Model of the Plasma Potential Distribution in the Plume of a Hollow Cathode

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In most gridded electrostatic thrusters, a single, centrally located hollow cathode supplies the electrons that produce ions by electron bombardment in the discharge chamber. In recent engine tests several cases of discharge cathode orifice plate and keeper erosion have been reported. Understanding how and why the erosion pattern changes with operating parameters is critical to predicting thruster life over years of operation anticipated for proposed electric propulsion missions, such as JIMO. In this paper we present results from a new model of the plasma potentials in the plume just downstream of the hollow cathode keeper. We examine the electron drift velocity as the hollow cathode plasma and neutral gas expand downstream of the keeper. If the drift velocity exceeds the thermal velocity a double layer potential structure develops that is the source of hot electrons. Ions are accelerated upstream through the double layer. The locations of the double layers are calculated using a simple model. It is shown that as the cathode gas flow increases, the location of the double layer moves farther downstream.

I. Introduction

In most gridded electrostatic thrusters, a single, centrally located hollow cathode supplies the electrons are produced ions by electron bombardment in the discharge chamber. Erosion of the discharge chamber hollow cathode keeper was one of the main results from the NASA Solar Electric Propulsion Technology Application Readiness (NSTAR) Extend Life Test (ELT). The mechanism for keeper erosion and the parameters that control it have been the subject of several experimental and theoretical studies. While it is assumed that sputtering from ion bombardment is the fundamental cause, presently there is no first principles predictive model of discharge chamber keeper erosion. The plasma density, temperature, and potential distribution in the vicinity of the keeper are critical parameters in determining the fluxes and energies of ions that cause keeper sputtering. Published studies have shown that wear on the keeper erosion occurs depends on the operating conditions. Under certain conditions, only the downstream face of the keeper erodes, under other conditions the keeper orifice enlarges. Understanding how and why the erosion pattern changes with operating parameters is critical to predicting thruster life over years of operation anticipated for proposed electric propulsion missions, such as the Jupiter Icy Moons Orbiter (JIMO).

In this paper we present a simple, preliminary, model that predicts that a plasma double layer must form downstream of the cathode orifice, and that ions counter streaming across the double layer are the source of the observed keeper erosion. The model predicts that double layer moves downstream with increasing gas flow, in agreement with recent measurements. We also show that for the laboratory cathode in the absence of magnetic field, that electron heating in the cathode orifice can prevent the conditions for double layer formation, and that the measured cathode plasma plume potentials, densities, and electron temperature are in good agreement with a 1-D fluid model.

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II. Background

During the first wear test of the NSTAR 30-cm ion thruster, two discharge chamber hollow cathode assemblies were used—one for the first 867 hours and the second for the last 1164 hours of the test, which was voluntarily terminated at 2031 hours. Examination of both cathode assemblies, after testing, revealed substantial erosion of the outer edge of discharge cathode orifice plate, heater wire and thermal radiation shielding. Several design changes were made to fix the erosion problem; changes included moving the heater wire so that it did not extend downstream of the orifice plate and the cathode assembly was enclosed within a cylindrical keeper electrode. After a 1000 hour test was conducted on the new design at ~25% higher cathode flow rate (due to a calibration error), no erosion was observed at the outer edge of the orifice plate but the keeper thickness changed by 25-75 mm. During the subsequent 8000 hour test, erosion was observed on the inner diameter and the downstream surface of the keeper; the maximum eroded depth on the keeper face was ~530 mm or about 1/3 of the keeper thickness. Both the design changes and the higher flow rate were used on the NSTAR thruster that flew successfully on Deep Space One.11

However, during the subsequent Extended Life Test of the NSTAR flight spare thruster, severe erosion of the keeper electrode was observed during the first 4693 hours of the test, which was conducted mostly at the full power point, photographs show surface texturing consistent with the keeper erosion observed during the 8000 hour test. Between 4693 and 10451 hours, the thruster was operated at the NSTAR half power point and the keeper orifice eroded to the point where the entire cathode orifice plate and about half of the heater wire were exposed to the discharge chamber plasma. Full power operation was resumed between 10451 and 15617 hours and the keeper orifice continued to erode; at the end of this full power segment, the entire heater wire and radiation shielding were exposed to the discharge chamber plasma. Testing at the NSTAR minimum power point was conducted between 15617 and 21306 hours, and the keeper orifice continued to erode, albeit at a lower rate. Testing of the thruster continued until 30,352 hours when test was voluntarily terminated. Posttest inspection showed that the keeper electrode face had been completely eroded away.

Several groups have performed investigations to try to understand the mechanisms that caused the erosion of the NSTAR discharge cathode keeper. Williams used laser-induced fluorescence (LIF) to measure relative erosion rates and plasma ion velocity vectors in the vicinity of the discharge cathode in a 30 cm ion engine.2 He concluded that a potential hill downstream of the cathode accelerated ions back to the cathode and caused the erosion. Foster measured radial plasma density and potential profiles near the keeper.3 Domonkos measured erosion of a copper keeper.4 Kolasinski used surface layer activation of molybdenum keepers to obtain keeper erosion rates for a range of operating parameters.5 Herman measured plasma densities and temperatures in the 30 cm discharge chamber.6,7,8

Wilbur and co-workers have previously discussed double layer formation downstream of hollow cathodes.3,14,15 Below we develop a model of double formation and apply it to the NSTAR cathode. Additionally we examine the plume from a higher current cathode and show that the conditions for double layer formation are found in the orifice for nominal gas flow, but move downstream as the gas flow increases.

A. Discharge Cathode Double Layer

In the discussion below, we calculate the approximate axial location for a double layer using several simplifying assumptions. A double layer is a region in a plasma where the potential changes rapidly between two relatively low field regions. Since the total change in fields is small, there is approximately no net charge in the region; that is, across the layer the ion charge density and the electron charge density integrate to zero. This leads to a relationship, first proposed by Irving Langmuir (1929),9 between the electron current density and the counter streaming ion current density,

\[ \frac{j_e}{\sqrt{m_e}} = \frac{j_i}{\sqrt{m_i}} \]  \hspace{1cm} (1)

This relation is known as the "Langmuir condition."

One way double layers form is when the electron drift velocity exceeds the electron thermal velocity. In the plume of a hollow cathode, the neutral gas falls with the inverse square of the distance downstream. This condition can occur when the neutral gas density drops and the ionization rate is insufficient to support the current. When this occurs, the double layer potential accelerates thermal electrons to energies in excess of the neutral gas ionization potential, increasing the ionization rate.
To solve for the axial location of the double layer, we calculate the location from which the counter streaming ions satisfy the Langmuir condition. We assume that the gas expands at a fixed cone angle, the tangent of whose half angle is $\alpha$.

$$n_0(z) = \frac{f_{\text{gas}}}{v_0 \pi (r_0 + \alpha z)^2}$$  \hspace{1cm} (2)

We assume that the ions generated by accelerated electrons within about one local radius downstream flow through flow back through the double layer

$$I_i \approx I_e \int_{-\infty}^{x + t_0 + \alpha z} \frac{f_{\text{gas}}}{\sigma n_0(z)} \, dz$$  \hspace{1cm} (3)

Substituting the expression for density we obtain

$$I_i \approx I_e \frac{f_{\text{gas}}}{v_0 \pi} \frac{1}{\alpha} \int_{t_0 + \alpha z}^{x + t_0 + \alpha z} \frac{1}{x^2} \, dx$$  \hspace{1cm} (4)

We define the "Langmuir ratio", $R$, as the ratio of the ion and electron current times the ratio of the square root of the masses,

$$R = \frac{j_i}{j_e \sqrt{m_i/m_e}}$$  \hspace{1cm} (5)

The stable double layer location is when this ratio equals one. Substituting for the ratio of the current densities we obtain

$$R = \frac{f_{\text{gas}}}{v_0 \pi} \frac{m_e}{m_i} \frac{1}{(t_0 + \alpha z) (1 + \alpha)}$$  \hspace{1cm} (6)

The Langmuir ratio decreases monotonically with axial distance (Figure 1). This leads to a stable double layer, because if the Langmuir condition isn't met in the stationary frame, it must be satisfied in the moving frame. If the location were too far upstream, the double layer would move out; if the double layer were too far downstream, the double layer would move in. Also shown in Figure 1 is how the Langmuir ratio increases with flow for two different cathode flow rates, and consequently, the double layer location move further downstream.
Figure 1. Langmuir ratio as a function of downstream axial distance for two flow rates.

Alternatively, we can solve for the axial location where the Langmuir condition equals one (Figure 2). The general behavior, that the double layer location moves axially with increasing gas flow, has been reported in Ref 7 by visual observation.

Figure 2. Axial location where the Langmuir condition equals one as a function of the cathode flow rate.

B. Cathode Plume Expansion Plasma
The above analysis showed that if a double layer was formed, the location of the double would depend on the neutral gas flow from the cathode. Recent tests by Goebel, et al., both double layer potential structures and visual indications of double layers (Figure 3).
Figure 3. Photograph of the plasma downstream of a hollow cathode showing the dark region and plume characteristic of a double layer.

To examine the behavior of this 25A laboratory cathode, we have assembled a 1-D fluid model. This model is similar to that in Ref.5, but has significant improvements in the plasma transport algorithms.

For simplicity, in this preliminary model we neglect ion and electron inertial in the momentum equations. In future studies, we expect to retain the ion inertia terms, but still neglect electron inertia. The combined ion-electron ambipolar diffusion equation is solved for plasma continuity,

\[ \frac{\partial n}{\partial t} = \nabla \cdot (D_a \nabla n) + \dot{n} \quad (7) \]

where resonant charge exchange between ionized and neutral xenon is assumed to be the principle ion scattering mechanism and the ambipolar diffusion coefficient is

\[ D_a = \frac{1}{n_0 \sigma_{\text{ceq}} v_1} \left( \frac{e(T_n + T_i)}{M} \right) \quad (8) \]

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The electric field is solved using Ohm’s law with classical resistivity. Ionization terms are neglected, since even if all the gas leaving the cathode were ionized, it would contribute at most a few percent of the total current.

$$j_e = \frac{1}{\eta} \left( E + \frac{\nabla nT_e}{n} \right), \quad E = \eta j_e - \frac{\nabla nT_e}{n}$$  \hspace{1cm} (9)

The electron scattering time is,

$$\tau_e = \frac{1}{\nu_{ei} + \nu_{e0}}$$  \hspace{1cm} (10)

where we use the classical electron-ion scattering frequency

$$\nu_{ei} = 2.9 \times 10^{-12} n\Lambda T_e^{\frac{1}{2}}, \quad \Lambda = 23 - \frac{1}{2} \ln \left( 10^{-6} n T_e^{-1} \right)$$  \hspace{1cm} (11)

and electron neutral scattering using a temperature averaged cross section\(^7\)

$$\nu_{e0} = n_0 \sigma_{e0}(T_e) \sqrt{\frac{eT_e}{m_e}}$$  \hspace{1cm} (12)

The plasma resistivity is

$$\eta = \frac{\alpha_{\text{Braginski}}}{\varepsilon_0 \tau_c \alpha_p}, \quad \alpha_{\text{Braginski}} = 0.51$$  \hspace{1cm} (13)

The electron energy equation includes convection, thermal conduction, Joule heating, pressure work, ionization losses, and energy transfer to heavy ions and electrons. The last term is small, and could have been neglected.

$$\frac{\partial}{\partial t} n_e T_e = -\nabla \cdot \left( -\frac{1}{2} j_e T_e - \kappa \nabla T_e \right) + \eta j_e^2 - j_e \cdot \nabla n_T - n_e e E_i - 3en \frac{m_e}{m_i} (\nu_{ei} + \nu_{e0})(T_e - T_i)$$  \hspace{1cm} (14)

The electron thermal conductivity term is given by,

$$\kappa = 3.2 \left( \frac{eT_e}{m_e} \right) \tau_e en$$  \hspace{1cm} (15)

Axial depletion of the neutral gas leaving the cathode is taken into account using a simple exponential model,
\[
\begin{align*}
\frac{d n_0}{dt} &= -n_0 n \sigma_{\text{ion}} v \\
n_0 &= n_r(z) + n_b \\
\frac{d n_r(z)}{dz} &= \frac{1}{v_0} \frac{d n_r}{dt} \\
&= \frac{1}{v_0} \left( \frac{d n_b}{dt} - \frac{d n_b}{dt} \right) \\
&= -\frac{1}{v_0} n_r(z) n \sigma_{\text{ion}} v 
\end{align*}
\] (16)

C. Discharge Cathode Double Layer

We have applied this model to a region starting at the keeper and extending 4 cm downstream, for a laboratory cathode at two different gas flows. The angle of the gas expansion was estimated from the plume photographs to be about 22.5° (a full cone angle of about 45°). Future studies will employ a more sophisticated gas model, using the algorithms developed in Ref. 18.

![Plume downstream of the laboratory hollow cathode running at 25 A and 5.5 sccm.](image)

The first set of calculations are for the cathode running at 25 A and 5.5 sccm. As seen in Figure 5, there is no apparent dark region. If there is a double layer it is upstream of the orifice plate. The 1-D code was run using the measured plasma density, electron temperature, and potential at the downstream surface of the orifice plate as boundary conditions. Results are shown in Figure 6 and Figure 7. In general, the 1-D model shows the same qualitative behavior at the data. The potential rises on a centimeter distance scale downstream of the keeper, and the electron temperature rises to just over 5 eV and remains relatively flat.
Figure 6. Density calculated using the 1-D model compared with laboratory measurements.

Figure 7. Plasma potentials, electron temperatures, and electron drift velocity calculated using the 1-D model compared with laboratory measurements for the case of 25 A and 5.5 sccm.

When the gas flow is increased to 10 sccm, the 1-D fluid model plasma potentials and temperatures no longer agree with the measured data. While there is clear evidence of a double layer in both the visual plume, Figure 3 and the plasma potential, Figure 8, the 1-D model does not capture this feature. The only evidence from the 1-D model that a double layer may be needed is that the electron drift velocity rises near double layer values just downstream of the keeper, a half centimeter short of the observed double layer. The electron drift velocity ratio is 50% higher in the 10 sccm case than in the 5.5 sccm, mainly a function of the lower boundary condition temperature. One can speculate...
that upstream of the cathode, drift instabilities or perhaps a double layer was responsible for heating the plasma electrons in the lower gas flow case.

![Graph](image)

**Figure 8.** Plasma potentials, electron temperatures, and electron drift velocity calculated using the 1-D model compared with laboratory measurements for the case of 25 A and 10 sccm.

### III. Conclusion

The gas density fall off just downstream of the hollow cathode orifice is seen to play a major role in the plasma potentials. When the flow is high, the potential remains below 20 volts for about a centimeter, and then discontinuously increases. At the lower flows, both calculations and measurement show a rapid increase in plasma potential near the keeper. We have performed additional calculations for larger angles of gas expansion. As the expansion angle increases, both the rate of potential rise and the relative electron drift velocities increase close to the keeper. This suggests that plasma ions that impact the keeper would fall through higher potentials as the gas flow is lowered. Also the rapid plasma rise could occur inside the keeper orifice, providing a source of energetic ions that would enlarge the orifice.

While some evidence of double layer behavior has been found, the approximate analysis and 1-D fluid model presented here, offer only clues, not a definitive, predictive theoretical treatment of this problem. To predict keeper erosion for the long duration missions being planned, detailed models are needed that will accurately predict in two dimensions the plasma ion fluxes and energies bombarding keeper surfaces. We are planning a serious modeling effort, coordinated with a program of hollow cathode plume plasma measurements, to develop the capability to predict keeper erosion for the wide range of ion thrusters envisioned for JIMO and follow-on Prometheus missions.

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### References
