All-optical Photonic oscillator with high-Q whispering gallery mode resonators

A. A. Savchenkov, A. B. Matsko, D. Strekalov, M. Mohageg, V. S. Hehenko, and L. Maleki Jet Propulsion Laboratory, California Institute of Technology,

4800 Oak Grove Drive, Pasadena, California 91109-8099

Abstract— We demonstrated low threshold optical photonic hyper-parametric oscillator in a high-Q (10¹⁰) CaF₂ whispering gallery mode resonator which generates stable 8.5 GHz signal. The oscillations result from the resonantly enhanced four wave mixing occurring due to Kerr nonlinearity of the material.

I. INTRODUCTION

Generation of high quality microwave signals directly from light has been of considerable interest, ever since the advent of photomixers and lasers. Here we propose to use a low threshold optical hyper-parametric oscillator based on a WGM crystalline resonator possessing a cubic nonlinearity for generation of spectrally pure high frequency microwave signals. The approach is designed to generate signals with high enough quality to satisfy applications such as high performance radar.

We describe such an oscillator and realize a proof of the principle experiment using a high-Q crystalline CaF₂ (calcium fluoride, fluorite) whispering gallery mode resonator. Calcium fluoride is a low loss crystalline material possessing cubic nonlinearity. Though the nonlinearity of CaF_2 is even smaller than that of fused silica, we were able to observe a strong nonlinear interaction among resonator modes resulting from the high Q-factor of the resonator. The nonlinearity results in a four-photon parametric process like $\hbar\omega_0 + \hbar\omega_0 \rightarrow \hbar\omega_1 + \hbar\omega_2$, which leads to the generation of coherent optical signals from vacuum fluctuations at frequencies ω_1 and ω_2 . In the resonator $\omega_1 \approx \omega_0 + \delta_{FSR}$ and $\omega_2 \approx \omega_0 - \delta_{FSR}$, where ω_0 is the frequency of the external pumping resonant with one of the resonator's modes, and δ_{FSR} stands for the free spectral range (FSR) frequency interval of the resonator.

High Q-factors are essential for realization of the efficient nonlinear optical interactions: oscillation threshold value for the optical pump power is proportional to Q^{-2} . Crystalline WGMs are promising here because pure optical crystals have extremely small losses compared with other materials. Moreover, dielectric WGM resonators belong to an open resonator type. Q-factors of such resonators suffer from atmospheric dust and water. Our experiments show that crystals, as materials for WGM resonators, are advantageous because their quality either does not degrade, or degrades much slower in time compared with, e.g., fused silica resonators. Finally, symmetry properties of crystal lattices allow to separate, enhance, and/or suppress selectively various nonlinear processes occurring in the materials.

High-Q WGMs were first observed in liquid droplets, as well as in solidified droplets of fused amorphous materials, such as fused silica. Liquids and amorphous materials form only a small part of high quality optical materials suitable for fabrication of WGM resonators. For instance, some crystals are transparent enough to sustain high-Q WGMs, on the one hand, and are nonlinear enough to allow continuous manipulation of the WGMs' characteristics, on the other. Until recently, the only remaining problem with the realization of crystalline WGM resonators was the absence of a fabrication process that would yield nanometer-scale smoothness of spheroidal surfaces for elimination of surface scattering. Recently this problem was solved and crystalline resonators with $Q > 10^7$ were fabricated using mechanical optical polishing techniques [1], [2]. We have further improved the technique for fabrication of ultrahigh-Q crystalline WGM resonators with Q approaching 10^9 [3], and, finally, exceeding 10^{10} [4].

Selection of a material for highest-Q WGM resonator must be based on fundamental factors such as the widest transparency window, high purity grade, and environmental stability. According to recently reported measurements on scattering in CaF₂ the value of linear attenuation of light in the material is $\alpha = 3 \times 10^{-5}$ cm⁻¹ at 193 nm, and, therefore, extremely small scattering can be projected in the near infrared band, corresponding to limitation of Q at the level of 10^{13} . Lattice absorption at near infrared wavelengths can be predicted from the position of the middle infrared multi-phonon edge, and yields even smaller Q limitations. Taking into account the above reasoning we fabricated CaF₂ WGM resonators with



Fig. 1. Typical WGM resonance of the calcium fluoride resonator. The experimental data are well fitted with a Lorentzian curve having 15 kHz full width at the half maximum (FWHM), which corresponds to $Q > 10^{10}$.



Fig. 2. Experimental setup.

Q factors exceeding 10^{10} (Fig.1), which allowed us to realize an efficient optical oscillator.

In what follows we describe our experiment and propose a theoretical model for the observed process.

II. EXPERIMENT

In this section we report on the observation of low threshold optical hyper-parametric oscillations in a high-Q (6×10^9) crystalline CaF₂ whispering gallery mode resonator. We send a monochromatic linearly polarized 1.32 μ m optical wave into the resonator and find that the outgoing light from the resonator is phase modulated with the frequency ~ 8 GHz which corresponds to the free spectral range of the resonator. The generated microwave signal has a narrow (≤ 40 kHz) linewidth. The modulation appears only after exceeding a distinct power threshold (~ 1 mW) for the optical pump.

The scheme of our experimental setup is shown in Fig. 2. Light from a 1.32 μ m YAG laser is sent into a CaF₂ WGM resonator with a glass coupling prism. The laser linewidth is less than 5 kHz. The maximum coupling efficiency is better than 50%. The coupling can be tuned by adjusting the gap between the prism and the resonator. CaF_2 resonators are fabricated with commercially available optical windows by polishing the cylindrical preforms. A typical resonator has a toroidal shape with a diameter of several millimeters and thickness in the range of several hundred microns. The resonators possess Q-factors on the order of $10^9 - 10^{10}$.

The output light of the resonator is collected into a single-mode fiber after the coupling prism, and is split into two equal parts with a 50/50 fiber splitter. One output of the splitter is sent to a slow photodiode D1 that produces a DC signal used for locking the laser to a particular resonator's mode. The other output is mixed with a delayed laser light that has not interacted with the resonator, and the mixed signal is directed to fast photodiode D2. With this configuration the disc resonator is placed into an arm of a tunable Mach-Zehnder interferometer. If the delay between the interferometer branches is correctly chosen we observe a narrowband microwave signal emerging from the photodiode.

The locking loop enables us to introduce any amount of optical power into the resonator, which would be a problem otherwise. This is because the spectrum of the resonator can drift due to thermal and Kerr effects. The larger the laser power, the larger do the modes shift, resulting in a reduced total power accumulation in the resonator. The feedback loop compensates for this shift and changes the laser frequency in such a way as to keep a significant amount of light in the mode. The new mode frequency is determined by the amount of the optical power absorbed in the resonator as well as by the heat exchange of the cavity with the external environment.

The Mach-Zehnder interferometer is important because it allows the study of the phase-modulated light. We found that if the light from the resonator is directly sent to a fast photodiode it would not generate any microwave signal above the noise floor of our spectrum analyzer. However, if the resonator is placed into an arm of the interferometer and the delay in the second arm of the interferometer is correctly chosen, the modulation appears. This is a distinct property of phase modulated light.

The results of our measurements are shown in Figs. 3 and 4. The typical spectrum of the generated microwave signal is depicted in Fig. 3. The nonlinear interaction of light and CaF_2 resonator results in a clean microwave signal. We found that the signal frequency is centered at the FSR frequency of the resonator, and is stable with tem-



Fig. 3. Microwave signal on the outlet of optical detector and 22 dB amplifier. The signal is generated by the light emerging CaF₂ disc. Carrier microwave frequency corresponds to δ_{FSR} .

perature, pump power, and coupling changes, because the FSR frequency of the resonator does not significantly changes with any of those parameters.

We measured the value of the microwave power as a function of the optical pump power (Fig. 4) to find the efficiency of the parametric process. Using this curve we can estimate the modulation efficiency. The transformation of the amplitude modulated light at power P_0 and modulation depth $\mu \left[P = P_0(1 + \mu \cos \omega_{mw} t)\right]$ into microwaves with power P_{mw} by means of a photodiode is determined by $P_{mw} = R^2 \rho \mu^2 P_0^2/2$, where R is a transformation coefficient of the optical power to a photocurrent, and ρ is the resistance at the output of the photodiode. The typical values are R = 0.7 A/V, and $\rho = 50 \Omega$. With this expression in hand we find that in our setup (Fig. 2) the maximum ratio of $\sim 7\%$ between a sideband power and the pump power is achieved at 4 mW input optical power. Increasing the pumping power results in a gradual decrease of the sideband power because of the generation of higher harmonics.

III. THEORY

To explain the results of our experiment we consider three cavity modes: one nearly resonant with the pump laser and the other two nearly resonant with the generated optical sidebands. We begin with the following equations for the slow amplitudes of the intracavity fields

$$\dot{A} = -\Gamma_0 A + ig [|A|^2 + 2|B_+|^2 + 2|B_-|^2]A + 2ig A^* B_+ B_- + F_0, \qquad (1)$$

$$B_{+} = -\Gamma_{+}B_{+} + ig[2|A|^{2} + |B_{+}|^{2} + 2|B_{-}|^{2}]B_{+} + igB_{-}^{*}|A|^{2}, \qquad (2)$$

$$\dot{B}_{-} = -\Gamma_{-}B_{-} + ig[2|A|^{2} + 2|B_{+}|^{2} + |B_{-}|^{2}]B_{-} + igB_{+}^{*}|A|^{2}, \qquad (3)$$



Fig. 4. Plot of the microwave power on the output of the optical detector versus 1.32 μ m pumping light power. The microwave frequency corresponds to the δ_{FSR} of the disc. The solid line is a guide for the eye.

where

$$\begin{split} \Gamma_0 &= i(\omega_0 - \omega) + \gamma_0, \\ \Gamma_+ &= i(\omega_+ - \tilde{\omega}_+) + \gamma_+, \\ \Gamma_- &= i(\omega_- - \tilde{\omega}_-) + \gamma_-, \end{split}$$

 ω_0, ω_+ , and ω_- are the eigenfrequencies of the optical cavity modes; ω is the carrier frequency of the external pump (A), $\tilde{\omega}_+$ and $\tilde{\omega}_-$ are the carrier frequencies of generated light $(B_+$ and B_- respectively). These frequencies are determined by the oscillation process and can not be controlled from the outside. However, there is a ratio between them (energy conservation law): $2\omega = \tilde{\omega}_+ + \tilde{\omega}_-$. Dimensionless slowly varying amplitudes A, B_+ , and B_- are normalized such that $|A|^2$, $|B_+|^2$, and $|B_-|^2$ describe photon number in the corresponding modes.

$$g = \omega_0 \frac{n_2}{n_0} \frac{\hbar \omega_0 c}{\mathcal{V} n_0},\tag{4}$$

is a coupling constant, n_2 is an optical constant that characterizes the strength of the optical nonlinearity, n_0 is the linear refractive index of the material, \mathcal{V} is the mode volume, c is the speed of light in the vacuum. Deriving this coupling constant we assume that the modes are nearly overlapped geometrically, which is true if the frequency difference between them is small. The force F_0 stands for the external pumping of the system $F_0 = (2\gamma_0 P_0/\hbar\omega_0)^{1/2}$, where P_0 is the pump power of the mode applied from the outside.

Our model is oversimplified because it does not take into account neither the temperature nonlinearity of the resonator nor the properties of the feedback loop locking the frequency of the pump laser to the frequency of the chosen mode of the resonator. However the model allows finding the oscillation threshold as well as the oscillation frequency of the system.

For the sake of simplicity we assume that the modes are identical, i.e. $\gamma_+ = \gamma_- = \gamma_0$, which is justified by observation of actual resonators. Then, solving the set (1-3) in steady state we find the oscillation frequency for generated fields

$$\omega - \tilde{\omega}_{-} = \tilde{\omega}_{+} - \omega = \frac{1}{2}(\omega_{+} - \omega_{-}), \qquad (5)$$

i.e. the beat-note frequency depends solely on the frequency difference between the resonator modes and does not depend on the light power or the laser detuning from the pumping mode. As a consequence, the electronic frequency lock circuit changes the carrier frequency of the pump laser but does not change the frequency of the beat-note of the pumping laser and the generated sidebands.

We considered only three interacting modes in the model, however the experiment show that a larger number of modes could participate in the process. The number of participating modes is determined by the equidistance of the modes of the resonator. Generally, modes of a resonator are not equidistant because of the second order dispersion of the material and the geometrical dispersion given by the mode structure. The geometrical dispersion is comparably small in our case. However the dispersion of the material is large enough. Using Sellmeier dispersion equation we find that approximately three sideband pairs can be generated in the system (we see only two in the experiment).

Threshold optical power can be found from the steady state solution of the set (1-3)

$$P_{th} \simeq \frac{\pi}{2} \frac{n_0^2 \mathcal{V}}{n_2 \lambda Q^2},\tag{6}$$

where we neglected by the influence of the selfphase modulation effects on the oscillation threshold. Theoretical threshold value for our experiment is $P_{th} \approx 0.2$ mW, where $n_0 = 1.44$ is the refractive index of the material, $n_2 = 3.2 \times 10^{-16} \text{ cm}^2/\text{W}$ is the nonlinearity coefficient for calcium fluoride, $\mathcal{V} \approx 10^{-4} \text{ cm}^3$ is the mode volume, $Q = 6 \times 10^9$, and $\lambda = 1.32 \ \mu\text{m}$.

We derive also the expression for the phase diffusion coefficient for the beat note of the carrier and sidebands generated in our system

$$D_{beat} \simeq \frac{\gamma_0^2}{4} \frac{\hbar\omega_0}{P_{B out}} \times$$

$$\left[1 + \left(\sqrt{\left(\frac{P_0}{P_{th}}\right)^2 - 1} + \frac{P_{B out}}{P_{th}} \right)^2 \right]$$
(7)

where $P_{B out} = \hbar \omega_0^2 |\langle B \rangle|^2 / Q_0$ is the output power in a sideband. Allen deviation is $\sigma_{beat}/\omega_{beat} = (2D_{beat}/t\omega_{beat}^2)^{1/2}$. We could estimate $\sigma_{beat}/\omega_{beat} \simeq 10^{-13}/\sqrt{t}$ for $\gamma_0 = 3 \times 10^5$ rad/s, $P_{B out} = 1$ mW, $\omega = 1.4 \times 10^{15}$ rad/s, and $\omega_{beat} = 5 \times 10^{10}$ rad/s.

It is easy to see from Eq.(6) that the efficiency of the parametric process increases with a decrease of the mode volume. We used a relatively large WGM resonator because of the fabrication convenience. Reducing the size of the resonator could result in a dramatic reduction of the threshold for the oscillation. Since the mode volume may be roughly estimated as $\mathcal{V} \approx 2\pi\lambda R^2$, it is clear that reducing the radius R by an order of magnitude would result in two orders of magnitude reduction in the threshold of the parametric process.

IV. CONCLUSION

We have reported on a new architecture for a photonic microwave source. This source, based on an ultra-high quality factor crystalline resonator, generates all-optically a narrow-band stable microwave signal. The linewidth of the signal depends on the Q-factor of the resonator. Measured Q of the CaF₂ resonator used in our setup exceeds 2×10^{10} , theoretically this value may be three orders of magnitude higher. Because crystals may possess high nonlinearity and low optical losses it is possible to further improve the quality of the fabricated oscillator by several orders of magnitude that makes the oscillator promising for many high performance microwave applications.

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