\hat{\mathbf{z}}_x - \hat{R}_x^{-1}\hat{R}_{xy}\delta\hat{\mathbf{y}}$$

$$\delta\hat{\mathbf{y}} = \hat{R}_y^{-1}\delta\hat{\mathbf{z}}_y$$

or

$$\delta\hat{\mathbf{x}} = \hat{R}_x^{-1}\delta\hat{\mathbf{z}}_x + S\,\delta\hat{\mathbf{y}} \tag{42}$$

where

$$S = -\hat{R}_x^{-1}\hat{R}_{xy} \tag{43}$$

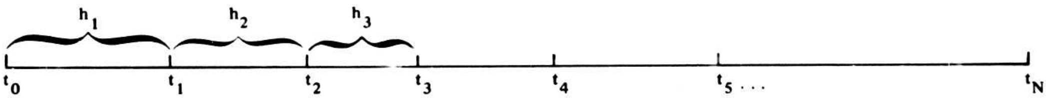
S is called the **sensitivity matrix** and from equation (42) it is easy to see that

$$S = \frac{\partial(\delta \mathbf{x})}{\partial(\delta \mathbf{y})} \tag{44}$$

In equation (42) the term $\hat{R}_x^{-1}\,\delta\hat{\mathbf{Z}}_{\mathbf{x}}$ is the computed estimate that would result if $\delta\hat{\mathbf{y}} = 0$.

- **The Batch Sequential Filter** (References 10, 16, 17, 18)

The current filter time span for estimation is divided into **one** interval, t_0 to t_N. This interval is in turn subdivided into batches of varying time increments, h_i, specified by the user. See Figure below. For portions of the interval containing no data (or where data have been deleted), only predicted (mapped) estimates and their statistics are computed at specified batch break times.



The system dynamics in terms of the current state are

$$\delta\begin{bmatrix} \mathbf{p}_{i+1} \\ \mathbf{x}_{i+1} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} M & 0 & 0 \\ V'_{i+1,i} & I & 0 \\ 0 & 0 & I \end{bmatrix} \delta \begin{bmatrix} \mathbf{p}_i \\ \mathbf{x}_i \\ \mathbf{y} \end{bmatrix} + \begin{bmatrix} \omega_{i+1} \\ 0 \\ 0 \end{bmatrix} \tag{45}$$

where

$\delta\mathbf{p}_i$ = a vector of stochastic parameters that are piecewise constant having discontinuities at the t_i time points.

$\delta\mathbf{x}_i$ = the vector of the spacecraft epoch state correction at t_i (corrections to the nominal)

$\delta\mathbf{y}_i$ = a vector of the remaining estimated parameters (corrections to the nominal). The value of y will be constant, though the estimate will be a function of the data used.

M = a diagonal transition matrix computed for each batch according to

$$m_j = \exp\left[-(t_{i+1} - t_i)/\tau_j\right] \qquad j = 1, n_p$$

τ_j = the user input correlation time for the j^{th} stochastic parameter

(If $\tau_j = 0$, $m_j = 0$).

n_p = the number of stochastic parameters

$V'_{j,k} = \dfrac{\partial(\delta\mathbf{x}_j)}{\partial(\delta\mathbf{p}_K)}$ the partial of the state at t_j with respect to the parameters p 'on' from t_K to t_j ($j > K$)

ω_{i+1} = the process noise which has non-zero mean

where the mean of the process noise is given by:

$$\overline{\omega}_{i+1} = \overline{\delta\mathbf{p}}_{i+1} - M\,\overline{\delta\mathbf{p}}_i \qquad (\overline{\omega}_i = 0 \text{ for the first iteration})$$

and covariance

$$\tilde{\Lambda}_\omega \delta_{ij} \qquad (\delta_{ij} \text{ is the Kronecker delta})$$

where the j^{th} diagonal element is given by

$$\sigma^2_{\omega(j)} = (1 - m_j^2)\sigma^2_{pi(j)}$$

for each i where the variances of the p error components vary from batch to batch and may be specified by the user in terms of standard deviations $\sigma_{pi(j)}$.

Remark: The advantage of the current sequential filter is compatibility with the classical batch processor (when referenced to the initial epoch, mapping is avoided).

- **Data Weighting**

The application of weights to individual points is currently performed in links ACCUME, ACCUM2 because it requires certain observational angles not available in links SOLVE1, SOLVE2. For radiometric data, the weight associated with a particular point has the form

$$W = \frac{1}{\sigma^2} \tag{46}$$

where

$$\sigma = \tilde{\sigma}\beta_1\beta_2 F \tag{47}$$

Here

$\tilde{\sigma}$ = the a priori sigma in units of that observable for data from that time at that station.

T_c = count time in seconds

$$\beta_1 = \begin{cases} \dfrac{1}{\cos\gamma} & \text{for } \mathbf{A} \\ \dfrac{1}{\cos\delta} & \text{for HA} \\ 1 & \text{for all others} \end{cases} \tag{48}$$

$$\beta_2 = \begin{cases} 1 & \text{for } \mathbf{A} \\ 1 + \dfrac{18}{(\gamma+1)^2} & \text{for all others} \end{cases} \tag{49}$$

$$\mathbf{F} = \begin{cases} \sqrt{\dfrac{60}{T_c}} & \text{doppler observables} \\ 1 & \text{other observables} \end{cases} \tag{50}$$

when **A** is the azimuth angle, γ is the elevation angle, and δ is the declination angle of the spacecraft.

II.1.3.7 Display Systems

The solution obtained by the above steps is supposed to be a better estimate of the parameters defining the spacecraft equation of motion and the computed observables than the values previously used. With the improved equations of motion, one would expect a different computed trajectory for the probe, which would mean different predictions of where the probe would be in the future as well as different explanations of where it had been.

From the standpoint of mission planners, future predictions are most important, especially if they can be given in terms of quantities the planners like to work with, like distances from a body, time of passage past a specific point, etc.

These quantities are derived by transforming or "mapping" the relative position and velocity of the probe with respect to the reference body into a variety of coordinate systems. The rather straightforward computations involved can also be used to transform the solution covariance on the appropriate parameters into uncertainties in the mission planners quantities. One of the most common coordinate systems used for interplanetary flight planning at JPL is the so-called "B-plane" system (known also as the **R-S-T** coordinate system).

To the OD engineer, it is important to see whether the new solution has improved the agreement of the computed with the actual observables. There are occasions, for example, with stochastic parameters, in which the ability to represent certain portions of the orbit are greatly improved while the agreement of the computed observables with the actual orbit decreases in other portions, but in general an improved orbit gives improved residuals. The ODP has the ability to produce plots of the residuals before and after the solutions to list the residuals and generate statistics on them.

- **B-Plane**

The B-plane system is a convenient way of expressing errors at the target planet as a linear function of errors in the orbit

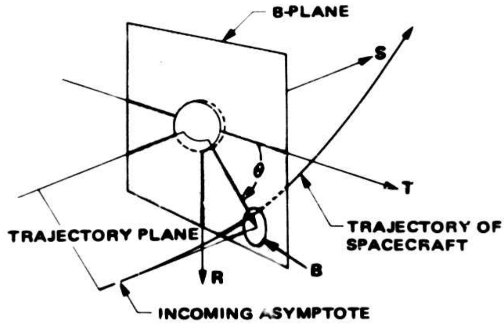


Fig. 9. B-Plane

where

B = Target parameter

S = Direction of the incoming nominal trajectory asymptote

T = Vector parallel to the ecliptic plane and perpendicular to **S**

R = Vector normal to **S**, **T** directed South

θ = Orientation angle of the **B**-vector

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II.1.4 Maneuver Planning

If the predicted arrival location and time determined by the OD process described above differs significantly from the desired aim point, as long as the spacecraft is able, it is conceivable to perform a maneuver that changes the expected arrival parameters to be the desired ones. Such a maneuver might be a ' midcourse" correction which by a slight change in spacecraft velocity early in the flight causes a large change in the eventual arrival parameters. It could also be an orbit insertion maneuver which slows the flyby velocity sufficiently to allow capture by the target planet. Whatever the case, the maneuver has to be designed carefully using some techniques already seen in the OD discussions.

The basic problem is to find that set of velocity changes which will change the probe's trajectory enough that it goes to the desired aiming point and arrives at the desired time. This is obviously analogous to the problem of determining that set of parameters which minimizes the discrepancy between observed and computed data. The solution, of course, involves computing the partial derivatives of the aiming point parameters with respect to the velocity increments, and iteratively solving for these increments.

The maneuver must in addition be subject to certain constraints to protect the spacecraft instruments and to retain communication with the ground. The TV camera, and other sensitive instruments, for example, should not be pointed at the Sun during the maneuver itself or the turns that orient the motor's direction before the actual "burn." The spacecraft antennas should not point in such a direction that communication with the ground becomes impossible. These and other constraints are very important in the maneuver and pre-maneuver turns planning.

The maneuver planning is done by project navigation team personnel on the GPCF Univac 1108's using a program called the Maneuver Operations Planning System (MOPS). This program was designed by Section 392 and was developed and is being maintained by Section 914. It makes use of some ODP software for part of its activities.

- **Aiming Zones**

The purpose of a mission is to perform certain scientific experiments (i.e., radio science experiment) near the target planet. This requires the definition of a guidance success (aiming) zone at the target planet, which means that if the spacecraft passes through this zone the scientific experiments can be performed satisfactorily (Reference 12). In the MVM'73 mission the aiming zone at Venus was a small region. The figure below shows the relative size of interplanetary aiming zones for different planetary missions.

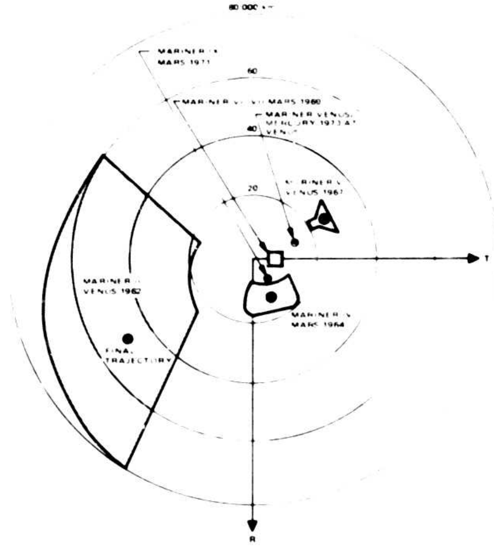

Fig. 10. Aiming Zone Requirements for Planetary Missions

II.2 Program Overview

II.2.1 History

The ODP currently in use at JPL is based on a conceptual design that began in the first half of the 1960's. During that time JPL was using another ODP, developed to support the Ranger, Surveyor, and Lunar Orbiter missions and used quite successfully for Mariners II and IV. This program had been implemented on the IBM 7094 and was upgraded from time to time to meet new project requirements. As time went on, however, it became apparent that the program would be less and less able to meet the accuracy requirements of forthcoming missions because of limitations in certain basic models as well as because there was insufficient precision in the single precision variables used by the code to represent fundamental quantities. Thus the concept of a new program, started from scratch, was born. The old ODP began to be referred to as the Single Precision ODP (SPODP), while the more accurate new program was dubbed the Double Precision ODP (DPODP).

The DPODP (Reference 1, 5, 13, 15) was to have a greater variety of solved for parameters, more refined observable equations, timing and polar motion models, advanced relativistic effects, and double precision computations throughout. The program was a-building for 5 years, but an operational version was delivered to the Mariner '69 project in May of 1969. This version ran on the IBM 7094-7044 Direct Coupled System (DCS), under the IBSYS operating system, and was used successfully as a demonstration throughout the MM'69 flight operations.

The present basic structure of the program depends quite heavily on this IBM 7094 heritage. Because of the large number of parameters it was designed to solve for at a given time (up to 50), and because of the provisions for an additional 20 consider parameters, the program used significant amounts of core just for data storage. When the core for the instruction code to support the complex equations of motions, observable equations and the variety of possible estimated parameters was included, it easily exceeded the IBM 7094's available core storage. As a consequence, the program was broken initially into a series of overlayable "links", which communicated among themselves by means of a variety of files. As it later developed, this design was quite fortuitous because it greatly facilitated interaction of the ODP with other programs, and has enabled much more efficient and cost effective use of the program in a variety of OD related applications. This will be discussed in subsection II.2.2 below.

The fact that the ODP was implemented on what was then the Scientific Computing Facility's (SCF) operating system has also been significant because as time went on, the program began to be used more and more by people not directly involved in flight operations. As will be seen later, the initial development philosophy was directed toward this end, but for the first time under IBSYS on the DCS the ODP could be run as a very large but nevertheless garden-variety user program with no special computer configuration or operator cognizance required.

When JPL decided to replace the DCS with 3rd generation hardware, there was considerable time spent studying whether the ODP might be converted to an IBM 32 bit machine. The SCF machine was to be a 36 bit Univac 1108 under the Exec 8 operating system. After a lengthy investigation it was decided to avoid the 32 bit machine for two basic reasons: the 64 bit double precision was not adequate for computing observables on the more demanding future missions, and the conversions of a significant amount of character-processing-related code would require more time and resources than were available before the MM'71 mission was launched.

It was at this same time that the DSN inherited a pair of IBM 360/75's for use in telemetry processing to replace its pair of IBM 7094's (which could be DCS or stand-a-lone configured). Once the decision was made to eliminate the IBM 7094's entirely, the SPODP went with them, and the DPODP became the mainstay of OD at JPL. There was an extensive effort to get the DPODP converted to run under Exec 8 on the Univac 1108, at the same time as JPL and Univac were trying to get JPL's hardware configuration running under Exec 8.

This conversion effort took the better part of a year, in 1970. The original implementation was a brute force affair resulting in one huge program with multiple overlays. The observable and partial derivative generation modules actually took longer to run on the Univac 1108 than on the DCS because of slower I/O devices used for overlaying. Once the program was certified on the 1108 however, an effort began immediately to optimize its execution by taking advantage of the larger core available. In addition, the many links of the one large overlayable program were broken off as separate programs to reduce the overall amount of core required to run the system, since the operating system insisted on allocating enough core for the longest link all the time, even when a short I/O-bound link was executing. To facilitate user execution of this sequence of programs with a myriad of interconnecting files where he formerly saw one program, a "monitor" was developed to generate the necessary runstreams and simplify tape and file assignments.

This optimization effort was directed toward producing a program for interplanetary operations on MM'71. The DPODP was officially rechristened the Interplanetary ODP (IPODP) to distinguish it from a version with enhanced solution capabilities being planned for the orbiter phase of the MM'71 mission. This satellite ODP (SATODP) included a complete redesign and restructuring of the information matrix accumulation, solution generation, output display and mapping sections of the program to take advantage of operational experience gained during the MM'69 mission. The intent was to have the SATODP changes ready in time for MM'71 launch, with the IPODP as an assured backup to support launch and cruise if necessary. In fact, a preliminary version of the SATODP was ready and was used for launch, and the IPODP was retired without ever having served on a flight project. None of its optimization was wasted however because all the changes were assumed into the SATODP.

With the SATODP being the only ODP left at JPL it was safe to rename it the plain old ODP which it has remained to this day. The program continues to be enhanced with new capabilities and further optimized for faster, more efficient operation on the Univac 1108. A few changes were made to support specific solar panel configurations on the MVM'73 mission and to include the ability to handle stochastic parameters in the solution algorithms. Some major modifications are being made under Viking auspices to combine the numerical integration of the trajectory and trajectory related partial derivatives and to introduce other coding modifications to minimize I/O and CPU time and core usage within each program.

II.2.2 Program Usage

II.2.2.1 General Introduction

The initial philosophy in developing the DPODP was to provide a single, all inclusive program which could do everything for everyone to fantastic accuracy. It was hoped that the inevitable inefficiencies of such a generalized program could be more than made up for by eliminating the coding and coordination effort involved in developing multiple independent specialized programs. Conceptually this was a good idea, but for the first few years the result was a program that in fact satisfied most requirements but was exorbitantly expensive to run and very difficult to modify to include relatively minor changes desired by non-project users. So much effort was being expended in the conversion from the DCS to the 1108, and from DPODP to IPODP to SATODP that little time was available to incorporate those features which might have mollified the general user somewhat.

The overall program capabilities were to include: covariance analysis, simulation, flight operations, post flight analysis, and celestial mechanics support for landed spacecraft, impact probes, earth and planetary orbiters, interplanetary cruise missions and planetary flybys. Characteristics of each of these applications will be described below, with an indication of the program's history and future plans in that area.

II.2.2.2 Covariance Analysis

One of the most important aspects of OD takes place long before the spacecraft is even built, when navigation personnel investigate how well they think they will be able to determine the spacecraft's orbit based on the tracking geometry associated with the planned trajectory and the uncertainties in certain trajectory and observable affecting parameters. To do this, they generate "pseudo" observables to take the place of actual data from the stations, with time tags appropriate to the expected tracking patterns, and process these data in the normal way through the program. A nominal trajectory is chosen which reproduces that planned for the mission and partial derivatives of the observables with respect to the various parameters of interest are computed. The agreement between the computed and the pseudo observables is immaterial because it is the covariance on the solution and not the solution itself that is of interest.

The analysts study the dependance of the covariance for the spacecraft state on the amount and types of data, the tracking pattern, the a-priori uncertainties in consider parameters, the solved for parameter set, etc. Their results are very important because they can cause a trajectory redesign to avoid pathological geometries or impose requirements on the amount of orbit adjustment fuel that must be carried to compensate for orbits determined poorly because of weak data or high sensitivity to badly modelled effects.

For a long time it was felt that covariance analysis could be quick and sloppy, with low precision, highly flexible study programs. It soon became apparent however that on the more demanding missions, the quality of the partial derivatives had as much an effect on the covariance analysis as they would on the solutions themselves, and the ODP began to be used more frequently for partial derivative generation. With the restructuring of the solution "links" (as the now separate programs are still called) and the incorporation of more sophisticated solution algorithms usage of the ODP for covariance analysis has increased significantly, and is expected to grow even further as the coding is modified to decrease core utilization and throughput time and thus operating costs.

There are still a series of specialized OD covariance analysis programs in Section 391 used for development and testing of new data types and processing techniques. Most notable among these is ATHENA, which was developed in the post-SATODP era and which consequently was designed to interchange files with the ODP. Thus partials produced by the ODP can be used to evaluate the new solution algorithms in ATHENA, and ATHENA-generated conditional equations can be processed with the more sophisticated data selection, weighting, and adjustment capabilities of the ODP.

II.2.2.3 Simulation

Another important aspect of preparing for flight operations is navigation team training and testing. This is a period in which artificial data are fed into the tracking system from within the SFOF instead of from the stations. These data are processed by the navigation team as though they were real; they are edited and used to update nominal values of parameters and the updated solutions are passed to the maneuver team, who compute the necessary orbit adjustments, etc. This exercise has always in the past been useful only for establishing operational procedures because the simulated data have never been of a quality befitting any ODP in use at the time. They were usually produced by a program developed as an afterthought when it was realized that the navigation team had to be trained and tested, and the program was implemented on a different machine and with less sophisticated models than the ODP for the sake of operational efficiency.

As the requirements for high precision orbit determination increased on the more recent missions, it becomes ever more apparent that the OD engineers could profit by training in an environment where they were confronted by high precision data which reflected realistic problems they might expect to encounter during flight operations. For example, the simulated data might be based on a trajectory in which the simulation engineers had included a spacecraft gas leak, or different timing polynomials from those available to the trainees might be used to represent sudden changes in the earth's rotation rate not easily detectable or modellable in near-real time.

Only the ODP itself could produce such precision observables, but for a long time it was prohibitively expensive to do so because of the amount of data that had to be simulated to allow for the full flexibility of the DSN. This is beginning to change with the development of the Simulation Output Program (SOP) system which works in concert with the ODP to generate only enough range observables within the ODP to allow fitting a polynomial to the station-probe round trip time as a function of time and still interpolate to a specified accuracy at intermediate points. The differenced-range approach is used to produce Doppler observables (and hence cycle counts) at any specified sample rate, and the troposphere and charged particle effects can be included as desired. Simulated data can be produced at real time rates from these polynomials, at any tracking pass pattern desired, which might even be changed as the test progresses.

II.2.2.4 Flight Operations

Flight Operations have always been the major application of the programs in the DPODP family. Core considerations had not been as important here as for the general "paying" customer because the projects usually had dedicated machines and block time. CPU time was not a major concern because most people were happy to have the full precision of the program rather than playing brinksmanship with lower accuracy but faster code. The major complaint had been throughput time occasioned by the basic structure of the ODP, which passes intermediate results on files from program to program. This turns out to be a problem only in very specific instances, where numerous iterations are performed back to back on the same data set, solving for the same parameters. Otherwise, the fact that intermediate files can be saved and revised has been a tremendous benefit and has greatly reduced overall computer requirements, or at least increased the amount of work that could be done in the previous amount of time. The problem situations are also being solved by development of a specialized full accuracy, limited solution parameter capability program for use in Viking orbiter operations.

A further modification being planned is the creation of an interactive monitor to further reduce the turnaround time in flight operations. This monitor will enable interactive data editing, display of critical information on a CRT in near real computation time, and quicker decisions and follow through in making new runs or more detailed analysis of old results.

II.2.2.5 Post Flight Analysis

The post flight analysis task is basically no different from flight operations, other than that it involves solutions for a greater variety of parameters, with somewhat more esoteric modelling than was possible in-flight. This application also includes comparison of results for common parameters from a number of missions. It is here that station locations are derived, acceleration models are refined, and that the benefit of hindsight is used to reanalyze problems that occurred in flight. New techniques are evaluated against old data so that the wheat can be separated from the chaff. New flight teams practice solving the most recent problems, so as to build up their repetoire of useful tricks.

II.2.2.6 Celestial Mechanics Support

Among the experiments on most deep space probes are those called celestial mechanics experiments, which use the high precision Doppler and range data to help measure various parameters like masses of the planets, fundamental constants, etc. To do this the experimenters need OD programs at least as precise as the data they receive, with the ability to solve for a number of rather esoteric parameters not normally estimated in mission operations. Such work usually taxes a program to its limits of flexibility and accuracy.

To support the celestial mechanics experimenters, the ODP has been equipped with models for determining constants in the relativistic light time equations, parameters describing extent and consistency of the solar corona, masses of planetary satellites, etc.

There have been, and still are, special purpose programs (like POEAS) written within Section 391 to perform OD related to specific celestial mechanics experiments outside the ODP. Such programs were initiated in response to the non-project users' dilemnas mentioned earlier, namely inability to get minor changes or non-project related capabilities implemented in a timely manner, and/or the exceptional expense involved in operating the earlier versions of the ODP. As time goes on, both of these problems are being ameliorated; by better familiarity with the program and longer range scheduling on one hand, and by continual optimization of the program's structure and coding on the other.

II.2.2.7 Radio Occultation Experiment Support

One of the ODP usages is in the so called radio occultation experiment. The precise orbit of the spacecraft and the computed doppler observables obtained from ODP are used to obtain refraction index profiles of the ionosphere and atmosphere of a planet.

II.2.3 Implementation

The ODP is currently implemented on a Univac 1108 under the Exec 8 operating system. Almost all the code is written in FORTRAN V, with the exception of a few routines used for ultra-efficient I/O on a group of high volume, special format files, and a few other system type routines for interacting with Exec 8.

For reasons mentioned above, the program is, and has always conceptually been, a series of programs. These programs intercommunicate via a number of files, some of which may be saved from one run for use on the next, and others of which are just scratch files used within a given job. The trend lately has been to write most new files in, and reconvert more and more old files to, a generalized format suggested some time ago by the designers of the SSDPS mentioned earlier. The type "66" format used for planetary ephemerides was applied to a number of ODP files, and renamed "Lawson's Generalized Format". The major non-FORTRAN routines referred to above handle this LGFIO. All non-LGF files contain FORTRAN "unformatted" binary records and until very recently, with the advent of the new routines, the LGF files had actually been implemented with FORTRAN unformatted I/O as well.

The program has always been divided into 2 major parts: that dealing only with trajectory generation, and that dealing with everything else. The trajectory portion is still being called DPTRAJ (References 1, 15), and has always been under the cognizance of Section 392. The remaining portion has always been under the cognizance of Section 391. Both portions are coded and maintained by Section 914.

Each of these two major parts was designed with a single program to handle control input for all the activities under its purview. There is the OD input A link (ODINA) for DPTRAJ, and the OD input B link (ODINB) for the rest of the ODP. Each of these links accept namelist formatted input which is stored on a set of three files, the "LOCK", "SAVE", and "USE" files for ODINA, and the "LOCK2", "SAVE2", and "USE2" files for ODINB.

The "Lock" files (LOCK and LOCK2) are designed to contain default values which would be the same for a given class of users, but might change from class to class. These would include parameters such as numerical integration tolerances, output format controls, planetary masses and gravitational field descriptors, etc. There is room in the Lock files for every input parameter. The Lock files would usually be created once, and read in by every user in that class. The program never overwrites the Lock files. The Save files (SAVE and SAVE2) are created from the Lock files and include any supplementary or replacement information supplied by the user. They are used as a means of providing initial default values within a particular job which are to be used in case after case within that job. The Use files are the means by which the control information is actually transferred from the input links to the appropriate programs. They initially contain the same information as is on the Save files, but can be updated by specific programs to include corrected parameters out of a solution.

Each of the input links can be directed to initialize itself by reading the Lock, Save, or Use files. Only under the lock file creation option can the Lock file be written. If the Lock or Save file is loaded, the input link will read additional namelist information and output a Save and a Use file containing the updated controls. If the Use file is loaded, as when one wishes to perform another iteration, new card input is allowed but only an updated Use file is written.

The major output of DPTRAJ comes from its program PATH. This probe ephemeris is used by maneuver personnel, mission planners, DSN station schedulers, etc. as well as by the OD engineer. Within the ODP the probe ephemeris (sometimes called the probe ephemeris tape, PET) is used to assist the program VARY derives its variational equations and to help REGRES compute observables and their partial derivatives.

VARY produces a file that contains the probe ephemeris merged with the variational equations. These are used by REGRES as described above, and also by the mapping generation program MAPGEN.

The best known file in the whole program is undoubtedly the REGRES file. This contains residuals and partial derivatives (the basic components of conditional equations) for every data point processed. These may be input to ACCUME, which determines weights, selects and/or adjusts data, and accumulates the information matrix for eventual solution. REGRES files are reused constantly to examine solutions based on different weighting schemes, data sets, media calibrations, etc.

Table 1 provides a brief description of the links currently comprising the ODP(MM'71 and MVM'73 versions). Figure 11 provides a functional flow of the links and also indicates the major inputs. It should be noted that the numerous files providing intra-link communication are not indicated. These files are as described on the ODP charts (Reference 14).

Table 1. ODP Link Description

TRAJECTORY LINKS

1. ODINA: This link reads trajectory input and stores it on files to be read by other links.

2. PATH: This link integrates numerically the equations of motion of a free probe (using the ephemeris time as the independent variable). The positions, velocities and accelerations are all referred to the 1950 Earth mean equatorial cartesian frame from epoch to the desired ending conditions.

3. LANDER: This link writes a probe ephemeris tape for a probe which is landed on one of eleven possible bodies.

4. POST: This is the trajectory output link. It prints the probe's state at each event (i.e., discontinuity or phase change) or at desired times.

5. TRIC: This link transforms the injection probe state from an input coordinate system into the trajectory system: Earth mean equator and equinox of 1950.0. centered at either the physical central body or the user's desired (central) body.

ORBIT DETERMINATION LINKS

1. ACCUME: This link reads the REGRES file and selects data according to input criteria. It adjusts residuals for media and other effects, and computes and applies data weights. It packs the A-matrix of partial derivatives into upper-triangular form. The residual vector is reduced from a M-vector (M = No. of observations processed) to an N-vector (N = No. of parameters). The packed A-matrix and residual vector are stored on a file. They can also be punched and later input to either ACCUME or SOLVE1/SOLVE2.

2. ACCUM2: This link can operate in the single batch mode exactly like ACCUME or it can output information arrays, etc., at several batch times with or without stochastic parameters. The data is edited in the same way as in link ACCUME. (If flag FILMOD=1 then the filtering mode of link ACCUM2 is used.)

3. FILTER: This link computes the forward filtering solutions. Starting with the earliest data point, it processes the observational partials and residuals to generate a forward filtered estimate of the state at epoch. Information arrays at type 2 times are saved for input to links MOPUP and SMOOTH. These arrays are used to compute both the forward filtering and the smoothed estimates and statistics at the type 2 times.

4. MAPGEN: For each given time-coordinate system pair (and set of parameters) this link computes the matrix for mapping to that time and transforming the probe's state into the specified coordinate system. The result is the product of these matrices. MAPGEN reads the vary file and writes its matrices on a file for MAPSEM.

5. MAPSEM: Given either an a-priori covariance or the covariance computed by SOLVE1, this link applies to it the mapping-transformation matrices from MAPGEN. (Actually, the square root of the covariance is mapped.) Further, MAPSEM transforms the correction (solution) vector into each time-coordinate system and updates the nominal transformed state.

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Table 1. ODP Link Description (Contd)

6.	MOPUP:	This link: 1) Tests to determine if the filtering solution at epoch has converged. 2) Prints the forward filtering solution at the type 2 times. 3) Updates the use files to prepare for executing another iteration with the new state estimate or for another iteration with the new state estimate or for processing the data in the next time span. Input to MOPUP is prepared by link FILTER.
7.	ODINB:	This link reads the OD input and stores it on files to be read by other links.
8.	OUTPUT:	This link prints and plots residuals. It reads the ACCUME file and solution from SOLVE1 or SOLVE2 (or SMOTH2) and forms estimated residuals. These can be printed and or plotted.
9.	REGRES:	This link reads the VARY output file (for a probe in free space) and either a tracking data file or a REGRES output file. It selects from the latter the desired observations, it computes corresponding observables, based on the input physical constants and probe ephemeris. It also computes the partial derivatives for each observation with respect to a set of (input) parameters. Its output is a file which contains names of parameters for which partials were formed, values of these parameters, the observables, the residuals (observed-minus-computed values) and the partial derivatives of the observables with respect to solve-for parameters.
10.	SMOOTH:	This link: 1) Computes backward filtered estimates and covariances by processing the observational partials and residuals starting with the final data point. 2) Combines the backward filtered estimates and covariances with the forward filtered results to compute smoothed estimates. 3) Computes the statistics of the difference between the forward and backward result. 4) Prints out the results at the type 2 times. Input to link SMOOTH is provided by link FILTER.
11.	SMOTH2:	This link computes smoothed solutions from epoch to the time of each batch based on filtered solutions from SOLVE1 and smoothing arrays from ACCUM2. The solution arrays computed are written for use in link OUTPUT.
12.	SOLVE1:	This link reads the output from ACCUME and forms a covariance matrix and solution for a set of estimated parameters. It used Householder algorithms to perform a singular-value decomposition to obtain the solution. An a-priori covariance matrix can be used. Consider parameters are also an optional input. SOLVE1 prints the solution and covariance matrices.
13.	SOLVE2:	This link reads the output from ACCUME and forms a non-linear solution for a set of estimated parameters. It employs Bogg's algorithm for partial-step solution to the non-linear estimation problem. No covariance matrices are formed. An a-priori covariance matrix can be used.
14.	UPDATE:	This link writes the new parameter values (from SOLVE1 or SOLVE2) on the use files. These new values can then be used in trajectory computation, mapping, etc.
15.	VARY:	This link numerically integrates the partial derivatives of the probe with respect to a set of dynamic parameters (those which affect the probe state). The integration method is identical to that used in link PATH. The output is a file of sum-and-difference arrays of the partial derivative of the probe's acceleration with respect to the initial probe's state vector.

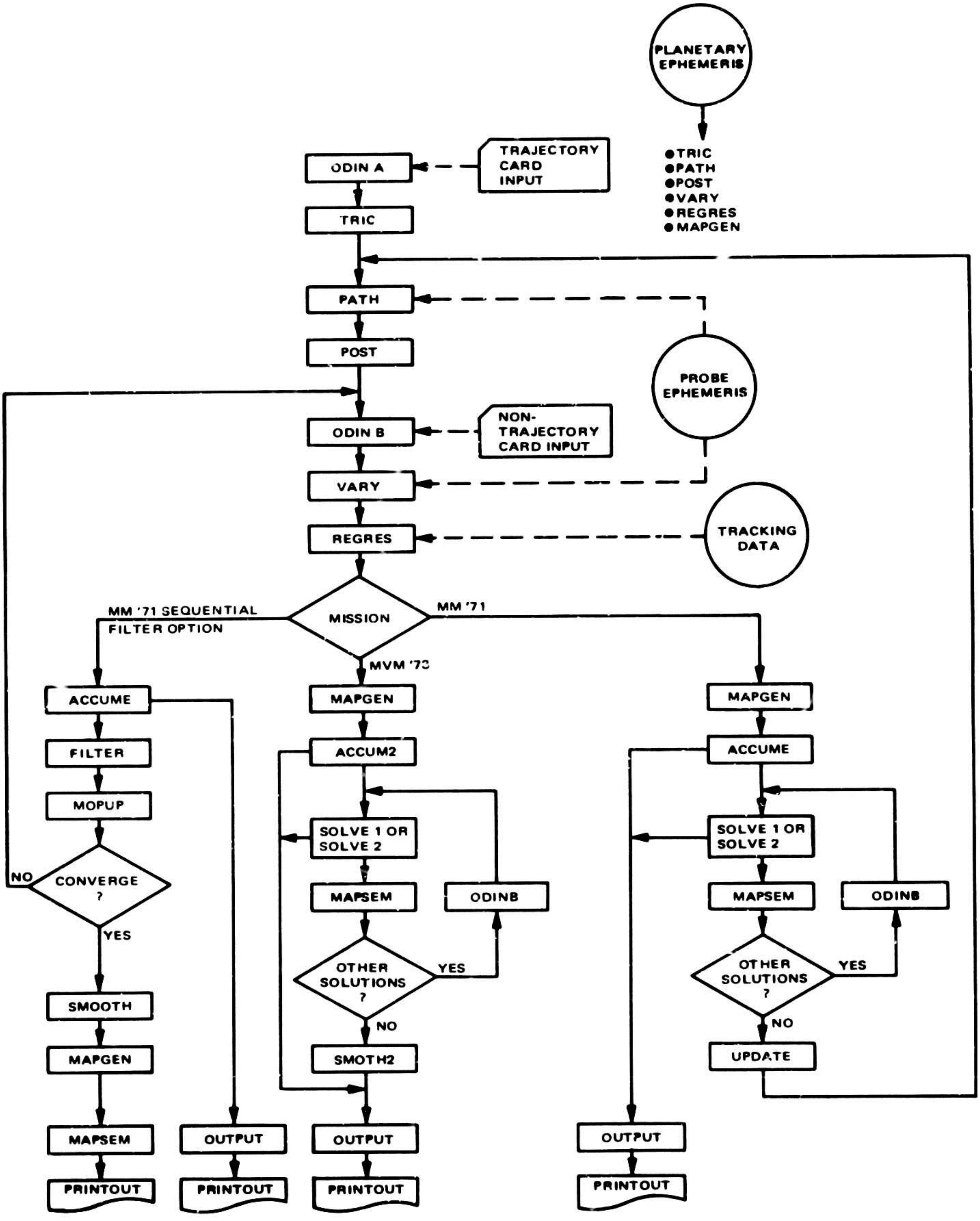

Fig. 11. ODP Functional Diagram

Major modifications being made under Viking direction include the following:

1. A new input link, INIT, has been developed to process ODP input data and generate an LGF file containing these inputs. In doing so it assumes the functions of ODP link ODINB.

2. A new link, PV, simultaneously computes the trajectory and solution to the variational equations using an extensive set of force models. In doing so it combines the functions of programs PATH and VARY and provides the user with a more efficient and accurate program.

3. PVRA is a link which shall have the same force models as PV but shall form an information matrix and residual vector for the solution to the variational equations and data partial derivatives for the probe state only. The functions of PV, REGRES, and ACCUM2 are combined in PVRA to produce a fast, state-only ODP. The file produced is formatted like an ACCUM2 output file.

4. A new batch sequential filter model has been developed which provides the capabilities to: modify filter information arrays between batches; provide for parallel sequential filter solutions; evaluates statistically the effects of errors on filter estimates; computes smoothed solutions for each intermediate batch; transforms intermediate solutions to user-selected coordinate systems; and iterates the sequential filtered solutions until a residual vector is minimized.

5. The entire ODP has been modified to provide for: selection of only the needed software to complete a task; utilization of the LGFIO direct access routines; and dynamic allocation of storage.

These types of modifications will continue to be made as the ODP is enhanced from mission to mission.

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