An Adaptive Cross-Correlation Algorithm for Extended-Scene Shack-Hartmann Wavefront Sensing

Erkin Sidick
Joseph J. Green, Catherine M. Ohara, and David C. Redding

Jet Propulsion Laboratory
California Institute of Technology

**Principle of Conventional SH-WFS**

- A Shack-Hartmann sensor places a lenslet array at a plane conjugate to the WF error source.
- Each sub-aperture lenslet samples the WF in the corresponding patch of the WF.
- When observing a star, the image is an array of spots, each of which is a sub-aperture PSF.
  - $\Delta x$ is proportional to local wavefront tilt.
  - Wavefront-sensing $\rightarrow$ Finding $\Delta x$ for all sub-images.
  - Use centroiding (center-of-mass) method to find $\Delta x$. 

---

**Diagram:**
- Wavefront
- Lenslet Array
- Incoming Beam
- Perfect
- Distorted
- Image Plane (CCD)
Extended-Scene S–H WFS

- The Shack-Hartmann Camera produces images as the convolution of the conventional image (limited by a tight field stop) with a regular grid of subaperture PSFs, as above.
- Each subaperture is much blurrier than the main image, as its diffraction limit is defined by the subaperture, not the full aperture.
- Subaperture image shown at right at full size (64x64).
- ACC algorithm finds the central 32x32 box, and then identifies the multi-pixel shift of the features in the inner 16x16 cell with respect to a reference subaperture.
- The subaperture-to-subaperture cell shifts give a measure of subaperture tilt.
Point-Source (Star) versus Extended-Scene

- Following images were measured at JPL SH-WFS Testbed
  - Can be used with both point-source and extended-scene
  - Each spot-image is replaced by a sub-image in extended-scene SH-WFS
  - Local wavefront distortion causes a sub-image to shift from ideal position
  - SH camera provides large capture range WFS&C
Autofocus Testbed

32x32 Actuator Deformable Mirror

Extended Scene Simulator
Object slides in filter wheel coupled with uniform arc-lamp illumination

Lenslet Array & Shack-Hartmann Camera

Obscuration Mask
Establishes F/# and Pupil Obscurations for PRC

Phase Retrieval Camera
Adaptive Cross-Correlation (ACC) Algorithm — How it Works

- Property of Fourier-transform:
  - Shift in time-domain $\leftrightarrow$ Linear-phase in frequency-domain
    - In Fourier optics, $t \rightarrow (x, v)$ and $f \rightarrow (u, v)$

- Fourier-transform pair—Shown as one-dimensional for simplicity:

  $s(x) \leftrightarrow \hat{s}(u)$

  $s(x - \Delta x) \leftrightarrow \hat{s}(u) e^{-j2\pi\Delta xu}$
ACC Algorithm — How it Works (con.)

- In JPL testbed, only those cells marked with red-circle are used:
  - \( r(x) \) = reference cell
  - \( s(x) \) = test cells

- In ideal case:

\[
\begin{align*}
  r(x) & \leftrightarrow \hat{r}(u) \\
  s(x) = r(x - \Delta x) & \leftrightarrow \hat{r}(u) e^{-j2\pi\Delta xu}
\end{align*}
\]
• Black square = usable sub-image (cell)
• Real world is very different from the ideal case:
• Following illustration was made in one-dimension only. In reality, everything is 2-dimensional: \((x, y) \leftrightarrow (u, v)\)

• In real world:

\[
\begin{align*}
  r(x) & \leftrightarrow \hat{r}(u) \\
  s(x) & \neq r(x - \Delta x) \\
  s(x) & \leftrightarrow \hat{s}(u) \\
  \hat{c}(u) & = \hat{r} \ast (u) \hat{s}(u) = |\hat{c}(u)| e^{j2\pi \varphi(u)} \\
  \varphi(u) & = \Delta xu + \varphi'(u)
\end{align*}
\]
ACC Algorithm — How it Works (con.)

16x16 matrix \( r(x,y) \) or \( s(x,y) \):
Used to find \((\Delta x, \Delta y)\) from CC-function

32x32 matrix \( S(x,y) \):
Used to shift \( s(x,y) \) by \((-\Delta x, -\Delta y)\) to match it with \( r(x,y) \)

- **Advantages of using smaller dimensions for \( r(x,y) \) & \( s(x,y) \):**
  - Avoids wrap-around error when performing sub-image multi-pixels shifting
  - Makes the ACC calculations much faster
  - Increases the WFS dynamic range

- **To shift \( S(x,y) \) by \((-\Delta x, -\Delta y)\):**
  - Obtain \( S(u,v) \) by FFT \( \rightarrow \) \( S(u,v)\exp[-j2\pi(-\Delta xu-\Delta yv)] \) \( \rightarrow \) (by IFFT) \( S(x+\Delta x, y+\Delta y) \)
\[
\hat{r}(u, v) = FFT\{r(x, y)\}
\]

\[
i \leq N_c
\]

Yes

\[
\hat{S}_i(u, v) = FFT\{s_i(x, y)\}
\]

\[
\hat{S}_i(u, v) = FFT\{S_i(x, y)\}
\]

16 \times 16 \text{ pixels}

32 \times 32 \text{ pixels}

For example:

\[
Tol = 0.01 \text{ pixels}
\]

\[
N_{\text{iter}} = 15
\]

STOP

\[
\Delta x = \Delta x + \hat{\Delta}x, \quad \Delta y = \Delta y + \hat{\Delta}y
\]

\[
\hat{S}_i(u, v) = \hat{S}_{i0}(u, v) \exp[-j2\pi(-\Delta xu - \Delta yv)]
\]

\[
S_i(x, y) = IFFT\{\hat{S}_i(u, v)\}
\]
32 x 32 Pixels Test Cells to be Analyzed

- Obtained 8 SH images with different integration time
- Used different cells for $r(x,y)$ & $s(x,y)$
- Red-square corresponds to a 16x16 pixels area

- Variations of IQM’s with iteration time:
  - $MSE = \text{Mean-Squared Error}$
  - $MFI = \text{Modified Fisher-Information}$

\[
MSE = \sum_{x,y} \left| g_i(x, y) - \bar{g}_i(x, y) \right|^2
\]

\[
\Psi_i(x, y) = g_i(x, y) / 4095
\]

\[
MFI = 4 \times \sum_{x,y} [\nabla a_i(x, y)]^* \nabla a_i(x, y), \quad a_i(x, y) = \sqrt{\Psi_i(x, y)}
\]
Speed versus Tolerance

- Shifted $s(x,y)$ by known amount $\Delta x$, and determined the relative offset between $r(x,y)$ and $s(x,y)$ with ACC
- Used different cells for $r(x,y)$ & $s(x,y)$
Examples: Point-Source Spot Image Analyzed with ACC

DM Poke Patterns

Offset Diagrams

Deformable-Mirror (DM)
Extended Scene versus Point-Source

- Same poke patterns are used in both cases, but the measurement are done on different dates.

- There are some differences in light path and actuator registration for point-source and extended scene, which is partially responsible for difference in OPD results.

RMS = 50, PV = 299.5nm
RMS = 54.7, PV = 327.4nm
OPD Map Measured with Extended Scene

DM Poke Pattern

Reconstructed OPD Map

Active Window

OPD is in nm
Example of Extended-Scene WFS&C

- There are 78 actuators inside the active window, but only 50 eigen-modes were used in this experiment.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>RMS</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td># = 0</td>
<td>54.7nm</td>
<td>327.4nm</td>
</tr>
<tr>
<td># = 1</td>
<td>40.6nm</td>
<td>295.2nm</td>
</tr>
<tr>
<td># = 2</td>
<td>38.5nm</td>
<td>261.9nm</td>
</tr>
<tr>
<td># = 3</td>
<td>37.5nm</td>
<td>292nm</td>
</tr>
<tr>
<td># = 4</td>
<td>34nm</td>
<td>277.8nm</td>
</tr>
</tbody>
</table>
Summary

- Extended-scene SH sensor is useful when point-source is not available but SH-WFS is needed.

- ACC requires only about 4 image-shifting iterations to achieve 0.01 pixel accuracy.

- ACC is insensitive to both background light and noise—much more robust than centroiding.

**Acknowledgement**
- We thank Rhonda Morgan at JPL for her assistance with the S-H testbed.