Estimation of cyclic error due to scattering in the internal OPD metrology of the Space Interferometry Mission

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Abstract: A common-path laser heterodyne interferometer capable of measuring the internal OPD with accuracy of the order of 10 pm was demonstrated at JPL. To achieve this accuracy, the relative power received by the detector that is contributed by the scattering of light at the optical surfaces should be less than -97 dB. A method has been developed to estimate the cyclic error caused by the scattering of the optical surfaces. The result of the analysis is presented.

Key words: Metrology, scattering, cyclic error

1 Introduction

The Space Interferometry mission (SIM) is a space-based optical Michelson interferometer. Its primary goal is to measure the positions and motions of celestial object. In its simplest form, SIM performs relative astrometry between two stars by taking a pair of internal optical path delay measurements: one for each star, as shown in Figure 1.[1] The two delay-line positions are determined by searching for the maximum fringe amplitude on each star. A key measured quantity is the change in internal delay Δd,

$$\Delta d = \vec{B} \bullet (\vec{s}_1 - \vec{s}_2),$$

where $\vec{B}$ is the baseline vector and $\vec{s}_1$ and $\vec{s}_2$ are unit vectors to the two stars. The angular distance between the two stars $(\vec{s}_1 - \vec{s}_2)$ can then be calculated once Δd and $\vec{B}$ are known. The function of internal metrology is to measure Δd. The baseline of SIM is 10 meters; Δd must be measured with accuracy on the order of 10 pm to achieve 1 micro arcsecond astrometry.

A Common-Path Heterodyne Interferometer (COPHI) was proposed at JPL as a multi-purpose and high-resolution distance measuring interferometer.[2-4] A similar scheme as shown in Figure 2 was proposed and adapted by SIM for the internal metrology of optical path difference (OPD) in the two arms of the stellar interferometer. The measurement beam (with frequency f_0) is split by the astrometric beam combiner into two beams. The beams propagate towards the star in the two arms of the stellar interferometer. Two masks are placed after the beam combiner to spatially separate the two beams, one with two rectangular apertures in the south-north orientation and the other with two rectangles in the east-west orientation. The two beams are retro-reflected by corner cubes #1 and #2. A second laser beam with frequency $f_0+\Delta f$, which serves as the local oscillator, then interferes with both measurement beams at the second beam splitter. The heterodyne signals are detected by the two detectors, respectively. The phase difference ($\Delta\phi$) between the two heterodyne signals is used to calculate the displacement of optical path between the corner cubes:

$$\Delta d = \frac{\Delta\phi}{2\pi}\frac{\lambda}{2},$$

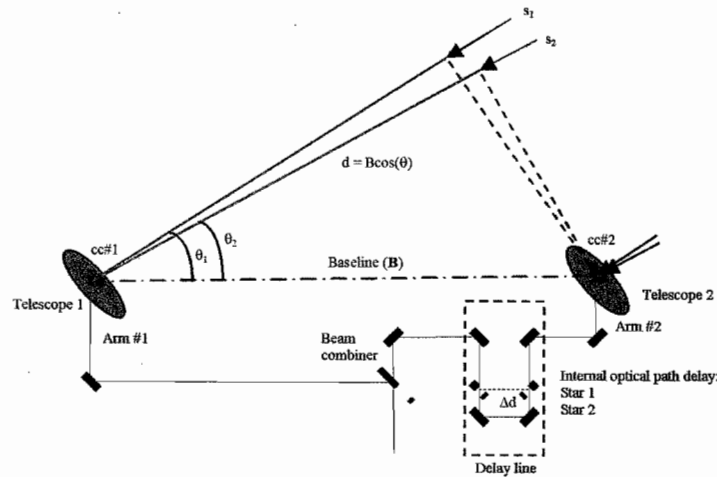where λ is the wavelength of the metrology laser.

Figure 1. Stellar Interferometer for astrometric measurements. The internal delay Δd between two star central fringes gives the angular distance between the two stars.
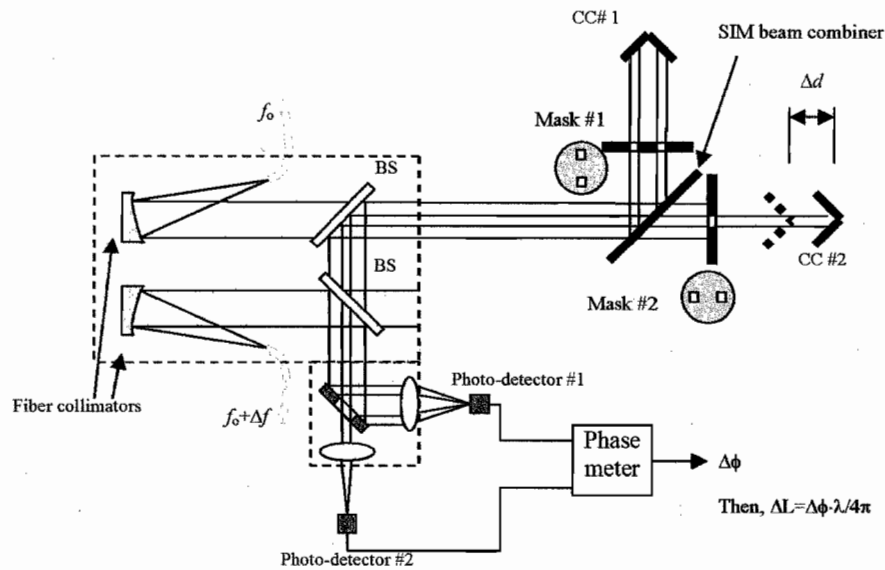

Figure 2. The layout of COPHI configured for the metrology of internal OPD in SIM.

One obvious advantage of using COPHI is that a single interferometer is used to measure the internal path difference between the two arms of the stellar interferometer. Other advantages include much smaller nonlinear errors for polarization leakage is not an issue in COPHI and much smaller thermal effects for the two measurement beams are of common path until the beam combiner.

2 Cyclic errors

There are two types of cross-talk in the measurement of the displacements in the two arms. Cross-talks manifest as cyclic errors in the measurements, which are errors with the same or multiples of the signal frequency as the delay (Δd) in the arms change. The first type is called the inter-arm cross-talk. Because of diffraction and scattering, light in first arm crosses over to the collection aperture of the detector for the second arm or vice verse. The second type is called the intra-arm cross-talk. A small amount of light power from scattering or multiple reflections is received by the detector of the same arm. The measurement error in distance x for both types of cross-talk is proportional to the relative amplitude of the electric field of the contaminated light collected by the detector. It is given as[5]

$$|dx| = \frac{\lambda}{4\pi}\left|\frac{dA}{A}\right| = \frac{\lambda}{4\pi}\sqrt{\left|\frac{dP}{P}\right|},$$

where A is the amplitude of the electric field and P is the power of the light received at the detector while dA and dP are that of the cross-talk received by the detector. For a cyclic error of 2 pm the relative power of scattered light is -97 dB.

The inter-arm cross-talk can not be reduced by the technique known as in-phase cyclic-averaging. Thus it must be small by design. Extensive modeling of diffraction effects has been carried out and optimization of the masks has been performed at JPL to minimize the cross-talk of diffraction to be < 1 pm.[6] The intra-arm cross-talk, however, can be reduced by cyclic-averaging. This means that the requirements on the intra-arm cross-talk do not have to be as tight as the inter-arm case, but still less than tens of picometers. We present here the results of modeling the cross-talk due to scattering.

3 Harvey model and ASAP

Harvey characterized the scattering of optical surfaces in great detail.[7] His result, verified by many other measurements, is often referred as the Harvey model and is widely used in modeling the scattering of light by optical surfaces. In the Harvey model, the bi-directional scattering distribution function (BSDF) is the same with respect to the specular direction. Figure 3 shows the BSDF for 0, 22.5, 45 and 67.5 degrees. The logarithm of BSDF is linearly related to the logarithm of the sine of the scattering angle, i.e. $\log_{10}(BSDF) = c*\log_{10}[\sin(\theta_s)]$, where c is a constant related to the RMS micro roughness of the surface and the wavelength of light. Furthermore, the logarithm of the BSDF is an invariant with respect to the logarithm of the difference of the sine of scattering angle and the sine of the specular angle. Figure 4 shows this relationship and the Harvey model predicts that every time the RMS roughness doubles the BSDF increases by the same amount (6 dB for the parameter that we choose) .

The optical analysis software, ASAP, has extensive capabilities of modeling scattering. One of its categories of scattering models is the Harvey model.[8] ASAP also has a versatile physical optics tool for coherent beam propagation.[9] It uses the Gaussian beam decomposition method. The electric field of any light beam (amplitude and phase) is decomposed into a number of Gaussian beams. The Gaussian beams are then propagated by trace the base ray and a few parabasal rays. At any plane of interest, the electrical field is constructed by coherently summing all fields of the Gaussian beams at that plane.

Angle: 1= 0.0 2= 22.5 3= 45.0 4= 67.5
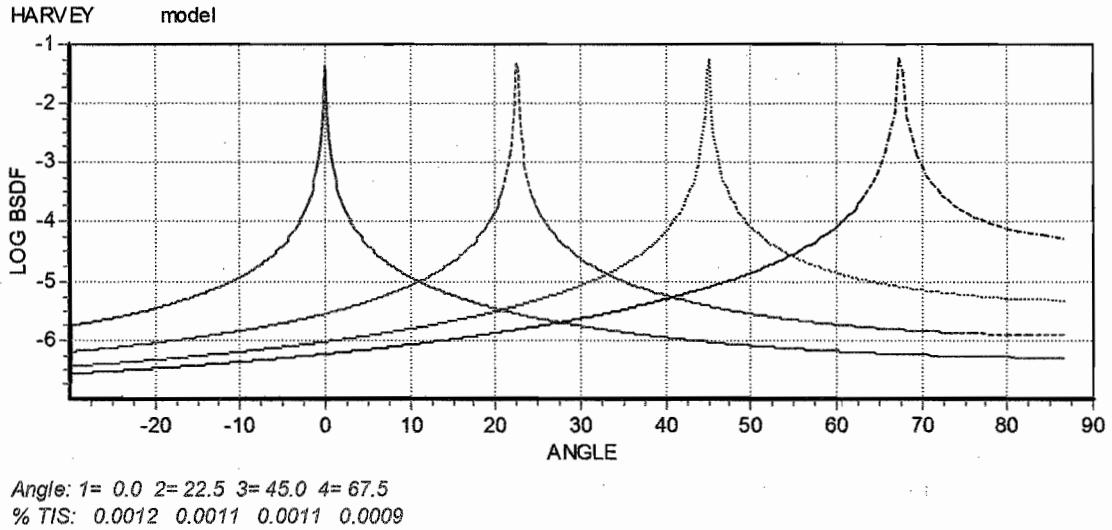% TIS: 0.0012 0.0011 0.0011 0.0009

Figure 3. The bi-directional scattering distribution function in Harvey model. The BSDF for angles of incidence 0 22.5 45 and 67.5 degrees are plotted. They all have the same functional forms.
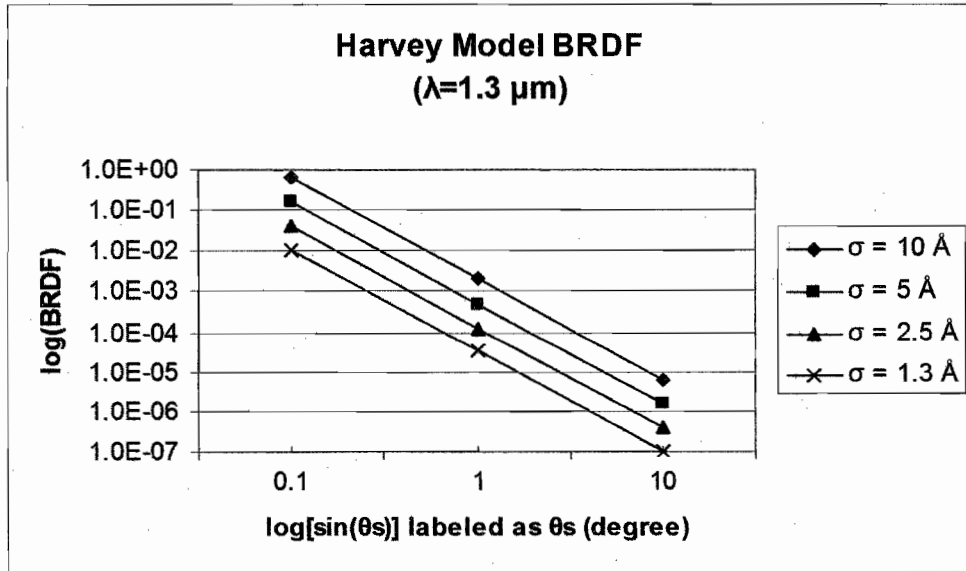

Figure 4. The logarithm of BSDF is linearly related the logarithm of the scattering angle. The magnitude of BSDF is related to the surface micro roughness. It increases by 6 dB (4 times) when the surface roughness doubles.

4 Scattering analysis

We have built a model of the metrology optical system in ASAP. The raytrace layout of the model is presented in Figure 5. Collimated laser light from the fiber is split first at BS #1. About 50% of the light is transmitted and 50% reflected. The transmitted light propagates to the beam combiner, where the laser beam is spatially divided into two groups of beams, one group traveling towards each corner cube. Both groups of beams are retro-reflected by the corresponding corner cubes. The returned beams co-propagate after the combiner. They encounter BS #1 and BS #2. There is a beam separator mirror that selects beam #2 and directs it towards
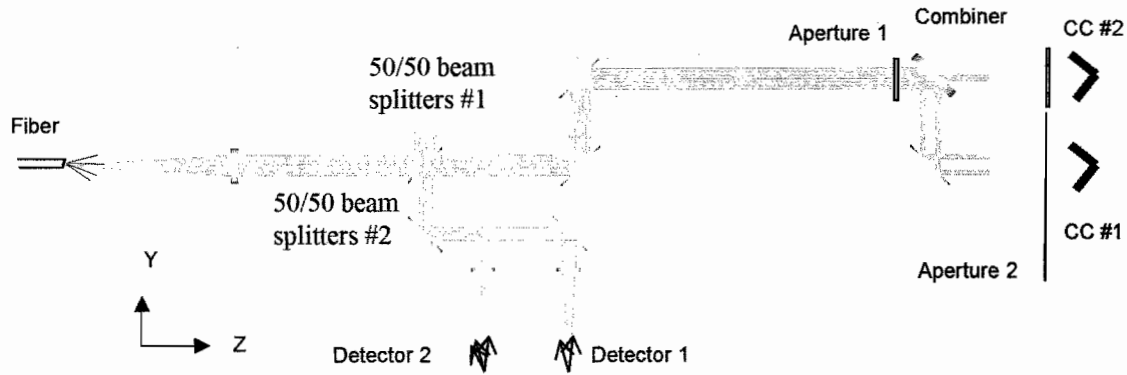
Figure 5. Optical layout in ASAP of the metrology system. Laser light from the fiber is collimated and split into two groups at the combiner, one group for each arm. Each group is retro reflected by its corner cube and received by its detector.

detector #2. The rest of the light is directed towards detector #1. Gaussian beam decomposition in ASAP is used to propagate the laser beam in our model. At the aperture in front of the combiner the laser beam is divided to four beams. The irradiance distribution of the light beams right after this aperture is shown in Figure 6. The beam combiner splits the four beams into two groups: the south-north (SN) beams travel in arm #1 and the east-west (EW) beams travel in arm #2. Let us follow the EW group of beams. They transmit through the beam combiner and propagate towards corner cube #2. The Irradiance distribution right at aperture #2 in front of CC #2 is presented in Figure 7. It shows the diffraction effects on the light beams. After the retro-reflector CC #2, the light travels backwards. It eventually is received by detector #2. The irradiance distribution at detector #2 is shown in Figure 8.

In ASAP, various scattering properties can be assigned to a surface of interest. As we mentioned above, Harvey model provides a good representation to the scattering of polished optical surfaces. We have first investigated three likely major contributors for the intra-arm cross-talk for EW beams in arm #2. They are: 1) scattering by the three surfaces of the corner cube (#2 for EW beams) received by detector #2; 2) scattering by the fiber tip of returned light from CC #2 and received by the detector # 2 after another round trip; 3) scattering by the detector #2 and received by the detector #2 after another round trip.

We use case 1 as an example to demonstrate how we derived the scattering power from ASAP analysis. In our applications, ASAP is used to propagate coherent laser beams through out our optical systems. ASAP is also used to trace the scattered light through the system and calculate the power of scattered light received by the detector. At each surface of the corner cube, ASAP can compute two kinds of power sums, coherent and incoherent. Since scattering is generally considered an incoherent phenomenon, ASAP uses the incoherent power incident upon a surface and calculate the angular distribution of scattering (BSDF). We know that the total power of our laser beam should be a coherent sum. A correction factor is used to account for the fact that the total power being scattered is that of a coherent sum. A factor of 5 is determined to provide good general estimation to convert the total incoherent power to coherent power. In our ASAP results, the portion of scattered light by the corner cube that is received by the detector is -95 dB for an corner cube with 5 Angstroms RMS surface roughness. The first correction we have to do is to add a factor of 5 (+7 dB) to indicate that the true total scatted power at the corner cube. Since the scattered light is very close to the optical axis in order for it to be received by the detector we assume that the scattered light is 100% coherent to the non-scattered light (the worse case). Thus, the sum at the detector should also be a coherent one. Therefore, there is another factor of 5.

This gives -81 dB for the total scattered power received by the detector, which corresponds to 9.4 pm cyclic error. Figure 9 shows the irradiance distribution of the light scattered by corner cube #2 and received by detector #2.

If we need to know how a corner cube with surface micro roughness of 10 Angstroms performs, we use the property of the Harvey model (see Figure 4). The BSDF increases by 6 dB (4 times) when the surface roughness doubles. The total scattered power received by the detector is then -81 + 18 dB for a corner cube has three surfaces. This translates to 74 pm cyclic error.
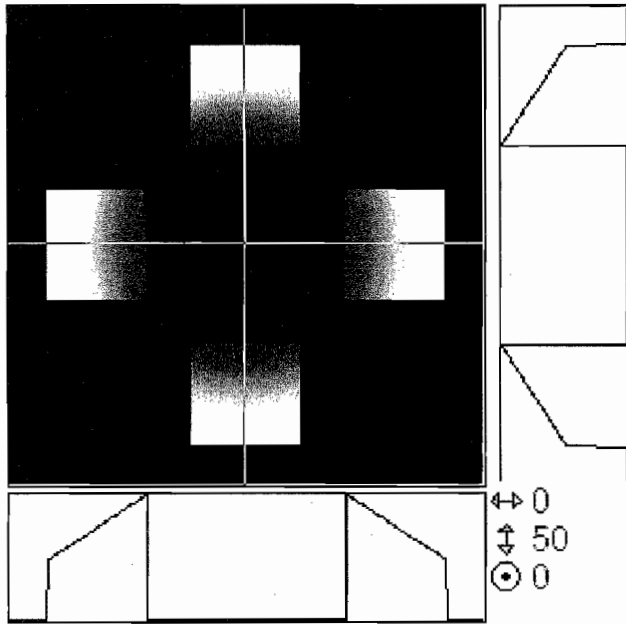

Figure 6. Irradiance distribution after aperture #1. The laser beam is divided into four beams. The south-north beams travel in arm #1 and east-west beams travel in arm #2.
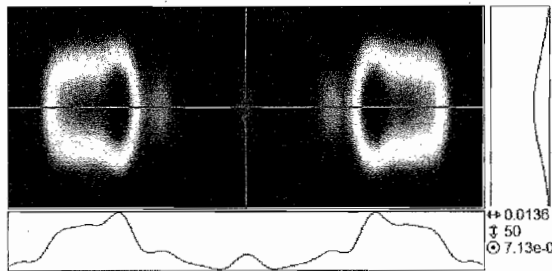

Figure 7. Irradiance distribution after aperture #2 in front of corner cube #2. The diffraction effects are clear present.
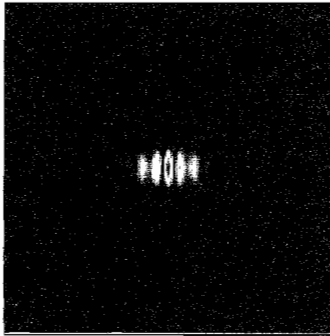
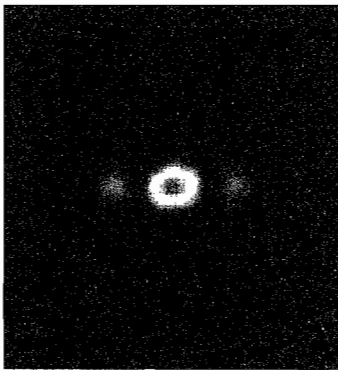Figure 8. Irradiance distribution of east-west (EW) beams at detector #2.

Figure 9. Irradiance distribution of light scattered by corner cube #2 and received by the detector #2.

In a similar fashion, we determined that the relative power scattered by the fiber tip received by detector #2 after another round trip is -110 dB. We assumed the fiber tip is of 5 Angstroms RMS roughness. For case 3, the relative power from the scattering of the detector is -85 dB for normal incidence. If we tilt the detector, the relative power from scattering can be reduced significantly. The reflectance of the detector is assumed to be 2% and the RMS roughness is 5 Angstroms. It appears that the cyclic error due to the scattering of the corner cube can be much greater than -97 dB, which is required to achieve less than 2 pm error. Additional reduction in cyclic error can be achieved by cyclic averaging. We have also determined that inter-arm cross-talk due to scattering is less than -110 dB, i.e. much less than 1 pm cyclic error. As we mentioned above, diffraction is the main source of inter-arm cross-talk.

5 Summary

A model is built in ASAP for the SIM internal metrology optical system. ASAP's beam-propagation tool is used to calculate the coherent power at the surface of interest. Scattering analysis is performed and the coherent correction factor is used to estimate the scattered power of the laser metrology beam. Our analysis assists the design and development of our metrology system. Our analysis shows that the cyclic errors due to scattering of key optical surfaces are below or near the acceptable levels if we use surfaces better than 5 Angstroms RMS micro roughness. There are other sources of scattering in our system yet to be investigated. They include various beam-dumps to collect stray-light and quadrant detectors to control the pointing of metrology beams.

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