Hyper-parametric oscillations in a WGM fluorite resonator

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Thanks: NASA, DARPA
... Come in various shapes and sizes.

We are mostly interested in the disk resonators. They:
- have cleaner spectrum;
- are ideal for electro-optical applications;
- may have very small mode volume (good for nonlinear optics).

Low microwave power EO modulators
Continuously tunable, narrow-band pass and stop optical filter

[References available]
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Typical WGM spectrum

\[ Q = \frac{\omega}{\gamma} > 2 \times 10^{10} \]
Important parameters

\[ Q \]

Cavity field build-up factor and linewidth

\[ \frac{Q^n}{V} \]

Small size and optical energy density
E.g. \( V = \pi \times 1 \text{ mm} \times 100 \text{ \mu m} \times 10 \text{ \mu m} \)

Figure of merit for nonlinear processes:

Purcell's factor: \( n=1 \)
SRS and FWM: \( n=2 \)
Frequency doubling: \( n=3 \)

[V.S. Ilchenko et al., JOSA B 20, 1304 (2003)]
For crystalline resonators, linewidth is ultimately determined by the material absorption $\alpha$:

$$2\gamma^{-1} = n_0 (\alpha c)^{-1}$$

$$\Rightarrow \quad Q = \frac{2\pi n}{\alpha \lambda}$$

For $\alpha \simeq \alpha_{UV} e^{\lambda_{UV}/\lambda} + \alpha_R \lambda^{-4} + \alpha_{IR} e^{-\lambda_{IR}/\lambda}$

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Hyper-parametric oscillations in fluorite resonators

\[ Q = 2 \times 10^{10} \text{ at } \lambda = 1310 \text{ nm} \]

**Selection rules**

*FWM: TE-TE*

*SRS: TE-TM*

**Transition diagram**

\[ \omega_0 \quad \omega_+ \quad \Omega_{\text{FSR}} \]

**Optical spectrum**

\[ \omega_- \quad \omega_0 \quad \omega_+ \]
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Microwave beat note observed

Second-order ($2\Omega_{\text{FSR}}$) beat note is insignificant

Raman scattering is not observed (expected at 322 cm$^{-1}$)

A.A. Savchenkov et al.,
Submitted to PRL (2004)
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Analysis

Kerr Hamiltonian: \( H = H_0 + V, \quad H_0 = \hbar \omega_0 a'^t a + \hbar \omega_+ b'^t_+ b_+ + \hbar \omega_- b'^t_- b_- \), where

\[
V = -\frac{\hbar g}{2} (a'^t a a + b'^t_+ b_+ b_+ + b'^t_- b_- b_-) - 2\hbar g (b'^t_+ b_+ b_- + a'^t_- b_+ a + a'^t_- b_- a)
\]

Self-phase modulation

\[
-\hbar g (b'^t_- b_+ a + a'^t_+ b_+ b_-)
\]

Cross-phase modulation

Four-wave mixing

Equations of motion in an open system:

\[
\begin{align*}
\dot{a} &= -(i\omega_0 + i\kappa(T) + \gamma_0 + \gamma_{c0})a + ig[a'^t a + 2b'^t_+ b_+ + 2b'^t_- b_-]a + 2iga'^t b_+ b_- + f_0 + f_{c0}, \\
\dot{b}_+ &= -(i\omega_+ + i\kappa(T) + \gamma_+ + \gamma_{c+})b_+ + ig[2a'^t a + b'^t_+ b_+ + 2b'^t_- b_-]b_+ + igb'^t a + f_+ + f_{c+} \\
\dot{b}_- &= -(i\omega_- + i\kappa(T) + \gamma_- + \gamma_{c-})b_- + ig[2a'^t a + 2b'^t_+ b_+ + b'^t_- b_-]b_- + igb'^t a + f_- + f_{c-}
\end{align*}
\]

Temperature tuning

SPM and CPM

FWM

Where \( \langle f_{c0} \rangle = \sqrt{\frac{2\gamma_{c0} P_0}{\hbar \omega_0}} e^{-i\omega t} \) and \( g = \omega_0 \frac{n_2 \hbar \omega_0 c}{n_0 \nu n_0} \).
Results of analysis

1. Complex dynamics of the system (laser lock helps)

2. Threshold power: \( P_{th} \approx 1.54 \frac{\pi \gamma_0 + \gamma_c}{2 \gamma_c^2} \frac{n_2^2}{\lambda Q^2} \)

3. Phase modulation (when critically coupled)

4. Beat note frequency \( \omega - \omega_0 = \omega_+ - \omega = \frac{1}{2}(\omega_+ - \omega_-) \)
   is independent on the nonlinear dynamics of the system

5. Phase diffusion is very low: \( D_{min} = \frac{(\gamma_c + \gamma_0)^2}{2} \frac{\tilde{\omega}}{P_{B_{opt}}} = \frac{\omega_0^2 \gamma_0 + \gamma_c}{2 \gamma_c} \frac{\hbar \omega_0 n_2 \lambda}{4 \pi \lambda n_0^2} \)

Experiment

To be tested
Summary and conclusions

Very high Q-factor and small volume allow us to reach the oscillation threshold for the four-wave mixing process in WGM disk resonators with low power DC optical fields.

The generated fields can be represented as the sidebands of the phase-modulated carrier (the pump). The sidebands frequency is very stable in spite of complex mode dynamics of the system, which suggests the possibility of its application as a secondary frequency standard.

The absence of the SRS in the pump polarization (TE) is a consequence of the selection rules; however its absence in the TM polarization is surprising and is in contradiction with the results for amorphous materials [S.M. Spillane, T.J. Kippenberg, K.J. Vahala, Nature 415, 621 (2002)]. This may be due to asymmetry of the Brillouin zone in crystals. Further research is needed.

Just like parametric down conversion, hyper-parametric conversion can produce nonclassical (e.g. entangled or squeezed) light. We plan on carrying out the research of quantum optical properties of our system.
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Earlier results on CW four wave mixing in amorphous materials


CS$_2$ droplets


Fused silica microspheres