Bayesian Approach to the Joint Inversion of Gravity and Magnetic Data, with Application to the Ismenius Area of Mars

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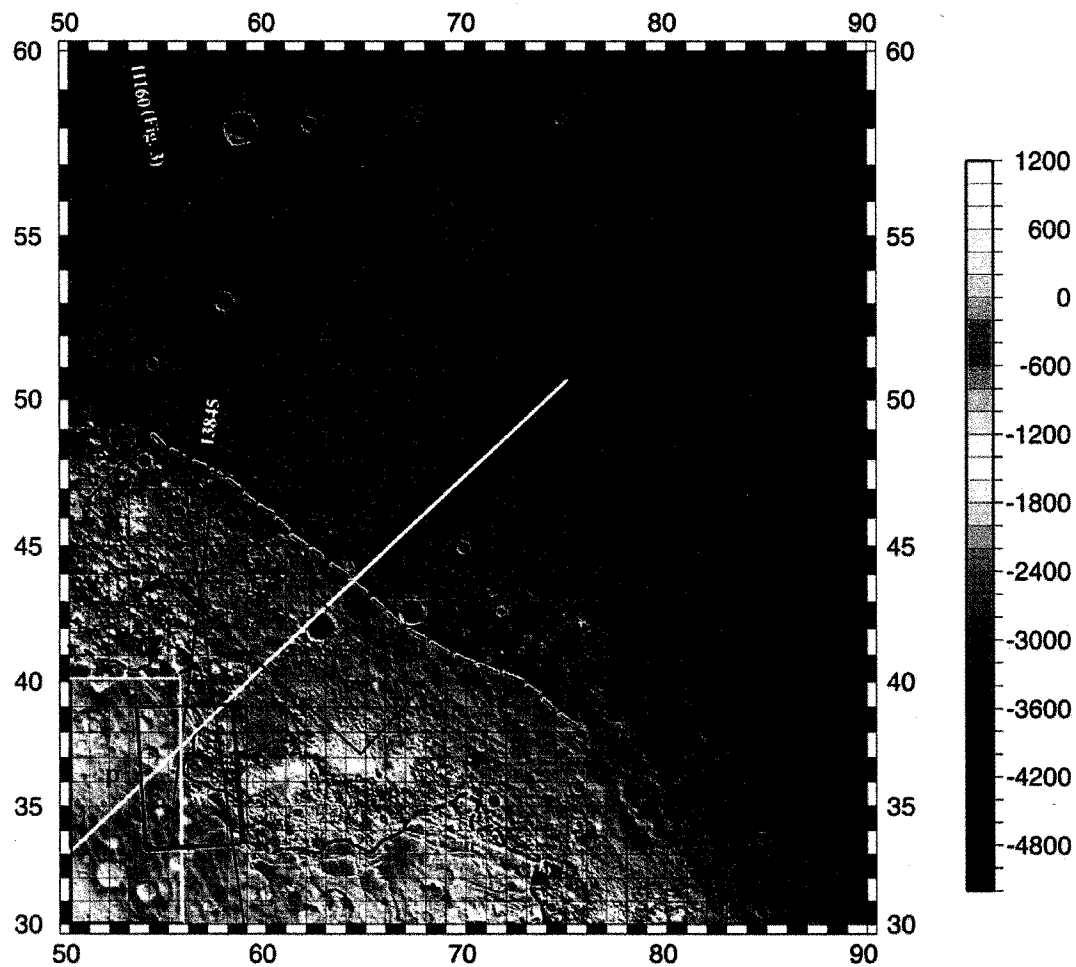
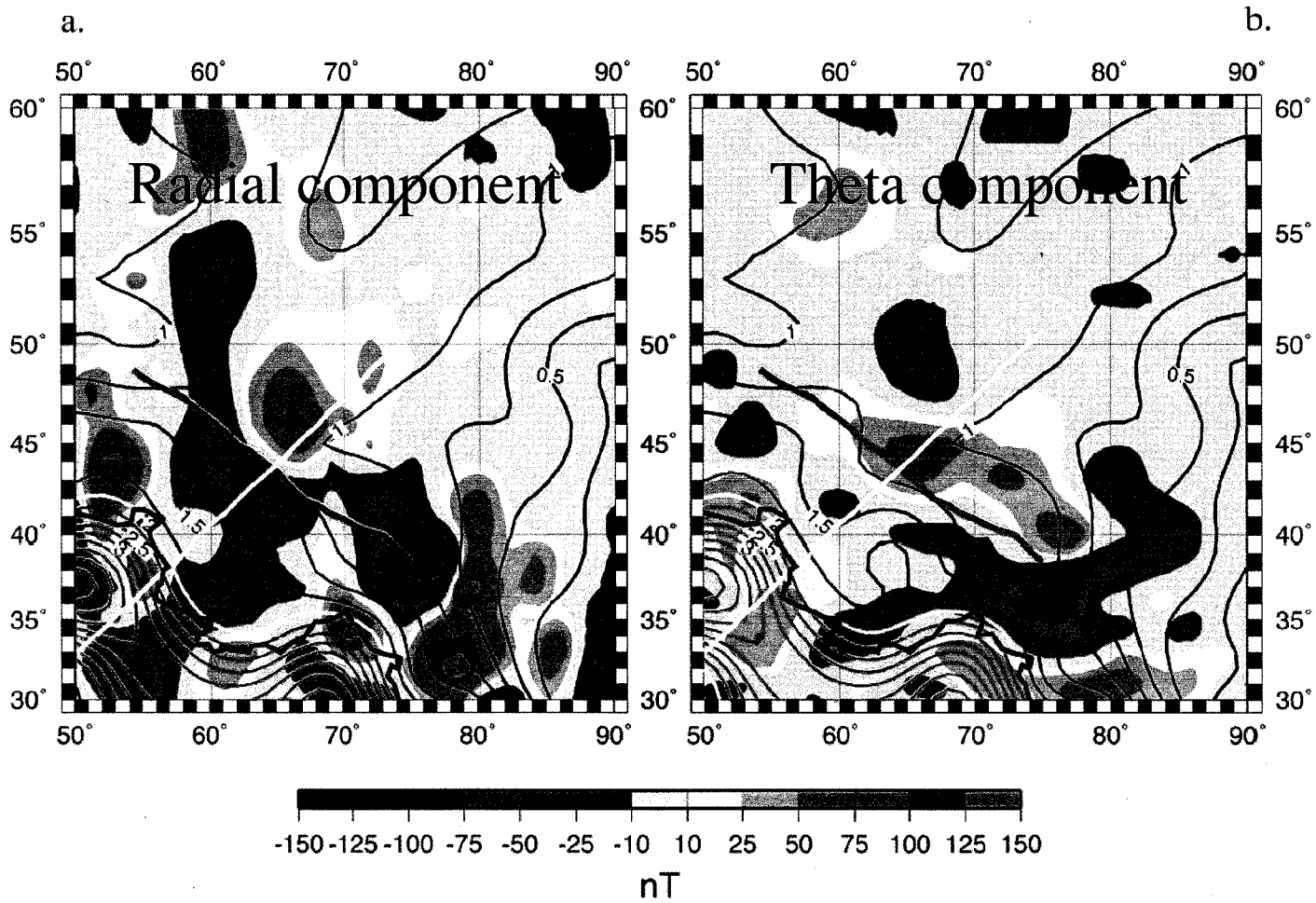
Highland-Lowland Dichotomy

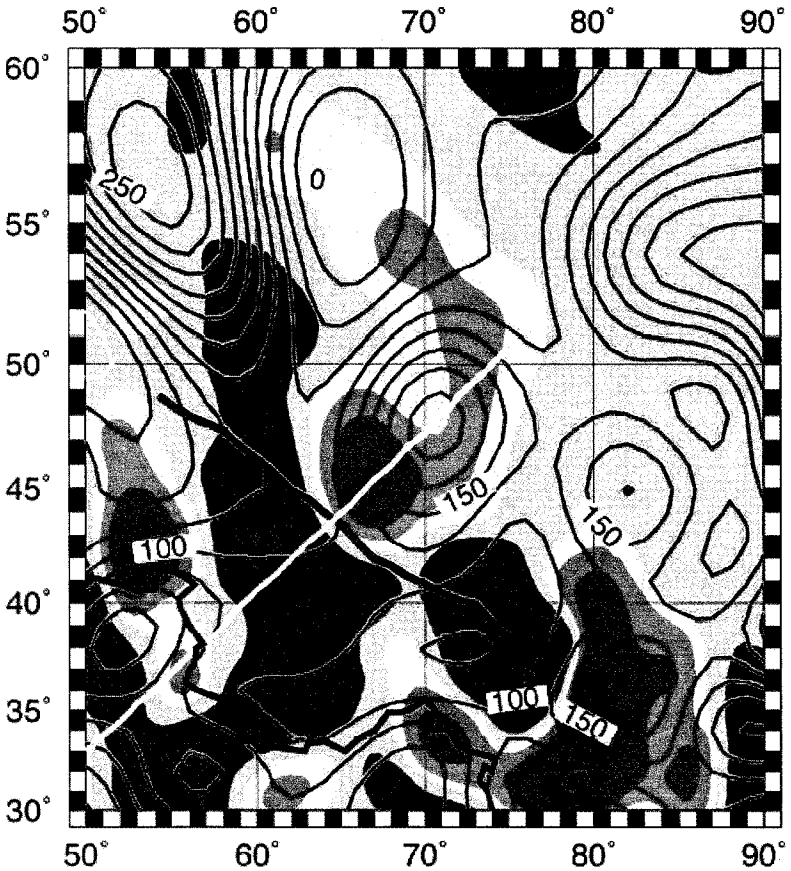


Topography and Magnetic Field Components in Local Area



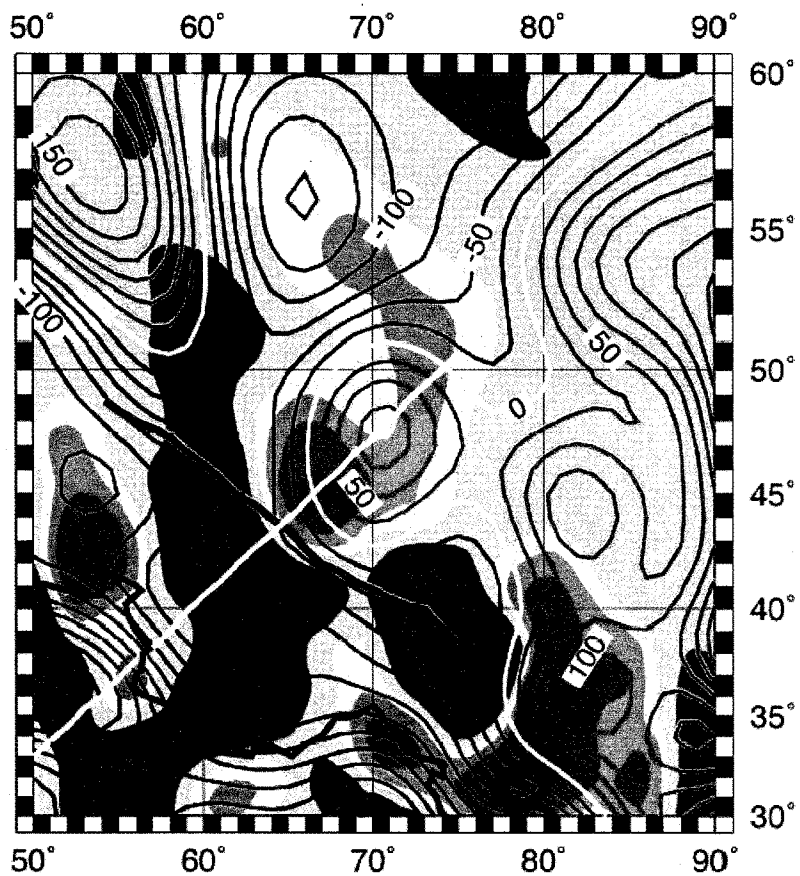
Total Magnetic Field, and Gravity Anomalies

c.



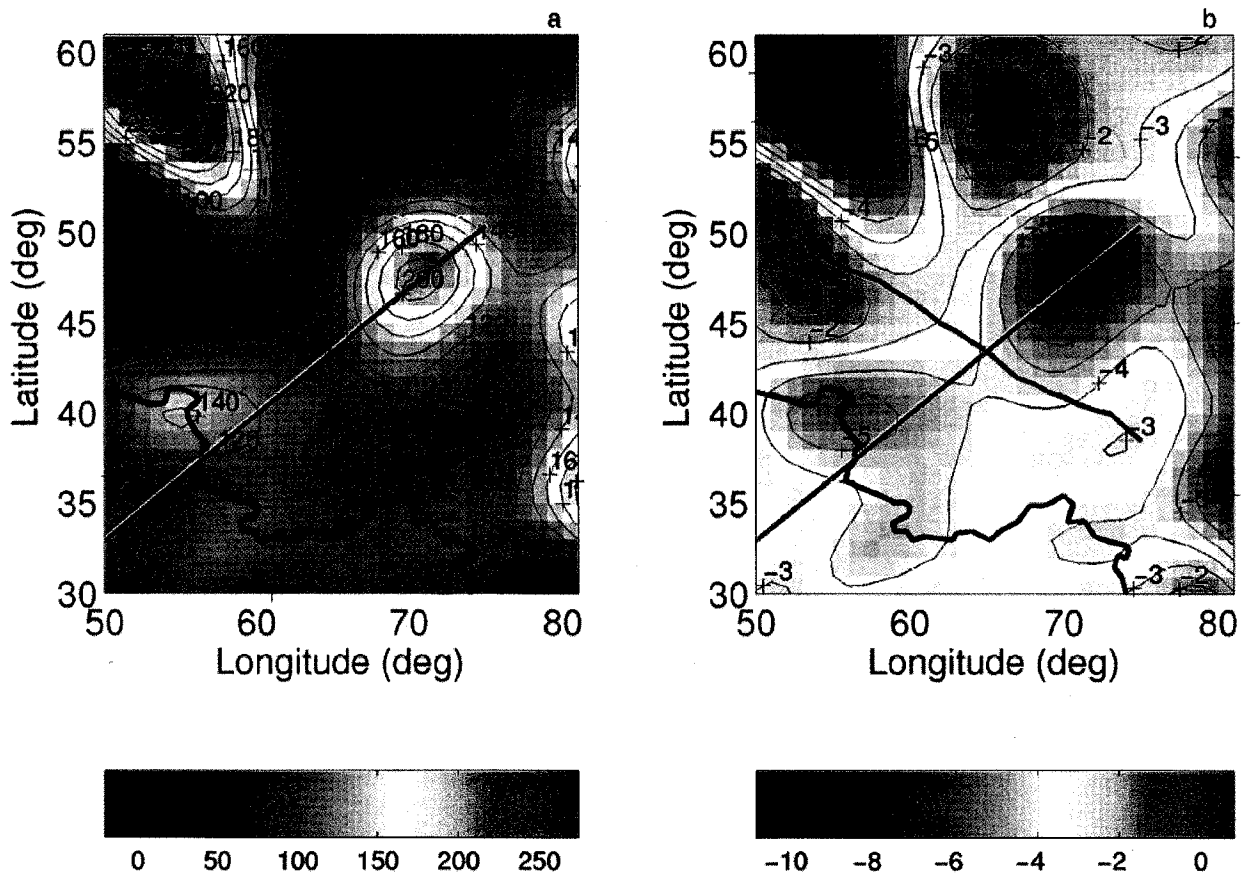
Free-Air Gravity

d.



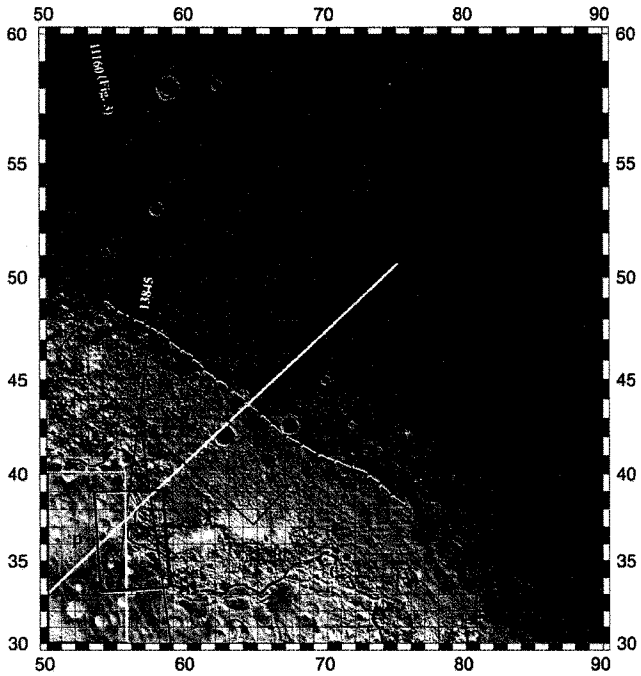
Bouguer Anomaly

Isostatic Anomaly

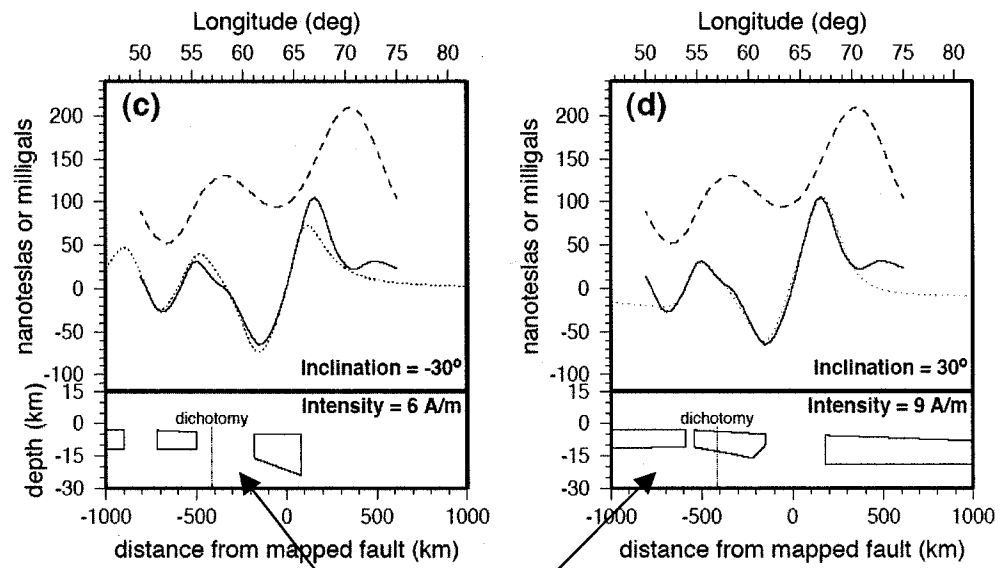


Topography and 50 km thick crust Equiv. variations in crust thickness

Summary of 1D Models…



Topography of study area



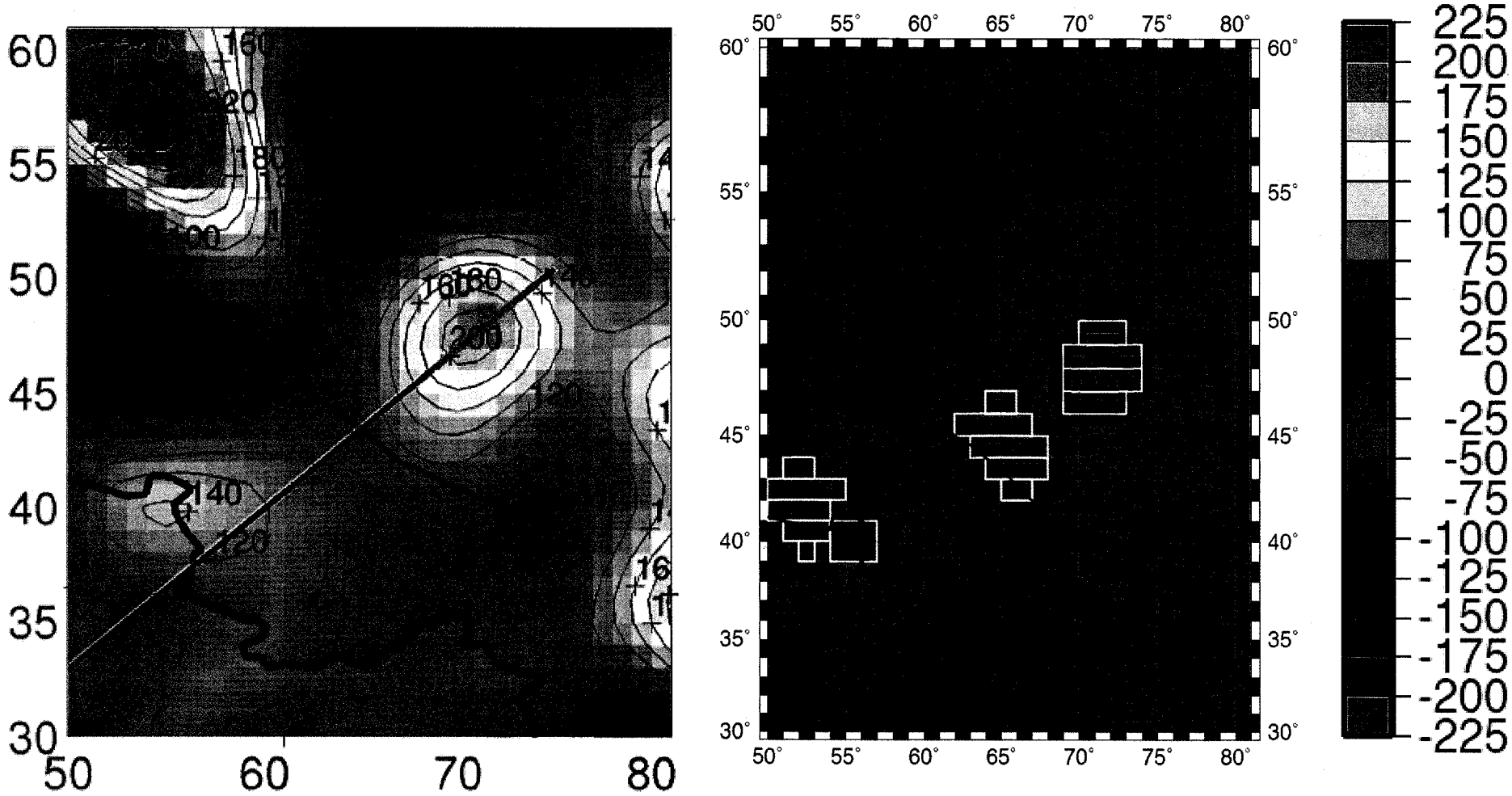
Two distinct models…

- Joint inversion of gravity and magnetic anomalies of a local region of the Martian highland/lowland dichotomy in the Ismenius area
- Challenge - models which fit the data are non-unique. Need to quantify uncertainty in models with Bayesian approach - allows prior information to be incorporated, and a stochastic search of all models which fit the gravity and magnetic data.

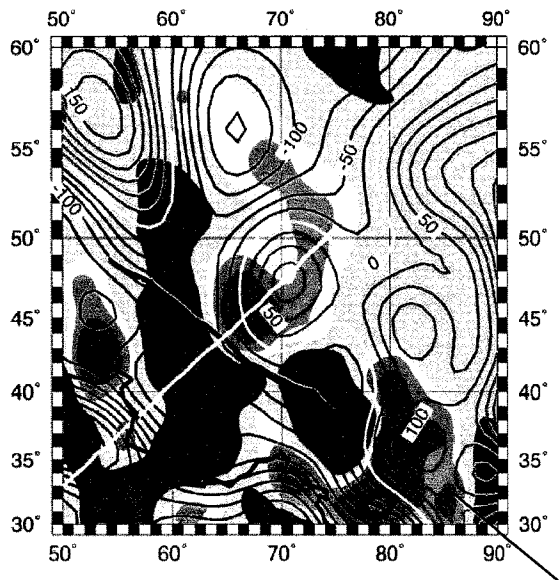
Inversion…

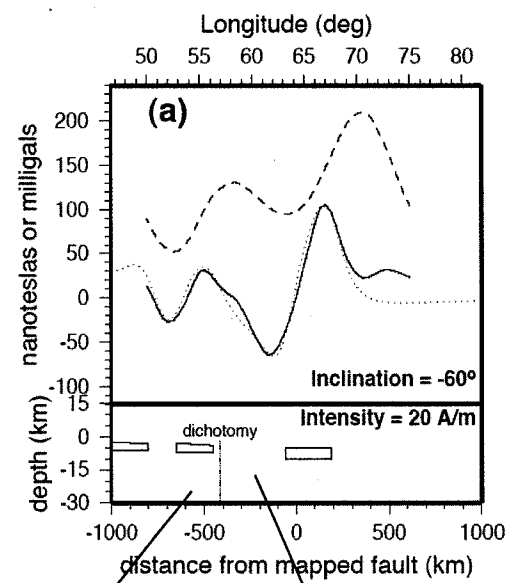
- Given the measurements, want to to infer the physical properties
- Methods of inversion: least squares (set up normal equations and solve a linear problem), conjugate gradient descent (will find a local minimum, and need a good initial guess for that minimum to be the globally "best" answer, if there even is a global minimum)
- Stochastic search - I.e. genetic algorithms, simulated annealing, etc.

Example Forward 3D Model



Joint Probability by "Forward" Problem…

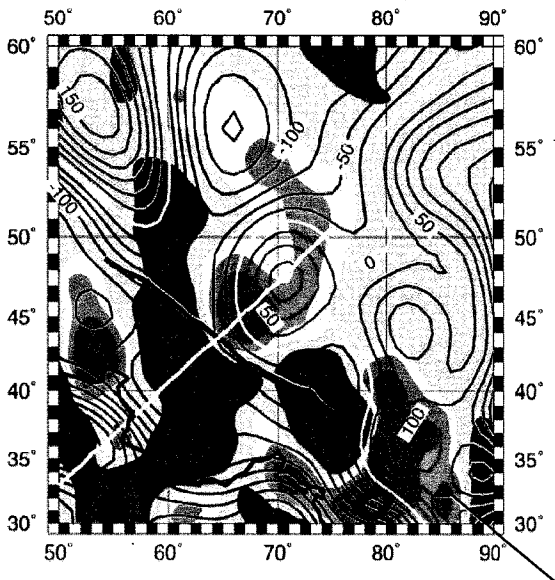


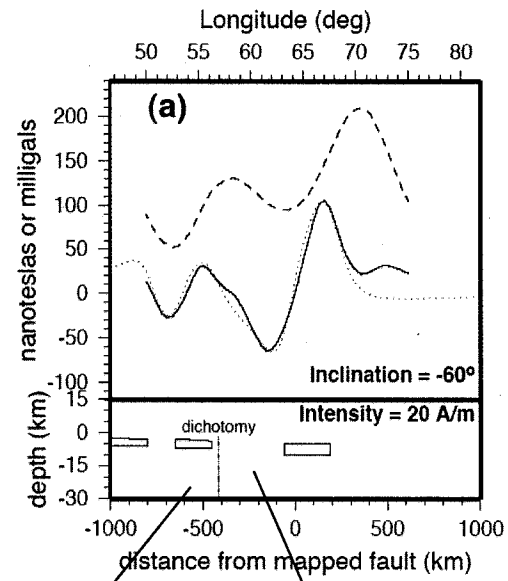


$$P\left[D,\{\rho_i,m_i,h_i,t_i\}\right] = P\left[D\middle|\{\rho_i,m_i,h_i,t_i\}\right]P\left[\{\rho_i,m_i,h_i,t_i\}\right]$$

Joint Prob. Data Given Model "Prior" prob.

Bayes Posterior for "Inverse" Problem

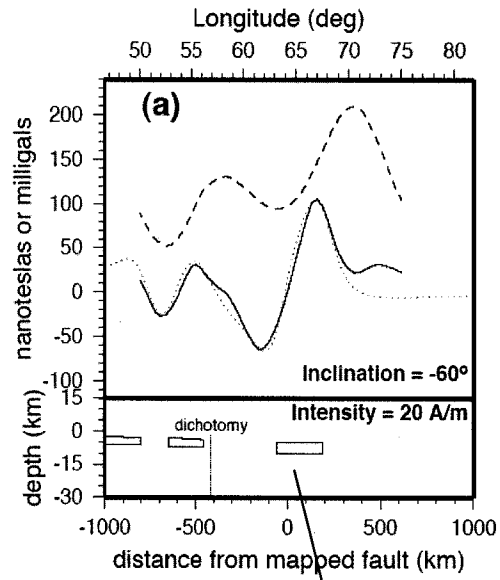


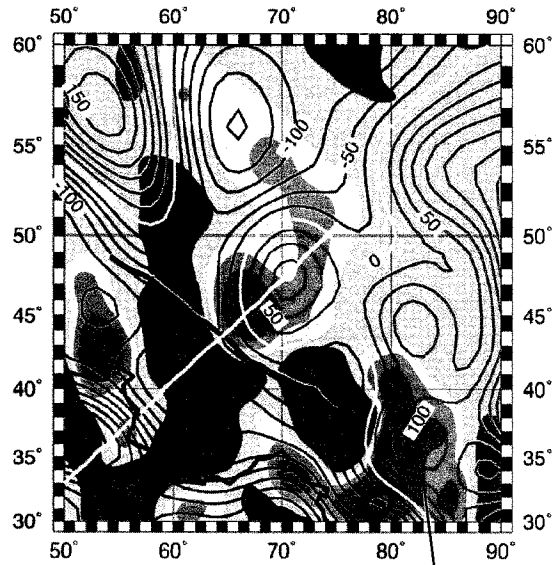


$$P\left[\{\rho_i, m_i, h_i, t_i\} \middle| D\right] \propto P\left[D \middle| \{\rho_i, m_i, h_i, t_i\}\right] P\left[\{\rho_i, m_i, h_i, t_i\}\right]$$

Bayes Posterior Data Given Model "Prior" prob.

Maximum Likelihood Solution Given (Depth, Thickness)





$$\left[C^{-1} + F^T N^{-1} F\right]\left(\hat{\rho}_i, \hat{m}_i\right) = F^T N^{-1} D$$

(for a Gaussian assumption on density and magnetization variations)

Conclusions, and Future Work

- Models are non-unique. Bayes inference provides a framework to characterize uncertainty in conclusions given data!
- Traditional "inverse" problems are seen as those solutions which maximize the Bayes posterior (or conditional posterior).
- Future work will include: more realistic density and magnetic structure (spectral elements), and joint inference of many model parameters from GLOBAL data