Abstract—The design, error budget, and preliminary test results of a 50-56 GHz synthetic aperture radiometer demonstration system are presented. The instrument consists of a fixed 24-element array of correlation interferometers, and is capable of producing calibrated images with 0.8 degree spatial resolution within a 17 degree wide field of view. This system has been built to demonstrate performance and a design which can be scaled to a much larger geostationary earth imager. As a baseline, such a system would consist of about 300 elements, and would be capable of providing contiguous, full hemispheric images of the earth with 1 Kelvin of radiometric precision and 50 km spatial resolution. An error budget is developed around this goal and then tested with the demonstrator system. Errors are categorized as either scaling (i.e. complex gain) or additive (noise and bias) errors. Sensitivity to gain and/or phase error is generally proportional to the magnitude of the expected visibility, which is high only in the shortest baselines of the array, based on model simulations of the earth as viewed from geostationary earth orbit (GEO). Requirements range from approximately 0.5% and 0.3 degrees of amplitude and phase uncertainty, respectively, for the closest spacings at the center of the array, to about 4% and 2.5 degrees for the majority of the array. The latter requirements are demonstrated with our instrument using relatively simple references and antenna models, and by relying on the intrinsic stability and efficiency of the system. The 0.5% requirement (for the short baselines) is met by measuring the detailed spatial response (e.g. as measured on the antenna range), and by using an internal noise diode reference to stabilize the response. This result suggests a hybrid image synthesis algorithm in which long-baselines are processed by Fast Fourier Transform (FFT), and the short baselines are processed by a more precise (G-matrix) algorithm which can handle small anomalies among antenna and receiver responses. Visibility biases and other additive errors must be below about 1.5 millikelvin on average, regardless of baseline. This requirement is largely met with a phase shifting scheme applied to the
local oscillator distribution of our demonstration system. Low mutual coupling among the horn antennas of our design is also critical to minimize biases caused by cross-talk of receiver noise. Performance is validated by a 3-way comparison between interference fringes measured on the antenna range, solar transit observations, and the system model.

Keywords- remote sensing; radiometer; interferometer.

I. INTRODUCTION

GeoSTAR is a concept to provide high spatial resolution soundings of the earth's atmosphere from geosynchronous earth orbit (GEO) in discrete microwave bands from 50 GHz to 180 GHz [1] (also see related papers in this issue). Images of the earth are synthesized by Fourier Transform of interferometric data collected with a Y-array of correlation interferometers. The concept eliminates the need for large mechanically scanned apertures, but it poses many new challenges-- particularly in the area of calibration. A large spaceborne system will involve hundreds of antennas and many tens of thousands of correlators. Costs associated with design choices will be high, so it is imperative to develop an error model and to demonstrate how requirements will be met with real hardware. This paper presents preliminary results of a small (24-element) 50-56 GHz system which has been built under NASA's Instrument Incubator Program to provide such a demonstration.

Our design is based on aperture synthesis techniques originally developed for radio astronomy, and applied more recently to earth remote sensing. The first such application was the Electronically Scanned Thinned Array Radiometer (ESTAR) [2], which was a 1-dimensional synthesis array operating at L-band to measure soil moisture. This small (5-element) aircraft system viewed a wide pushbroom swath, and was subject to a high degree of mutual coupling and array embedding effects which were difficult to model. This problem lead to the so called G-matrix calibration, where images are synthesized by inversion of interferometric fringes measured on the antenna range [3]. With this approach, the accuracy of the images depend on the quality of the antenna range measurements and the degree to which such measurements accurately represent the operational configuration (i.e. as installed in an aircraft or spacecraft structure). More recently, the European Space Agency has advanced the Microwave Imaging Radiometer by Aperture Synthesis (MIRAS) instrument for the Soil Moisture Ocean Salinity (SMOS) mission. Scheduled for launch in 2007, this 69-element system is a 2-D imager configured as a Y-array of closely spaced patch antenna elements with a wide field of view (FOV) appropriate for low earth orbit. Like ESTAR, the broad antenna pattern of this system is subject to significant array embedding effects and mutual coupling which must be precisely measured and then accounted for in the inversion [4, 5]. These measurements are quite costly, and possibly impractical for GeoSTAR. A large system with 300 elements or more is envisioned for each of three observation bands of GeoSTAR. It will be difficult to measure the G-matrix with very high precision for such a large array. Moreover, the inversion of such a large data set-- which would involve the inversion of a 30,000 x 30,000 matrix-- poses a major challenge in itself.
Our approach in GeoSTAR is to seek a design which does not depend so heavily on measurements of the antenna responses, nor on the inversion of such large matrices. To the extent possible, we seek a design which can be characterized by a single well matched antenna pattern which is predictable and uniform among all elements of the array. If this can be achieved, then the synthesis problem becomes much simpler, and can possibly be performed by much more efficient Fast Fourier Transform-- rather than the numerically intensive G-matrix. One advantage with GeoSTAR is that observations of the earth are made in a relatively narrow, 17 degree wide FOV. This allows for a larger elemental antenna aperture, which offers more design options to reduce mutual coupling and other array embedding problems.

Following an overview of the instrument concept in Section II, we present an error budget in Section III. This establishes some priorities for the design which is presented in Section IV. Processing algorithms are discussed in section V along with some preliminary test results. The end-to-end performance is measured on the antenna range in Section VI. Some fundamentals of the interferometer mathematics and of the synthesis process are summarized in the Appendix, for reference.

II. INSTRUMENT CONCEPT

GeoSTAR consists of a Y-array of receivers configured in the geometry of Figures 1 and 2. The antennas share the same field of view (FOV) and the IF signals of all receivers are simultaneously cross-correlated against one another in a digital subsystem. Each correlated antenna pair forms an interferometer which measures a particular spatial harmonic of the brightness temperature across the FOV. When expressed as a function of antenna spacing-- or “baselines” with dimensions u and v by astronomy convention-- this complex cross-correlation is called the visibility function. The visibility function is the Fourier transform of the brightness temperature image in the FOV, as weighted by the elemental antenna pattern. The mathematics of this technique are well established, and summarized in Appendix I for reference. With sufficient sampling of visibility over a range of spacings one can reconstruct, or synthesize, a 2-D image by inverse transform. The “Y” configuration provides the needed samples using a minimum number of antennas and with a fixed geometry-- in a so called thinned array. As illustrated in Fig. 2, the spacings between the various antenna pairs yield a uniform hexagonal grid of visibility samples in the u-v plane. There are 8 elements in each arm of Fig. 2, and this yields 64 unique u and v spacings when the x and y positions of arm 1 are subtracted from arms 2, for example. Another 64 conjugate-symmetric samples are derived by subtracting arm 2 from arm 1. In all, the 24-element system produces 384 UV samples (=6*64). Note, in this particular layout, that all of the UV samples are formed between elements in different arms, and that none are necessarily formed between elements within an arm. This scheme (we call the “staggered-Y”) simplifies the electrical and mechanical design, as detailed in Section IV.

The smallest spacing of the sample grid in Fig. 2 determines the unambiguous field of view (UaFOV), which for GEO observations has a special interpretation. For the
hexagonal u-v sample grid with spacing $d$ in Fig. 2, sources in the FOV are aliased periodically every $\frac{2}{\sqrt{3}} \frac{\lambda}{d}$ radians in the image plane along three axes: one horizontal and two diagonals separated by 120 degrees. This establishes a hexagonal region within which images are synthesized. In our application, we fit this region to match the earth disk diameter of 17.5° when viewed from GEO. This sets both the antenna element spacing and diameter at about 3.75 wavelengths, or 2.25 cm at 50 GHz. At 56 GHz the wavenumber spacing will increase, so this spacing does imply that the earths limb will impinge slightly on the aliased regions. This is a reasonable tradeoff, however, since these regions are of little value for atmospheric sounding given the shallow incidence angles involved. But note that, strictly speaking, there is no “unambiguous” FOV for GeoSTAR. The elemental antenna patterns do not end abruptly at the edge of this region, so the brightness in the surrounding aliased regions must be known and corrected in the image processing. In space this does not pose a problem since the temperature of the cosmic background is well known. But this does play a role in our ground based demonstrator instrument, which must be tested in an ambient environment.

The longest spacing determines the smallest spatial scale that can be resolved. The synthetic aperture diameter of Fig. 1 is 60 cm, which yields about 0.8 degrees of angular resolution for the demonstrator system. 50 km spatial resolution on the earth will require about 100 receiving elements per array arm in a GEO system. This will produce 60,000 UV samples and 60,000 linearly independent image pixels within the FOV. Note that the Fourier Transform provides a 1-to-1 mapping, which ensures that there are N linearly independent pixels in an image which is synthesized from N linearly independent u-v samples. This property provides an alternative calculation of the spatial resolution: the area covered by the hexagonal image plane is $\frac{2}{\sqrt{3}} \frac{\lambda^2}{d^2}$, so N linearly independent pixels within this region implies a linear resolution of $\frac{\lambda}{d} \sqrt{\frac{2}{N\sqrt{3}}}$ radians near the center of the FOV. At a geostationary altitude of 36,000 km and with N=60,000, this calculation estimates a resolution of 42 km. This is consistent with the array factor of reference [2].
III. ERROR BUDGET

Our design is based on an overall calibration requirement of 1 Kelvin error in the synthesized brightness temperature image of the earth using a large array of 300 elements. Our analysis arbitrarily divides the error budget equally between categories of “gain” and of “additive” errors. Gain and additive errors are presumed independent, so an equal split of the 1 K overall error implies 0.7 K (= 1 K / (SQRT(2))) allocations for each of these categories. Gain errors include anything that results in an uncertain amplitude scaling or phase shift in the visibility measurements. These include uncertainties in elemental antenna patterns and array alignment, as well as uncertainties of the gain, efficiency, and phase response of the correlators. Additive errors include correlator biases (null offsets) and measurement noise (set by system noise, bandwidth, and integration time). Additive errors are measured in units of Kelvin, whereas gain errors are expressed as a percentage and/or degrees of phase and must be scaled to Kelvin by the magnitude of the expected signal. Gain is a complex value, so gain and phase error specifications are often redundant: 0.01 radian of phase error usually has the same impact as 1 % of gain magnitude error since they both represent the same displacement of visibility in the complex plane.
In the simplest analysis of gain errors, we can divide the 0.7K budget allocation by an approximate 260K mean earth temperature to arrive at a requirement of 0.3% to be applied uniformly to the entire array. Yet this is a difficult requirement to meet, and we know that most of the signal in GeoSTAR is contained in the shortest baselines of the visibility function. A better analysis takes this into account. Fig. 3 plots the RMS magnitude of visibility versus u-v baseline, as computed from an AMSU 52.8 GHz brightness temperature map of the earth with the current GeoSTAR antenna model. Here we see that only the zero-baseline channel exceeds 100 Kelvin, and that only the shortest baselines of less than 20 wavenumbers are in the 1 to 5 Kelvin range. Past about 100 wavenumbers the visibilities are below 0.1 Kelvin\(^1\). This indicate that GeoSTAR should be much less sensitive to gain errors for larger baselines. This is good from a hardware standpoint, since for example, it permits greater mechanical error towards the ends of the array arms. However, it is not appropriate to allocate too much error to the longer baselines since there are a great many more visibilities with the longer baselines, and this will tend to weight these errors more heavily the image. A more judicious distribution of errors is needed which balances the practical hardware limitations with the sensitivities and numbers of correlators in the overall array.

Table 1 provides a gain budget that accounts for both the magnitudes and the numbers of visibility samples. Here, we have grouped visibilities by their distance from the center of the u-v plane, and distributed the errors by applying the rule that image errors are the RSS of visibility errors\(^2\). In order to distribute error allocations in a reasonable manner, Table 1 subjectively divides the u-v plane into eight annular regions, centered on the origin, as specified in the first column. These regions are progressively larger (roughly in powers of 2 with each region) and encompass ever greater numbers of visibility samples (in column 2) with distance from the origin. The RMS visibilities from Fig. 3 are summarized in column 3, but it is the RSS visibility in column 4, and the 0.25 K delta-T (=0.7 K /\text{SQRT}(8)) allocation in column 6 that determines the delta-G error in column 5. The delta-G requirement is computed as 0.5 * delta-T divided by the RSS visibility. The factor of 0.5 is a nominal number which is needed to account for the antenna pattern scaling which occurs during image synthesis. For GeoSTAR, and as discussed in the next section and Appendix I, this amounts to a scaling by a factor of about 1.6 near the

\(^1\) GeoSTAR visibilities are dominated by the contrast at the earth’s limb in all observation bands. The dashed line of Fig. 3 represents the same data after subtracting the a constant temperature from the earth disk. This dashed line shows that the contribution from variability within the earth is nearly an order of magnitude smaller than the contribution from the limb. This is interesting because it shows how requirements might change if GeoSTAR were provided an initialization from other sources (e.g. low earth orbit observations, or climate averages). However, our present goal is an absolute (not relative) calibration, so we do not yet consider this in our error budget.

\(^2\) This is a basic property of the Fourier Series (energy conservation), and worth noting because it is central to the discussion: To a first order, the root-mean-square (RMS) brightness of the image is always equal to the root-sum-square (RSS) of the visibilities. One Kelvin applied to N visibility samples always adds SQRT(N) Kelvin to the RMS brightness temperature. It does not matter whether the visibilities are coherent or random in phase, provided that the RMS temperature is evaluated over the entire image plane. This relation is only modified if we weight the aperture (e.g. to change sidelobe levels) or scale the image to compensate for the antenna patterns or the mismatches among receivers.
center of the FOV, and about 3 near the earth limb. We use a nominal factor of 2 here, but note that there is an inherent degradation of delta-T near the edges of the image plane where the antenna beam tapers off.

Table 1 shows that a 4% gain error is acceptable for the great majority of visibility samples. This corresponds to about 2.3 degrees of phase error in the complex plane (or about 0.2 mm of mechanical alignment for the 0.6 cm wavelength). Only a few baselines near the center of the array need a more precise calibration. This establishes top level gain requirements for the correlators, and must be further divided into allocations for array distortions, receiver gain, and antenna pattern errors. The zero-baseline is the only channel requiring a 0.1% calibration. This will actually be measured with a conventional Dicke radiometer (not shown in Fig. 1 or 2) using an identical antenna to those in the rest of the array. The error allocations of Table 1 change only slightly if the array size increases or decreases (note that the last row of Table 1 represents a large part of the array, but a relatively small part of the overall budget).

![Fig. 3. RMS visibility versus UV baseline from GeoSTAR model and AMSU 52.8GH brightness temperatures. The dashed line represents the variability within the earth disk after having removed the mean earth temperature and the contrast at the earth limb.](image)
Additive errors include correlator biases (null offsets) and the basic measurement noise set by system noise, bandwidth, and integration time. The latter is relatively constant among all visibility samples, so it is necessary to keep this noise below 0.5*0.7 K /\sqrt{N}, where N is the number of visibility samples in the Fourier Series. The factor of 0.5 again accounts for the scaling of the earth image by the antenna pattern. With N=60,000 (real valued\(^3\)) samples, the RMS visibility errors must be less than about 1.5 mK. The visibility noise (delta-V) for bandwidth, B, integration time, \(\tau\), and system noise temperature \(T_s\) using a 1-bit correlator is

\[
\Delta V = \frac{\pi T_s}{2\sqrt{2B\tau}} . \tag{3}
\]

The system noise is about 500K in our instrument, and the double sideband bandwidth is 200 MHz. So 1.5 mK implies a minimum of \(\tau = 740\) seconds. This is a minimum which does not yet allocate any of the additive budget to the visibility biases. We wish to avoid increasing this integration time any further, so we will pursue a design which keeps

\(^3\) Note that we use N=60,000-- which is the number of independent real-valued visibilities, whereas Table 1 has N=30,000 complex correlations. Complex gain gas two parts: magnitude and phase, and it is the RSS of these two that must meet the delta-G requirement of Table 1. This is perhaps an inconsistency in notation, but it does have some basis in the way we process the data. In our processing algorithms we calibrate visibilities as complex numbers, and then re-cast the visibilities as real valued (odd and even pairs) to save computer time in the final image synthesis.

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<table>
<thead>
<tr>
<th>range (wavenumber)</th>
<th>count</th>
<th>RMS (K)</th>
<th>RSS (K)</th>
<th>(\Delta G) (%)</th>
<th>(\Delta T) (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 3</td>
<td>1</td>
<td>108.8</td>
<td>108.8</td>
<td>0.1</td>
<td>0.25</td>
</tr>
<tr>
<td>3 - 9</td>
<td>9</td>
<td>5.7</td>
<td>17.0</td>
<td>0.7</td>
<td>0.25</td>
</tr>
<tr>
<td>9 - 21</td>
<td>45</td>
<td>1.2</td>
<td>8.2</td>
<td>1.5</td>
<td>0.25</td>
</tr>
<tr>
<td>21 - 45</td>
<td>177</td>
<td>0.5</td>
<td>6.6</td>
<td>1.9</td>
<td>0.25</td>
</tr>
<tr>
<td>45 - 93</td>
<td>765</td>
<td>0.18</td>
<td>5.1</td>
<td>2.4</td>
<td>0.25</td>
</tr>
<tr>
<td>93 - 189</td>
<td>3126</td>
<td>0.056</td>
<td>3.1</td>
<td>4.0</td>
<td>0.25</td>
</tr>
<tr>
<td>189 - 381</td>
<td>12639</td>
<td>0.024</td>
<td>2.7</td>
<td>4.6</td>
<td>0.25</td>
</tr>
<tr>
<td>381 - 683</td>
<td>13239</td>
<td>0.024</td>
<td>2.7</td>
<td>4.6</td>
<td>0.25</td>
</tr>
<tr>
<td>net: 30,001</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.7</td>
</tr>
</tbody>
</table>

Notes:
1. Range of radii from center of u-v plane specify annular rings on u-v plane.
2. “count” of independent complex visibility samples which fall in each range.
3. RMS visibility in this u-v range (from Fig. 3).
4. RSS visibility in this range (=RMS*\sqrt{count}).
5. Gain error allocation for these visibilities which result in a net 0.7K \(\Delta T\) in the image, as computed from \(0.7K/(2*RSS*\sqrt{8})\). The factor of 2 accounts for antenna pattern beam efficiency within the earth disk, and the root(8) represents the number of u-v range bins in this table.
6. Image \(\Delta T\) allocated to this range bin (=0.7 K gain allocation / \sqrt{8})
biases well below the measurement noise. Our results, as discussed below, show that extremely small biases of 0.5 mK are achievable for back-end errors (e.g. digitizer null offsets and common mode noise from the LO). The mutual coupling of front-end receiver noise between antennas ultimately dominates the biases otherwise. But this problem is found to be prevalent only in the short baselines of the array, where the coupling is highest. This is permissible since these short baselines represent a small fraction of the complete array.

IV. DEMONSTRATOR INSTRUMENT

The GeoSTAR demonstrator instrument operates at 4 AMSU channels between 50 and 56 GHz. A simplified block diagram is given in Fig. 4. From left to right in Fig. 4 (or front to back in Fig. 1) the signal starts at the horn apertures with horizontal polarization. The horns are a variant of a Potter horn which adds a parabolic profile section to broaden the useful RF band to 50-56 GHz. The design permits very close spacing in the array while maximizing the aperture area. This is important for the GEO observation as it maximizes the fraction of the antenna energy which is received from the earth disk. This fraction (we call “earth disk beam efficiency”) is about 42% with this design. We also tested a straight taper conical horn with uniform E-plane illumination which increases this fraction to 48%. However, the mutual coupling of that horn is high, and when tested on the antenna range we found that this design was subject to significant array embedding effects which perturbed the radiation patterns at the 5% to 10% level. The aperture taper provided by the parabolic Potter design reduces these effects, which simplifies the antenna modeling problem. The low coupling is also crucial for controlling correlation biases caused by the leakage of receiver noise from one antenna to another. To further suppress biases from coupling, ferrite isolators (not shown in the diagram) are added to the six closest elements at the center of the array where the coupling is greatest (about -60 dB between adjacent elements). Beyond these elements, the majority of the array operates without isolators. This strategy lowers the overall noise figure of the system.

Each horn incorporates a circular to rectangular transition followed by a waveguide twist of either 0, +60, or -60 degrees to match the orientation to the three arms in Fig. 1. These twists provide a simple and precise polarization alignment. We considered circular polarizers, but found they were not easily balanced through the 10% bandwidth.

The signal in Fig. 4 next passes through an 8-way calibration feed manifold which periodically injects a noise signal into all receivers from a common noise diode source. This signal provides a reference to stabilize the system against gain, phase, and receiver noise drifts. The injected signal reaches the receiver inputs with about 5 K equivalent noise temperature. The noise diode signal is distributed to the three arms via phase shifters. These shifters were intended as a means to resolve the quadrature balance of each correlator, but later proved redundant with other circuits described below.

Next, the antenna signal passes into the MMIC receiver module where it is amplified using InP FET low noise amplifiers and then double-sideband downconverted by subharmonic quadrature mixers to two IF baseband signals. Receiver noise is about 400
K. Each receiver also contains a programmable bias circuit which can adjust the gate and drain voltages of each amplifier stage to affect gain and noise figure. This circuit was originally envisioned as another calibration tool (e.g. to switch off a receiver and thereby measure correlator biases), but it proved more useful to balance and tune the receivers during production and tests. The gain of the RF section is about 50 dB. We operate with such high gain to minimize the impact of common mode noise from the local oscillator, which will otherwise bias the correlations.

The local oscillator of Fig. 4 operates from 25 to 28 GHz to tune from 50 to 56 GHz at RF. The LO is distributed via three 2-bit phase shifters and amplifiers which incorporate power levelers. These periodically shift the phase to each arm by 45, 90, or 135 degrees, which results in shifts of 90, 180, and 270 degrees, respectively at RF. Constant LO power is ensured by an active circuit consisting of a coupled detector with feedback to control amplifier gain. As discussed in Section V, this circuit proved superior to the above noise diode phase shifters and to the amplifier controls when estimating quadrature balance and correlator biases.

The in-phase (I) and quadrature (Q) IF signals from each mixer are next amplified and low-pass filtered at 100 MHz. These are small and inexpensive lumped element filters. The phase match among these filters is excellent across the band, resulting in very high (>99%) efficiency in the correlators. The IF signals are then digitized at a clock rate of 200 MHz. For reasons of product availability the analog to digital converters are presently 8-bit devices, but these could be replaced with one-bit converters (i.e. comparators). The correlators only use 1-bit (the sign bit). One-bit correlators require the least power with a relatively minor penalty in sensitivity, which is a fair tradeoff given the great number of correlators required by GeoSTAR. The correlator of Fig. 4 is implemented in an FPGA. This system calculates all possible correlations to be formed between the 24 elements, but we actually only use correlations which are formed between different array arms. We do not use the correlations formed among elements within each arm since these are redundant in Fig. 2.

![Fig 4. Receiver system block diagram – one receiver of 24 shown](image-url)
V. DATA PROCESSING AND EARLY TEST RESULTS

The 1-bit correlations are first mapped to linear correlations using the Van Vleck formula [8]. This removes the nonlinearity of a 1-bit correlator when the input signals are known to be Gaussian. This step is applied to all four correlators associated with each antenna pair. Each antenna is associated with an “I” and a “Q” IF signal, so each antenna pair is associated with four correlators: “II”, “QQ”, “QI” and “IQ”. This represents a two-fold redundancy in our data which we use to reduce measurement noise. If there were no biases, and if the subharmonic mixers of Fig. 4 were perfectly balanced in quadrature, then these four correlations could be immediately combined into a single complex correlation. Yet the quadrature balance is known to be poor-- on the order of 10 degrees of phase-- and the raw correlations are known to contain large biases due to digitizer null offsets and leakage of correlated noise from the LO. To fix this, the LO phases are shifted in a sequence that rotates all correlations to all four phase quadrants. The exact phase shifts are determined from network analyzer measurements made prior to system integration. These are applied in a linear regression to resolve the amplitude, offset, and phase of each correlator. This yields four redundant complex correlations which are averaged to form the final estimate. This process ensures very precise quadrature balance, and virtually eliminates biases caused by anything other than direct leakage of the RF signals among the antennas. At present, we have observed total biases ranging from about 3 mK to 40 mK in the shortest baselines, due almost entirely to leakage between antennas. This has contributed a net 0.5 K to the raw synthesized image errors, but is very stable and readily corrected to the 0.1 K level. We expect that these biased will continue to diminish to acceptable levels for the larger array, so we do not anticipate a problem. We have also conducted separate tests with isolated receivers which show back-end biases at the 0.5 mK level after many thousands of seconds of integration time, which meets our goals for the larger array.

The above correlations are next scaled to visibility using an estimate of the system noise temperature, and then aligned in phase to the aperture plane. We have thus far used LN2 and ambient targets to estimate receiver noise temperature, and point sources on an antenna range to align the phase. These references are transferred to operations by at least two methods: the first uses the internal noise diode to deflect the correlation and system noise by a reliable amplitude and phase. This provides a convenient and steady reference, but there are noise penalties due to the time required to measure the noise diode. The second method relies on the intrinsic stability of the receivers. The receiver noise temperatures of GeoSTAR are quite stable at the ~2K level, which represents about 0.4% of the ~500 K system noise. The observed phase stability is better than ~1 degree. These stabilities readily satisfy the phase and amplitude needs of most correlators in Table 1. To meet the stricter phase requirements for correlations near the center of the UV plane, we will likely need the noise injection. This is an ongoing study, but we now envision a hybrid scheme which uses long running averages of low duty cycle noise diode injection-- applied only to those correlators near the center of the UV plane. The larger baselines should not need this reference circuit, which comes at considerable costs.
The visibilities are next transformed into an image. Ideally, this step is a Fourier Transform followed by a scaling within the earth disk by the elemental antenna pattern. Details are referred to Appendix 2.

VI. ANTENNA RANGE AND SOLAR TRANSIT TESTS

In October of 2005 the completed GeoSTAR demonstrator was tested on a compact range (CR) at the NASA Goddard Space Flight Center facility in Greenbelt, MD. These tests were conducted with the complete system to validate the end-to-end model of interference fringes. The configuration of GeoSTAR in the CR is shown in Fig. 5. The test fixture placed the antenna array directly over the azimuth drive at the base of the fixture, and directly in line with a polarization drive located behind the array in Fig. 5. As shown, the antenna is rotated 90 degrees in polarization relative to Figures 1 and 2. The signal source for the tests consisted of an amplified noise diode which was switched on and off under the control of the GeoSTAR data system. This was placed at the focal point of a large paraboloid reflector (not shown) of the CR which collimates a wavefront-creating in effect an infinitely long antenna range. Data processing involved computing a noise diode deflection (NDD) from the “on” and “off” states of the source at each test position. Measurements were made every 2 degrees of azimuth, and every 5 degrees of polarization. The deflections were calculated for all 192 complex correlators of the GeoSTAR array. The amplitude response of each of the 24 elements were then isolated by applying the closure constraint

$$V_{AA} = \frac{V_{AB}V_{CA}}{V_{BC}}$$

(4)

where the subscripts denote the responses formed between any three elements, A, B, and C, of the array. For any single element, there were 64 such combination in our array, so an average was used. This equation is only valid for the response to a 100% correlated point source; passband and polarization mismatches among elements will also degrade this relation. Data indicate that (4) is reliable to the 0.1% level, based on the consistency among the 64 solutions available for each antenna.

Fig. 6 presents an example antenna response from preliminary analysis of the antenna range data. An average has been formed from 36 different azimuth scans versus polarization. An overlay of the model is also provided which evidently agrees quite well with the measurements. To date, our model is based entirely on the designed geometry of the Y-array, and on a spherical wave expansion of the elemental horn. A third measurement, labeled “tot” in the figure is also provided; this data was recorded by a device (which has not been discussed yet) called a totalizer which is, in effect, a power detector.\(^4\) This measurement also agrees with the correlator data.

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\(^4\)This device is a power detector, in effect, which measures the power from the receiver indirectly by counting the number of digitized samples which exceed a set threshold. This scheme takes advantage of the remaining 7 bits of our 8-bit digitizers. The Error Function is used to map the total counts exceeding the threshold to a linear scale—under the assumption that the voltage is Gaussian.
The logarithmic scale of Fig. 6 is insufficient to view errors below a few percent. A more detailed look at our data reveals typical errors on the order of 2% to 4% relative to the model antenna pattern. When we first examined these errors we suspected anomalies associated with the CR. The CR design specified only 1 dB of amplitude uniformity within the collimated wavefront, and we indeed observed such anomalies—particularly as the antenna was rotated in polarization. These anomalies were about 100 times larger than our measurement goal. We were able to recover from these effects by the fact that the array was centered on the azimuth and polarization axes of the range: this minimized the array displacements during each azimuth scan and thereby mitigated the large amplitude and phase errors of the CR wavefront. To compensate for large variations versus polarization, as well as drift in source power over the many hours required by these tests, each azimuth scan was adjusted to maintain a constant power at the center position. The beam center position was also adjusted slightly in azimuth to correct apparent phase anomalies of the CR. These adjustments brought the measurements to a state that appeared to agree with the models at the 2% level, but these results were still suspect since the CR anomalies were poorly understood. We therefore needed an independent confirmation of the measurement precision, and this was achieved by observing the sun in outdoor tests which were conducted several months later.

The solar transit observations were conducted at JPL by simply pointing GeoSTAR at the sky and allowing the sun to pass across the FOV for several hours. These data were then processed by subtracting an atmospheric background which was recorded at the end of the test, and then by using the known elevations from ephemeris data to fit the response to the atmospheric opacity versus elevation above the horizon. These responses were further normalized to a model of the sun visibility versus baseline using a Bessel Function. This correction was small since the sun diameter of 0.6 degrees was small relative to our largest baseline—which resolves 0.8 degree.

Fig. 7 summarizes the solar and antenna range responses—each as compared to the model. The scale here is the percentage error relative to the model. The color image represents the complete measurement of the antenna range, and the three black traces in this image indicate the path of the sun through the FOV as observed on three different days. The solar responses are plotted together with the extracted antenna responses in the three remaining graphs of Fig. 7. The agreement between the solar and antenna range responses here is very good, and is typical of the 24 elements of our array. Overall, the RMS difference between the antenna range measurements and the solar responses are typically 0.3%, with a few outliers with errors around 0.6%. This agreement is really quite good, and it indicates that the antenna range data is very reliable. The overall RMS errors of either of measurements (sun or the range) with respect to the model are in the 0.8% to 1.5% range when calculated along the solar transit lines. Peak antenna gains are found to be slightly higher on average than the model would predict, but are in agreement with one another at the 1% level, with outliers at 2% relative to the mean. This result confirms that our system will meet requirements in the majority of the array in Table 1 by modeling alone, and that it will be possible to measure the antenna patterns with sufficient accuracy for the few elements near the center of the array.
Fig. 5: GeoSTAR as tested on the Compact Range. The “Y” array can be seen here rotated by 90 degrees in polarization. The system is otherwise wrapped in absorber material to reduce reflections on the range.

Fig. 6: GeoSTAR antenna patterns as modeled (blue) and measured by the correlators (green) and totalizers (red). An average of 36 different azimuth sweeps at difference polarizations are presented here.
Fig. 7: GeoSTAR antenna pattern error example. Errors are given as a percentage relative to the model. The upper left graph is the complete antenna range result, coded such that red means too much gain. The three black traces represent the paths recorded on various days as the sun passed the field of view. The solar responses are plotted in the three graphs, below and right. The green traces represent the sun responses, and the blue traces are the extracted antenna range responses.

VII. CONCLUSION

We have developed a comprehensive error budget for a future Geostationary microwave imager, and have demonstrated a practical system and a calibration approach that will meet a goal for 1 Kelvin accuracy in a large imager with 50 km resolution on the earth. Our design paid close attention to controlling the antenna patterns and interference fringes. We found that it is possible to build a system which meets requirements by design, without an extensive campaign to precisely measure a G-matrix for the majority of the array. Only the short baselines of the array need the more precise calibration, and we demonstrated that these measurements are straightforward. These results show that the inversion of the image will also be straightforward, since they show that majority of the synthesis processing can be handled by a conventional FFT.

Our design also needed to address some very challenging issues related to biases, quadrature balance, noise performance, and stability. We designed several circuits into our demonstrator so that we could evaluate the merits of various approaches. These included the phase-modulated noise injection circuitry, the amplitude-modulated LNA’s, and the phase-switched LO. In the end, the phase-switched LO won out, since it
simultaneously solved the null-offset and quadrature balance problems. A single-phase noise diode is the only other calibration device that appears to be necessary to stabilize the system— and only in the short baselines of the array. For the majority of the array, where gain requirements are relaxed, the intrinsic phase and receiver noise are stable enough to meet requirements without this circuit. This works our advantage in terms of both cost and noise performance. The noise figure of GeoSTAR, like any radiometer, is paramount. By eliminating the ferrite isolators and directional coupler in the larger baselines of the array, we will easily gain a dB or so of improved noise figure.

We plan to continue our work this year with a field demonstration that uses a large temperature controlled disk target which will be deployed above GeoSTAR to simulate the earth disk as viewed from GEO with a cold background. This will provide a more convincing demonstration and a more comprehensive data base with which to illustrate various recalibration options for our system. These include the use of the sun as a phase reference as it passed into the aliased regions of the image, and the use of the limb itself as a reference. The utility of a ground beacon is also being considered, and will be tested, as a means to provide a continuous phase reference for the system.

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REFERENCES


APPENDIX I

Ideally, the visibility function is the Fourier transform of the brightness temperature image as weighted by the elemental antenna pattern in the equation

\[ V(u, v) = \iiint_{r^2 + s^2 \leq 1} T(r, s) \left[ \frac{f(r, s)}{\Omega \sqrt{1 - r^2 - s^2}} \exp[-j2\pi(ru + sv)] \right] r dr ds \]  

where \( r \) and \( s \) are horizontal and vertical coordinates of the image plane (also referred to as the directional cosines) defined in terms of spherical coordinates by

\[ r = \sin \theta \cos \phi \]
\[ s = \sin \theta \sin \phi \]  

\( T(r, s) \) is the brightness temperature image, \( f(r, s) \) represents the normalized antenna voltage pattern, \( \Omega \) is the beam solid angle of the antenna, and \( u \) and \( v \) are the respective horizontal and vertical spacings between elements of the array- given here as wave numbers. For the moment, (1) neglects other terms detailed by Camps, et al [5], and by Corbella [7], which are necessary to model antenna mismatches, mutual coupling, passband mismatches, fringe wash, array alignment, etc.. These are discussed below.

By identity of (1) with the Fourier series, the image synthesis takes one of two forms. The first applies to the GEO earth observation where the earth is contained within the UaFOV and surrounded by, approximately, zero Kelvin. In this case the integral of (1) is limited precisely to one interval of the hexagonally periodic Fourier Series such that

\[ T(r, s) = \frac{\Omega \sqrt{1 - r^2 - s^2}}{A_u} \left[ V(0,0) + 2 \sum_i \Re[V(u_i, v_i)] \cos[2\pi(r u_i + s v_i)] + \Im[V(u_i, v_i)] \sin[2\pi(r u_i + s v_i)] \right] \]  

where \( i \) is an index for the u-v sample of Fig. 2, and \( A_u \) is the area of one hexagonal period in the r-s plane given by

\[ A_u = \frac{2\lambda^2}{d^2 \sqrt{3}} \]  

which equals 0.081 in the present design at 50.3 GHz and 2.25 cm element spacing. Our antenna has a beam solid angle of 0.135, so we see that the fraction on the left side of (3) is 1.66 near boresight. This is a factor which amplifies errors in our system (i.e. noise and biases).
The other synthesis case applies to ground based observations where the brightness temperature surrounding the main synthesis region is unknown. In this case, the aliased regions are superimposed (added) to the main region. This has been the case for most of the GeoSTAR observations thus far, and in these instances the ‘best’ synthesis approach appears to be the simplest case of

\[
T(r, s) \simeq V(0, 0) + 2 \sum_i \text{Re}[V(u_i, v_i) \cos(2\pi (ru_i + sv_i))] + \text{Im}[V(u_i, v_i) \sin(2\pi (ru_i + sv_i))]
\]

APPENDIX II

The G-matrix approach lumps all the gain errors into a single empirical function, g, which (in complex form) replaces (1) with

\[
V(u, v) = \iint_{r^2 + s^2 \leq 1} T(r, s)g(r, s, u, v)drds
\]

The matrix form of this is

\[
V = GT
\]

where V is a vector of M visibility samples, and T is a vector of P image pixels, with \(P>M\). This is the original formulation of reference [3]. In essence, G is treated as an arbitrary function to be determined entirely by measurement and then inverted by the Orthogonal Projection Theorem according to

\[
G' = G^i (GG^i)^{-1}
\]

One problem with this formulation is that the matrix inversion of (5) is very large, and very sensitive to small sampling errors. This has already become apparent even in our small demonstrator instrument, with \(M=385\). We anticipate that this inversion may be impractical in this form, and probably unnecessary in a large spaceborne system (\(M=60,000\)). More recently, we have reformulated the problem with what we call the “flat” G-matrix by casting (1) in the form

\[
V(u, v) = \iint_{r^2 + s^2 \leq 1} T(r, s) \frac{|f(r, s)|^2}{\sqrt{\Omega^2 - r^2 - s^2}} g(r, s, u, v) drds
\]

which takes the matrix form

\[
V = GFT
\]

where F is a PxP diagonal matrix which represents the ideal elemental antenna pattern, and G is composed of discrete samples of the complex exponent in (1). This new formulation has the advantage that the product GG^i in (5) is diagonal in the ideal case. This reduces the inversion of (5) to a transpose operation if the array is well behaved. If not-- if the elemental patterns are not well behaved-- we propose to lump all complex gain errors (antenna errors, fringe wash, etc.) into G, while maintaining a common ideal model of the antenna pattern F. This form is readily adapted to a hybrid synthesis in which large baselines of the array (which are sufficiently well behaved) may be processed with a conventional FFT. The short baselines (where sensitivity to error is high) can be processed by the G-matrix. These two solutions can be merged onto a common image plane, and then normalized with the common, ideal antenna pattern (as in
(3)). This approach reduces the size of the inversion in (5) to an appropriate level, and eases the processing load for the overall system.