

# Efficient generation of truncated Bessel beams using cylindrical waveguides

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**Abstract:** In this paper we address efficient conversion between a Gaussian beam (a truncated plane wave) and a truncated Bessel beam of a given order, using cylindrical optical waveguides and whispering gallery mode resonators. Utilizing a generator based on waveguides combined with whispering gallery mode resonators, we have realized Bessel beams of the order of 200 with a conversion efficiency exceeding 10 %.

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**OCIS codes:** (230.5750) Resonators, (260.1960) Diffraction theory, (140.3300) Laser beam shaping

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## 1. Introduction

Bessel beams obtain as propagation invariant solutions of the Helmholtz equation [1]. Propagation of arbitrary electromagnetic waves in space can be described using Bessel beams. An interesting distinctive feature of the Bessel beams is that they carry orbital angular momenta. Generation of Bessel beams of a given order is an important and interesting task of classical wave optics, for which there are a number of practical approaches. Low order Bessel beams have been realized with both passive and active schemes. Existing methods are also suitable for obtaining very high order beams. However, the technical difficulties increase significantly with increase of the required order. The highest orbital angular momenta of photons observed so far are approximately  $25 \hbar$  [2],  $30 \hbar$  [3], and  $300 \hbar$  [4] per photon.

One of the earliest developed techniques for the generation of Bessel beams requires an annular slit placed in the back focal plane of a positive lens [5, 1, 6, 7]. The efficiency of this method is low because an appreciable amount of the intensity of the illuminating beam is blocked by the slit structure. The most successful technique to generate a zeroth-order Bessel beams involves axicons [8, 9, 10, 11, 12, 13, 14, 15], either standing alone or inserted into active resonators. This method has nearly 100% efficiency. Higher-order Bessel beams can also be generated by illuminating the axicon with Gauss – Laguerre beams [12], but the efficiency is limited here because of limitations of holograms that produce the Gauss – Laguerre beams. The direct generation of higher-order Bessel beams is straightforward with diffractive optics, and the beams can be produced directly from a Gaussian beam by use of computer generated diffraction gratings and holograms [16, 17, 18, 19, 20, 3, 4]. The efficiency of this approach is nevertheless restricted because of the fabrication errors of the holograms. Finally, Bessel beams can also be directly obtained from a laser [21, 22, 23, 24, 25, 26]. The disadvantage of the method is in the difficulty of obtaining narrow, high-order beams at the laser output.

It was predicted theoretically that very high order Bessel beams can be generated using whispering gallery mode (WGM) resonators [27]. Generation of a mixture of Bessel beams of var-

ious orders has been demonstrated with this technique [29], however an efficient generation of high order beams with given quantum number has not been achieved there. In this work we modify the idea proposed in [27] and create a possible version of an orbital momentum generator able to convert a Gaussian beam into a high-order Bessel beam with 100% efficiency. To achieve this efficiency the well known phenomenon of critical coupling should be utilized. We demonstrate generation of Bessel beams of order exceeding 200 with the transformation efficiency better than 10%, as an example. Our generator allows creating Bessel beams of arbitrary order in the wide range of optical wavelengths keeping the conversion efficiency on the same high level. In what follows we present details of the experimental realization of the generator.

## 2. Theory

Let us consider a WGM resonator coupled to a cylindrical waveguide together with an external coupler (a tapered or angle cut fiber, prism, etc), as shown in Fig. 1. A Gaussian beam of light enters the resonator through the coupler and exits through both the coupler and the waveguide. A part of the incoming light is absorbed in the resonator. Our goal is to achieve a complete transformation of the incoming Gaussian beam into the Bessel beam propagating in the cylindrical waveguide. The Bessel beam can be extracted from the waveguide into free space without additional losses with an optical horn emitter [27].

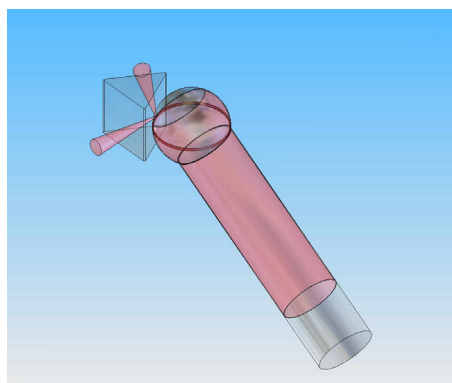


Fig. 1. A schematic of the orbital momentum generator.

The distribution of electric field amplitude in the spherical/nearly spherical WGM resonator is given by

$$\Psi_{WGM} = \frac{\bar{\Psi}_{WGM}}{\sqrt{r}} P_l^m(\cos \theta) J_{l+1/2}(k_{l,q}r) e^{im\phi}, \quad (1)$$

$\theta$ ,  $\phi$ ,  $r$  are the spherical coordinates; indexes  $l$ ,  $m$ , and  $q$  determine the spatial distribution of the field,  $m = 0, 1, 2, \dots$  and  $q = 1, 2, \dots$  are the azimuthal and radial quantization numbers, respectively, and  $l = 0, 1, 2, \dots$  is the mode number,  $\bar{\Psi}_{WGM}$  is a scaling parameter, and  $k_{l,q} = \omega n_0 / c$  is the mode wave number,  $n_0$  the index of refraction of the resonator material,  $\omega$  is the circular frequency of the incoming light. For the high order WGMs we have

$$k_{l,m,q} \simeq \frac{1}{R_{WGM}} \left[ l + \alpha_q \left( \frac{l}{2} \right)^{1/3} \right], \quad (2)$$

where  $R_{WGM}$  is the resonator radius, and  $\alpha_q$  the  $q$ th root of the Airy function:  $Ai(-\alpha_q) = 0$ . It should be mentioned that for an ideal sphere  $k_{l,m,q}$  does not depend on  $m$ . The WGMs with different  $m$ s are degenerate. This degeneracy is lifted in a spheroid. However, Eq. (2) can be used for high-order WGMs for the case of an ideal sphere.

The electric field distribution for the Bessel modes propagating in the cylindrical waveguide coupled to the WGM resonator is given by:

$$\Psi_W = \bar{\Psi}_W J_\nu(k_{\nu,\bar{q}} r) e^{i\nu\phi} e^{i\beta z}, \quad (3)$$

where  $\phi$ ,  $r$ , and  $z$  are the cylindrical coordinates;  $\nu = 0, 1, 2, \dots$  and  $\bar{q} = 1, 2, \dots$  are the angular and radial quantization numbers,  $\Psi_W$  is a scaling factor,  $\beta$  is the propagation constant of the Bessel beam,  $k_{\nu,\bar{q}}$  is the Bessel mode wave number

$$k_{\nu,\bar{q}} \simeq \frac{1}{R_W} \left[ \nu + \alpha_{\bar{q}} \left( \frac{\nu}{2} \right)^{1/3} \right], \quad (4)$$

where  $R_W$  is the radius of the waveguide. The WGM and the Bessel beam interact strongly if frequencies of the modes coincide

$$\epsilon_0 \left( \frac{\omega}{c} \right)^2 = k_{l,m,q}^2 = k_{\nu,\bar{q}}^2 + \beta^2. \quad (5)$$

Phase matching between the WGM and the mode of the waveguide requires the equivalence of the azimuthal numbers  $m = \nu$ . Using these conditions as well as the assumption  $k_{l,q} \gg \beta$  we derive the condition for the quantum numbers of the WGM will be excited:

$$\frac{m}{l} \simeq \frac{R_W}{R_{WGM}}. \quad (6)$$

Interestingly, this condition coincides with the condition for the spatial overlap of the whispering gallery and the waveguide modes, which is also necessary for the interaction. The WGM resonator is slightly aspherical, which is always the case in the experiment, so WGM modes can precess [30]. Assuming that modes with  $q = 1$  are excited and using the analysis from [30] we obtain (6).

A procedure for the Bessel beam generation and identification is the following: i) excite a WGM that leaks into the waveguide; ii) identify the WGM quantum numbers; iii) the angular number of the generated Bessel beam,  $\nu$ , must be equal to number  $m$  of the WGM; iv) the propagation constant of the Bessel beam is to be found from Eq. (5).

There is another way for the identification of the generated Bessel beam. It is enough to measure the radius of the waveguide and find the parameter  $\beta$  with an accuracy better than  $\sqrt{2\nu}/R_w$ . Then the angular momentum of the beam can be directly found from Eq. (5). A practical way for estimating  $\beta$  is based on the measurement of the dispersion of the Bessel mode or the group velocity  $V_g$  of light interacting with the mode of the waveguide. In fact,  $\beta\lambda = 2\pi n_0 V_g n_0 / c$ ,  $n_0 = \sqrt{\epsilon_0}$ .

Let us now discuss the efficiency of the transformation of the Gaussian beam into the Bessel beam. The reflected and transmitted electric field amplitudes for a resonator with two couplers are given by

$$E_{out} = \xi \left[ -\eta \frac{\gamma_{in} - \gamma_{out} - \gamma + i(\omega - \omega_0)}{\gamma_{in} + \gamma_{out} + \gamma - i(\omega - \omega_0)} + \sqrt{1 - \eta^2} e^{i\phi} \right] E_{in}, \quad (7)$$

$$E_{BB} = -\eta \frac{2\sqrt{\gamma_{in}\gamma_{out}}}{\gamma_{in} + \gamma_{out} + \gamma - i(\omega - \omega_0)} E_{in}, \quad (8)$$

where  $1 \geq \xi \geq 0$  describes the efficiency of collection of the light exiting the prism coupler by the photodiode,  $1 \geq \eta \geq 0$  stands for the input coupling inefficiency resulting from the phase mismatch of the input coupler (the prism, in our case) and the resonator,  $\phi$  is a phase shift specific to each setup,  $\omega_0$  is the frequency of a mode of the resonator,  $\omega$  is the carrier frequency of the incoming light,  $\gamma_{in}$  and  $\gamma_{out}$  are the attenuation rates for the loading of the resonator with the entrance coupler and exit cylindrical waveguide,  $\gamma$  is the decay rate describing the intrinsic attenuation in the resonator. Those expressions are valid if the frequency of the incoming light is located in the vicinity of a resonance of the resonator (i.e., the frequency detuning is much less than the free spectral range of the resonator). As follows from Eq. (8) the maximum conversion efficiency approaches unity. This can be achieved when both  $\gamma_{in} = \gamma_{out} + \gamma$  and  $\gamma_{out} \gg \gamma$ . Therefore, according to our simplified model, the orbital angular momentum generator can result in a nearly perfect transformation of a Gaussian beam into a high-order Bessel beam.

Finally, let us discuss the selectivity of the technique. A WGM resonator is able to distinguish between Bessel beams having the same longitudinal wave vector  $\beta$ . It can also generate or separate Bessel beams that have different carrier frequencies. It is also interesting to distinguish between photons with the same carrier frequency, but different angular momenta. In this case  $\beta$  is a function of  $\nu$ . An array of low contrast, different length resonators allows us to make such a selection. The resonators with different dimensions interact only with specific Bessel beams running in the waveguide.

### 3. Experiment

We have realized a prototype of the Bessel beam generator experimentally (see in Fig. 2). The generator was fabricated from 125  $\mu\text{m}$  SMF28 optical fiber. On one end of the 2 mm long structure is a 122.81  $\mu\text{m}$  diameter fused silica WGM resonator. As evident from the figure, the resonator geometry is not entirely spherical, and it adiabatically extends along the  $z$ -axis to a cone of minimum diameter  $2R_W = 62.78 \mu\text{m}$ . The conical shape is essential for emission of the Bessel beams into the free space [29]. Beyond the position of minimum diameter, the cone diameter increases adiabatically until it reaches the  $2R_{exit} = 125 \mu\text{m}$  diameter of the initial optical fiber. The end of the structure that is opposite to the resonator was cleaved perpendicular to the  $z$ -axis and polished.

Light from a 979 nm laser was slowly scanned and evanescently coupled to the WGM structure using a glass prism. Resonator modes with high orbital angular momenta coupled to the cylindrical waveguide were selected. This was achieved by controlling the input angle of the incoming beam of light, as well as selecting the input position to be far from the equatorial plane of the resonator structure. We simultaneously monitored the absorption spectrum of WGMs in a photodetector behind the prism (or output of angled polished fiber coupler [28]), and the transmission spectra of Bessel beams excited in the cylindrical waveguide. The scattered light from the resonator was collected using a 3 mm diameter plastic light pipe, and was observed with a photodiode. A second photodiode was placed at the cleaved end of the horn structure to measure the amount of light transformed into the Bessel beam. An example of the spectra taken with both photodiodes is shown in Fig. 2.

The high-Q mode  $A$  is nearly critically coupled to the waveguide. It results in the efficient generation of a Bessel beam with a well defined number. Mode  $C$  is strongly overcoupled by the waveguide and the emission of the Bessel beam is suppressed. Finally, mode  $B$  is not coupled to any Bessel modes as phase matching conditions were not met. Our experiment allows estimating the angular momentum of the generated Bessel beams: Mode  $A$  has a wavelength equal to  $\lambda = 979.098 \text{ nm}$ . Therefore the Bessel beam order is  $\nu = 2\pi R_W / \lambda = 195$ .

We found parameters describing our generator at the point of maximal conversion efficiency by varying the input coupling efficiency through changing the gap between the prism and the

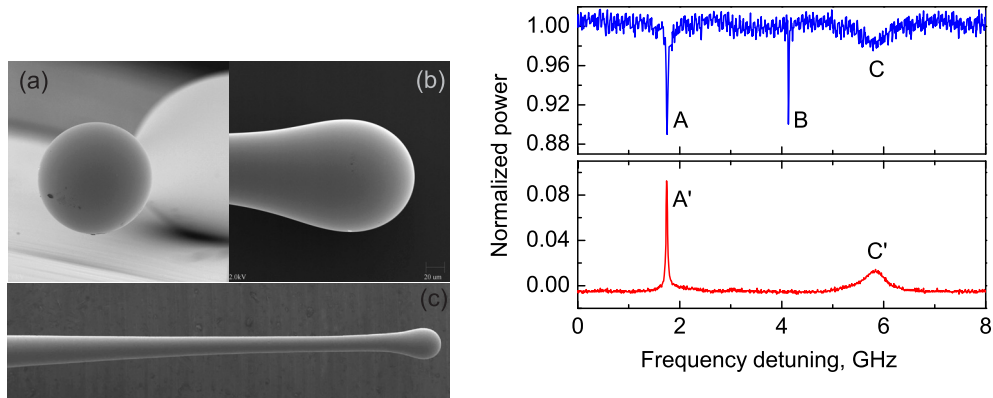


Fig. 2. Left: Scanning electron microscope images of the experimental realization of the generator shown in Fig. (1). Pictures (a) and (b) show different projections of the generator. Picture (c) shows a side image of the entire generator. Right: Power spectra of the WGM resonator (top) and Bessel mode (bottom) normalized to the input power. The efficiency of transmission of the input Gaussian beam into the Bessel mode (A') through WGM (A) is 9% for the particular plot. The coupling of the WGM to the Bessel mode is nearly critical (11% of the input power is absorbed from the input light, 9% is emitted into the Bessel mode).

resonator, and by using Eqs. (7) and (8):  $\eta = 0.5$ ,  $\phi = 0$ ,  $\gamma = 7.5$  MHz, and  $\gamma_{in} = 13.5$  MHz, and  $\gamma_{out} = 6$  MHz. The maximum conversion efficiency from the WGM to Bessel beam is on the order of 80%. The propagation distance [29] of the Bessel beam in free space was less than  $R_{exit}^2/R_W \simeq 1$  mm.

#### 4. Conclusion

In conclusion, we have proposed several designs for resonator/waveguide systems that are able to transform a plane wave into a Bessel beam and support propagation of high order Bessel beams. The efficiency of the generation can reach 100% if proper coupling conditions are fulfilled. We also have demonstrated such a Bessel beam generator experimentally, and converted a Gaussian beam into a Bessel beam with a discrete number of the order of 200. The conversion efficiency from WGM to a Bessel mode has been better than 80%, and from the Gaussian beam into the Bessel beam – better than 10%. The last number can be significantly improved if one uses a more efficient coupling using, e.g., tapered fibers [31]. Possible practical applications of the high order Bessel beams include demonstration of opto-mechanical effects and trapping of cold atoms (longer propagation distance should be achieved for the latter application).

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