PARALLEL ESTIMATORS AND COMMUNICATION IN SPACECRAFT FORMATIONS

Roy S. Smith * and **Fred Y. Hadaegh** **

* *Electrical & Computer Engr. Dept., Univ. California, Santa Barbara, CA 93106, USA.* roy@ece.ucsb.edu
** *Jet Propulsion Laboratory, California Inst. Technology, 4800 Oak Grove Dr., Pasadena, CA 91109, USA.* Fred.Y.Hadaegh@jpl.nasa.gov

Abstract:
This paper investigates the closed-loop dynamics of systems controlled via parallel estimators. This structure arises in formation flying problems when each spacecraft bases its control action on an internal estimate of the complete formation state. For LTI systems a separation principle shows that the necessary and sufficient conditions for overall system stability are more stringent than the single controller case; the controllers' open-loop dynamics necessarily appear in the closed-loop dynamics. Communication amongst the spacecraft can be used to specify the complete system dynamics and a framework for integrating the design of the communication links into the formation flying control design problem is presented.

Keywords: Formation flying control; Parallel estimation; Separation principle; Communication;

1. INTRODUCTION

Formation flying spacecraft are able to perform science missions that are infeasible with monolithic spacecraft. For example a formation of interferometric imaging spacecraft can realize an optical imaging system with an aperture of kilometers giving the resolution required to image planets in other solar systems. Precision in the spacecraft control is critical to the performance of such formations, and motivates this work.

Each spacecraft in the formation bases its control action on an estimate of the observable part of the entire formation. Note that in formations defined only by relative measurements, not all states are observable. Smith and Hadaegh (2004) illustrates how the unobservable states may be removed from the problem and for notational simplicity this paper simply assumes observability.

There are two major advantages in having each spacecraft estimate the entire formation state. The first is that it is then possible to implement a control which is optimal from the point of view of the entire formation. The second advantage is that each spacecraft also has sufficient information to implement higher level functions in the system hierarchy. Examples of such functions include: collision avoidance, path planning, formation reconfiguration, and communication network reconfiguration.

We contrast this with alternative approaches, based on leader-follower architectures (see for example: Wang and Hadaegh (1996); Robertson et al. (1999); Kapila et al. (1999); Tillerson et al.

(2003)) or local controllers networked via a communication system (see for example: Mesbahi and Hadaegh (2001); Fax and Murray (2003)). Such controller strategies may be simpler to implement but do not have the advantages listed above. The issue of decentralized estimation for formations has been considered by Carpenter (2000). The theory on parallel estimation given in this paper is has a much wider applicability, and early work in this area can be found in Khalil and Kokotović (1978) and Speyer (1979).

2. NOTATION

Vector subscripting has one of two meanings; when discussing a vector, $u \in \mathcal{R}^{n_u}$, u_i will refer to the ith component. This paper deals with parallel estimators, each estimating a state vector, $x \in \mathcal{R}^{n_x}$. In this case the subscripted vector, $\hat{x}_i \in \mathcal{R}^{n_x}$, refers to the ith estimate of x. Note also the use of $\hat{x}$ to denote an estimate of x.

The identity matrix of dimension $N \times N$ is denoted by I_N. The use of the Kronecker product simplifies a great deal of notation. Given $X \in \mathcal{R}^{m\times n}$ and $Y \in \mathcal{R}^{p\times q}$, the Kronecker product, $X \otimes Y \in \mathcal{R}^{mp\times nq}$, is defined by

$$X \otimes Y := \begin{bmatrix} X_{1,1}Y & X_{1,2}Y & \cdots & X_{1,n}Y \\ X_{2,1}Y & X_{2,2}Y & & \\ \vdots & & \ddots & \vdots \\ X_{m,1}Y & X_{m,2}Y & \cdots & X_{m,n}Y \end{bmatrix}.$$

3. PROBLEM DEFINITION

Consider the collected (observable) dynamics of the formation in a discrete-time[1] LTI framework,

$$x(k+1) = A\,x(k) + B\,u(k), \qquad (1)$$

where $x(k) \in \mathcal{R}^{n_x}$. A network of N controllers—one on each spacecraft—collectively applies the actuation signals via,

$$u(k) = \left[u_1(k) \cdots u_N(k)\right]^T.$$

For notational simplicity define projections, Q_i, onto each controller's "local" actuation signal via, $u_i(k) = Q_i u(k)$. Note that Q_i is also considered to be a partition of the identity,

$$\sum_{i=1}^{N} Q_i = I,$$

effectively augmenting $u_i(k)$ with zeros to expand it to the dimension of $u(k)$. The controllers have the same measurement of the plant output, although they may have different noise contributions,

$$y_i(k) = C\,x(k) + n_i(k), \quad i = 1, \ldots, N. \qquad (2)$$

The formulation in (2) assumes that each spacecraft has access to the same set of measurements. Smith and Hadaegh (2004), showed that equivalent controllers can be constructed from an over-parametrized set of relative measurements in a formation. This approach can be applied here to relax the assumption of identical measurements. In the more general case each spacecraft would have a different C_i matrix, and in this case the results cannot be stated as succinctly, although the relevant formulae are given.

Each controller is assumed to consist of an estimator and a static state feedback gain. A stabilizing state-feedback, $u(k) = K\,x(k)$, has been designed to give the desired formation closed-loop dynamics, $A_{\mathrm{cl}} := A + BK$. The task is now to implement this state-feedback via parallel estimators, with each estimator providing the state-feedback control for their components of the input.

In this scenario, the ith controller is given by,

$$\hat{x}_i(k+1) = A_{\mathrm{ctrl}}\,\hat{x}_i(k) - Ly_i(k)) \qquad (3)$$
$$u_i(k) = Q_i K\,\hat{x}_i(k), \qquad (4)$$

where $A_{\mathrm{ctrl}} := A+BK+LC$. This control is based on an estimate of the full system state, denoted by $\hat{x}_i(k)$. A wide range of methods are available for the design of the estimator error dynamics, given by the eigenvalues of $A+LC$. For notational simplicity, $A_{\mathrm{est}} := A + LC$. The structure of the system is illustrated in Figure 1.

In this structure $BK\hat{x}_i(k)$ is used as the plant input contribution to each of the estimates. This is actually an estimate of the actuation input applied by each of the other spacecraft and will not, in general, be correct for the non-local components of $u(k)$. The motivation for doing this is to consider structures that do not require that complete actuation information to be communicated amongst the spacecraft in the formation. The consequence of a lack of complete information about the plant input is that the estimator error dynamics become coupled.

4. PARALLEL ESTIMATOR DYNAMICS

The main result of this section is a separation principle giving the complete dynamics of the system illustrated in Figure 1. Define, for each estimator, an estimation error,

$$e_i(k) := x(k) - \hat{x}_i(k).$$

[1] Application to the continuous-time case is straightforward.

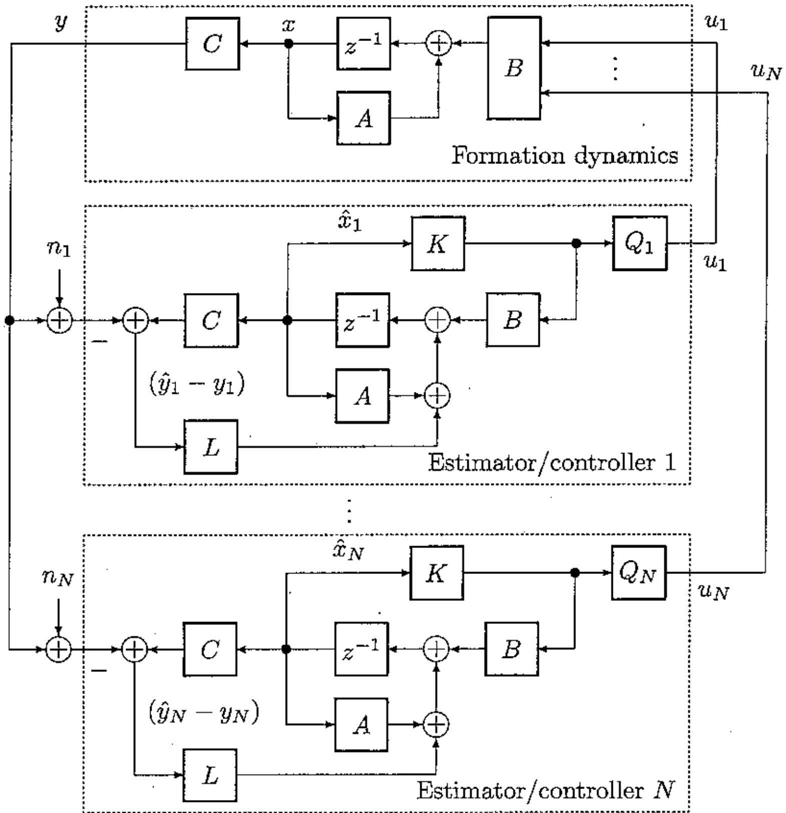

Fig. 1. System structure with N parallel estimators

The closed-loop plant dynamics are given by,

$$x(k+1) = A_{\text{cl}}\, x(k) \; - \; B \sum_{i=1}^{N} Q_i K e_i(k).$$

There are n_x states in the plant and n_x in each of the estimators, giving a total of $(N+1)n_x$ states in the formation. The remaining $N \times n_x$ state dynamics can be expressed in terms of the error dynamics for each of the estimators,

$$e_i(k+1) = A_{\text{ctrl}}\, e_i(k) - B \sum_{j=1}^{N} Q_j K e_j(k) \\ - L\, n_i(k).$$

Note that for stability analysis the $Ln_i(k)$ driving term in the error update equations can be neglected. The complete system closed-loop dynamics are therefore,

$$\begin{bmatrix} x(k+1) \\ e_1(k+1) \\ \vdots \\ e_N(k+1) \end{bmatrix} = \left(\begin{bmatrix} A_{\text{cl}} & 0 \\ 0 & I_N \otimes A_{\text{ctrl}} \end{bmatrix} - G \right) \begin{bmatrix} x(k) \\ e_1(k) \\ \vdots \\ e_N(k) \end{bmatrix}. \quad (5)$$

where,

$$G = \begin{bmatrix} 0 & BQ_1K & \cdots & BQ_NK \\ 0 & BQ_1K & \cdots & BQ_NK \\ \vdots & \vdots & & \vdots \\ 0 & BQ_1K & \cdots & BQ_NK \end{bmatrix}. \quad (6)$$

These equations form the basis of the parallel estimator separation theorem given below.

Theorem 1. Given a plant, defined by (1), and N parallel estimators, each defined by (3) and (4), the closed-loop system eigenvalues are:

$$\text{eig}(A_{\text{cl}}) \; \cup \; \text{eig}(A_{\text{est}}) \cup \underbrace{\text{eig}(A_{\text{ctrl}}) \; \cup \cdots \text{eig}(A_{\text{ctrl}})}_{N-1 \;\; \text{times}}.$$

This result extends the standard separation principle for LTI state-feedback and estimator designs in an interesting way. In the single controller case the necessary and sufficient conditions for closed-loop stability are that both $A+BK$ and $A+LC$ represent stable state dynamics. In the parallel estimator case this is no longer sufficient; we must also have each controller's open-loop dynamics, $A+BK+LC$, stable. The standard state-feedback and estimator design methods do not specify the eigenvalues of $A+BK+LC$ and must be used with extreme caution.

The formulation is easily generalized to the case where each estimator/controller uses a different measurement matrix,

$$y_i(k) = C_i\, x(k) + n_i(k), \quad i = 1, \ldots, N.$$

The design of each of the estimators now involves the design of a specific estimation gain, L_i, $i = 1, \ldots, N$. The closed-loop error dynamics are still decoupled from the closed-loop plant dynamics and are given by,

$$I_N \otimes (A + BK) - \begin{bmatrix} I \\ \vdots \\ I \end{bmatrix} \begin{bmatrix} BQ_1K & \cdots & BQ_NK \end{bmatrix} + \begin{bmatrix} L_1C_1 & & 0 \\ & \ddots & \\ 0 & & L_NC_N \end{bmatrix} \quad (7)$$

Equation 7 can be used as the basis for estimator design. For a fixed state-feedback design, K, the heterogeneous estimator design problem can be viewed as using $L_1, \ldots, L_N$ to place the eigenvalues of the matrix given in (7).

5. COMMUNICATION AMONGST ESTIMATORS

As the previous section made clear, the error dynamics of a parallel estimator configuration will exhibit open-loop controller dynamics. This problem can be addressed by communication amongst the estimators and a structure for doing this is now formulated. The original estimator error dynamics, $A + LC$ can be recovered—for all controller/estimators—by communicating each controller's actuation output, u_i, to all of the other controllers. This requires $N(N-1)$ communication links. However, the complete parallel estimator error dynamics can be specified with as few as $N - 1$ links; any fewer and the open-loop controller dynamics must appear in the combined estimator error dynamics.

5.1 A Communication Framework

Consider M unidirectional communication links, with the communicated signal defined as a linear function of the estimated state of a particular estimator,

$$v_l(k) = H_l\, \hat{x}_l(k) + w_l(k), \quad l = 1, \ldots, M.$$

The maximum singular value of H_l is proportional to the maximum communication power amplification. Noise, w_l, is assumed to corrupt the received signal. This is a simplistic model of a communication channel but it serves to determine the fundamental relationship between the communicated state information and estimator error dynamics.

The estimator receiving the noisy signal, v_l, updates its estimate in the following manner. Assume that the ith estimator receives the lth communicated signal. The $\hat{x}_j(k+1)$ update is performed as follows,

$$\hat{x}_i(k+1) = A_{\mathrm{ctrl}}\, \hat{x}_i(k) - Ly_i(k) + F_l v_l(k) - F_l H_l \hat{x}_i(k). \quad (8)$$

Note that noise enters the estimator through both the plant measurement, $y_i(k)$, and the received signal, $v_l(k)$. The matrix F_l specifies how the received signal is applied to the estimated state update and is proportional to the receiver's sensitivity. The effect of the communicated signal is applied to the update differentially; as a function of the difference between the estimated states of the transmitting and receiving estimators. This has the effect of maintaining the separation between the plant's closed-loop poles and the estimators' error dynamics.

If the ith estimator receives M_i communicated signals the state estimate update will be,

$$\hat{x}_i(k+1) = A_{\mathrm{ctrl}}\, \hat{x}_i(k) - Ly_i(k) + \sum_{l=1}^{M_i} F_l H_l(\hat{x}_l(k) - \hat{x}_i(k)) + \sum_{l=1}^{M_i} F_l w_l(k),$$

where $\hat{x}_l(k)$ is the estimated state of the lth transmitter received by the ith spacecraft estimator. Note that, $\hat{x}_j(k) - \hat{x}_i(k) = e_i(k) - e_j(k)$, giving the estimation error dynamics,

$$e_i(k+1) = A_{\mathrm{ctrl}}\, e_i(k) - B\sum_{j=1}^{N} Q_j K e_j(k) - L\, n_i(k) + \sum_{l=1}^{M_i} F_l H_l(e_l(k) - e_i(k)) - \sum_{l=1}^{M_i} F_l w_l(k). \quad (9)$$

The error dynamics do not explicitly involve $x(k)$; the plant's closed-loop dynamics and the parallel estimators' dynamics therefore remain decoupled under this communication scenario.

Communicating all actuator signals to all other spacecraft is sufficient to restore N copies of the designed estimator error dynamics to the closed-loop system.

Theorem 2. If each estimator/controller communicates its actuation input signal,

$$v_i(k) = u_i(k) = Q_i K \hat{x}_i(k), \quad i = 1, \ldots, N,$$

to all other estimator/controllers, then with the choice of

$$F_l = B, \quad l = 1, \ldots, N(N-1),$$

the closed-loop system eigenvalues are:

$$\mathrm{eig}(A_{\mathrm{cl}}) \cup \underbrace{\mathrm{eig}(A_{\mathrm{est}}) \cup \cdots \cup \mathrm{eig}(A_{\mathrm{est}})}_{N \text{ times}}.$$

This result also holds in the case where each estimator is designed with different error dynamics; $A + L_1C_1, \ldots, A + L_NC_N$.

This result makes it clear that the source of the coupled estimator error dynamics is the use of an incorrect estimated input signal $u_j(k)$, for $j \neq i$. The communication strategy used in Theorem 2 is no more than subtracting off the incorrect estimated actuation signals and replacing them with the correct communicated actuation signals.

The practical advantages of this scheme are limited; $N(N-1)$ communication links are required. A viable alternative may be to implement a single estimator which calculates and then communicates the required actuation signal to each of the other $N-1$ controllers. This approach requires only $N-1$ bidirectional links but introduces a latency in the actuation.

5.2 A Graph Theory Communication Specification

The more interesting case of incomplete communication (every estimator does not communicate with every other estimator) is now considered. The approach taken here is similar to that used by Fax and Murray (2003) in their work on cooperative control of vehicle formations. Our definition of the Laplacian differs and our work is conceptually closer to that of Pecora and Carroll (1998) on synchronized coupled systems.

The communication links are specified by an *adjacency matrix*, $A \in \mathcal{R}^{N \times N}$, where $A_{i,j} = 1$ if there is a communication link from estimator j (transmitter) to estimator i (receiver). This corresponds to a directed graph where each estimator/controller is a node, and each communication link is an arc. The *in-degree* of each node (estimator/controller) is the number of links for which that node is a receiver. The *Laplacian* of the graph is defined as,

$$L := D - A, \qquad (10)$$

where D is a diagonal matrix with the in-degree of each node on the diagonal. It is easy to show that every Laplacian has at least one zero eigenvalue.

Each communication link in our network can be defined by a simple Laplacian. Note that the sum of such links is also a Laplacian. Define, L_l, $\quad l = 1, \ldots, M$ as the Laplacian associated with the lth communication link. Tedious algebra shows that this has the effect of replacing the state transition matrix for the full system dynamics (given in (5) with,

$$\left(\begin{bmatrix} A_{\mathrm{cl}} & 0 \\ 0 & I_N \otimes A_{\mathrm{ctrl}} + \sum_{l=1}^{M} L_l \otimes F_l H_l \end{bmatrix} - G \right) \qquad (11)$$

where, G is given by (6). In the case where the communication update term, F_lH_l, is the same for each link, a significant and illustrative separation result can be proven. Define,

$$L := \sum_{l=1}^{M} L_l, \quad \text{and} \quad FH := F_lH_l, \quad l = 1, \ldots, M. \qquad (12)$$

This gives,

$$L \otimes FH = \sum_{l=1}^{M} L_l \otimes F_l H_l,$$

in the above. Denote the eigenvalues of L, ordered by increasing real part, by,

$$\mathrm{eig}(L) = \{\gamma_0, \gamma_1, \ldots, \gamma_{N-1}\},$$

where $\gamma_0 = 0$.

Theorem 3. Given a plant, defined by (1), and N parallel estimators/controllers, defined by (8) and (4), and with the communication between estimators defined by a Laplacian, L, as in (12), the closed-loop system eigenvalues are:

$$\mathrm{eig}(A_{\mathrm{cl}}) \cup \mathrm{eig}(A_{\mathrm{est}}) \cup \mathrm{eig}(A_{\mathrm{ctrl}} + \gamma_1 FH) \cup \cdots \cup \mathrm{eig}(A_{\mathrm{ctrl}} + \gamma_{N-1} FH) \qquad (13)$$

It is immediately apparent that in order to control the complete error dynamics the Laplacian must have $N-1$ non-zero eigenvalues. This means that it must have at least $N-1$ receivers in the network. The result given in Theorem 3 uses identical communication matrices (F and H) for each of the links. It is a simple matter to generalize this to a sum of Laplacians, each with different communication matrices, in the case where the Laplacians are simultaneously diagonalizable. The details are omitted for brevity.

An interesting case occurs when we have a "star" topology with one transmitter and $N-1$ receivers. If each communication link has transmission and receiver matrices, H_l, F_l, $l = 1, \ldots, N-1$, respectively, then the closed-loop eigenvalues are given by,

$$\mathrm{eig}(A_{\mathrm{cl}}) \cup \mathrm{eig}(A_{\mathrm{est}}) \cup \mathrm{eig}\left(A_{\mathrm{ctrl}} + F_1 H_1\right) \cup \cdots \cup \mathrm{eig}\left(A_{\mathrm{ctrl}} + F_{N-1} H_{N-1}\right). \qquad (14)$$

6. NETWORK TOPOLOGY AND SYSTEM DESIGN

The first point to note is that unlike the case of the "classical" separation principle, the controllers' open-loop dynamics appear in the closed-loop system. One can consider designing a parallel estimator network without communication but in such cases the open-loop dynamics of each of the controllers must be taken into consideration. This is a non-trivial problem.

A communication framework has been introduced that allows the design of communication links via a communication network defined through Laplacians, L_l, transmitter matrices, H_l, and receiver matrices, F_l, $l = 1, \ldots, M$. The designer can use communication to modify the open-loop controllers' dynamics that would otherwise appear in the closed-loop system. Note that the communication design can be performed independently of the control and estimator designs and will not affect the closed-loop eigenvalues that arise from those designs. The separation with respect to communication is not complete as the converse is not true: redesign of either the state feedback gain or the estimator gains will affect the eigenvalues placed via the communication system gains.

Design of the communication links has several components, all of which contribute to the closed-loop eigenvalue locations. The first aspect is the design of the network topology itself. This may be specified as a single Laplacian with multiple communication links, or via several Laplacians, each specifying a subset of the communication links. The eigenvalues of the Laplacian determine how the communication matrices, $F_l H_l$, modify the open-loop controller dynamics that appear in the closed-loop system.

Every Laplacian has at least one eigenvalue at zero and to completely remove the open-loop loop controllers' eigenvalues from the dynamics the Laplacian must have $N-1$ non-zero eigenvalues. This observation has significant engineering consequences; the functioning of receivers is more critical than the functioning of transmitters. A network in which open-loop controller dynamics do not appear in the error dynamics must have $N-1$ functioning receivers. It need have only one functioning transmitter.

At first glance it appears that the minimum communication required to specify the complete closed-loop dynamics is the same as that required to implement a centralized controller with all calculations carried out by a single estimator/controller. This is not the case; the dimension of the communicated signals, v_l, may be smaller than the dimension of the actuation signals, u_i. The centralized control option also has the disadvantage of introducing a latency into the feedback loop as actuation commands must be communicated and then applied. An additional advantage of the parallel estimator structure is that the communication links specified by F_l and H_l may be designed specifically for the communication noise levels, w_l.

7. ACKNOWLEDGMENTS

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