ANTENNA LQG CONTROLLERS: PROPERTIES, LIMITS OF PERFORMANCE, AND TUNING PROCEDURE

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Abstract: The LQG controllers significantly improve antenna tracking precision, but their tuning is a trial-and-error process. A control engineer has two tools to tune an LQG controller: the choice of coordinate system of the controller, and the selection of weights of the LQG performance index. The paper selects the coordinates of the open-loop model that simplify the shaping of the closed-loop performance, and analyzes the impact of the weights on the antenna closed-loop bandwidth, disturbance rejection properties, and antenna acceleration. Finally, it presents the LQG controller tuning procedure that rationally shapes the closed-loop performance.

Keywords—Antennas, Control technology, Control system design, Servo systems, Tracking systems

1. PROBLEM STATEMENT

The pointing and tracking requirements are increasingly stringent for new and existing antennas and radiotelescopes. For example, the Ka-band (34 GHz) communication of the NASA Deep Space Network (DSN) antennas requires pointing accuracy of 1 mdeg (rms) (Gawronski *et al.* 1995; Gawronski 2001); the Large Millimeter Telescope built at Cerro La Negra (Mexico) by the University of Massachusetts and Instituto Nacional de Astrofisica, Optica y Electronica requires pointing of 0.3 mdeg, see (Gawronski and Souccar 2004). These requirements forced the implementation of LQG controllers. The control system of the 34-meter DSN antenna shown in Fig.1 includes the LQG algorithm. It meets the Ka-band requirements and is used to track the Cassini spacecraft on its journey to Saturn. This paper presents principles of the LQG controller design, allowing for shaping the tracking and disturbance rejection properties of antennas or radiotelescopes.

The tuning of LQG controllers for the antenna tracking purposes is a tricky process. The controller shall address the antenna tracking requirements (such as minimization of the antenna servo error in wind gusts, and fast responding to commands) and antenna limitations (such as acceleration limits). The LQG closed loop properties, defined through the LQG performance index, are shaped by LQG weights. The requirements are not directly reflected in the LQG weights. Thus, the relationship between LQG weights and antenna requirements needs to be established. This paper answers this question indirectly. It explains the properties of a simple (PI) controller and a simple (rigid) antenna, and next, by analogy, extends these properties to a real antenna with an LQG controller. This connection leads to the development of a controller tuning method that addresses the antenna tracking performance criteria.

The antenna controller tuning procedure introduced in this paper is developed in three steps: the adjustment of the open-loop model (through selection of the coordinate system and auxiliary components), the analysis of a simple (rigid) antenna and simple (PI) controller (to derive basic properties of the closed-loop system), and finally, the extension of the properties of a simple system to the real (flexible) antenna and a complex (LQG) controller.

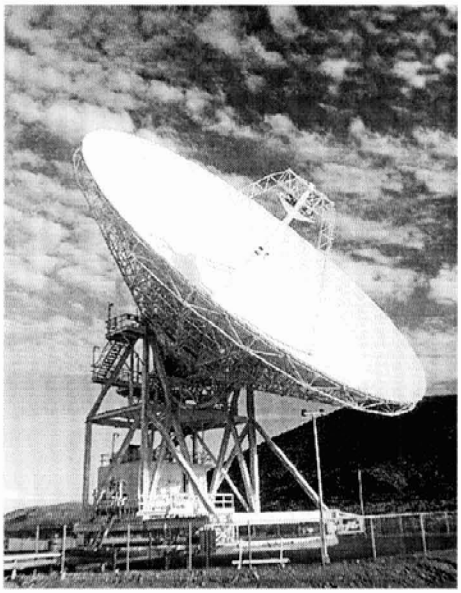


Fig.1. The 34-meter NASA/JPL Deep Space Network antenna, at Goldstone, California.

2. OPEN-LOOP MODEL

Antenna control system monitors azimuth and elevation axes. Since motions in both axes are uncoupled, in the following only a single axis is analyzed. The antenna control system is shown in Fig.2. It consists of the antenna open-loop system, position controller, and rate and acceleration limiters. The controller output u represents the commanded rate, and its derivative is the commanded acceleration, a. An antenna open-loop system consists of antenna structure, motors, gears, amplifiers, and the rate loop feedback. The antenna position (measured at the encoder) is the output of the open-loop system. The rate command is its input. is the state-space triple.

The antenna open-loop model (A,B,C), is obtained from field tests and the system identification. It is transformed into modal coordinates, for details see (Gawronski 2004). The weak coupling of modal states allows adjusting each modal state independently that simplifies the controller tuning process.

The modal model is transformed further, to obtain antenna position y as its first state; the new state is $x_p^T = \{y \quad x_p^T\}$, where x_f are the remaining (unchanged) states. The new state-space representation (A_p, B_p, C_p) is obtained using the transformation $x_p = Px_m$, where $P = \begin{bmatrix} C_{m1} & C_{m2} \\ 0 & I \end{bmatrix}$.

Note that $y = C_m x_m$, where $C_m = [C_{m1} \quad C_{m2}]$.

Next, the model is augmented with an integrator, see (Johnson 1968; Athans 1971), to eliminate the steady state errors in a constant-rate tracking. The new state is

$$x_o^T = \{y_i \quad x_p^T\} = \{y_i \quad y \quad x_f^T\}. \qquad (1)$$

The state y_i satisfies the following equation $\dot{y}_i = y = C_p x_p$ so that the new representation (A_o, B_o, C_o) is

$$A_o = \begin{bmatrix} 0 & C_p \\ 0 & A_p \end{bmatrix}; \quad B_o = \begin{bmatrix} 0 \\ B_p \end{bmatrix}; \quad C_o = \begin{bmatrix} 0 & C_p \end{bmatrix} \quad (2)$$

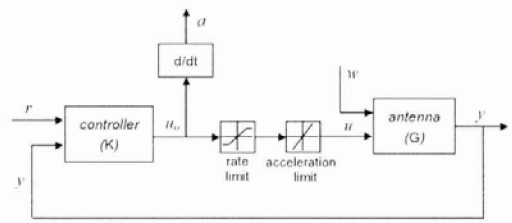


Fig. 2. Antenna control system: G – open-loop system, K – controller, r – command, y – antenna position, u – limited rate command, u_o – unlimited rate command, w – disturbance, a – acceleration

The state x_o is the desired state vector of the antenna open-loop system. It consists of the integral of the position, the position, and the flexible deformations in modal coordinates.

The open-loop system is designed such that it represents an integrator at low frequencies. The magnitude of the transfer function of a perfect integrator is shown in Fig.4, dashed line as a straight line sloping at –20 dB/dec. The magnitude of the transfer function of the 34-meter antenna is shown in Fig.4, solid line, as a straight line sloping at –20 dB/dec for low frequencies (up to 1 Hz), and showing flexible deformations (resonances) at higher frequencies.

Fig. 4. Magnitudes of the transfer function of the open-loop model of the 34-meter antenna (solid line), and a rigid antenna (dashed line).

3. PI CONTROLLER AND RIGID ANTENNA

In the closed-loop system shown in Fig.2 K denotes the controller transfer function, and G is the antenna transfer function. A rigid antenna is a pure integrator, and the controller is assumed a proportional-and integral (PI) controller thus

$$K = k_p + k_i / s \text{ and } G = 1/s, \qquad (3)$$

where k_p is the proportional gain, and k_i is the integral gain.

3.1. Closed-loop transfer functions

Consider the following transfer functions: T_{ry} (from command to encoder); T_{wy} (from disturbance to encoder); T_{ra} (from command to acceleration); and T_{wa} (from disturbance to acceleration). From the block diagram in Fig.2 we obtain $T_{ry} = GK/H$, $T_{wy} = G/H$, $T_{ra} = sK/H$, $T_{wa} = -sGK/H$, where $H = 1 + GK$ and introducing (3) to the latter equations we get $H = s^2 + k_p s + k_i$ and

$$T_{ry} = (k_p s + k_i)/H, \quad T_{wy} = s/H$$
$$T_{ra} = (k_p s + k_i)s^2/H, \quad T_{wa} = -(k_p s + k_i)s/H \quad (4)$$

3.2. The proportional gain analysis

The controller tuning starts with the selection of the proportional gain, thus we assume $k_i = 0$ in the above transfer functions, obtaining $H = s/k_p + 1$

$$T_{ry} = 1/H, \; T_{wy} = 1/k_p H,$$
$$T_{ra} = s^2/H, \; T_{wa} = -s/H \quad (5)$$

The magnitudes of the above transfer function are shown in Fig.6. showing that the increase of the proportional gain: (1) increases the bandwidth of the transfer function T_{ry} (from the command to the antenna position), see Fig.6.a, (2) improves the disturbance rejection properties of the antenna by lowering the magnitude of the disturbance rejection transfer function T_{wy}, see Fig.6.b, (3) increases the impact of the command on the antenna acceleration (increases the magnitude and the bandwidth of the acceleration transfer function T_{ra}), see Fig.6.c, (4) increases the impact of disturbances on the antenna acceleration (increases the magnitude and the bandwidth of the acceleration transfer function T_{wa}), see Fig.6.d.

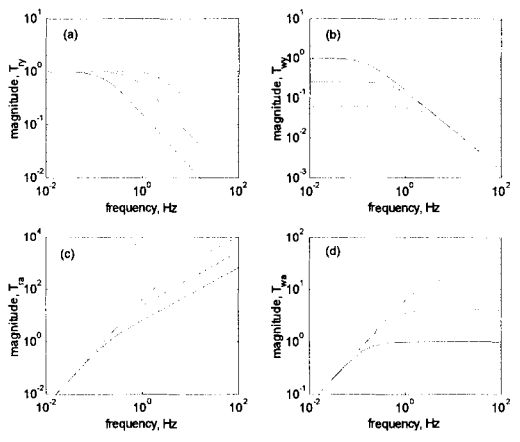


Fig. 6. Magnitude of transfer functions of the proportional controller for k_p=1 (solid line), k_p=4 (dashed line), and k_p=16 (dotted line): (a) T_{ry}, (b) T_{wy}, (c) T_{ra}, and (d) T_{wa}.

The first two transfer functions show the improvement of the antenna performance with the increase of the proportional gain. However the last two show a potential problem: antenna acceleration increases at high frequencies, both due to command and due to disturbances. The increased acceleration indicates that the antenna can hit the acceleration limit, and enter a nonlinear regime; consequently its performance will deteriorate, leading even to instability. Thus, the proportional gain increase is limited by the acceleration limits imposed at the antenna drives.

3.3 The integral gain analysis

First, we introduce two critical values: the critical integral gain, and the critical frequency.

Critical integral gain. Large integral gain causes oscillations of the closed-loop system. The poles of the closed-loop system are the roots of $s^2 + k_p s + k_i$: $s_{1,2} = 0.5\left(-k_p \pm \sqrt{k_p^2 - 4k_i}\right)$. The system is non-oscillatory if poles are real, i.e., for $k_i \le 0.25k_p^2$;

$$k_{ic} = 0.25k_p^2 \quad (6)$$

is the upper limit of the integral gain, called the critical integral gain.

Critical frequency. For the critical integral gain the denominator of the transfer functions is: $(s + 0.5k_p)^2$. At frequency $\omega_o = 0.5k_p = \sqrt{k_i}$ the slope of the transfer function drops by –40 dB/dec. This is the critical frequency of the closed loop system that determines the antenna bandwidth. In the following, the frequencies significantly smaller than ω_o are called low frequencies; frequencies significantly larger than ω_o are called high frequencies; and frequencies in the neighborhood of ω_o are medium frequencies.

The following analysis shows how the transfer functions depend on the integral gain, by considering low, medium, and high frequencies in Eqs.(4). Note first that for medium frequencies the variations of all four transfer functions are minimal (see Fig.7) since the integral gain is smaller than the critical integral gain. For low and high frequencies the transfer functions behave as follows: (1) T_{ry} does not depend on k_i, since for low frequencies $T_{ry} \cong 1$ and for high frequencies $T_{ra} \cong k_p/s$, see Fig.7a; (2) T_{wy} is inverse proportional to k_i for low frequencies, $T_{wy} \cong s/k_i$; and for high frequencies it does not depend on k_i, $T_{wy} \cong 1/s$, see Fig.7b; (3) T_{ra} does not depend on k_i, since for low frequencies $T_{ra} \cong s^2$, and for high frequencies $T_{ra} \cong k_p s$, see Fig.7c; (4) T_{wa} does not depend on k_i, since for low frequencies $T_{wa} \cong -s$, and for high frequencies $T_{wa} \cong -k_p$ see Fig.7d.
The above analysis showed that the integral gain impacts the disturbance rejection transfer function T_{wy} only, at low frequencies.

3.4. PI controller tuning procedure

The PI controller tuning procedure involves

1. Tuning the proportional gain. Increase the gain until antenna hits acceleration limits at typical commands and at expected disturbances.
2. Tuning the integral gain. Increase the gain until oscillations or undershoot appear. It should be smaller than the critical integral gain.

The proportional gain shapes the bandwidth of the transfer function T_{ry}. The larger the gain, the wider is the bandwidth. The proportional gain limit is set by the antenna acceleration limits, since the increase of proportional gain increases antenna acceleration, see Fig.6c,d.

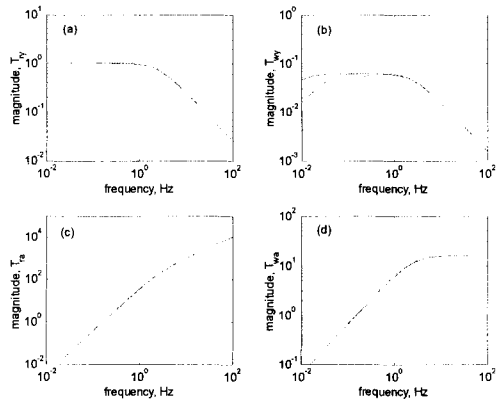


Fig. 7. Magnitude of transfer functions of the PI controller: k_p=16, and k_i=1 (solid line), k_p=16, and k_i=4 (dashed line), k_p=16, and k_i=16 (dotted line), and k_p=16,: (a) T_{ry}, (b) T_{wy}, (c) T_{ra}, and (d) T_{wa}.

The integral gain improves the disturbance rejection properties. But there is a limit to the increase: the integral gain should be smaller than the critical integral gain, to prevent antenna oscillations.

4. LQG CONTROLLER AND FLEXIBLE ANTENNA

The closed-loop system with the LQG controller has the same structure as in Fig.2; the controller has the structure as in Fig.8.

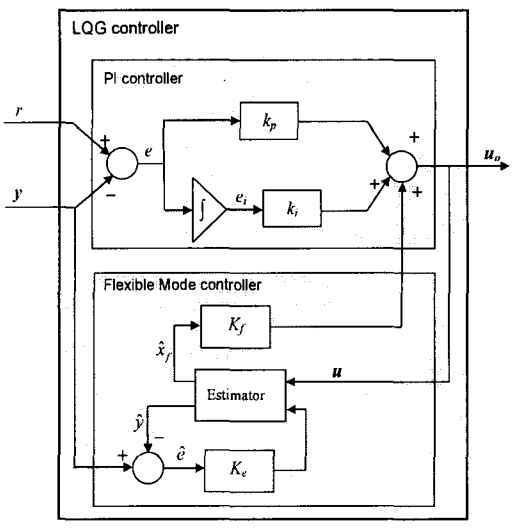


Fig. 8. LQG controller: k_p – proportional gain, k_i – integral gain, K_f – flexible mode gain, K_e – estimator gain, r – command, y – antenna position, e – servo error, e_i – integral of the servo error, u_o – rate command, $\hat{y}$ – estimated position, $\hat{e}$ – estimation error, and $\hat{x}_f$ – estimated flexible mode states.

4.1. LQG controller description

The controller gains are obtained by minimizing the performance index J,

$$J^2 = E\left(\int_0^\infty (x_o^T Q x_o + u_o^T R u_o)\, dt\right),$$

where Q is a positive semidefinite weight matrix and R is a positive scalar. We assume R=1 which is equivalent to $R\neq1$ with the scaled weight matrix Q/R. The minimum of J is obtained for

$$u_o = -K_c \hat{x}_o\,, \tag{7}$$

with the gain $K_c = B_r^T S_c$, and S_c is the solution of the controller algebraic Riccati equation

$$A_o^T S_c + S_c A_o - S_c B_o B_o^T S_c + Q = 0.$$

We see that the controller gain K_c depends solely on the weight matrix Q (A_o and B_o are fixed).

Similarly to the antenna state x_o, the controller gain is divided into the proportional gain k_p, integral gain k_i, and flexible mode gain K_f, i.e.,

$$K_c = \begin{bmatrix} k_i & k_p & K_f \end{bmatrix} \tag{8}$$

Introducing (1) and (8) to (7) one obtains

$$u_o = -k_i e_i - k_p e - K_f \hat{x}_f\,. \tag{9}$$

The missing part of the controller is the estimated state $\hat{x}_o$. It is obtained from

$$\dot{\hat{x}}_o = A_o \hat{x}_o + B_o u_o + K_e (y - C_o \hat{x}_o). \tag{10}$$

where $K_e = S_e C_o^T$, and S_e is the solution of the estimator algebraic Riccati equation

$$A_o S_e + S_e A_o^T - S_e C_o^T C_o S_e + V = 0. \tag{11}$$

For antenna controller tuning purposes we assume $V = Q$ to obtain the balanced gains of the controller and the estimator (Gawronski 2004).

4.2 LQG weights in modal coordinates

The LQG weight matrix is selected as a diagonal matrix (due to independence of states in modal coordinates), $Q = diag(q_i, q_p, q_f)$, where q_i is the integral weight, q_p is the proportional weight, and q_f is a vector of flexible mode weights. Just, it is convenient to present the LQG weights in the vector form as the LQG weight vector q

$$q^T = \left\{ q_i \quad q_p \quad q_f^T \right\}. \tag{12}$$

For the antenna model the modal states are weakly coupled. They are also almost independent from the antenna position and integral of the position. Thus the corresponding weights act independently on each flexible mode, and almost-independently on position, and on the integral of the position states. This adds to the flexibility to the controller tuning.

4.3 Resemblance of LQG and PI controllers

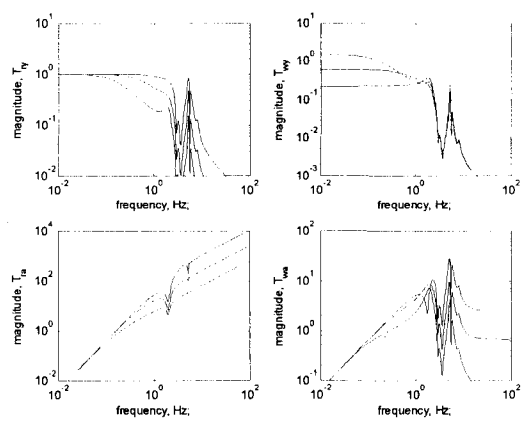
Notice that for the rigid antenna the increase of the proportional gain improves antenna bandwidth and the disturbance rejection properties. However, an increase of proportional gain, when applied to a flexible antenna, is drastically limited: even a moderate gain can excite structural vibrations and cause instability, (Gawronski *et al.* 1995). However, the LQG controller includes the flexible mode part, which is able to restrain antenna vibrations. In this

way, the increased proportional gain does not excite vibrations: a flexible antenna behaves approximately as a rigid one. Therefore the controller tuning approach used for rigid antenna with PI controller can be also used for tuning the LQG controller of a flexible antenna. The limitations are formulated as follows: the flexible mode gains should not be excessive – they should be large enough to assure vibration suppression, but not larger. Such controller is called a low authority LQG controller, (Gawronski 2004).

Consider the 34-meter antenna open-loop model with transfer function shown in Fig.4, solid line. At lower frequencies the transfer function is identical with the transfer function of an integrator, and at higher frequencies it shows flexible mode resonances. To this antenna we apply an LQG controller as follows. First, we select its weights of three LQG controllers, such that their integral gain is zero, and proportional gains are 1, 4 and 16, respectively. For these cases the plots of magnitudes of the transfer functions T_{ry}, T_{wy}, T_{ra}, and T_{wa} are shown in Fig.9. Comparing Fig.9 and Fig.6 we see similarities between the rigid antenna with PI controller and flexible antenna with LQG controller. Namely, the plots of T_{ry} show the expanding bandwidth with the increase of the proportional gain. The plots of T_{wy} show the decreasing antenna response to disturbances with the increase of the proportional gain. The plots of T_{ra} and T_{wa} show increased acceleration response at high frequencies.

Fig.9. Magnitude of transfer functions of the LQG controller for k_p=1 and k_i=0 (blue line), k_p=4 and k_i=0 (green line),and k_p=16 and k_i=0 (red line): (a) T_{ry}, (b) T_{wy}, (c) T_{ra} , and (d) T_{wa}.

Next, we select the weights of the LQG controller to obtain a fixed proportional gain, $k_p = 16$ and to obtain the integral gains 1, 4, and 16, respectively. Note from Eq.(6) that the critical integral gain is 64 in this case. The plots T_{ry}, T_{wy}, T_{ra}, and T_{wa} for the



above three cases are shown in Fig.10. Comparing Fig.10 and Fig.7 we see similarities between the rigid antenna with PI controller and flexible antenna with LQG controller. The integral gain impacts significantly the disturbance rejection properties (T_{wy}) only, and there is no significant impact on the closed loop bandwidth (see T_{ry} plot) and on the system acceleration, see the plots of T_{ra} and T_{wa}.

Finally, we analyze the impact of flexible mode weights on antenna dynamics. Figure 11 presents the magnitudes of the transfer functions T_{ry}, T_{wy}, T_{ra}, and T_{wa} for fixed proportional and integral gains ($k_p = 9.5$ and $k_i = 6.3$) and for small flexible mode weights (blue lines) and for large flexible mode weights (green lines). The plots show that the excessive flexible mode weights reduce the closed loop bandwidth (Fig.11a), and deteriorate the disturbance rejection properties (Fig.11b).

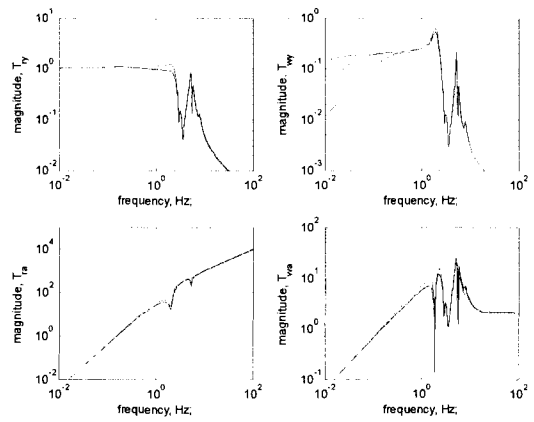


Fig.10. Magnitude of transfer functions of the LQG controller for k_p=16 and k_i=1 (blue line), k_p=16 and k_i=4 (green line), and k_p=16 and k_i=16 (red line): (a) T_{ry}, (b) T_{wy}, (c) T_{ra} , and (d) T_{wa}.

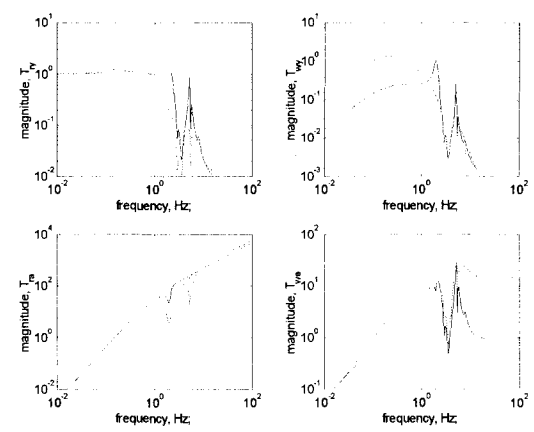


Fig. 11. Magnitude of transfer functions of the LQG controller for k_p=9.5 and k_i=6.3: flexible weights small (blue line), and large – overdamped modes (green line): (a) T_{ry}, (b) T_{wy}, (c) T_{ra} , and (d) T_{wa}.

4.4. Properties of the LQG weights

The above comparison shows that the LQG weights have similar impact on a flexible antenna performance as PI gains on a rigid antenna performance. The following list summarizes the properties of the LQG weights: (1) The increase of the flexible mode weights causes antenna vibration suppression. A single mode weight impacts only states corresponding to this particular mode (the flexible mode coordinates are weekly coupled); (2) The increase of the proportional weight increases the closed-loop bandwidth and improves the disturbance rejection properties; (3)The increase of the integral weight improves the disturbance rejection properties, but does not impact the bandwidth.

4.5 Limits of the LQG weights

The i the proportional, integral, and flexible mode gains have their limits. (1)Large flexible mode weights lead to the overdamped dynamics, (reduced bandwidth, depreciated disturbance rejection properties). (2) Large position weight causes excessive antenna acceleration that leads to non-linear dynamics and deterioration of the performance. (3) Integral weight should not exceed the critical weight in order to prevent low frequency oscillations.

4.6 LQG controller tuning procedure

Based on the above analysis the following sequence of the LQG controller tuning is recommended:
(1) Tuning the flexible mode weights. Apply small weights of the integral of the position and position (which result in small PI gains), and also apply small flexible mode weights. Check the closed-loop transfer function for the appearance of flexible mode resonances. If they are excessive, increase the corresponding flexible mode weights.
(2) Tuning the proportional weight. Increase the position weight: the proportional gain increases accordingly. The increase of the position gain causes the expansion of the closed-loop bandwidth. Increase the weight till bandwidth reaches the antenna fundamental frequency.
(3) Tuning the integral weight. Increase the integral of the position weight, causing the increase of the integral gain. The weight should increase until oscillations appear. The integral gain should satisfy the condition (6).
(4) Correct the flexible mode weights. Check the flexile mode dynamics. If resonances resurface after tuning the proportional and integral parts, increase the corresponding flexible mode weights.

The procedure described above lead to the development of the LQG controller tuning tool as a Matlab graphical user interface (GUI), (Maneri and Gawronski 2000).

5. CONCLUSIONS

The paper shows how to select the coordinates of the controller for simple tuning of the PI and LQG controllers. Also, it shows how the controller gains of the PI controller and the controller weights of the LQG controller impact the antenna closed-loop performance. Finally, it shows the limits of the LQG weights. These features allow to improve antenna performance in wind disturbances.

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