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ABSTRACT

G-Guidance is a robust G&C (guidance and control) algorithm developed under the small-body GN&C task. The G-Guidance scheme utilizes a model predictive control approach, along with a convexification of the governing dynamics, control constraints, and trajectory/state constraints. This report details an enhancement to the FY2005 G-Guidance algorithm; the addition of a fire-second approach helps to nullify velocity errors and hit desired final velocities much more precisely than the original fire-first scheme developed in FY2005. The enhancement preserves the primary benefit of the algorithm, which is to ensure required thruster silent times during trajectory maneuvers. The fire-second scheme increases the versatility of the algorithm for missions employing G-Guidance. For instance, a landing sequence could employ the fire-second scheme to ensure a null final velocity at landing, whereas an ascent sequence could utilize a fire-first scheme to ensure there is no drift of the spacecraft toward the ground at the beginning of the maneuver. Examples are provided within to demonstrate a fire-first versus fire-second guidance scheme. As in the existing G-Guidance algorithm, the examples and schemes incorporate gravity models and thruster firing times into discrete dynamics that are solved as a optimal control problem to minimize fuel consumption or thruster energy expenditure.

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1 Introduction

The G-Guidance algorithm has been developed from the FY2005 small-body GN&C task, based on work in JPL D-32948 and JPL D-32947 (both documents are cleared for U.S. and foreign release with clearance numbers 05-2804 and 05-2805, respectively) [1, 2]. G-Guidance provides a guidance and control (G&C) method for not only determining an optimal trajectory for landing, hopping, proximity maneuvering, or take-off on a small-body but also provides a means to guarantee the continued feasibility (i.e. the resolvability) of the guidance problem once initial feasibility is established. The guidance portion of the problem termed PWG (pseudo way-point generation) was used to autonomously generate state-space models at discrete time intervals during a trajectory maneuver, along with firing sequences that obeyed required thruster silence times and thruster firing limits. The original development of the PWG scheme [1] assumed a fire-first, followed-by-silence sequence. The disadvantage of this scheme is that upon final firing, there is a silence period in which velocity cannot be curtailed. Thus, for maneuvers with a specific final-velocity requirement, the final velocity based on the prior PWG scheme can only be reached within the accuracy of the model; final velocity errors cannot be taken out with this scheme, a local controller must instead be activated. By augmenting the PWG scheme with a fire-second technique, the final velocity can be established with a much higher precision because both the guidance and feedback firing can be performed and ceased at the final maneuver time, thus taking out velocity errors.

The algorithmic development herein is confined to a restatement of the earlier fire-first scheme, along with a development of the newer fire-second scheme. Comparison of these algorithms is discussed, along with simulations that demonstrate the fire-first versus fire-second schemes. The simulations also validate the fire-second technique within the G-Guidance framework. For a more thorough algorithmic development of the full G-Guidance method, including how feedback and resolvability are incorporated, refer to the earlier work in JPL D-32948 [1]. For the theoretical basis of the algorithm, refer to JPL D-32947 [2] or [3].

2 Pseudo Way-Point Guidance Scheme

2.1 Governing Dynamics

The state dynamics of a spacecraft orbiting, landing, or conducting proximity operations at a small-body are governed by the following equations of motion, expressed in a rotating frame (i.e. a frame fixed to the small-body):

$$\ddot{r} + \omega \times \dot{r} + 2\omega \times r + \omega \times (\omega \times r) = u + g(r)$$  (1)

where $r \in \mathbb{R}^3$ is the radius vector from the small-body center of mass to the spacecraft, $\omega \in \mathbb{R}^3$ is the rotation rate of the body, $u \in \mathbb{R}^3$ are applied specific forces (i.e. thruster firings, other control inputs, or disturbances with units of force per unit mass), and $g(r)$ is the gravity. Linearized models of gravity are used in the scheme:

$$g(r) \approx g(r_k) + \frac{\partial g}{\partial r} \bigg|_{r_k} (r - r_k) + \delta(r, r_k) = G r_k r + g_k + \delta_k$$  (2)

where $r_k$ is a reference radius, $\delta_k$ is a norm-bounded, higher-order gravity perturbation (neglected from here onward), $g_k = g(r_k) - \frac{\partial g}{\partial r} \bigg|_{r_k} r_k$, and $G r_k = \frac{\partial g}{\partial r} \bigg|_{r_k}$.
The class of small-bodies considered in the scheme have stable and constant rotation rates \( \dot{\omega} = 0 \), which is a necessary assumption to cast the dynamics of equation 1 in state space form:

\[
x = \begin{pmatrix} r \\ \dot{r} \end{pmatrix}
\]

\[
\dot{x} = A_0 x + Bu + B g(r)
\]

\[
\approx A_k x + Bu + B g_k
\]

where

\[
A_0 = \begin{bmatrix} 0 & I \\ -\tilde{\omega}^2 & -2\tilde{\omega} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix}
\]

\[
A_k = A_0 + B G r_k E, \quad E = \begin{bmatrix} I & 0 \end{bmatrix}.
\]

and \( \tilde{\omega} \) is the \( 3 \times 3 \) matrix representation of the vector cross product \( \omega \times (\cdot) \). Note, the subscript \( k \) on \( A_k \) and \( g_k \) indicates a dependence on chosen reference radius \( r_k \).

### 2.2 Discrete Dynamics Model

A discrete model of the dynamics is necessary for development of the PWG scheme. An assumption is made that finite burns are used in the guidance and that they are constant during each burn interval (though not necessarily the same constant), this facilitates the use of zero-order hold in the discretization. The time interval \( \Delta t \) used in the discrete dynamics is based on required thruster silent times of duration \( \delta_s \) and finite burns of duration \( \delta_f \) such that \( \Delta t \geq \delta_s + \delta_f \).

The solution to the equations of motion in equation 5 is

\[
x(t) = e^{A_k(t-t_0)} x(t_0) + \int_{t_0}^{t} e^{A_k(t-\tau)} B(u(\tau) + g_k) \, d\tau
\]

which can be used to develop the discrete models over fixed time interval \( \Delta t = t_{k+1} - t_k \).

#### 2.2.1 Fire First (Original PWG Firing Sequence)

For the fire-first implementation, the firing portion occurs during \( t \in [t_k, t_k + \delta_f] \), where \( t_k + \delta_f < t_{k+1} \). For the scheme, all discrete firings (i.e. \( u = u(t_k) \)) are considered constant, finite burns. For this discrete firing, the discrete solution becomes

\[
x(t_k + \delta_f) = e^{A_k((t_k + \delta_f) - t_k)} x(t_k) + \int_{t_k}^{t_k + \delta_f} e^{A_k((t_k + \delta_f) - \tau)} B(u(\tau) + g_k) \, d\tau
\]

\[
= e^{A_k \delta_f} x(t_k) + \int_{0}^{\delta_f} e^{A_k(\delta_f - \tau)} B d\tau \cdot u(t_k) + \int_{0}^{\delta_f} e^{A_k(\delta_f - \tau)} B d\tau \cdot g_k.
\]

The silent portion of the fire-first sequence follows (i.e. \( t \in (t_k + \delta_f, t_k + 1) \) with \( u = 0 \)) with the discrete solution being

\[
x(t_{k+1}) = e^{A_k((t_k + 1) - (t_k + \delta_f))} x(t_k + \delta_f) + \int_{t_k + \delta_f}^{t_k + 1} e^{A_k((t_k + 1) - \tau)} B d\tau \cdot g_k
\]

\[
= e^{A_k(\Delta t - \delta_f)} x(t_k + \delta_f) + \int_{\delta_f}^{\Delta t} e^{A_k(\Delta t - \tau)} B d\tau \cdot g_k
\]
Combining these two equations provides the discrete equations over each time interval $\Delta t$:

$$x(t_{k+1}) = e^{A_k \Delta t} x(t_k) + e^{A_k (\Delta t - \delta_f)} \int_{t_k}^{t_k + \delta_f} e^{A_k (\delta_f - \tau)} B d\tau \cdot u(t_k) + \int_{t_k}^{t_k + \delta_f} e^{A_k (\delta_f - \tau)} B d\tau \cdot g_k$$

$$\Rightarrow x_{k+1} = A_{d,k} x_k + B_{d,k} u_k + E_{d,k} g_k$$

(6)

where $A_{d,k} = e^{A_k \Delta t}$, $B_{d,k} = \int_{0}^{\delta_f} e^{A_k (\delta_f - \tau)} B d\tau$, $E_{d,k} = \int_{0}^{\Delta t} e^{A_k (\Delta t - \tau)} B d\tau$, $g_k$ fixed over each $\Delta t$,

$$u(t_k) = \begin{cases} 
  u_k, & t \in [t_k, t_k + \delta_f] \\
  0, & t \in (t_k + \delta_f, t_{k+1})
\end{cases}$$

with $u_k$ constant, and $k = 0, ..., N - 1$. Note, $g_k$ can be chosen based on the state $x(t_k)$ or $x(t_{k+1})$ to provide discrete updates to gravity, thereby increasing the accuracy of the discrete solution. Additionally, the dependence on the reference radius $r_k$ used to linearize gravity translates into dependencies on $k$ for all three matrices $A_d$, $B_d$, and $E_d$.

The above algorithm is the based of the PWG scheme implemented in JPL D-32948 [1], which also lays out the remainder of the PWG algorithm, along with the feedback control scheme for ensuring resolvability of the G&C algorithm.

2.2.2 Fire Second (Enhancement to PWG Scheme)

For the fire-second implementation, the silence portion occurs first (i.e. $t \in [t_k, t_k + \delta_s]$ where $t_k + \delta_s < t_{k+1}$ and $u = 0$). For this first, silent portion, the discrete solution is

$$x(t_k + \delta_s) = e^{A_k ((t_k + \delta_s) - t_k)} x(t_k) + \int_{t_k}^{t_k + \delta_s} e^{A_k ((t_k + \delta_s) - \tau)} B g_k d\tau$$

$$= e^{A_k \delta_s} x(t_k) + \int_{0}^{\delta_s} e^{A_k (\delta_s - \tau)} B d\tau \cdot g_k.$$

During the subsequent firing portion (with $t \in [t_k + \delta_s, t_{k+1}]$ and $u = u_k$ a constant, finite burn) the discrete solution becomes

$$x(t_{k+1}) = e^{A_k (t_{k+1} - (t_k + \delta_s))} x(t_k + \delta_s) + \int_{t_k + \delta_s}^{t_{k+1}} e^{A_k (t_{k+1} - \tau)} B (u_k + g_k) d\tau$$

$$= e^{A_k (\Delta t - \delta_s)} x(t_k + \delta_s) + \int_{\delta_s}^{\Delta t} e^{A_k (\Delta t - \tau)} B d\tau \cdot g_k + \int_{\delta_s}^{\Delta t} e^{A_k (\Delta t - \tau)} B d\tau \cdot u_k.$$

Reminder, $g_k$ is assumed constant over a $\Delta t$. Combining these two equations provides the discrete equations over each time interval $\Delta t$:

$$x(t_{k+1}) = e^{A_k \Delta t} x(t_k) + \int_{\delta_s}^{\Delta t} e^{A_k (\Delta t - \tau)} B d\tau \cdot u_k + \int_{0}^{\Delta t} e^{A_k (\Delta t - \tau)} B d\tau \cdot g_k$$

$$\Rightarrow x_{k+1} = A_{d,k} x_k + B_{d,k} u_k + E_{d,k} g_k$$

(7)

where $A_{d,k} = e^{A_k \Delta t}$, $B_{d,k} = \int_{\delta_s}^{\Delta t} e^{A_k (\Delta t - \tau)} B d\tau$, $E_{d,k} = \int_{0}^{\Delta t} e^{A_k (\Delta t - \tau)} B d\tau$, $g_k$ fixed over each $\Delta t$,

$$u(t_k) = \begin{cases} 
  0, & t \in [t_k, t_k + \delta_s) \\
  u_k, & t \in [t_k + \delta_s, t_{k+1})
\end{cases}$$

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with \( u_k \) constant, and \( k = 0, \ldots, N - 1 \). Again, \( g_k \) can be chosen based on the state \( x(t_k) \) or \( x(t_{k+1}) \), and the dependence on the reference radius \( r_k \) causes dependencies on \( k \) for all three matrices \( A_d \), \( B_d \), and \( E_d \).

The development of this fire-second enhancement fits seamlessly into the PWG Algorithm, along with the full G&C scheme presented in JPL D-32948 [1]. No further algorithmic development or any modification is necessary for utilizing this enhancement to the earlier G&C scheme.

3 Comparison of Fire-First Versus Fire-Second Scheme

The two simulations provided herein utilize similar small-body and spacecraft characteristics as those utilized in the simulations for JPL D-32948 [1]. For completeness, some of the simulation setup is repeated here.

The gravity environment used in the simulations is based on Eros data [4]. The spacecraft utilized has the following properties: mass of 400 kg, maximum available thrust magnitude of 125 N, maximum feedback thrust of 20 N, thruster specific impulse of \( I_{sp} = 300 \text{ sec} \), desired thruster firing time of \( \delta_f = 10 \text{ sec} \), required minimum silence time of \( \delta_s = 20 \text{ sec} \), and maneuver time of flight of \( T = 300 \text{ sec} \).

The initial state \( x_0 \) (position, in meters, and velocity, in meters/second) and the desired final state \( x_F \) are

\[
x_0 = \begin{bmatrix} 8950 \\ 100 \\ 0 \\ 1.5 \\ 2 \\ 0 \end{bmatrix} \quad \text{and} \quad x_F = \begin{bmatrix} 8450 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.
\]

The simulation shown in figure 1 is of the original fire-first sequence. As seen in the firing portions (bottom two subplots), both the guidance and feedback firings overlap and initiate at the beginning of the maneuver. Note also the silence time at the end of the maneuver, where nothing can be done to take out any velocity errors. This effect would be more problematic in a significant gravity environment.
The simulation shown in figure 2 utilizes the fire-second enhancement to the PWG scheme. Note additionally that the feedback scheme has been adapted to coincide with the fire-second scheme so that required silence times will be preserved. The maneuvers now initiate with a silence sequence, followed by the firing. Additionally, note that the ending of the maneuver has a coincident firing, which can be used to effectively null out velocity errors.
Figure 2: G&C Implemented with **Fire-Second** PWG scheme

4 Conclusions

The G-Guidance algorithm was enhanced to allow the use of a fire-second approach that helps to nullify velocity errors and hit desired final velocities much more precisely than the original fire-first PWG scheme developed in FY2005. This enhancement increases the versatility of the PWG scheme as a mission employing this form of G&C can choose between the fire-first and fire-second scheme depending upon maneuver objectives. For instance, a landing sequence could employ the fire-second scheme to ensure a null final velocity at landing, whereas an ascent sequence could utilize a fire-first scheme to ensure there is no drift of the spacecraft toward the ground at the beginning of the G&C maneuver.

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References


