Problem Formulation for Optimal Array Modeling and Planning[1]

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Abstract— To support NASA's vision to increase the DSN communications capability by at least 100 times the current capability of the 70m antennas, the option of large arrays of thousands of 12m antennas is being studied at JPL, and an operational prototype is planned for in the 2010 timeframe. The flexibility of dynamically subdividing a large antenna array into smaller array clusters of various sizes to support different concurrent missions, and the ability to add or remove antenna elements from an array cluster without interrupting the signal tracking enable new network operation concepts. Yet it poses unique challenges to the modeling and planning of the large array. Current DSN antennas planning and scheduling is done based on network support requests from individual missions that perform their own communication link analysis and ground-in-view period analysis. The largely manual 'horse-trading' among missions is done, by and large, based on antenna tracking-time metrics and does not take into account antenna network performance and reliability. Recognizing the iterative nature of negotiations for resources, mission tends to grossly overestimate their required coverage time, thus reducing the overall network efficiency. The above manual approach will break down in the operation of the large array of thousands of antennas. To efficiently utilize the large array, the modeling and planning process needs to 1) be highly automated, 2) take into account link capability and antenna element reliability, and 3) support long-term, short-term, and instantaneous planning. In this paper we describe an optimal modeling and planning framework for the future large array of DSN antennas. This framework takes into account the array link performance models, reliability models, constraint models, and objective functions, and determines the optimal sub-array clusters configuration that will support the maximum number of concurrent missions based on mission link properties, antenna element reliabilities, mission requests, and array operation constraints. Array cluster size can vary dynamically during the support of a mission. Larger number of small antennas may be needed during the acquisition/calibration phase. Some of these antennas may be released for other uses during the tracking phase. Thus, resources will be efficiently allocated to achieve full utilization.

TABLE OF CONTENTS

1. INTRODUCTION
2. KEY ASSUMPTIONS
3. A HEURISTIC APPROACH TO GENERATE AN INITIAL PLAN
4. MATHEMATICAL FRAMEWORK
5. HANDLING OPERATIONAL AND COMMUNICATIONS CONSTRAINTS
3. NUMERICAL RESULTS
4. SUMMARY
5. REFERENCES
6. BIOGRAPHY

1. INTRODUCTION

To support NASA's vision to increase the DSN communications capability by at least 100 times the current capability of the 70m antennas, the option of large arrays of thousands of 12m antennas is being studied at JPL, and an operational prototype is planned for in the 2010 timeframe. The flexibility of dynamically subdividing a large antenna array into smaller array clusters of various sizes to support different concurrent missions, and the ability to add or remove antenna elements from an array cluster without interrupting the signal tracking enable new network operation concepts. For example, during launch and when the spacecraft is in Earth's vicinity, the spacecraft signal power is so strong that it saturates the front end of the sensitive deep space signal receiving equipment, thus providing erroneous monitor reading. A smaller antenna or a cluster of smaller antenna can alleviate this problem. Another example is that Ka-band performance is sensitive to weather condition, which is difficult to predict way in advance. An antenna array enables a mission to dynamically allocate the number of smaller ground antennas during a pass based on the instantaneous weather condition so as to maintain a fix link margin to support the planned data rate for that pass. The new capabilities pose unique challenges to the modeling and planning of the large array operations.

Current DSN antennas planning and scheduling is done based on network support requests from individual missions

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In this paper we describe the problem formulation of optimal modeling and planning for the future large array of DSN antennas. We investigate an array network planning and operation concept that integrates link capabilities and telecom performances with scheduling to improve the communication efficiency between spacecraft and ground network. The operational setting for the proposed mission support paradigm assumes that an individual mission provides a predefined set of inputs that may include: (a) spacecraft trajectory and pointing information, (b) allowable data rates, (c) required data volume, (d) data priority, (e) navigation requirements, (f) onboard planned activities, and (g) any time constraints of uplink and downlink data delivery. The planning scheme also takes into account ground operation factors like (a) maximum number of antennas allocated to a spacecraft, (b) weather condition, (c) pre-calibration and post-calibration time, (d) operational spares to anticipate unexpected events, and (e) ground maintenance activities. Based on the constraints, mission requests, and mission/event priorities, the network determines a resource allocation plan that can best support the flight missions with the existing set of ground array antennas. This approach in general provides better link configuration and schedule timing information which results in more favorable elevation angles and higher supportable data rates, thus requiring less track time per spacecraft on the average. Advantages for our approach are higher supportable data transmission rates, shorter communicating time per pass and thus a larger number of missions can be supported with the existing set of ground network resources. This lowers the missions' operation cost in tracking, and helps to alleviate DSN future communication congestion.

The array planning scheme consists of two steps. First an initial plan is computed using straight-forward allocation of array antennas to each mission based on the requested data volume, an upper limit of array cluster size per spacecraft, and supportable data rate for the spacecraft. An upper limit of array cluster per spacecraft is to ensure that no single spacecraft will tie up the majority or the whole array at any one time. In the current Large Array Operation Concept [Dergi], this upper limit is set to be 50% of the overall array size. This initial plan ensures highest energy-efficient[5] data return for each mission. If this plan can meet all the operation constraints, this plan will be used as a baseline for the array network support plan. If this plan cannot meet all the operation constraints, this plan will be used in step 2 as the initial condition for one or more constrained optimization algorithms that are used to generate conflict-free plans. As this initial plan is usually 'close to' the optimal solution, if exists, the constrained optimization algorithms will converge quickly to the optimal solution and deliver a good plan.

This paper is organized as follows: Section 2 describes the key assumptions of the array planning problem. Section 3 discusses step 1 of the array planning scheme - a heuristic approach generates the initial plan. Sections 4 and 5 apply to step 2 of the planning process. Section 4 discusses the problem formulation and mathematical descriptions of resource and constraint models that constitute the constraint optimization process of array network planning. Section 5 provides the Monte Carlo simulation and analysis results. Section 6 discusses the concluding remarks and future work.

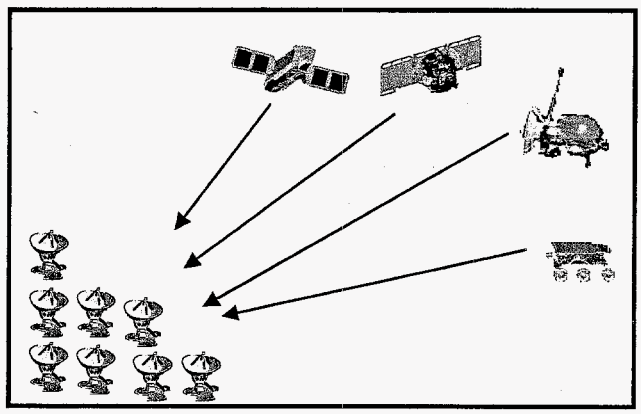

Figure 1 – An array communication link model of N spacecrafts and M identical ground antennas

2. Key Assumptions

In this paper, we make the following assumptions on the spacecraft and antenna array models and planning process:

a. Each spacecraft transmits at a fixed output power.
b. An antenna cluster supports one data rate per pass when tracking a spacecraft.
c. An antenna cluster supports one continuous track per pass.
d. An antenna cluster can support more than one spacecraft downlink when they are in the same field of view of the antenna cluster (multiple spacecraft per antenna).

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e. All antennas have the same G/T for a given elevation angle and weather condition.
f. The antennas are spaced far enough that there is no obstruction between the antennas when tracking above a minimum elevation angle of TBD degree.
g. Assume all antennas are in close proximity and the spacecraft is at far field, all antennas tracking the same spacecraft has same elevation angle.

3. A HEURISTIC APPROACH TO GENERATE AN INITIAL PLAN

For an individual spacecraft, the most energy-efficient way of sending back a fixed volume of data is to transmit at the highest supportable data rate. This, to a first order, is also a time-efficient way for ground antenna array planning as this approach ties up the cluster of antennas required supporting this data rate with the minimal time. To ensure that no single spacecraft will tie up the majority or the whole array at any one time, an upper limit of array cluster size is set. When Doppler tracking is required for a pass, the array cluster size and the data rate are chosen in such a way that the time span $[T_s, T_f]_{data}$ to send back a given volume of data is comparable to the time span $[T_s, T_f]_{Trk}$ to achieve a given Doppler measurement precision requirement.

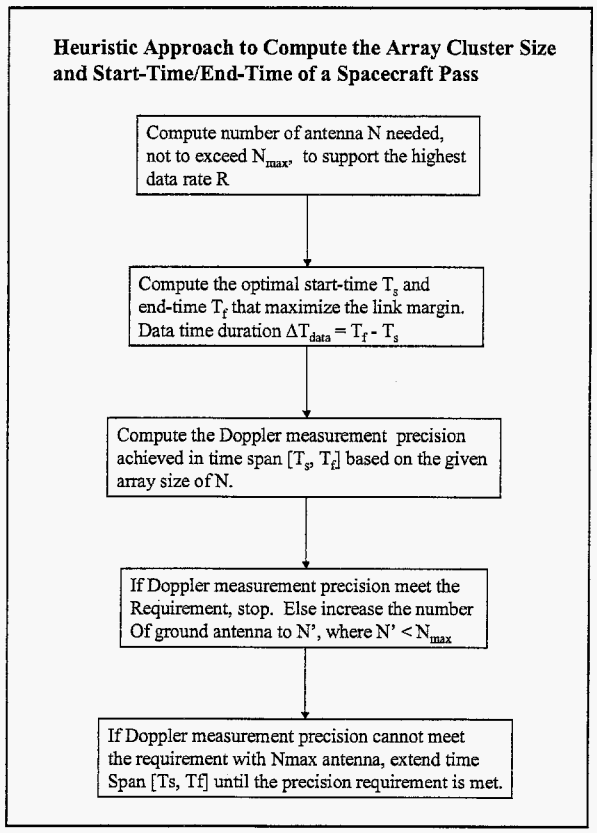

Figure 2 – Process flow to compute array cluster size and start-time/end-time of a pass

Using the above heuristic planning approach, we compute the start-time and end time of each pass and the corresponding number of antennas as a function of time. We then overlay the array size profiles of all the passes and compute the number of antennas required at each time point within the planning horizon. If this plan meets all operation constraints and the number of antennas required is lower than the maximum number of antennas in the array at all time points, this initial plan is deemed to be the baseline plan of the array network for the planning horizon. If the number of antennas exceeds the maximum at one or more time points, this plan will be used in step 2 as the initial condition for one or more constrained optimization algorithms that are used to generate optimal conflict-free plans. As this initial plan is usually 'close to' the optimal solution, if exists, the constrained optimization algorithms will converge quickly to the optimal solution and deliver a good plan.

4. MATHEMATICAL FRAMEWORK

We consider an array communication link model, which consists of a set of N spacecrafts and M identical ground antennas (Figure 1). Based on a planning horizon, say a week, the dynamic ephemeris of the spacecrafts, and the elevation mask angle, the lines of sight between the array and the spacecrafts can be established, which in turn yield K passes. We denote the passes by $\{P_k \mid k = 1,2,\ldots K\}$. Associate with each pass are the spacecraft number, the starting and ending times of the pass, n, T_0^k and T_f^k are respectively. By adding in the slanted range, the elevation angle, the bore-sight angles and the telecom configurations of each spacecraft such as transmitting power, frequency, antenna pattern, gains and losses, etc., a communication link budget can be calculated to attain the time-dependent profile of receiving powers at the array front from the spacecraft (Figure 3). The result is then combined with the mission link margin to generate the number of required antennas from the array, defined by $NORA(R(k),t)$, needed to close the link for a transmitting data rate $R(k)$. Depending on the design of the spacecraft $n(k)$, the data rate $R(k)$ can be selected from a list of the different supportable rates $\{R_{i,n(k)}\}_{i=1}^{L(n(k))}$ (Figure 4). Note that the higher the data rate is the larger number of antennas is needed, especially at low elevation angles when the atmospheric noise is high.

If communication is scheduled for the pass P_k, the actual transmitting data rate $R(k)$, the communication starting time t_0^k and ending time t_f^k must be specified. Thus the received data volume can be computed as

$$DV_k = R(k)\cdot(t_f^k - t_0^k) \quad for \quad 1 \le k \le K\,. \qquad (1)$$

Because $R(k)$ was selected for the pass k, a cluster of $NORA(R(k),t)$ antennas must be allocated to support spacecraft $n(k)$ for the time period from t_0^k to t_f^k. When all the considered passes are lined up over the time horizon, the cumulative number of required antennas can be defined as

$$NORA(t) = \sum_{k=1}^{K} NORA\ (R(k),t). \qquad (2)$$

An example of the cumulative number of required antennas is displayed in Figure (4).

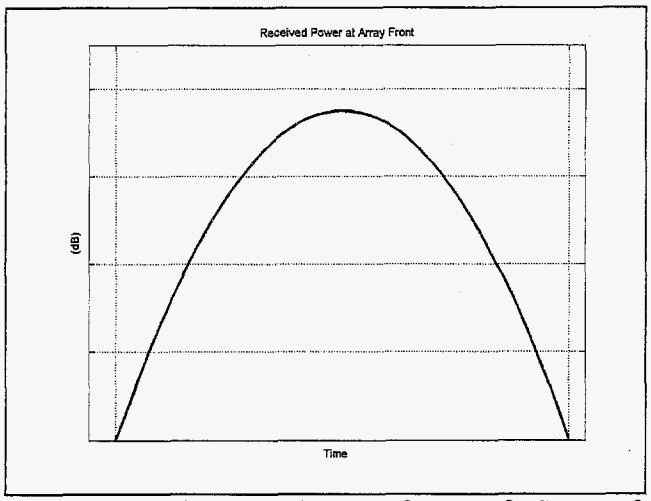

Figure 3 – Sample Received Power of a Pass of a Spacecraft at the Array Front

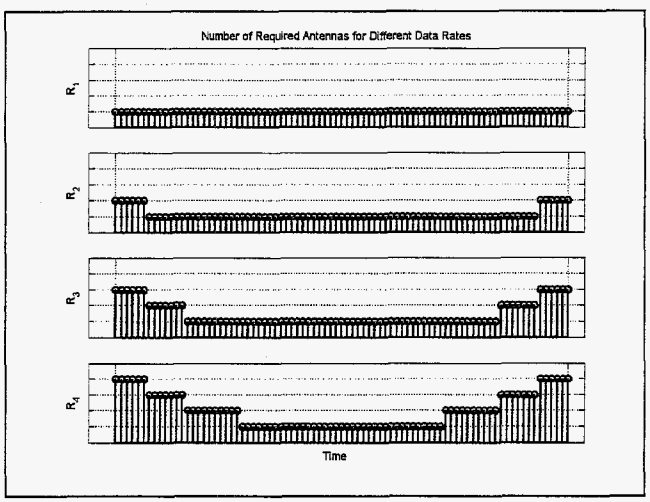

Figure 4 – The Required Number of Antennas Needed to Close the Link at Different Data Rates Based on the Received Power

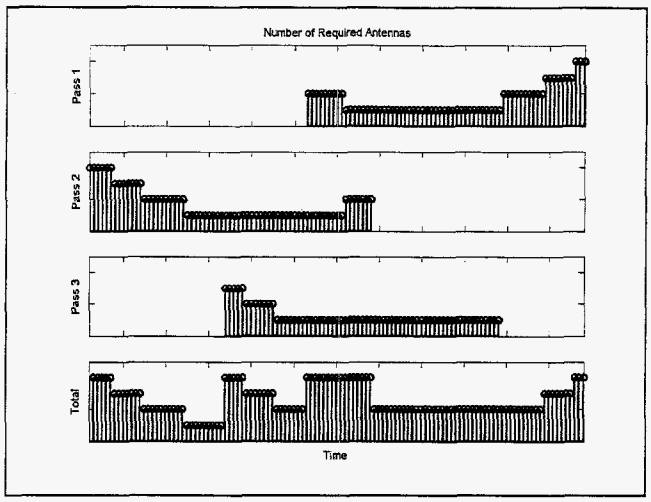

Figure 5: The Total Number of Required Antennas (bottom) Versus Those of the Different Passes (tops)

Our goal in optimizing the array planning is to search the best combination of starting times, ending times, and transmitting data rates $(R(k), t_0^k, t_f^k)$ for the passes $P_k, k = 1,2,\ldots K$ so that:

(a) The array's allocated time to the missions is as efficient as possible. Thus our planning objective is to minimize the array's total service time to all the consider passes or specifically

$$\min \sum_{k=1}^{K} (t_f^k - t_0^k) \qquad (3)$$

(b) The resulting data volume from the pass must meet or exceed the required data volume for the pass. This requirement can be expressed as,

$$R(k) \cdot (t_f^k - t_o^k) \geq RDV_k \qquad (4)$$

(c) The data rate $R(k)$ must be selected from a discrete set of mission supportable data rates $\{R_{i,n(k)}\}_{i=1}^{L(n(k))}$, i.e.

$$R(k) \in \{R_{1,n(k)}, R_{2,n(k)}, \ldots, R_{L(n(k)),n(k)}\} \qquad (5)$$

(d) The total number of required antenna should never at any time exceed M, the number of array antennas. That is,

$$NORA(t) \leq M \qquad (6)$$

(e) Communication in a pass must happen within the pass itself,

$$T_0^k \leq t_0^k \leq t_f^k \leq T_f^k \quad for \quad k = 1,2,\ldots K \qquad (7)$$

It should be pointed out that these are a few mission and operational constraints that we have considered. Additional ones can be incorporated in a similar fashion.

In summary, our array planning and optimization problem involve minimizing the cost function in (3) subject to the communications and operational constraints (4)-(7). The problem we are trying to solve falls into a class of mixed integer nonlinear constrained optimization. In particular, we seek an optimal solution

$$X = \begin{bmatrix} R(1) \\ t_o^1 \\ t_f^1 \\ \vdots \\ R(K) \\ t_o^K \\ t_f^K \end{bmatrix}_{3K\times 1}, \tag{8}$$

that minimizes the cost functional

$$C(X) = \sum_{k=1}^{K} (t_f^k - t_0^k), \tag{9}$$

and satisfies the constraints (4)-(7), which can be formulated as follows. The constraints (4) and (6) are employed as a nonlinear constraint

$$G(X) = \begin{bmatrix} G_1(X) \\ G_2(X) \end{bmatrix} \le 0, \tag{10}$$

where

$$G_1(X) = \begin{bmatrix} R(1)\cdot(t_f^1 - t_0^1) - RDV_1 \\ \vdots \\ R(K)\cdot(t_f^K - t_0^K) - RDV_K \end{bmatrix}, \tag{11}$$

and

$$G_2(X) = NORA(t) - M. \tag{12}$$

The constraints (7) can be transformed into a set of bound and linear constraints

$$AX \le B, \tag{13}$$

$$L_B \le X \le U_B, \tag{14}$$

where

$$A = \begin{bmatrix} 0 & 1 & -1 & & & \\ & & & \ddots & & \\ & & & 0 & 1 & -1 \end{bmatrix}_{K\times 3K}, B = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}_{K\times 1} \tag{15}$$

and

$$L_B = \begin{bmatrix} R_{1,n(1)} \\ T_o^1 \\ T_o^1 \\ \vdots \\ R_{1,n(k)} \\ T_o^K \\ T_o^K \end{bmatrix}_{3K\times 1}; \quad U_B = \begin{bmatrix} R_{L(n(1)),n(1)} \\ T_f^1 \\ T_f^1 \\ \vdots \\ R_{L(n(K)),n(K)} \\ T_f^K \\ T_f^K \end{bmatrix}_{3K\times 1} \tag{16}$$

There are many commercial of the shell software tools that are capable of solving such problem. Specific software, which we have used include MATLAB optimization toolbox and the ILOG Optimization Suite.

5. Numerical Results

Note:
This section is not finished yet. The numerical simulation of the array planning scheme is ongoing and does not make it into the paper for this version. The numerical results will be provided in the next version.

6. Summary

In this article, we have modeled a framework for optimizing the allocation of antennas resources of an array to support multiple concurrent missions. Our design takes into account the actual mission dynamics, spacecraft telecom configurations as well as those of the ground antennas. Based on the telecom predicts, the capabilities of the array, and the mission and operational requirements, our model can assess how efficient a schedule is and whether any mission requirements or operational constraints are not satisfied. Our model of framework belongs the class of constrained optimization problems, which can be solved using a number of commercial off the shelf software. Finally, our framework thus far has included various objectives and constraints, but its framework is quite flexible to allow easy adaptation for future modification or additional needs and requirements.

ACKNOWLEDGEMENTS

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Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not constitute or imply its endorsement by the United State Government, the CSUF, or the Jet Propulsion Laboratory, California Institute of Technology.

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BIOGRAPHIES

Dr. Kar-Ming Cheung *is a Technical Group Supervisor in the Communications Systems Research Section (331) at JPL. His group provides telecom analysis support for JPL missions, and develops the operational telecom analysis and predict generation tools for current and future JPL missions and the DSN. He received NASA's Exceptional Service Medal for his work on Galileo's onboard image compression scheme. He was the Manager of the Network Signal Processing Work Area of NASA's Deep Space Network (DSN) Technology Program. He has authored or co-authored 6 journal papers and over 20 conference papers in the areas of error-correction coding, data compression, image processing, and telecom system operations. Since 1987 he has been with JPL where he is involved in research, development, production, operation, and management of advanced channel coding, source coding, synchronization, image restoration, and link analysis schemes. He got his B.S.E.E. degree from the University of Michigan, Ann Arbor in 1984, his M.S. degree and Ph.D. degree from California Institute of Technology in 1985 and 1987 respectively.*

Dr. Charles H. Lee *is an associate professor of mathematics at the California State University Fullerton (CSUF) and a faculty part time staff in the Communications Systems Research Section (331) at the Jet Propulsion Laboratory. Before becoming a faculty member, he spent three years as a Post-Doctorate fellow at the Center for Research in Scientific Computation, Raleigh, North Carolina, where he was the recipient of the 1997-1999 National Science Foundation Industrial Post-Doctorate Fellowship. His research has been Computational Applied Mathematics with emphases in Control, Fluid Dynamics, Smart Material Structures and Telecommunications. He received his Doctor of Philosophy in Applied Mathematics in 1996 from the University of California at Irvine.*

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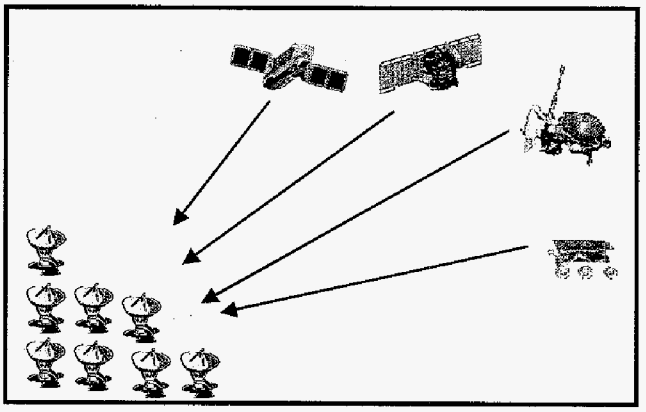

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3. A HEURISTIC APPROACH TO GENERATE AN INITIAL PLAN

For an individual spacecraft, the most energy-efficient way of sending back a fixed volume of data is to transmit at the highest supportable data rate. This, to a first order, is also a time-efficient way for ground antenna array planning, as this approach ties up the cluster of antennas required to support such data rate with the minimal time. To ensure that no single spacecraft will tie up the majority or the whole array at any one time, an upper limit of array cluster size is set. When Doppler tracking is required for a pass, the array cluster size and the data rate are chosen in such a way that the time span $[T_s, T_f]_{data}$ to send back a given volume of data is comparable to the time span $[T_s, T_f]_{Trk}$ to achieve a given Doppler measurement-precision requirement.

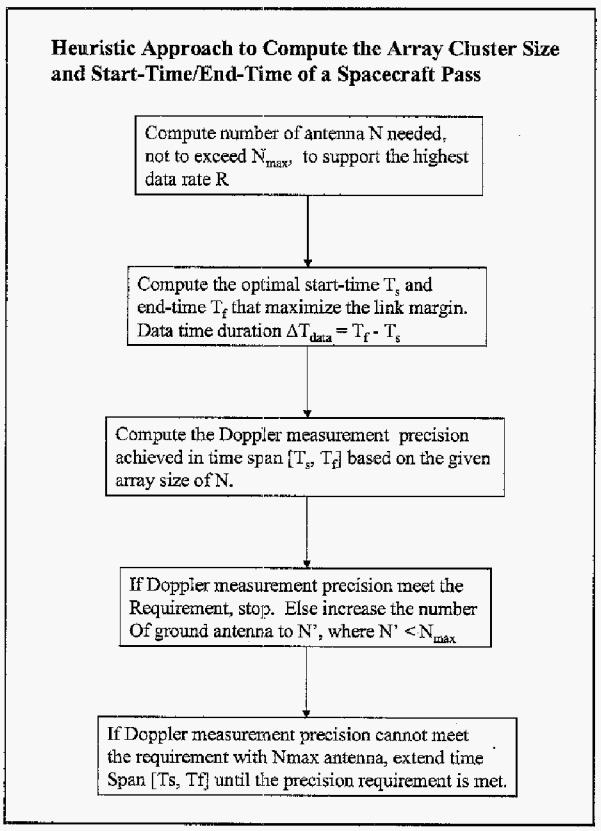

Figure 2 – Process flow to compute array cluster size and start-time/end-time of a pass.

Using the above heuristic planning approach, we compute the start-time and end time of each pass and the corresponding number of antennas as a function of time. We then overlay the array size profiles of all the passes and compute the number of antennas required at each time point within the planning horizon. If this plan meets all operational constraints, and the number of antennas required is lower than the maximum number of antennas in the array at all time points, this initial plan is deemed to be the baseline plan of the array network for the planning horizon. If the number of antennas exceeds the maximum at one or more time points, this plan will be used in step 2 as the initial condition for one or more constrained optimization algorithms that are used to generate optimal conflict-free plans. As this initial plan is usually 'close to' the optimal solution, if exists, the constrained optimization algorithms will converge quickly to the optimal solution and deliver a good plan.

4. MATHEMATICAL FRAMEWORK

We consider an array communication link model, which consists of a set of N spacecrafts and M identical ground antennas (Figure 1). Based on a planning horizon, say a week, the dynamic ephemerides of the spacecrafts, and the elevation-mask angle, the lines of sight between the array and the spacecrafts can be established, which in turn yield K passes. We denote the passes by $\{P_k \mid k = 1,2,\ldots K\}$. Associated with each pass are the spacecraft number and the starting and ending times of the pass, n, T_0^k, and T_f^k, respectively. By adding in the slant range, the elevation angle, the bore-sight angles and the telecom configurations of each spacecraft such as transmitter power, frequency, antenna pattern, gains and losses, etc., a communication link budget can be calculated to attain the time-dependent profile of received powers at the array front (Figure 3). The result is then combined with the mission link margin to generate the number of required antennas from the array, defined by $NORA(R(k), t)$, needed to close the link for a transmitter data rate $R(k)$. Depending on the design of the spacecraft $n(k)$, the data rate $R(k)$ can be selected from a list of the different supportable rates $\{R_{i,n(k)}\}_{i=1}^{L(n(k))}$ (Figure 4). Note that the higher the data rate, the larger the number of antennas is needed, especially at low elevation angles when the atmospheric noise is high.

If communication is scheduled for the pass P_k, the actual transmitter data rate $R(k)$, the communication starting time t_0^k and ending time t_f^k must be specified. Thus the received data volume can be computed as

$$DV_k = R(k)\cdot(t_f^k - t_0^k) \quad for \quad 1 \le k \le K\,. \qquad (1)$$

Because $R(k)$ was selected for the pass k, a cluster of $NORA(R(k),t)$ antennas must be allocated to support spacecraft $n(k)$ for the time period from t_0^k to t_f^k. When all the considered passes are lined up over the time horizon, the cumulative number of required antennas can be defined as

$$NORA(t) = \sum_{k=1}^{K} NORA\ (R(k),t). \qquad (2)$$

An example of the cumulative number of required antennas is displayed in Figure (4).

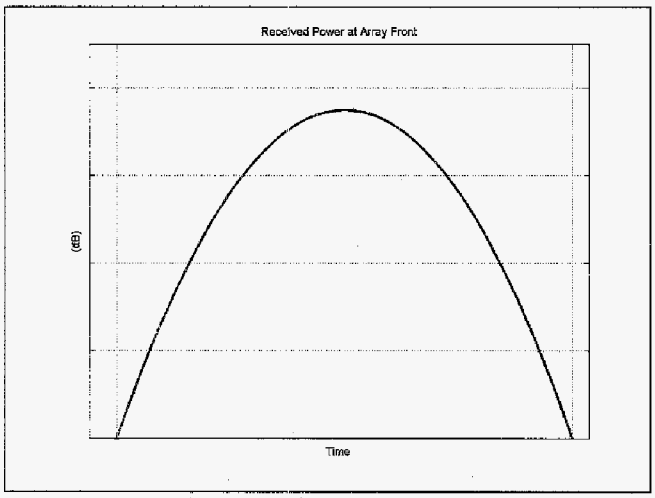

Figure 3 – Sample received power of a pass of a spacecraft at the array front

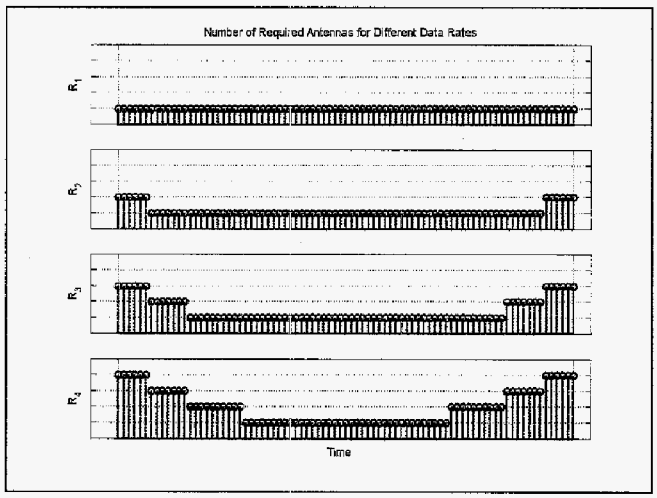

Figure 4 – The required number of antennas needed to close the link at different data rates based on the received power

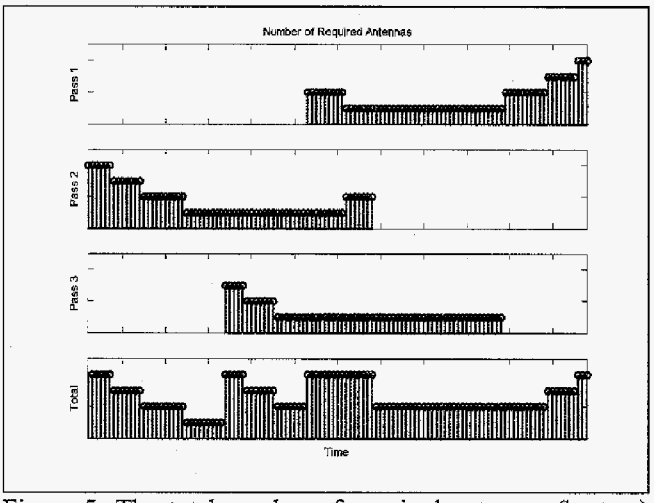

Figure 5: The total number of required antennas (bottom) versus those of the different passes (tops)

Our goal in optimizing the array planning is to search the best combination of starting times, ending times, and transmitting data rates $(R(k), t_0^k, t_f^k)$ for the passes $P_k,\ k = 1,2,\ldots K$ so that:

(a) The array's allocated time to the mission is as efficient as possible. Thus, our planning objective is to minimize the array's total service time to all the considered passes, or specifically

$$\min \sum_{k=1}^{K} (t_f^k - t_0^k)\,. \qquad (3)$$

(b) The resulting data volume from the pass must meet or exceed the required data volume for the pass. This requirement can be expressed as

$$R(k)\cdot(t_f^k - t_o^k) \ge RDV_k\,. \qquad (4)$$

(c) The data rate $R(k)$ must be selected from a discrete set of mission supportable data rates $\{R_{i,n(k)}\}_{i=1}^{L(n(k))}$, i.e.

$$R(k) \in \{R_{1,n(k)}, R_{2,n(k)}, \ldots, R_{L(n(k)),n(k)}\}\,. \qquad (5)$$

(d) The total number of required antennas should never at any time exceed M, the number of array antennas. That is,

$$NORA(t) \le M\,. \qquad (6)$$

(e) Communication in a pass must happen within the pass itself,

$$T_0^k \le t_0^k \le t_f^k \le T_f^k \quad for \quad k = 1,2,\ldots K\,. \qquad (7)$$

It should be pointed out that these are a few missions and operational constraints that we have considered. Additional ones can be incorporated in a similar fashion.

In summary, our array planning and optimization problem involve minimizing the cost function in (3) subject to the communications and operational constraints (4)-(7). The problem we are trying to solve falls into a class of mixed integer nonlinear constrained optimization. In particular, we seek an optimal solution

$$X = \begin{bmatrix} R(1) \\ t_o^1 \\ t_f^1 \\ \vdots \\ R(K) \\ t_o^K \\ t_f^K \end{bmatrix}_{3K\times 1}, \tag{8}$$

that minimizes the cost functional

$$C(X) = \sum_{k=1}^{K} (t_f^k - t_0^k), \tag{9}$$

and satisfies the constraints (4)-(7), which can be formulated as follows. The constraints (4) and (6) are employed as a nonlinear constraint

$$G(X) = \begin{bmatrix} G_1(X) \\ G_2(X) \end{bmatrix} \le 0, \tag{10}$$

where

$$G_1(X) = \begin{bmatrix} R(1)\cdot(t_f^1 - t_0^1) - RDV_1 \\ \vdots \\ R(K)\cdot(t_f^K - t_0^K) - RDV_K \end{bmatrix}, \tag{11}$$

and

$$G_2(X) = NORA(t) - M. \tag{12}$$

The constraints (7) can be transformed into a set of bound and linear constraints

$$AX \le B, \tag{13}$$

$$L_B \le X \le U_B, \tag{14}$$

where

$$A = \begin{bmatrix} 0 & 1 & -1 & & & \\ & & & \ddots & & \\ & & & 0 & 1 & -1 \end{bmatrix}_{K\times 3K}, B = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}_{K\times 1} \tag{15}$$

and

$$L_B = \begin{bmatrix} R_{1,n(1)} \\ T_o^1 \\ T_o^1 \\ \vdots \\ R_{1,n(k)} \\ T_o^K \\ T_o^K \end{bmatrix}_{3K\times 1}; \quad U_B = \begin{bmatrix} R_{L(n(1)),n(1)} \\ T_f^1 \\ T_f^1 \\ \vdots \\ R_{L(n(K)),n(K)} \\ T_f^K \\ T_f^K \end{bmatrix}_{3K\times 1}. \tag{16}$$

There are many commercial-off-the-shelf software tools that are capable of solving such a problem. Specific software, which we have used, includes MATLAB's optimization toolbox and the ILOG Optimization Suite.

5. Numerical Results

For the proof of concept, we consider in this simulation an array of 30 antennas supporting five different concurrent missions during a time period. The numbers of antennas required to support the missions at the highest supportable data rates are displayed in Figure 6. Clearly when all missions are supported at these highest possible data rates, the cumulative number of required antennas certainly exceeds the array size (see bottom of Figure 6).

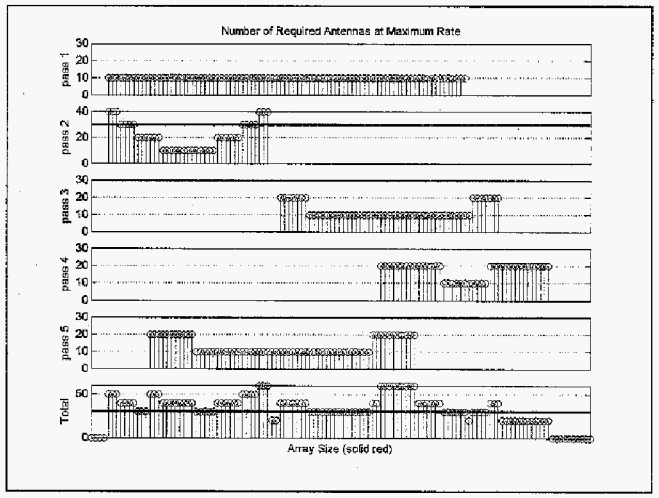

Figure 6 –The required numbers of antennas needed to close the links at maximum data rates for five concurrent missions (top 5) and the corresponding cumulative required number of antennas (bottom)

Our objectives in this simulation are to seek for each pass the optimal starting time and data rate so that (i) the cumulative required number of antennas never exceeds the array capacity, (ii) the achieved data throughput meets the mission data volume requirement and (iii) the total array's tracking time is smallest possible. In our case, we assume the required data volume for each mission is 60% of the largest possible data volume (highest data rate throughout the entire pass). The following procedures are implemented to verify whether the objectives are met. First, the stopping time for each pass is selected according to the selected

starting time, selected data rate, the required data volume, and the ending time of the pass. That is, once data rate is selected, the transmitting time is computed by taking the required data volume divided by the data rate. As a result, the stopping time is the starting time plus the transmitting time, provided it is within the pass. In the event that the starting time plus the transmitting time is beyond the pass's ending time, the corresponding data throughput will not be able to meet the required data volume for the pass. In that case, the pass's stopping time and ending time coincide. When the data rate, starting time, and stopping time for each pass are known, the total number of required antennas as a function of time to support all the missions can be found, which then can be compared with array's size to check for any violations. Finally, we sum up the tracking time for the passes to get the total array's time.

In seeking for an optimal solution, we use the genetic algorithm with all the objectives folded into the fitness function as penalties with appropriate weights. Our initial population size of 100 was assumed. The convergence to the optimal solution for the first 25 iterations is displayed in Figure 7. The blue and red curves indicate the average fitness value and the best fitness value, respectively. Their final values are also shown in the legend.

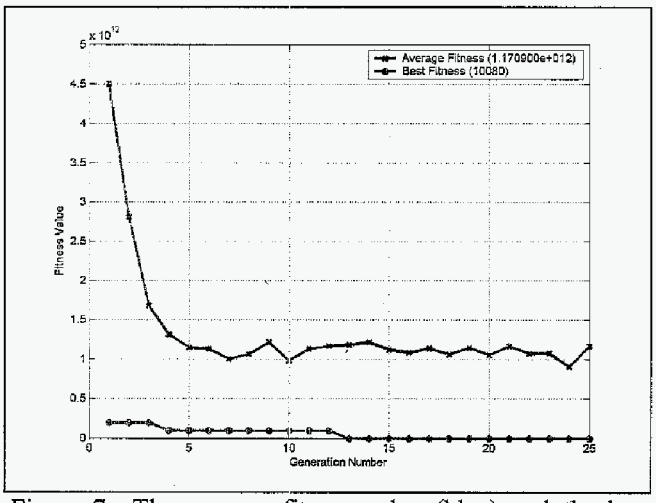

Figure 7 –The average fitness value (blue) and the best fitness value (red) for different iterations used in the genetic algorithm

The optimal starting time and data rates for the different passes are shown in Figure 8. The results indicate that the data volume requirements for all missions are satisfied, the total number of required number of antennas are always at the array's capacity and the total array time to support these missions is 10,080 seconds. We have repeated the optimization process multiple times, each time with a different set of initial populations. The results are consistent and seem to converge with less than 15 iterations every time.

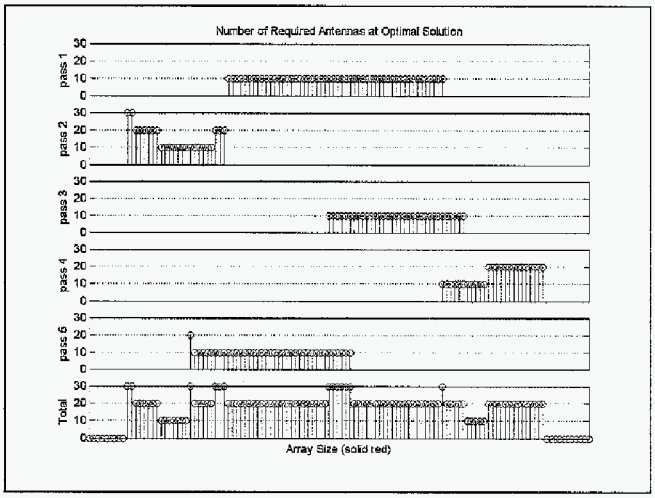

Figure 8 –The timeline of the required number of antennas based on the optimal solution to support the various missions

As mentioned earlier, this is a proof of concept simulation and optimization for array modeling and planning. Here we emphasize on the generality of the structure of the problem and its mathematical formulations, which we believe provide the necessary foundation to extend to reflect actual mission operations and larger size array.

6. SUMMARY

In this article, we have modeled a framework for optimizing the allocation of antenna resources of an array to support multiple concurrent missions. Our design takes into account the actual mission dynamics, spacecraft telecom configurations, as well as the telecom configurations of the ground antennas. Based on the telecom predicts, the capabilities of the array, and the mission and operational requirements, our model can assess how efficient a schedule is and whether any mission requirements or operational constraints are not satisfied. Our model of framework belongs to a class of constrained optimization problems which can be solved using a number of commercial-off-the-shelf software packages. Finally, our framework thus far has included various objectives and constraints, but its framework is quite flexible and allows easy adaptation for future modification or additional needs and requirements.

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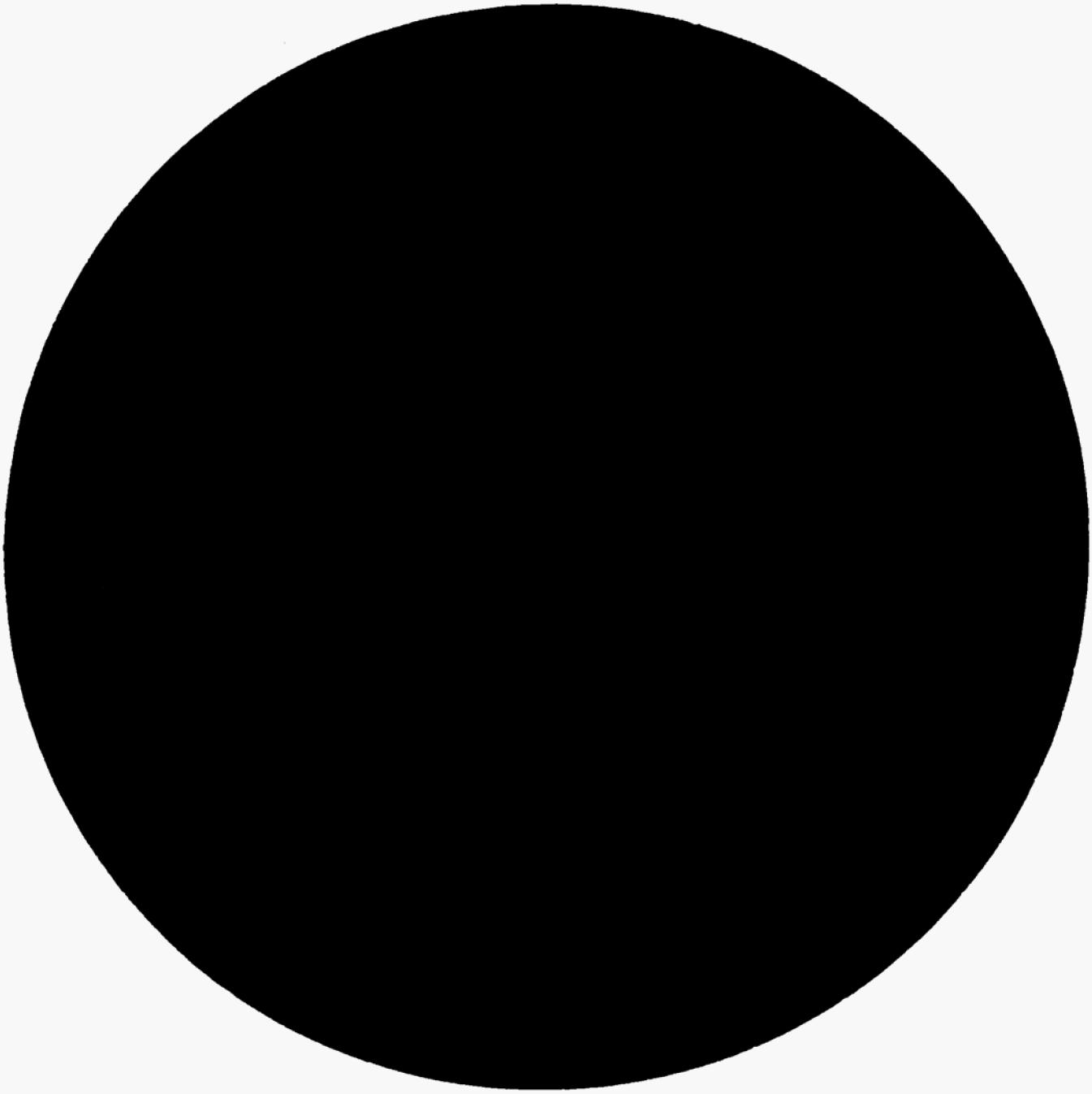
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