Bayesian Analysis of the Cosmic Microwave Background

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Overview

- The Cosmological significance of the Microwave Background
- Overview of experimental results and cosmological implications - geometry, composition, and age of the universe!
- Analysis challenges…
- Formulation of a Bayesian Approach
- Application to WMAP 1 and 3 year temperature and polarization data
- Current work in preparation for Planck
- Conclusions…

From the Proceedings of the National Academy of Sciences
Volume 15 : March 15, 1929 : Number 3

A RELATION BETWEEN DISTANCE AND RADIAL VELOCITY AMONG EXTRA-GALACTIC NEBULAE

By Edwin Hubble
Mount Wilson Observatory, Carnegie Institution of Washington

Communicated January 17, 1929

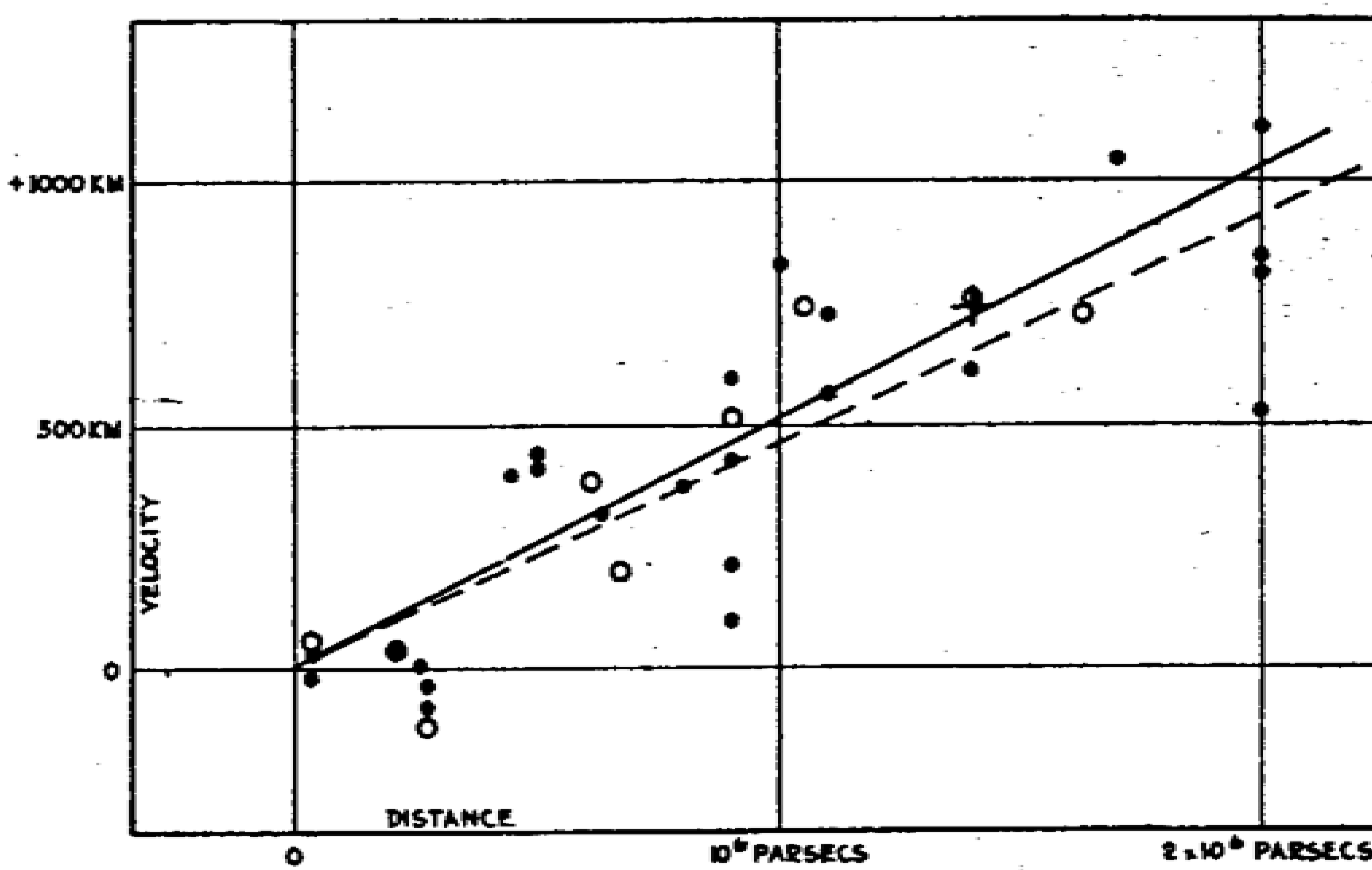
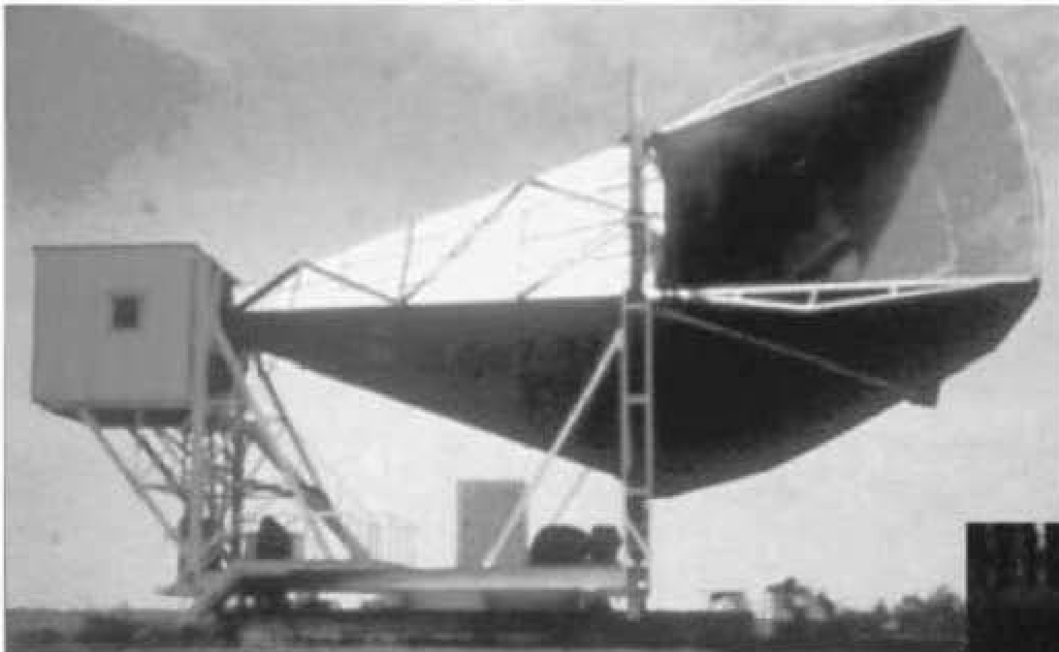


FIGURE 1

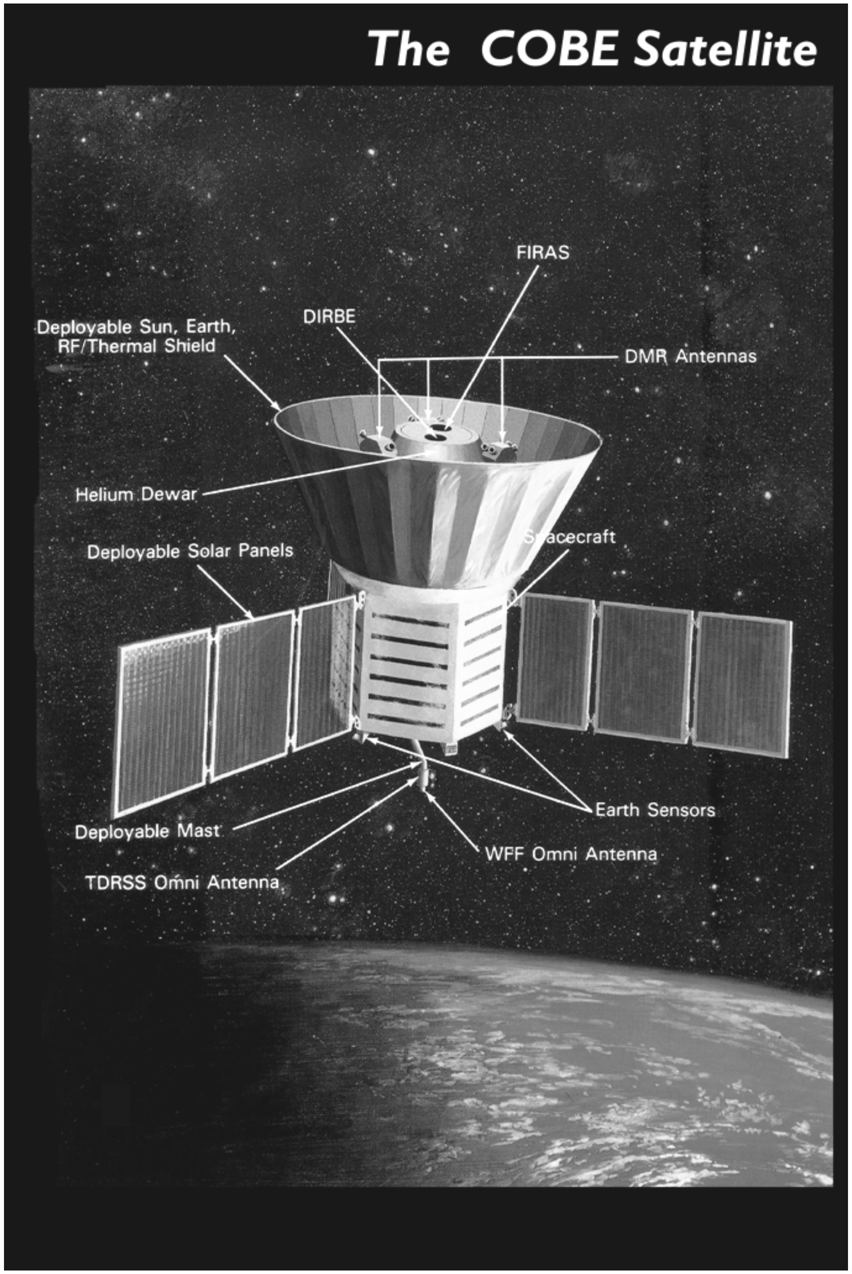
Discovery of Cosmic Background

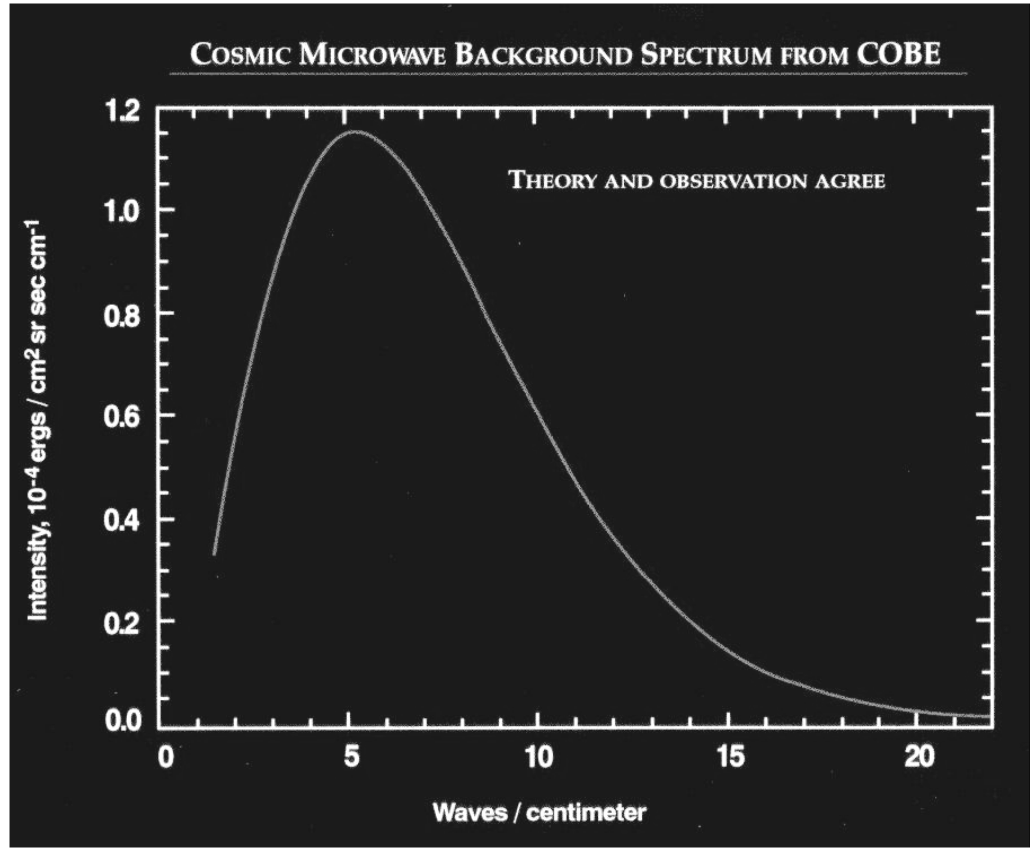


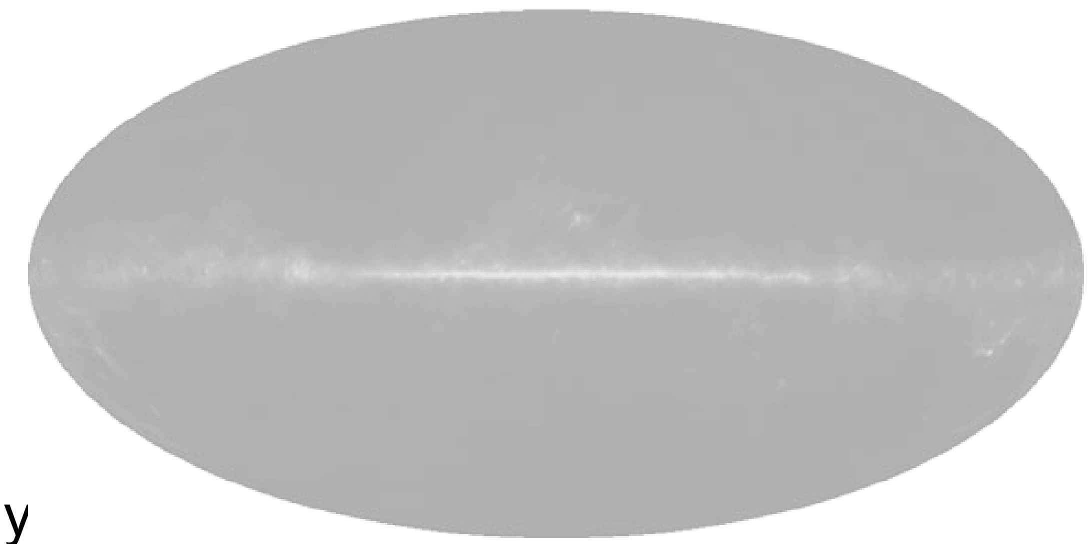
Microwave Receiver

Robert Wilson

Arno Penzias

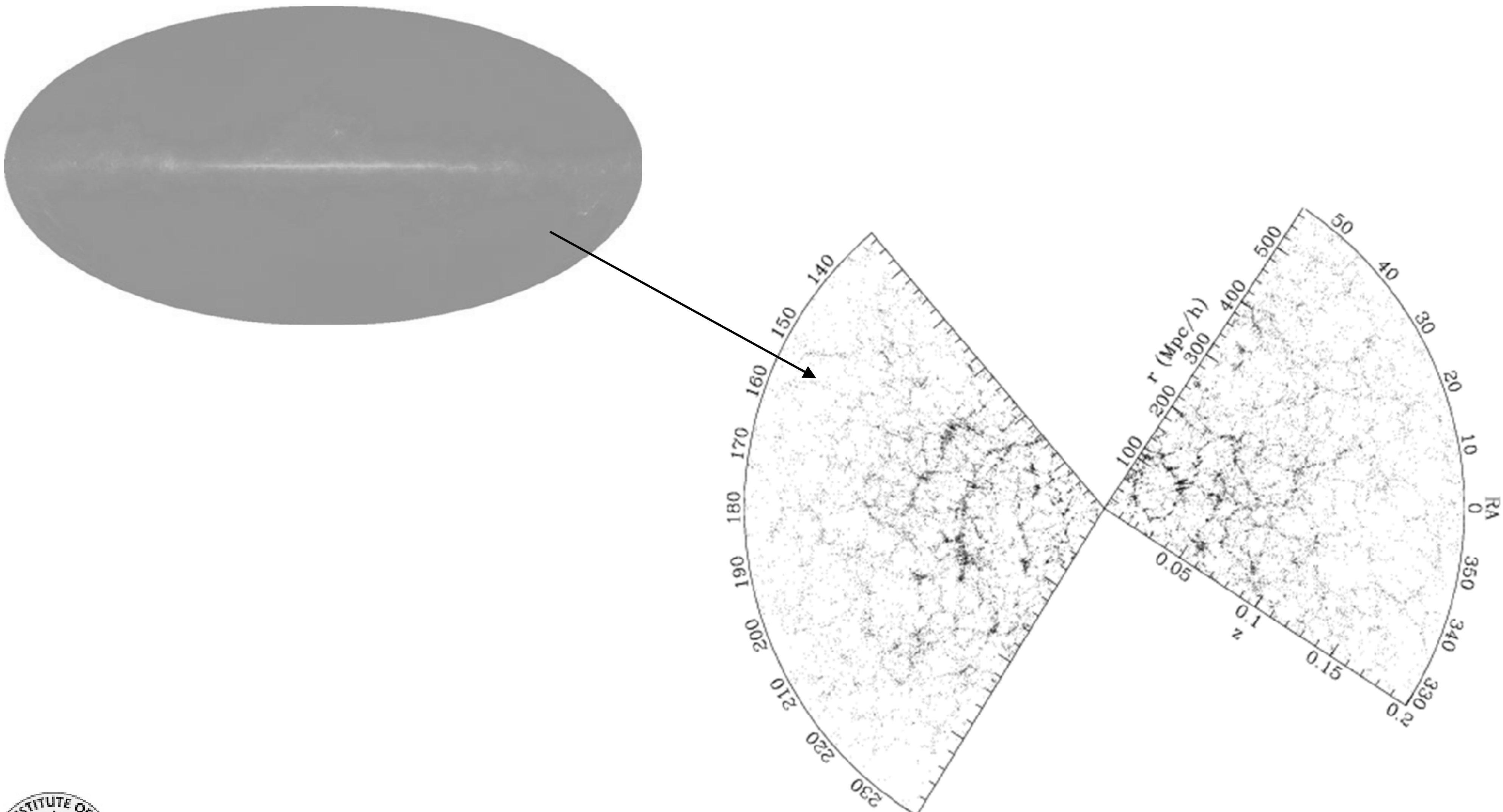




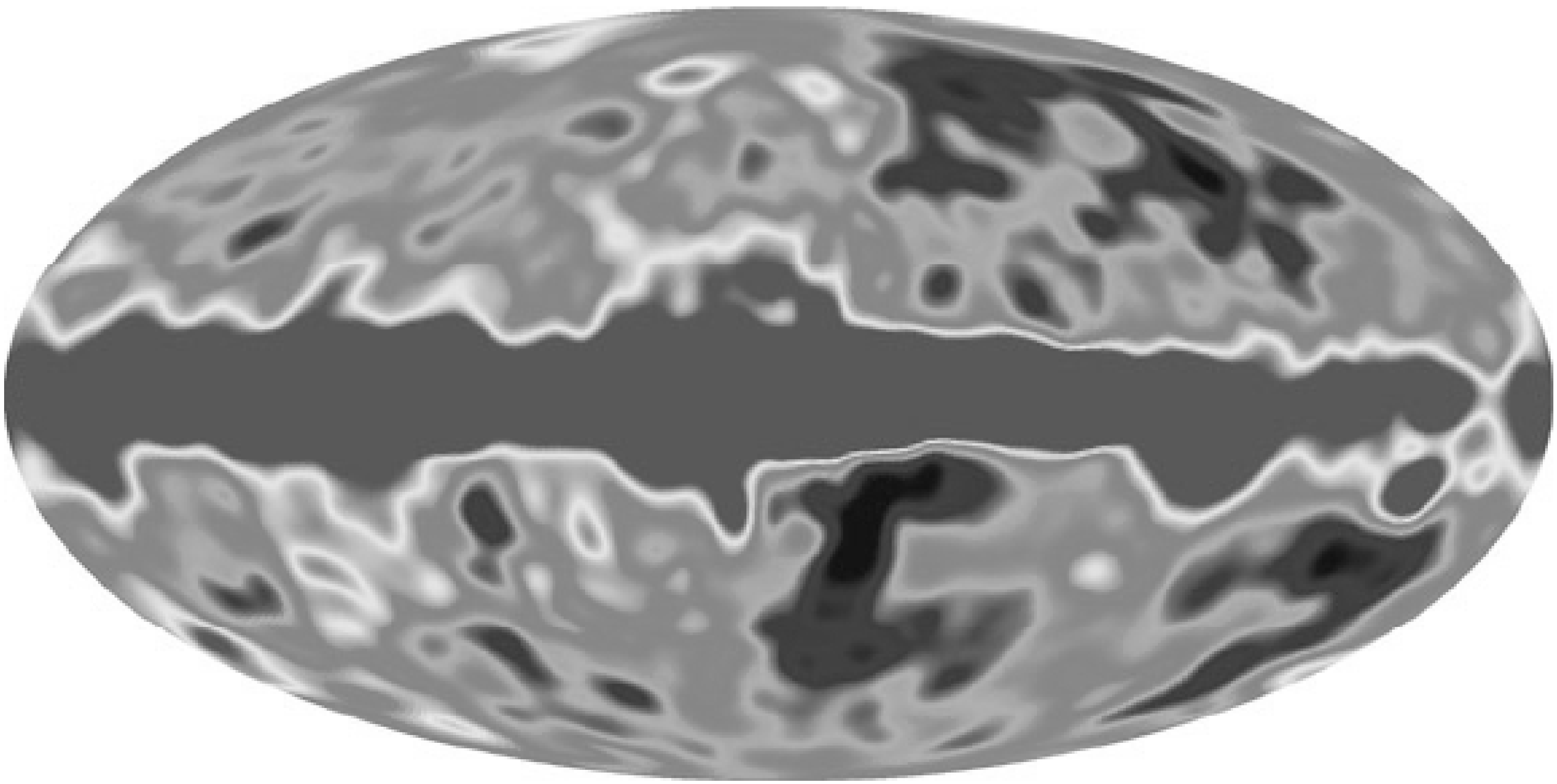


- CMB is a perfect blackbody
- Why is the CMB temperature so uniform across the sky??

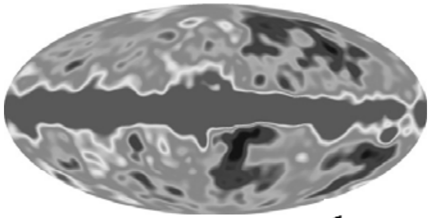
How Did Structure Form?



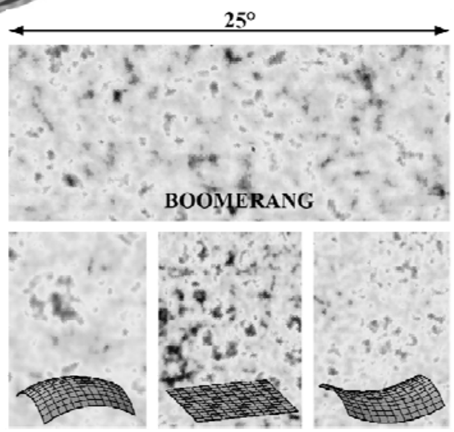
Seeds of Large-Scale Structure Detected!



Recent Results: From COBE to WMAP

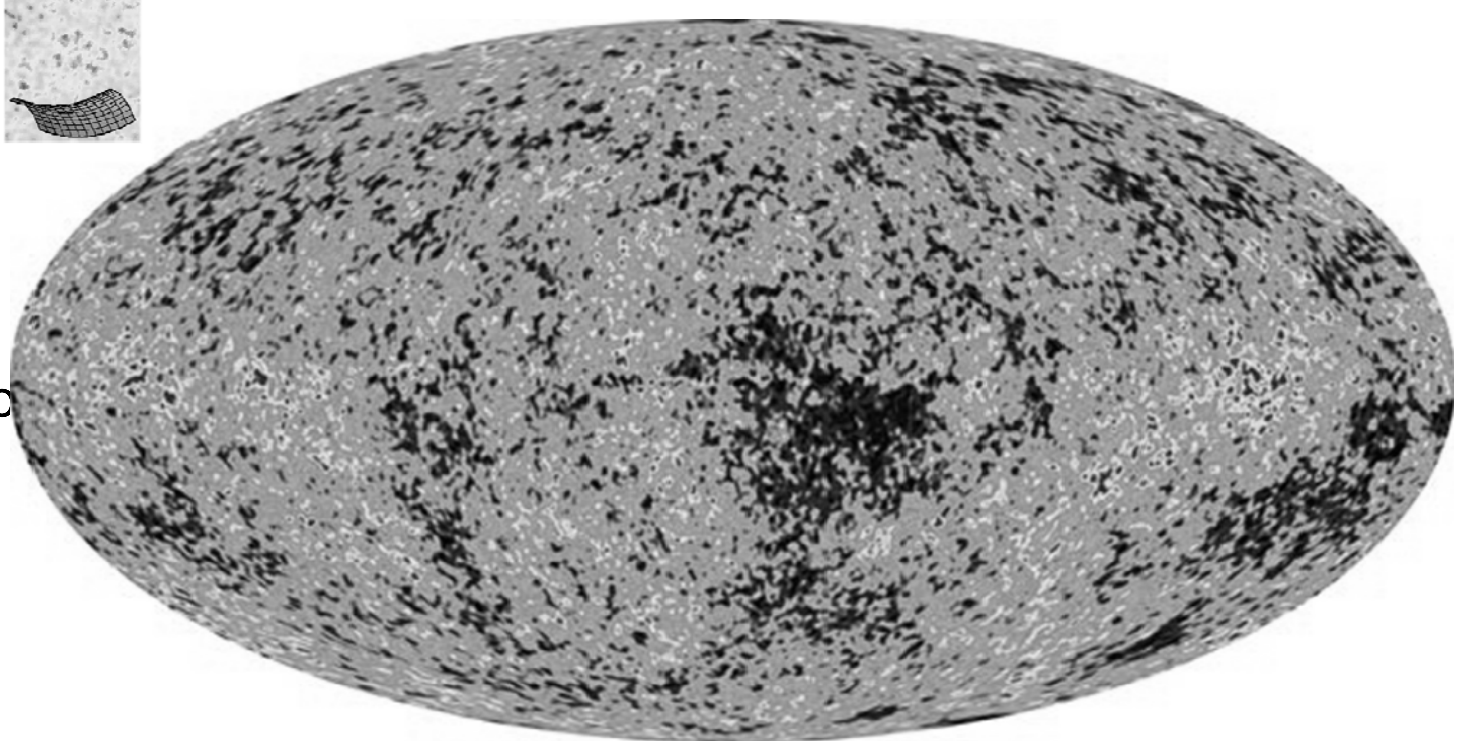


COBE, 7 degree resolution

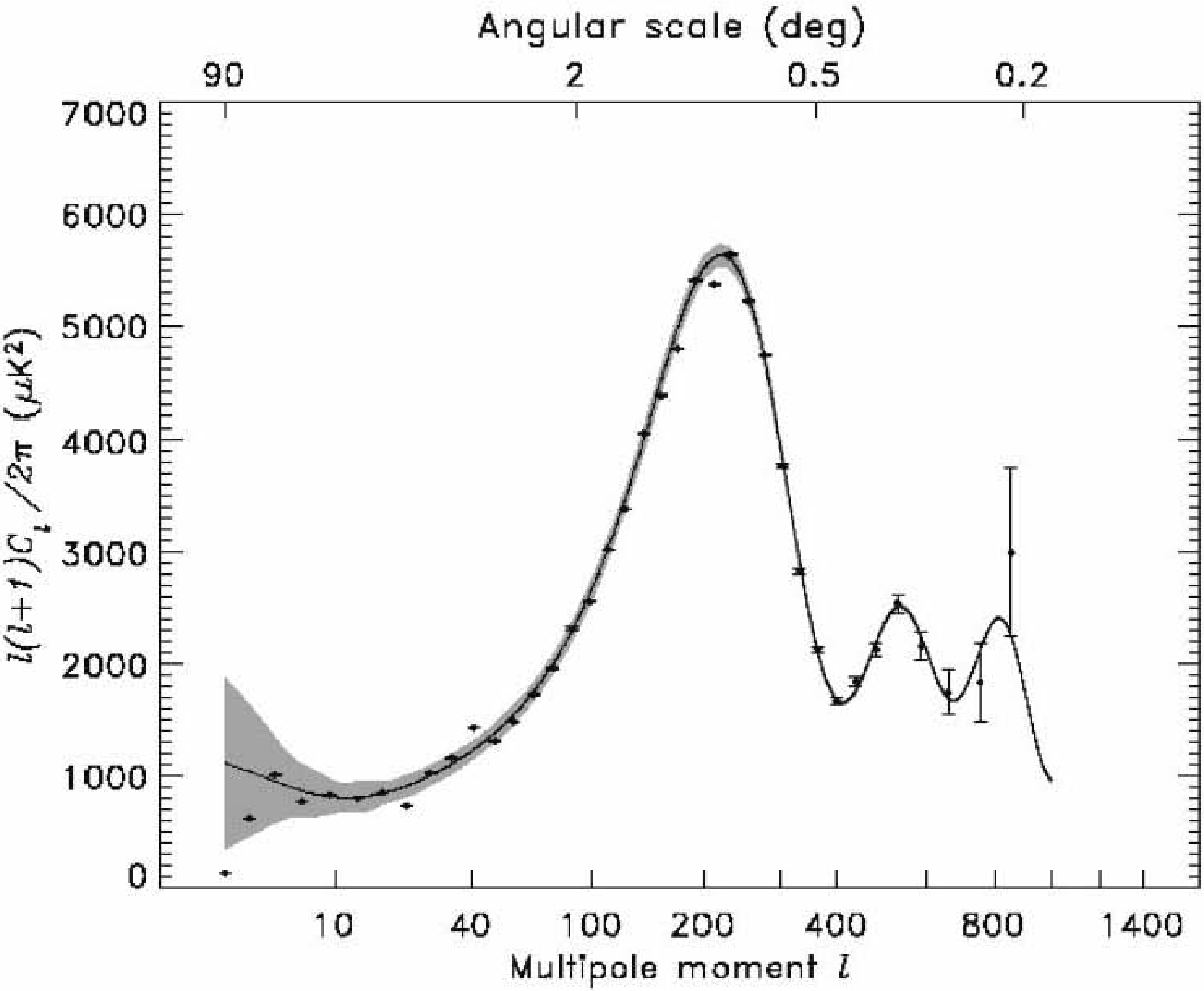


Boomerang, 1 degree resolution

WMAP, 1/2 degree resolutio

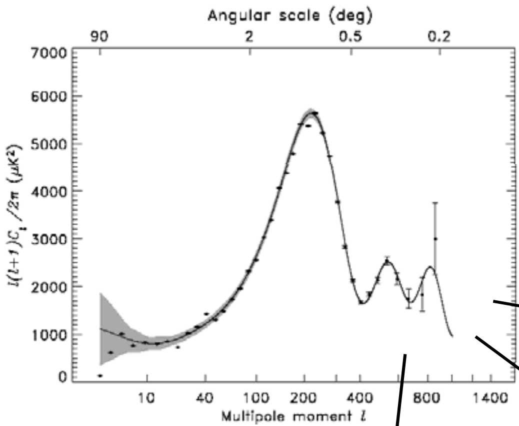


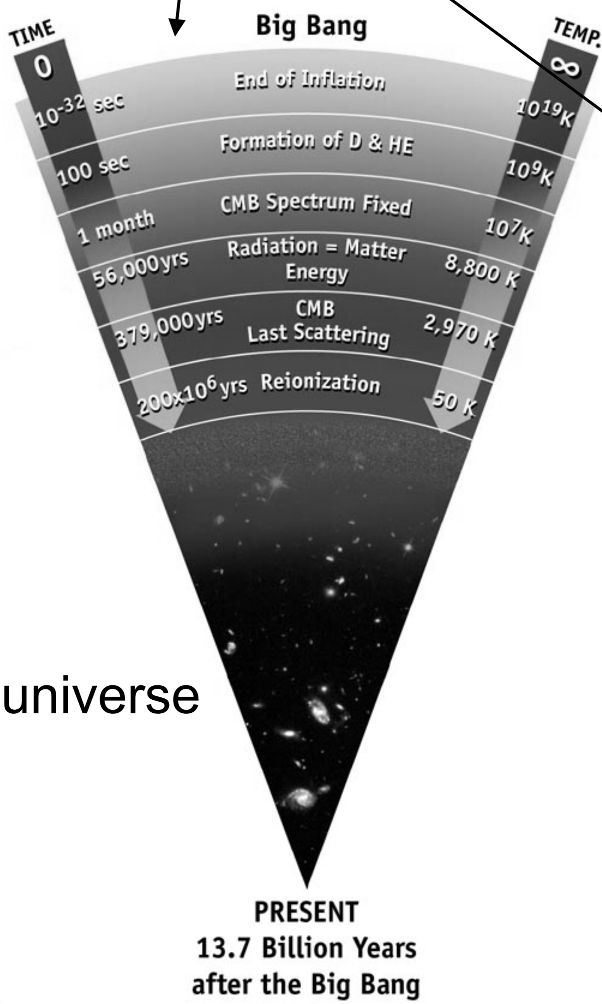
http://map.gsfc.nasa.gov



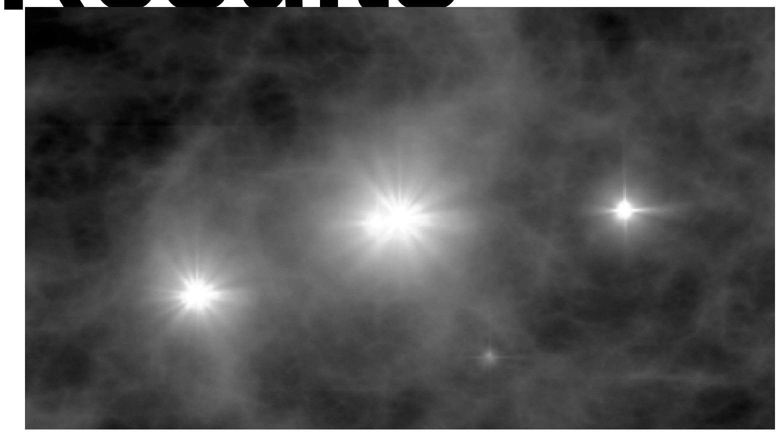
Angular scale (deg)
90
2
0.5
0.2
7000
6000
5000
4000
3000
2000
1000
0
10
40
100
200
400
800
1400
Multipole moment l
$l(l+1)C_l/2\pi$ (μK^2)

WMAP 1 Yr Results

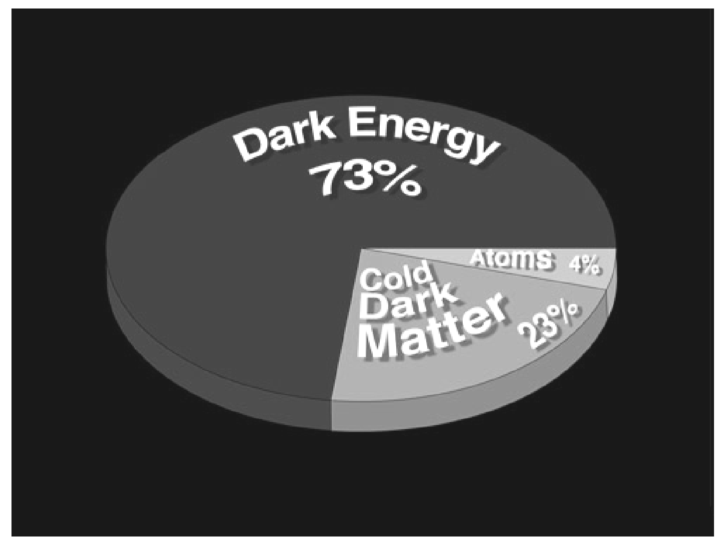




Age of universe



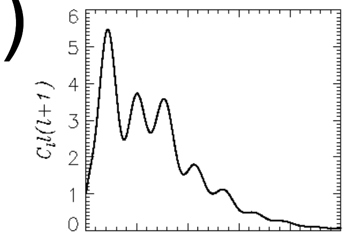
First stars turned on earlier than expecte



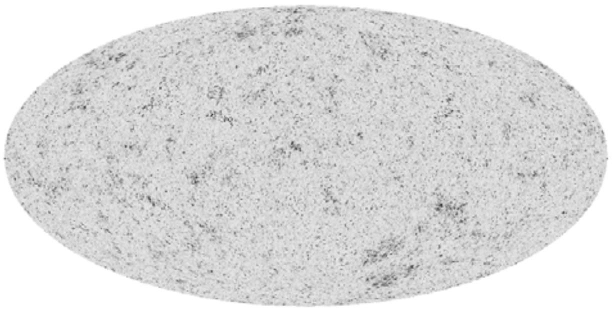
Composition of the universe

What are the data?

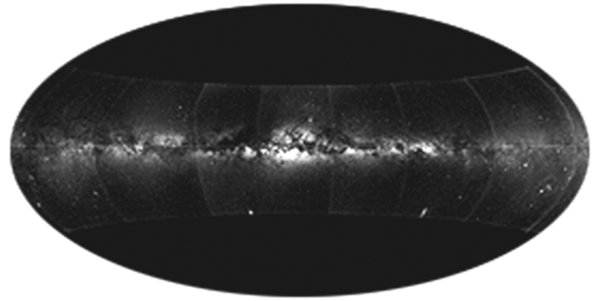
$\Omega \longrightarrow C_l(\theta)$

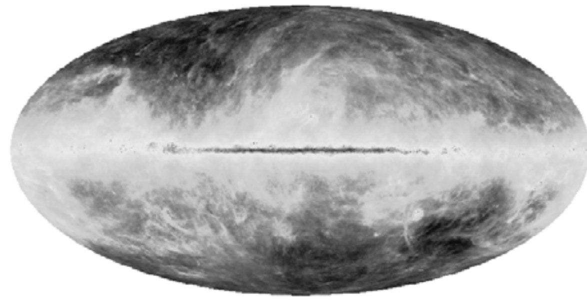


Signal



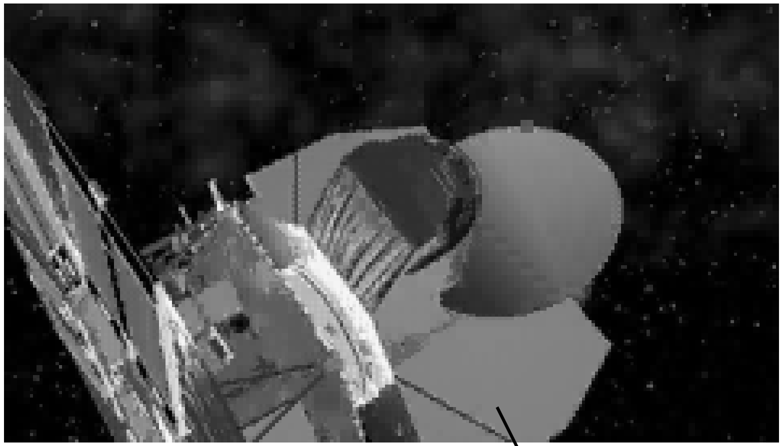
Foregrounds...



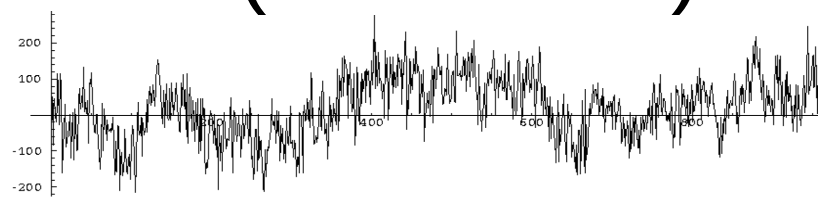


Sky

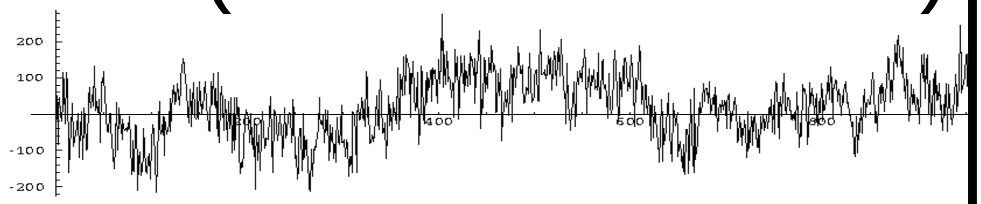
Frequency bands



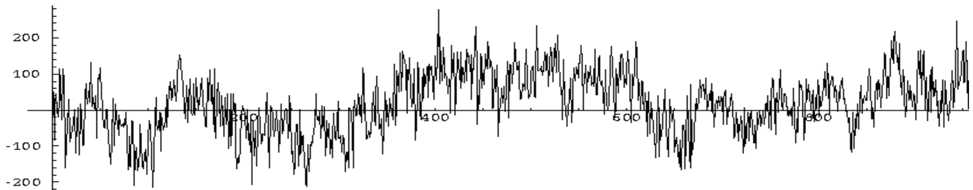
Noise (correlated?)

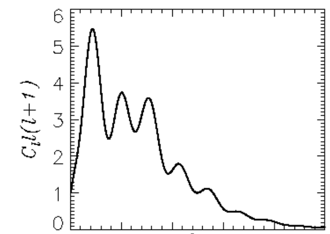


TOD (Time-ordered Data)

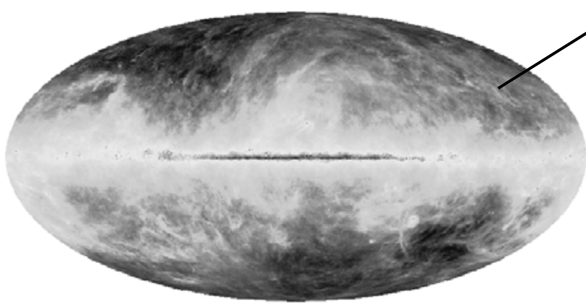


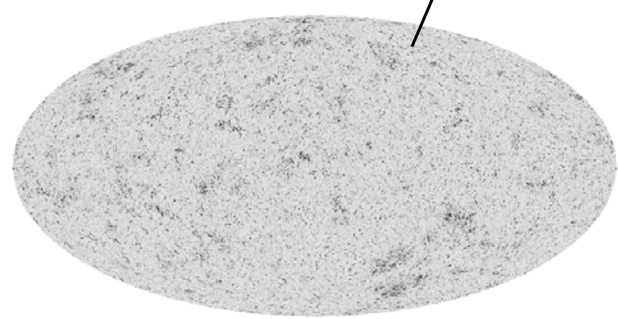
Simulation and Inference





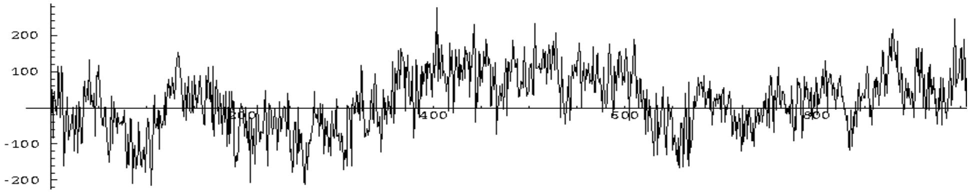
$$p(\Omega,\theta,s,f,d) = p(d \mid f,s)\,p(f \mid \theta)\,p(s \mid \Omega)\,p(\theta,\Omega)$$

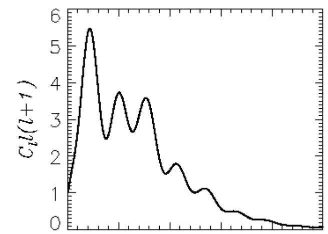




$$p(\Omega,\theta \mid d) = p(\theta,\Omega) \int d(s,f)\; p(d \mid f,s)\,p(f \mid \theta)\,p(s \mid \Omega)$$

Implementation of Bayesian Approach





$$p(\Omega,\theta \mid d) = p(\theta,\Omega)\int d(s,f)\; p(d \mid f,s)\,p(f \mid \theta)\,p(s \mid \Omega)$$

Direct evaluation: Computational Expense: $O[N^3]$

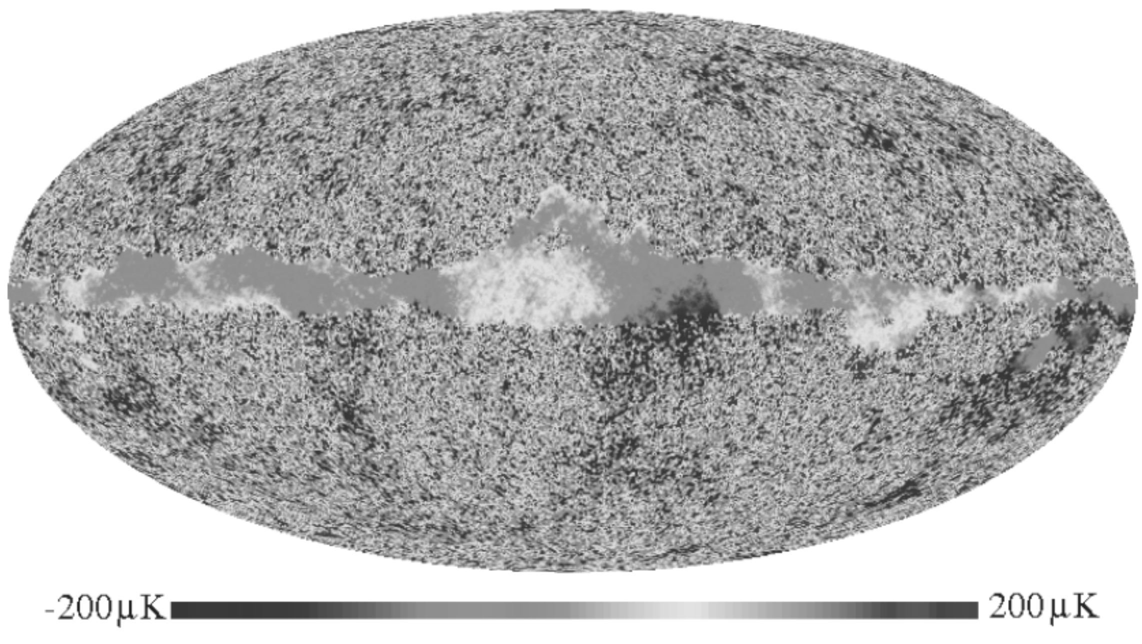
$$-\log\frac{p(\Omega \mid d)}{p(\Omega)} = \hat{s}(d)[C(\Omega)+N]^{-1}\hat{s}(d) + \log\left\|C(\Omega)+N\right\|$$

Gibbs Sampling: Computational Expense: $KO[N^{3/2}]$

$$\boxed{p(\Omega \mid d) = \int d\Omega' \left[\int ds\; p(\Omega \mid s,d)\,p(s \mid \Omega',d)\right] p(\Omega' \mid d)}$$

$$p(\Omega \mid d) = \int ds \; p(\Omega \mid s)\, p(s \mid \Omega', d)\, p(\Omega', s \mid d)$$

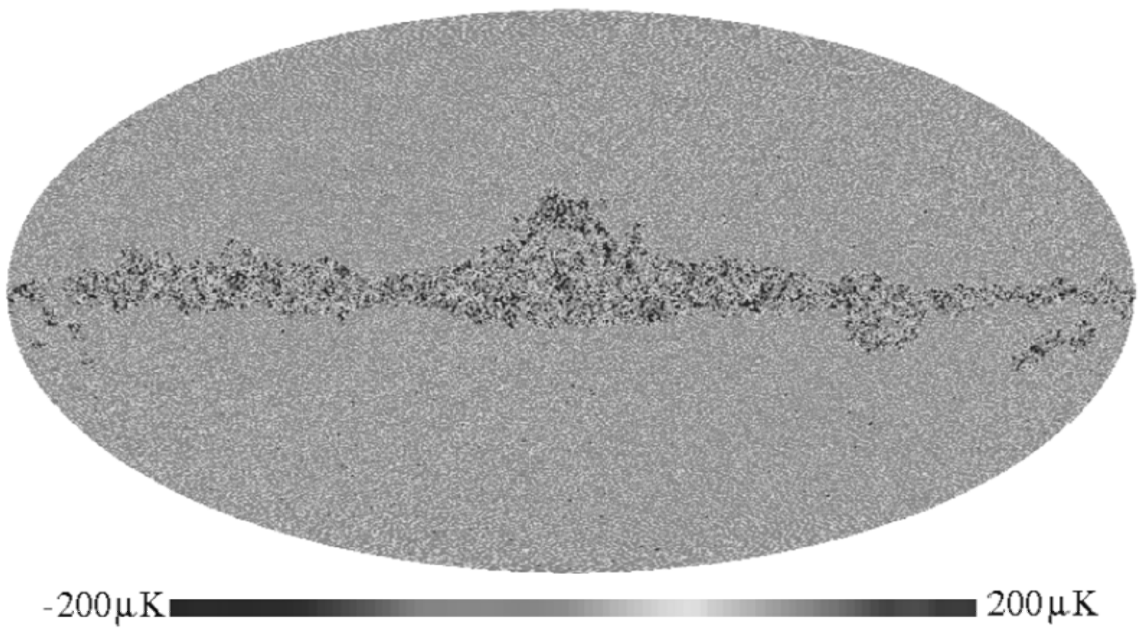
$$\boxed{[I + C^{+1/2} A^T N_{tt'}^{-1} A C^{+1/2}](C^{-1/2}\hat{s}) = C^{+1/2} A^T N_{tt'}^{-1} d}$$



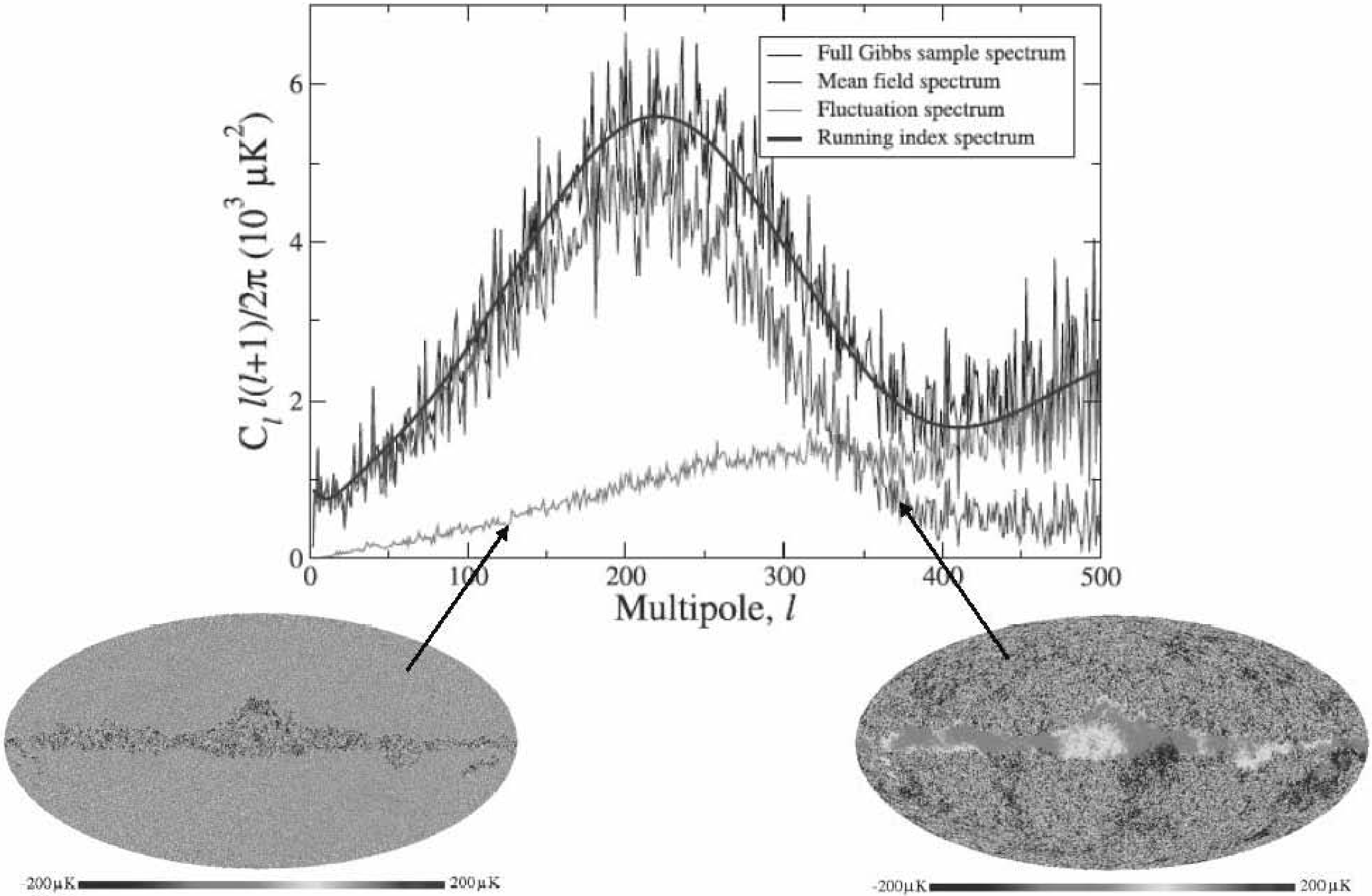
Mean Field map (given power spectrum estimate and data)

$$p(\Omega \mid d) = \int ds\ p(\Omega \mid s)\, p(s \mid \Omega', d)\, p(\Omega', s \mid d)$$

$$\boxed{[I + C^{+1/2} A^T N_t^{-1} A C^{+1/2}]\,(C^{-1/2}\xi) = \omega + C^{+1/2} A^T N_t^{-1/2} \tau}$$



Fluctuation map (given power spectrum estimate)

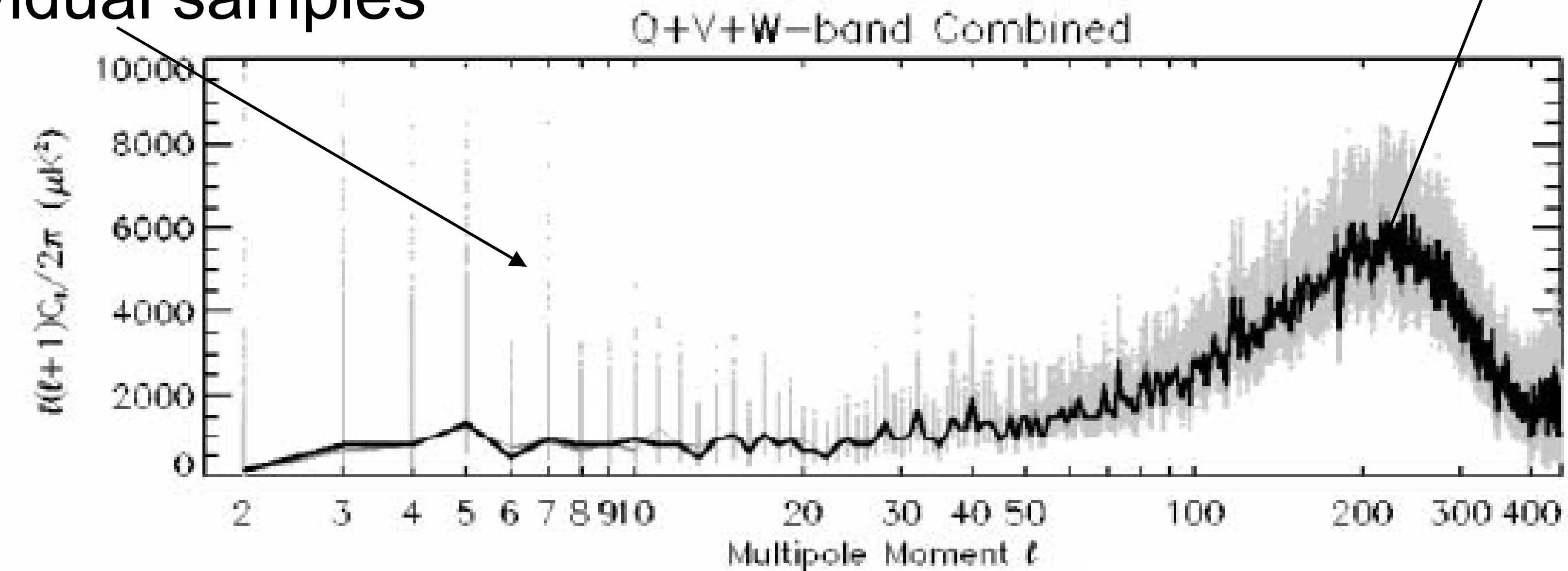


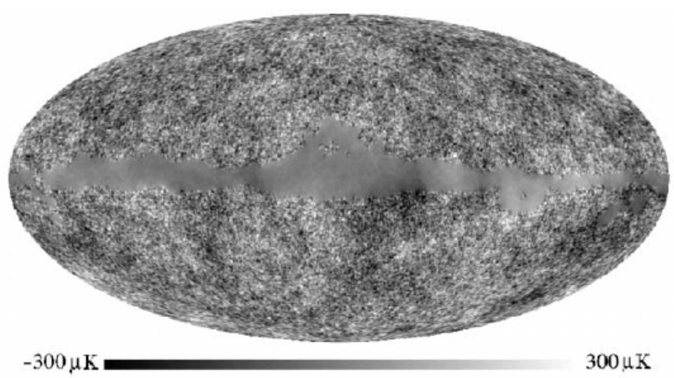
Full Gibbs sample spectrum
Mean field spectrum
Fluctuation spectrum
Running index spectrum
$C_l\, l(l+1)/2\pi\ (10^3\ \mu K^2)$
Multipole, l
0
2
4
6
0
100
200
300
400
500
-200μK
200μK
-200μK
200μK

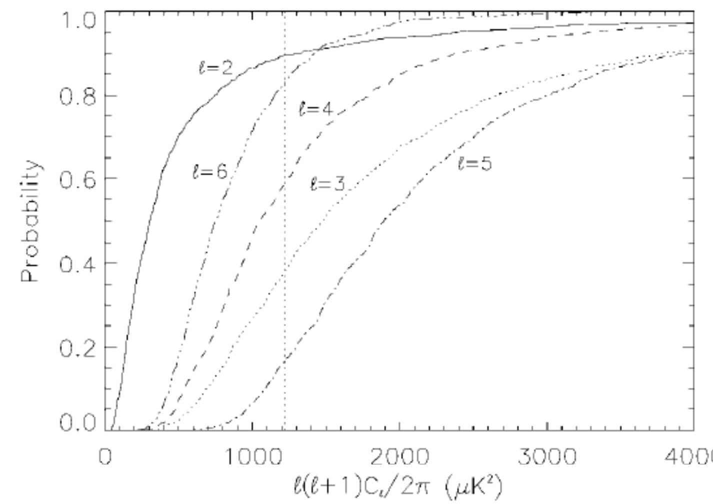
Gibbs Sampling Applied to WMAP I

Individual samples

Posterior mode







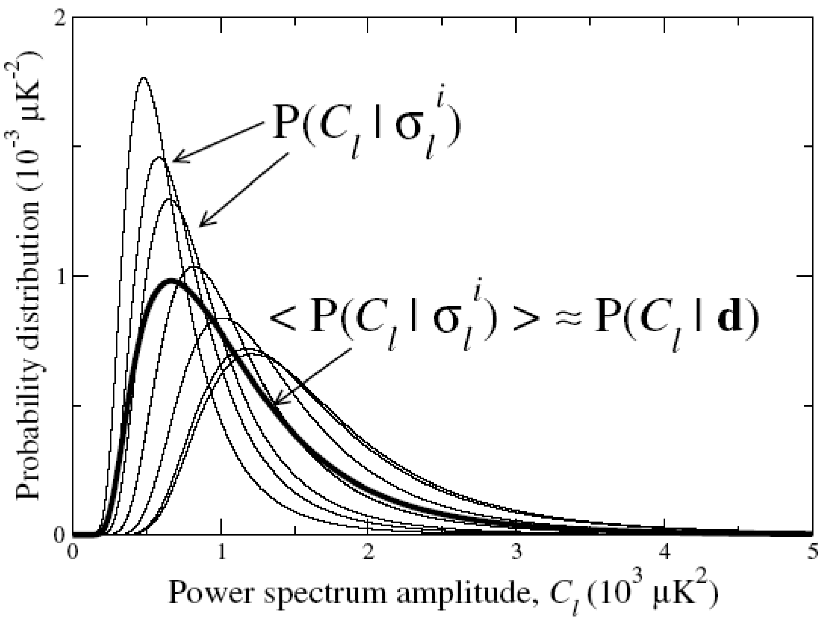
I.J. O'Dwyer et al, ApJ,617,L99,Dec. 2004

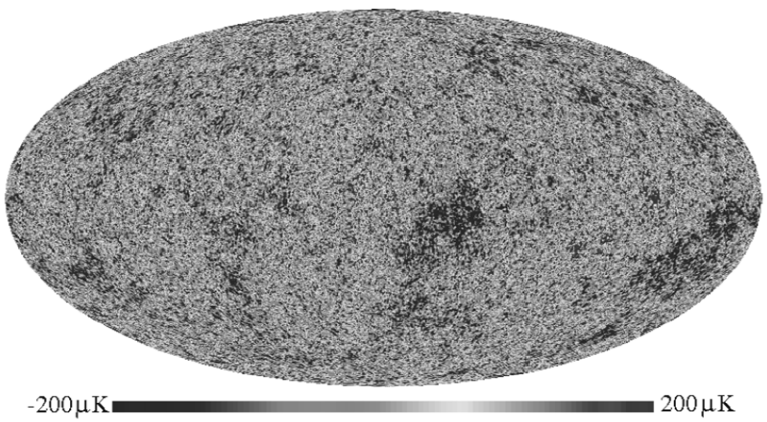
Blackwell-Rao Estimator For Parameter Posterior...

$$p(\Omega \mid d) \approx \int ds\, p(\Omega \mid s)\left[\int dC_l\; p(s \mid d, C_l)\, p(C_l \mid d)\right]$$

$$p(\Omega \mid s) \propto p(\Omega) \prod_{lm} \frac{e^{-\sigma_l / 2C_l(\Omega)}}{\sqrt{2\pi} C_l^{1/2}(\Omega)}$$

Sampled Maps from Gibbs chain, marginalized Over power spectra…





M. Chu et al, Phys.Rev. D, 71,2005

WMAP 3 Year Temperature Re-Analysis Results

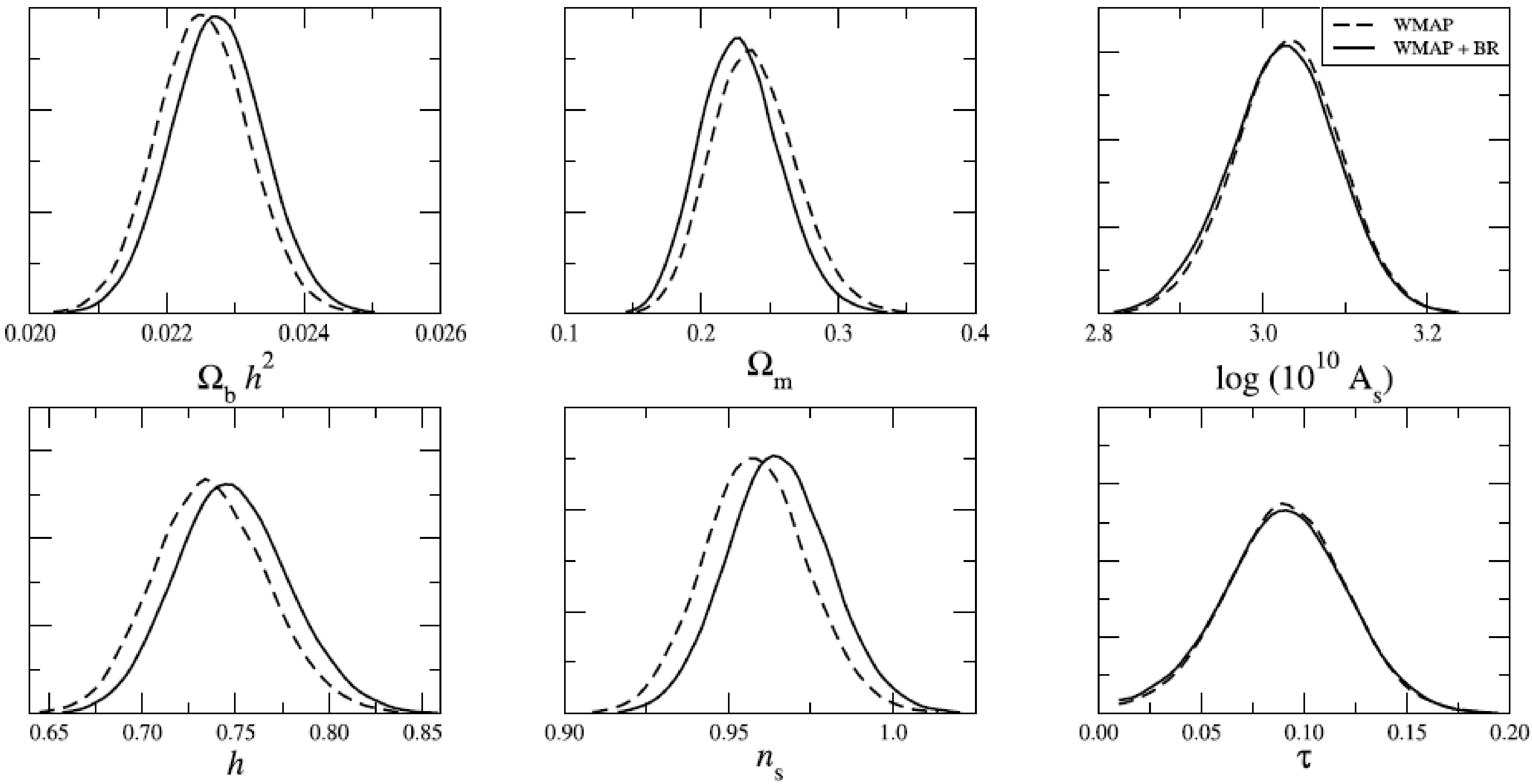
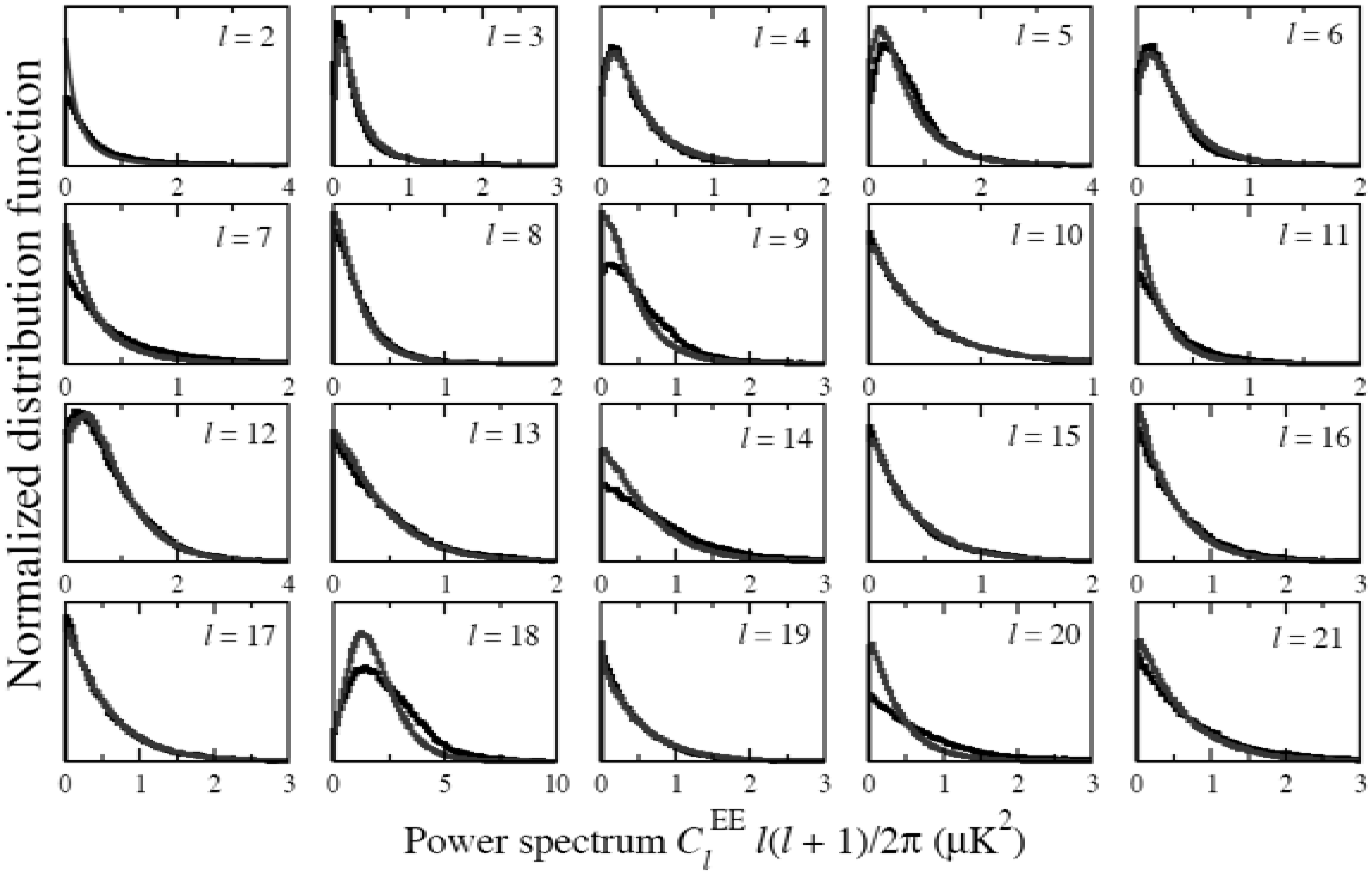
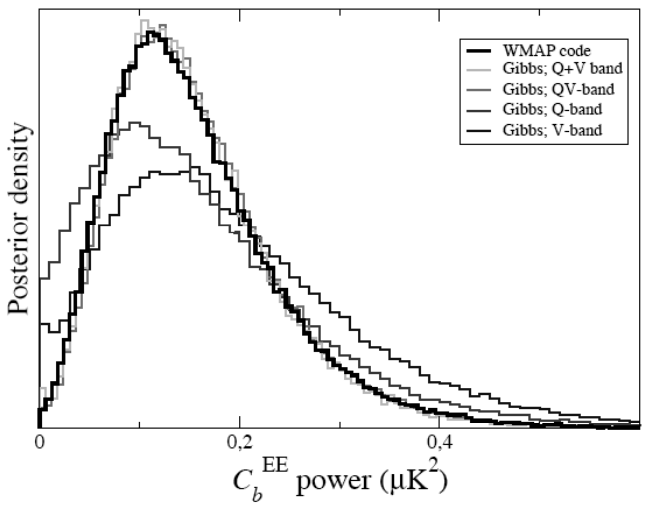


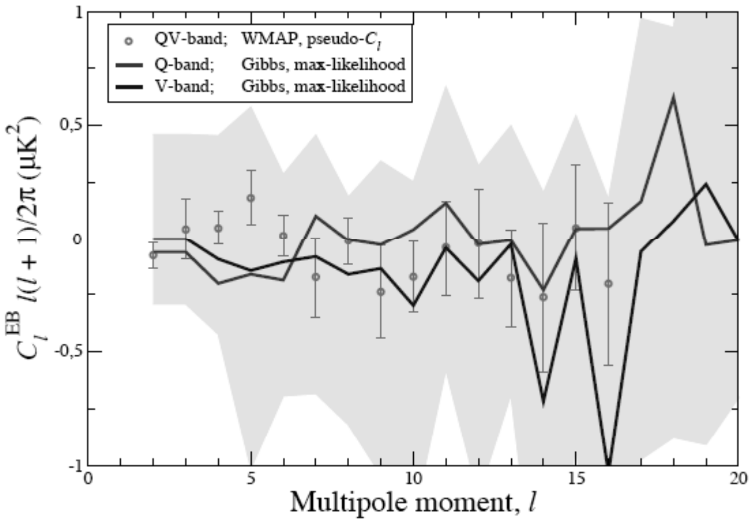
Fig. 7 of Eriksen et al, *A Reanalysis of the Three-Year Wilkinson Anisotropy Probe Temperature Power Spectrum and Likelihood*

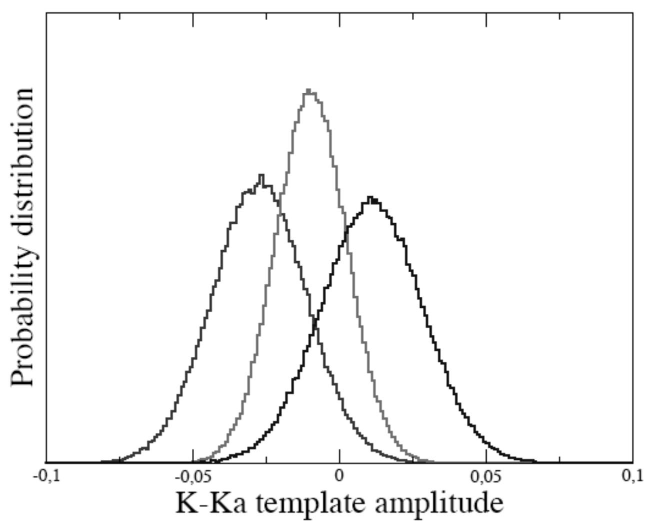
WMAP 3 Year Polarization

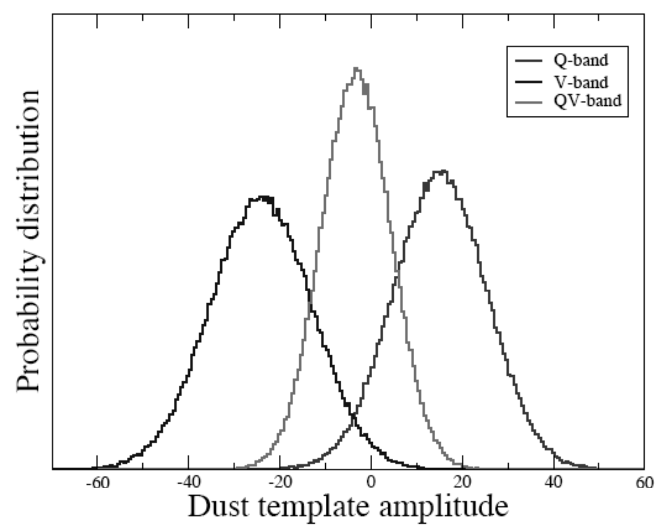


WMAP - Frequency Channel Dependence…







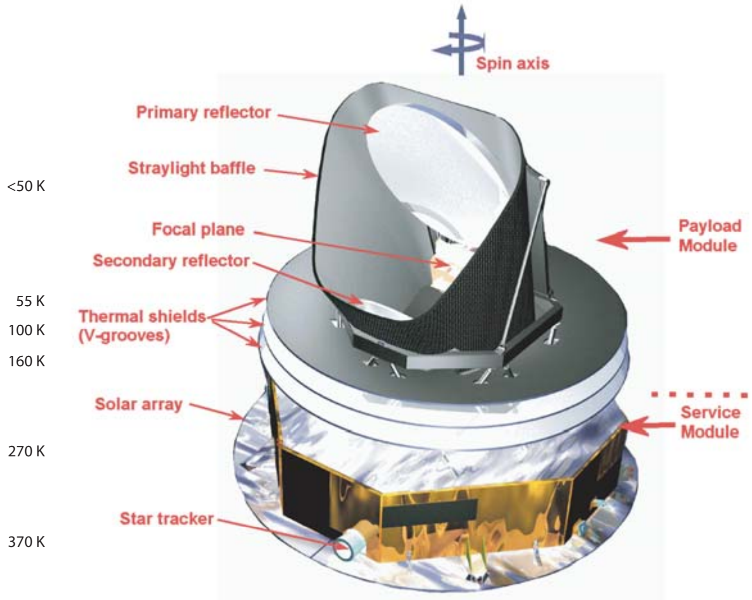


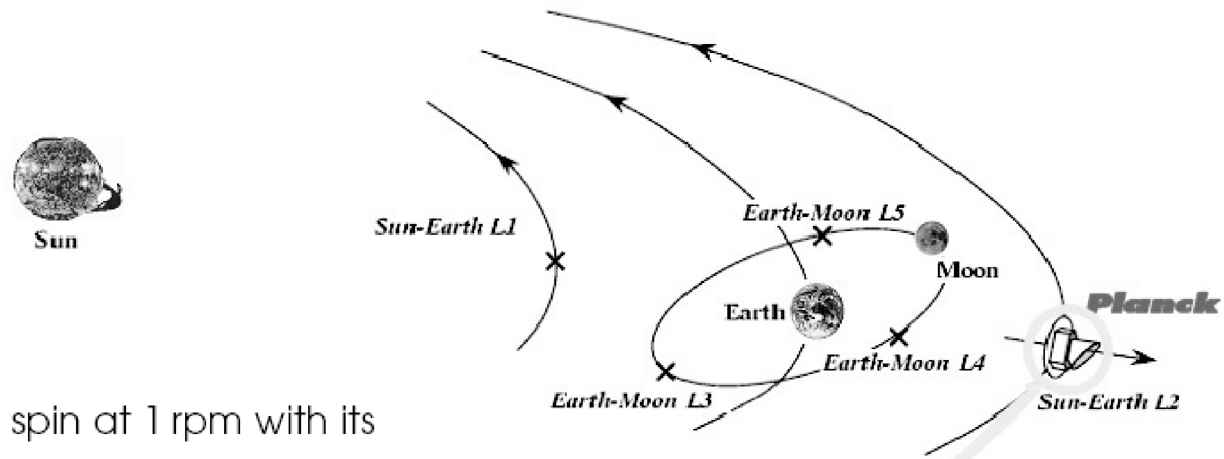
WMAP 3 Year Polarization Band Powers

TABLE 1
Marginalized EE and BB band powers

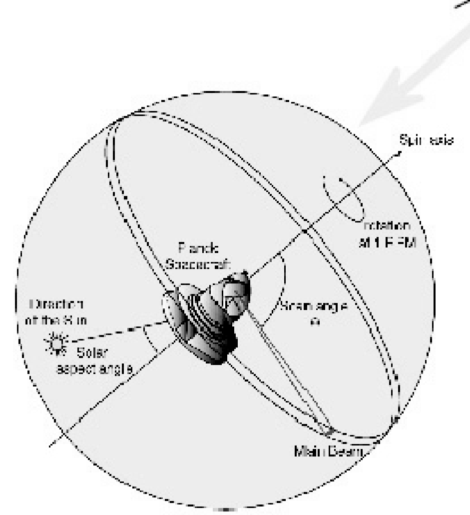
Data set	EE power ($10^{-1}\ \mu K^2$) $\ell = 2$–6	EE power ($10^{-1}\ \mu K^2$) $\ell = 2$–20	BB power ($10^{-1}\ \mu K^2$) $\ell = 2$–6	BB power ($10^{-1}\ \mu K^2$) $\ell = 2$–20
		EE free; EB = BB = 0		
WMAP	$1.1^{+0.9}_{-0.5}$	$0.64^{+0.46}_{-0.34}$	...	...
QV-band	$1.2^{+0.9}_{-0.6}$	$0.67^{+0.39}_{-0.38}$	...	...
Q+V-band	$1.1^{+0.8}_{-0.6}$	$0.65^{+0.38}_{-0.35}$	...	...
Q-band	$1.0^{+1.0}_{-0.8}$	$0.36^{+0.67}_{-0.36}$	...	...
V-band	$1.3^{+1.2}_{-0.9}$	$1.2^{+0.9}_{-0.7}$	...	...
		EE, BB free; EB = 0		
WMAP	$0.94^{+0.76}_{-0.58}$	$0.63^{+0.44}_{-0.37}$	< 0.70	< 0.40
QV-band	$1.1^{+0.8}_{-0.6}$	$0.61^{+0.38}_{-0.37}$	< 0.57	< 0.26
Q+V-band	$1.1^{+0.8}_{-0.6}$	$0.57^{+0.40}_{-0.31}$	< 0.58	< 0.30
Q-band	$0.3^{+1.3}_{-0.3}$	$0.23^{+0.68}_{-0.23}$	$0.3^{+1.2}_{-0.3}$	< 0.71
V-band	$1.4^{+1.4}_{-0.9}$	$1.1^{+1.0}_{-0.7}$	< 0.94	< 0.51
		EE, EB, BB free		
QV-band	$1.4^{+0.9}_{-0.7}$	$0.65^{+0.41}_{-0.30}$	$0.30^{+0.60}_{-0.30}$	$0.1^{+0.3}_{-0.1}$
Q+V-band	$1.3^{+0.9}_{-0.7}$	$0.66^{+0.43}_{-0.30}$	$0.31^{+0.59}_{-0.31}$	$0.1^{+0.3}_{-0.1}$
Q-band	$1.1^{+1.1}_{-0.8}$	$0.54^{+0.65}_{-0.36}$	$0.7^{+1.3}_{-0.6}$	$0.5^{+0.7}_{-0.4}$
V-band	$1.8^{+1.6}_{-1.1}$	$1.5^{+0.9}_{-0.9}$	$0.47^{+0.93}_{-0.47}$	$0.4^{+0.6}_{-0.4}$

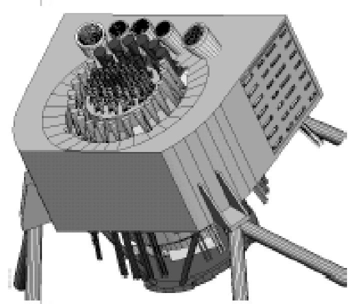
The Future of CMB Studies - Planck



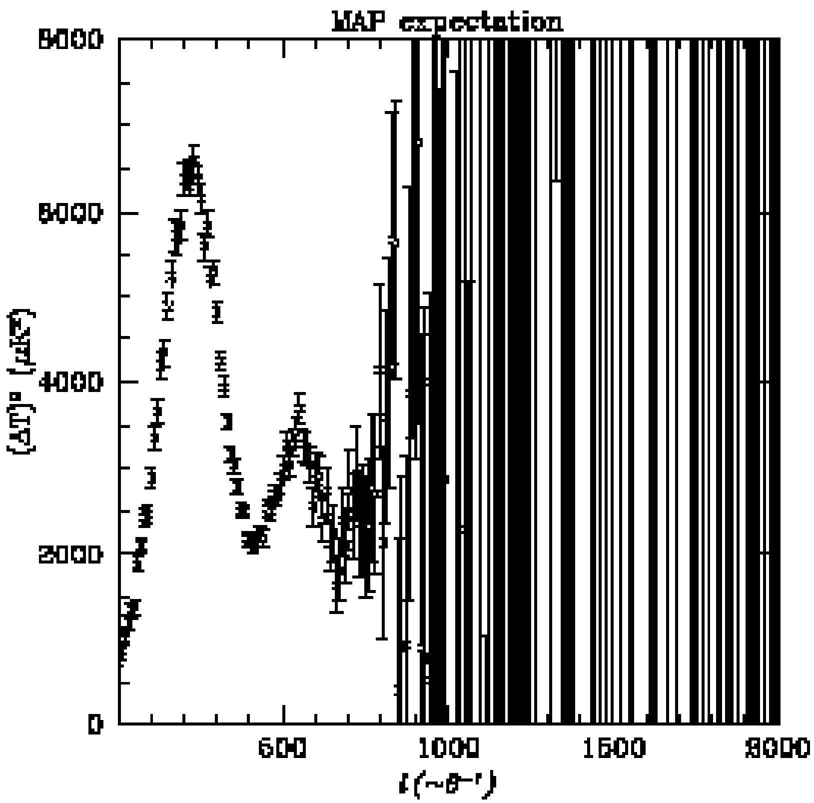


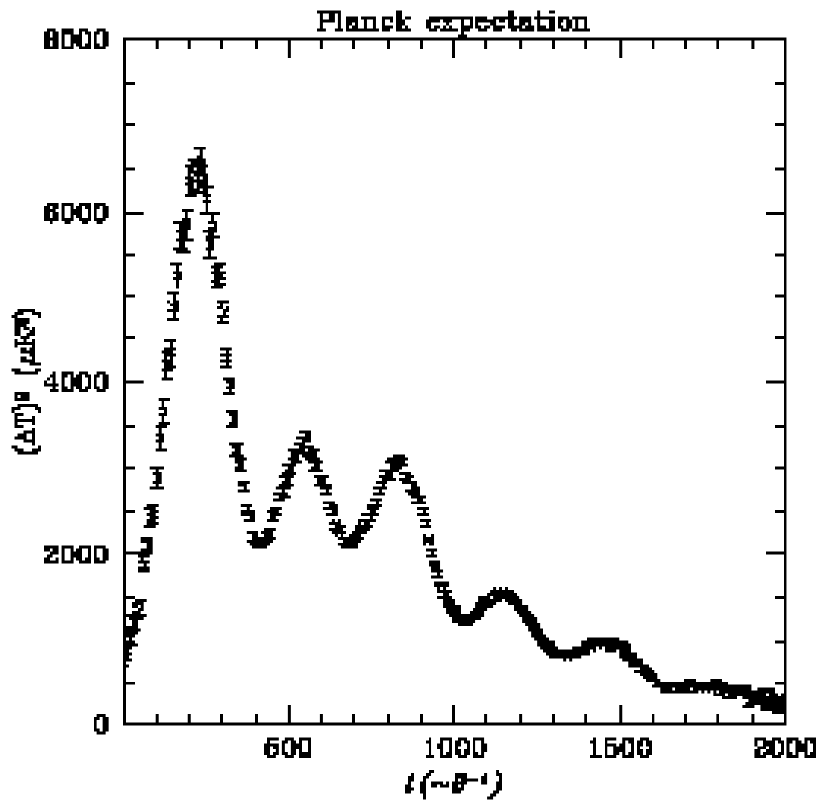
- Planck will spin at 1 rpm with its spin axis aligned with the Sun
  - Instruments scan nearly great circles on the sky
  - Entire sky observed every six months



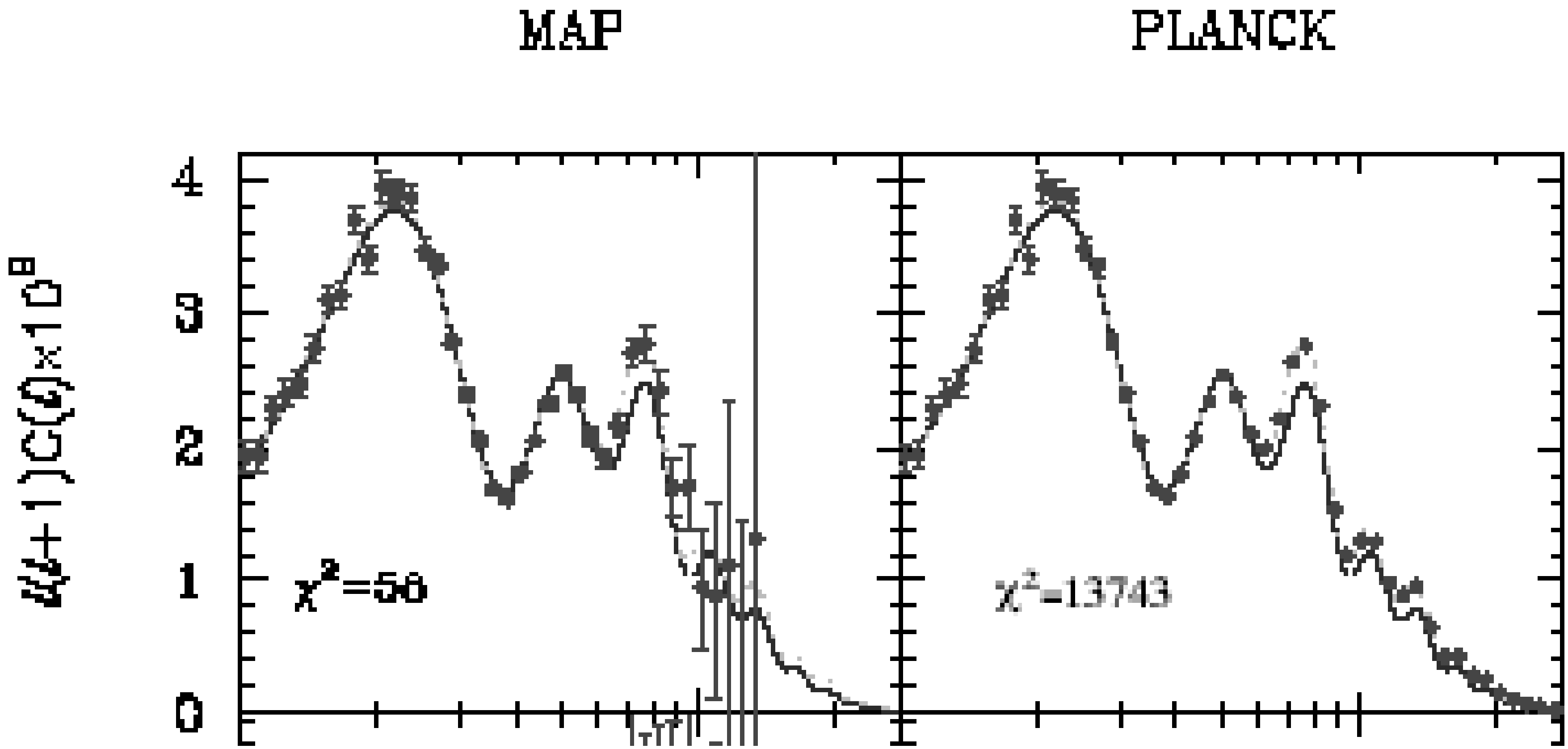


MAP vs. Planck - Expected 4 Year Sensitivity

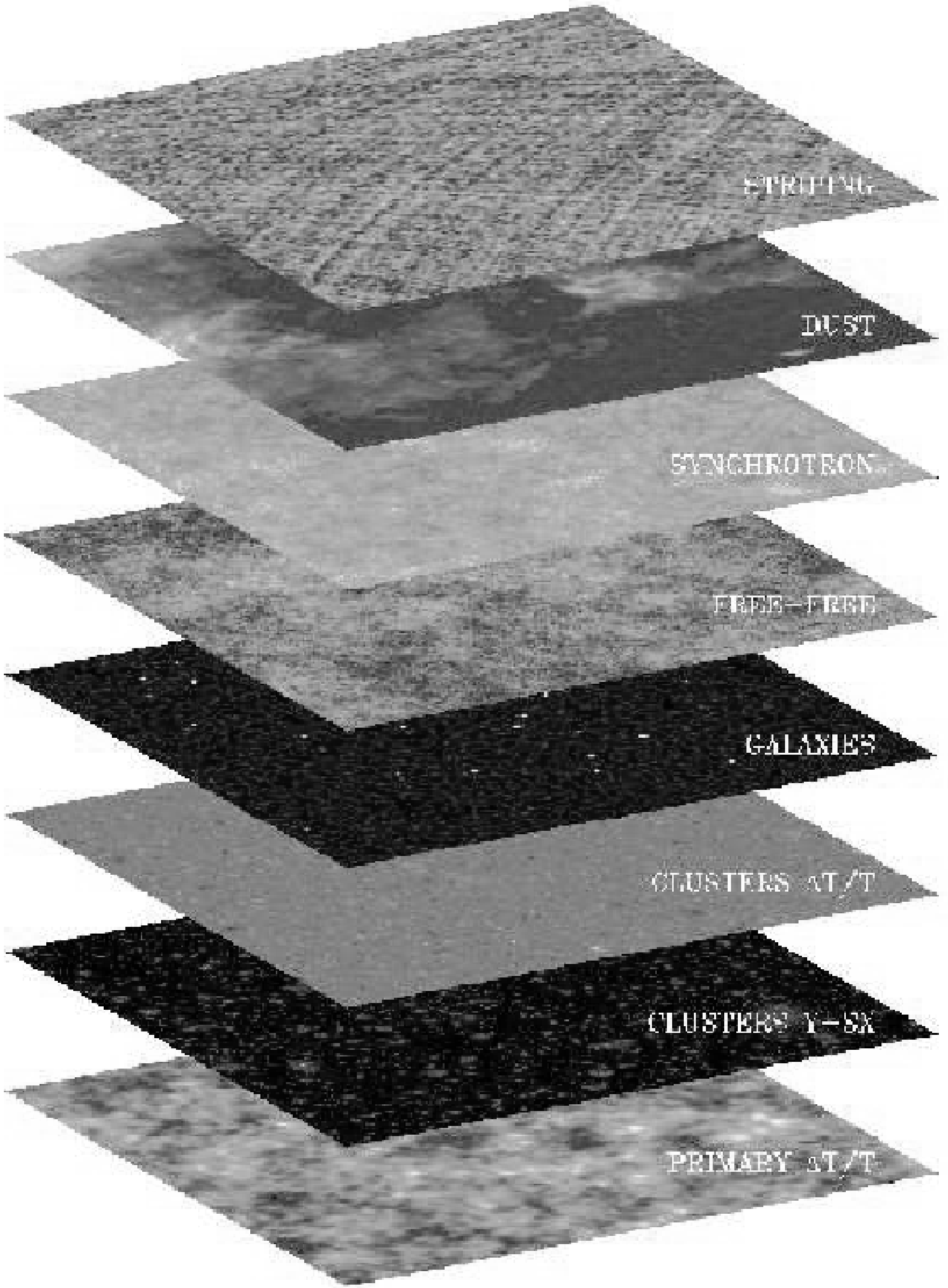




Planck: Resolving the Composition of the Universe!



- Solid line shows a spatially flat model
- Dotted line shows a model with 24% variation in baryonic density, and 5% variation in cold dark matter density…



Striping

Dust

Synchrotron

Free-Free

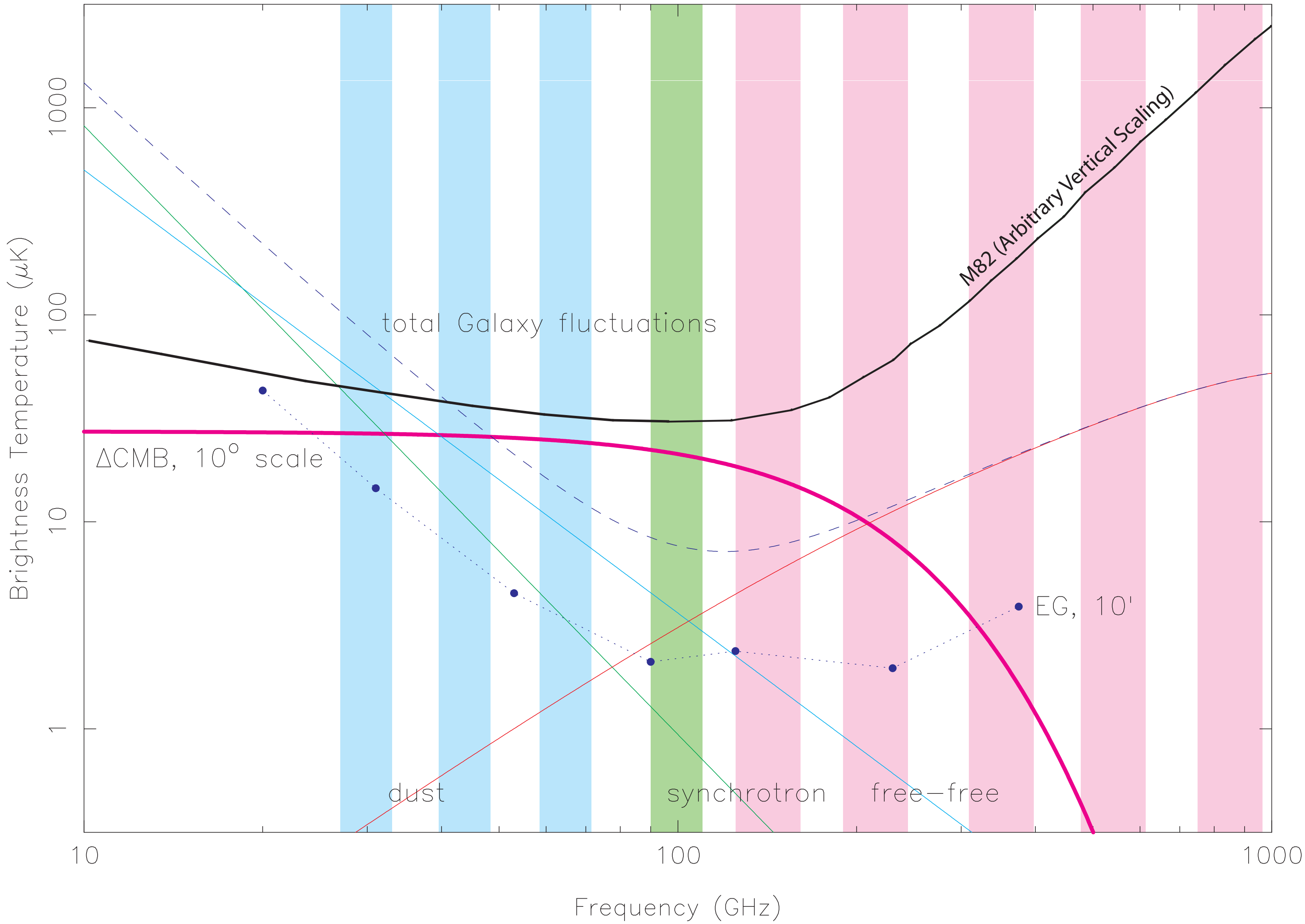
Galaxies

Clusters SZ Thermal

Clusters SZ Kinetic

CMB

Foreground Frequency Response



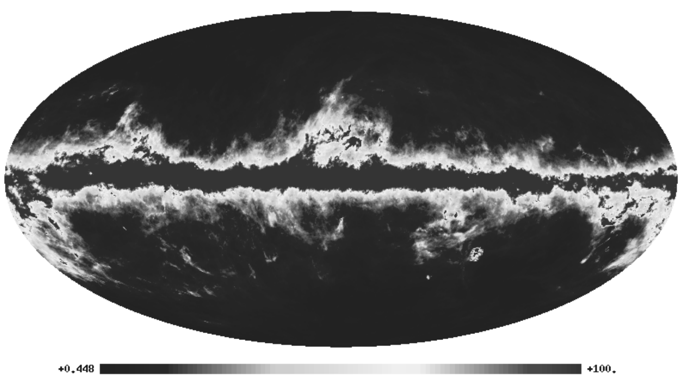
Including Foregrounds

$$d(\mathbf{p},\nu) = B(\mathbf{p},\nu)\circ s + \underbrace{\left(\sum_j a_j(\nu)\, t_j(\mathbf{p})\right)}_{} + \underbrace{\left(\sum_k b_k \left(\frac{\nu}{\nu_{0,k}}\right)^{\beta_k} f_k(\mathbf{p})\right)}_{} + \underbrace{\left(c(\mathbf{p})\left(\frac{\nu}{\nu_{ref}}\right)^{\alpha(\mathbf{p})}\right)}_{} + n(\mathbf{p},\nu)$$

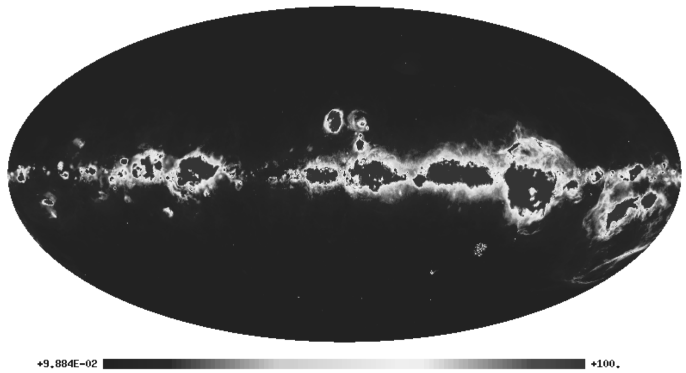
Monopole, Dipole, Dust:
Fit for amplitude at each frequency

Free-Free:
Fit for amplitude

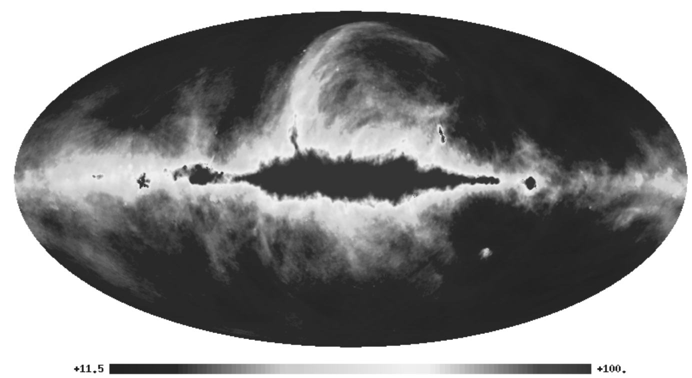
Synchrotron:
Fit pixel independent amplitudes, and spatially varying spectral index



FDS



H-alpha



Haslam

Joint Sampling CMB and Foreground Amplitudes

Previous Gibbs Implementation:

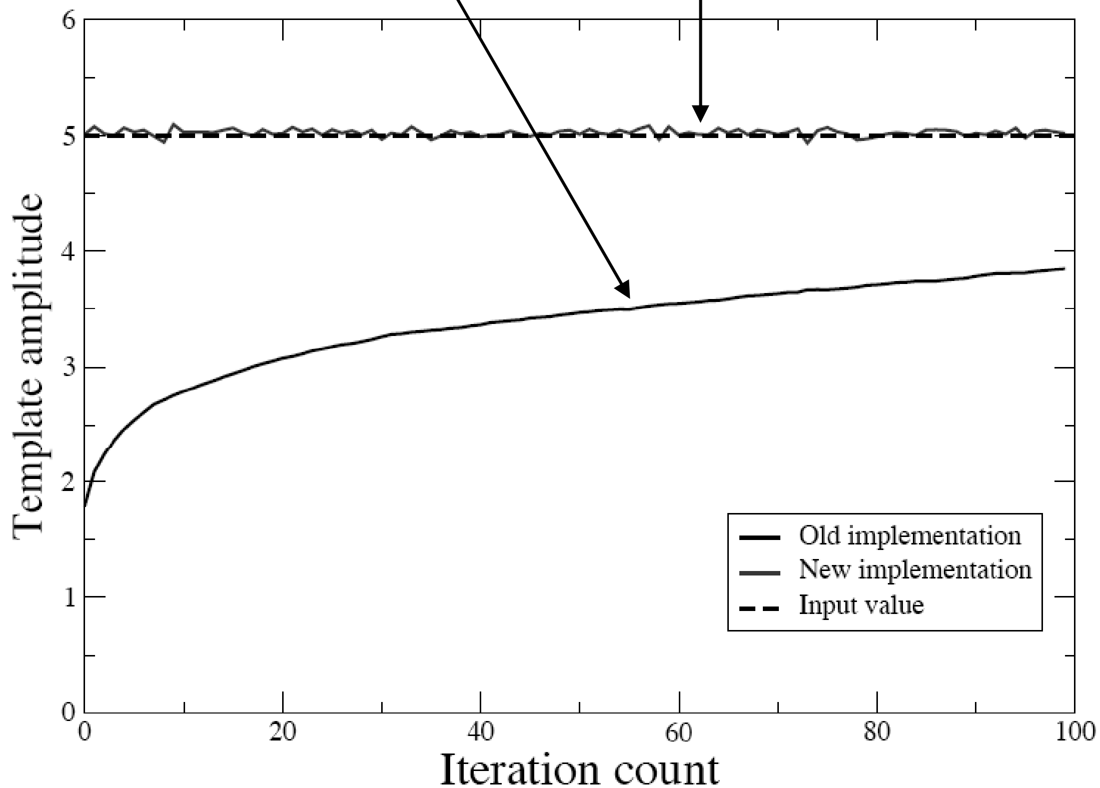
$$s \leftarrow p(s \mid a, C_l, \beta, d)$$

$$a \leftarrow p(a \mid s, C_l, \beta, d)$$

Improved Implementation:

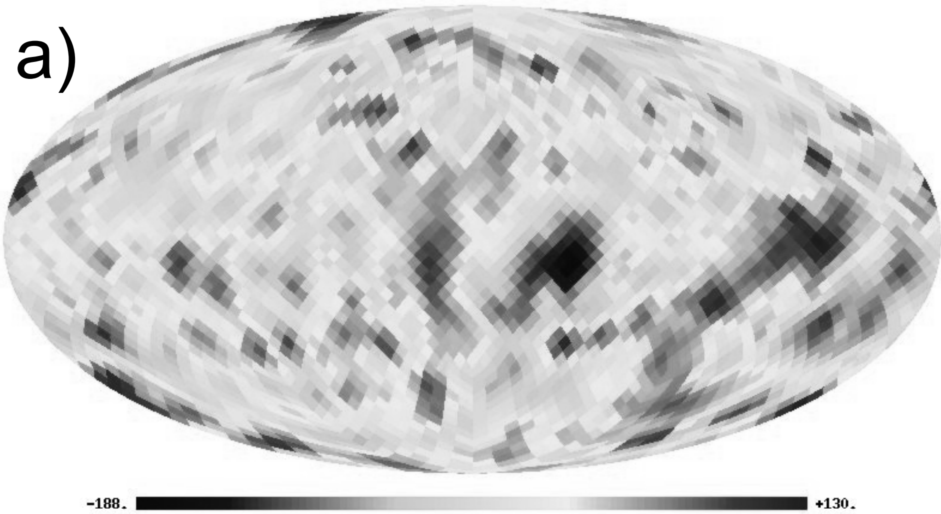
$$(s, a) \leftarrow p(s, a \mid C_l, \beta, d)$$

- Instant "burn-in"
- Virtually uncorrelated samples

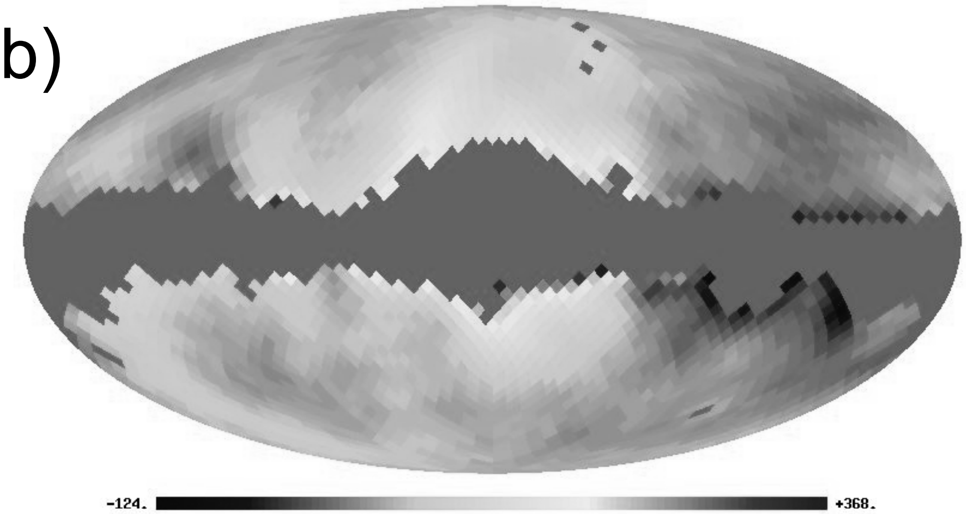


WMAP Synchrotron and CMB Map Samples

a)



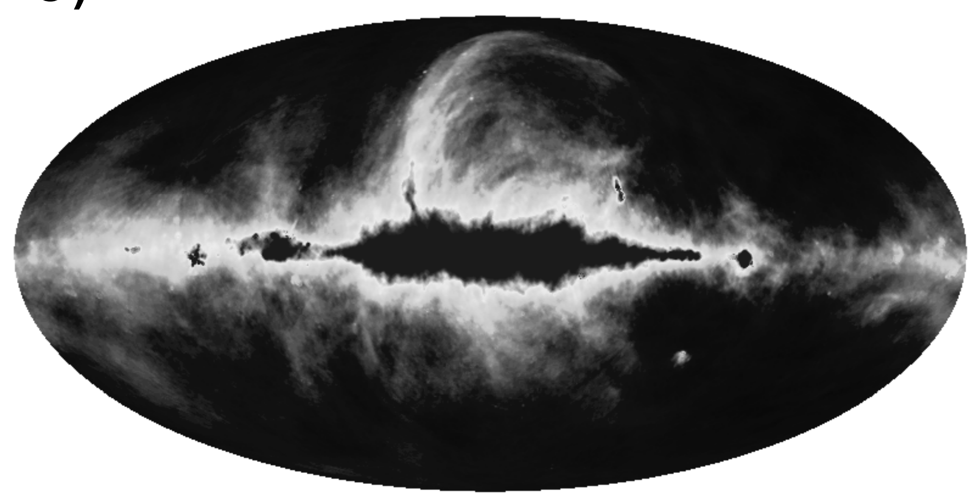
b)



Gibbs Sampled Maps:

- WMAP K,Ka,Q,V,W bands
- CMB+Synch. Map+Monopole
  + dipole+dust (FDS)+ff(H-alpha)
- Shown are:
  a) sampled CMB,
  b) sampled pixel-based synch.
  map (spectral index = -2.7),
  c) Haslam template for comparison

c)



Conclusions

- There is a wealth of cosmological information encoded in the spatial power spectrum of temperature anisotropies of the cosmic microwave background!
- Experiments designed to map the microwave sky are returning a flood of data (time streams of instrument response as a beam is swept over the sky) at several different frequencies (from 30 to 900 GHz), all with different resolutions and noise properties.
- The resulting analysis challenge is to estimate, and quantify our uncertainty in, the spatial power spectrum of the cosmic microwave background given the complexities of "missing data", foreground emission, and complicated instrumental noise.
- Bayesian formulation of this problem allows consistent treatment of many complexities including complicated instrumental noise and foregrounds, and can be numerically implemented with Gibbs sampling.
- Gibbs sampling has now been validated as an efficient, statistically exact, and practically useful method for low-resolution (as demonstrated on WMAP 1 and 3 year temperature and polarization data)
- Continuing development for Planck - *goal is to exploit the unique capabilities of Gibbs sampling to directly propagate uncertainties in both foreground and instrument models to total uncertainty in cosmological parameters*

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- "*Bayesian Power Spectrum Analysis of the First-Year Wilkinson Microwave Anisotropy Probe Data*", I.J. O'Dwyer et al, Astrophysical Journal, Volume 617, Issue 2, pp. L99-L102.
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