On the Detection of Energetically Efficient Trajectories for Spacecraft

Michael Dellnitz[*], Oliver Junge[*†], Martin Lo[‡] and Bianca Thiere[*§]

July 19, 2001

Abstract

We propose a new method for the detection of energy-efficient trajectories for spacecraft. Via a so called *target-shooting* approach a pseudo-orbit between the relevant points in space is constructed in a simple model of the problem. This approximate trajectory is meant to serve as input for a more sophisticated direct method in order to compute a true trajectory in the full model. We demonstrate the applicability of the new method by considering the redesign of part of the trajectory of the NASA/JPL Genesis discovery mission.

1 Introduction

The Genesis Discovery Mission [1] will collect and return solar wind sample from an L1 halo orbit for research on Earth to address the formation of the Solar System, a key question of NASA's Origins Program. The Genesis trajectory is unique; it is the first space mission to be designed using modern dynamical systems theory. A spectacular result of this approach is the fact that: if everything is performed perfectly with perfect knowledge and precision, the optimal Genesis trajectory requires *no deterministic maneuvers* after launch to complete its three-year mission. This encompasses transfer and insertion into an L1 halo orbit, remain in the halo orbit for four revolutions (two years) to collect the solar wind samples, depart the halo orbit, and return the samples to the Utah Test and Training Range (UTTR) before noon where a helicopter will capture the capsule containing the samples.

The original Genesis launch was scheduled for Feburary of 2001. But, in December 2000, the Genesis launch was delayed to July of 2001. For Genesis this required

[*]Department of Mathematics and Computer Science, University of Paderborn, D-33095 Paderborn, Germany; dellnitz@upb.de

[†]junge@upb.de

[‡]Navigation and Mission Design Section, Jet Propulsiona Laboratory, California Institute of Technology, Pasadena, CA 91109-8099; Martin.Lo@jpl.nasa.gov

[§]thiere@math.upb.de

a complete redesign of the delicate trajectory which depended on the heteroclinic connection between the L1 and L2 region and the stringent requirement to return to UTTR during the day. Initial redesign produced trajectories which required an extra loop around the Earth which lengthened the return time by roughly 5 months, cf. Figure 1. This is no surprise as studies of heteroclinic connections between L1 and L2 in the Sun-Earth system frequently required such a loop. In the end, the loop was removed by going to a completely different family of halo orbits using JPL's LTool.

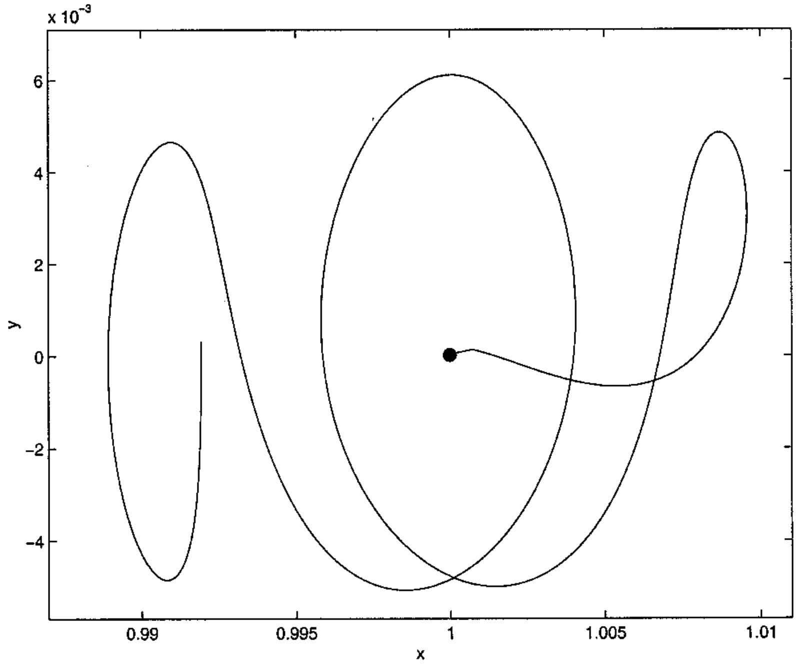

Figure 1: Initial redesigned return trajectory for the Genesis discovery mission. The Earth is located near $(1, 0)$.

In this paper, we explore a systematic approach to removing the undesired loop whithout changing to a different halo orbit. The technique is inspired by recently developed set-oriented methods for the global numerical analysis of dynamical systems [3, 4] – in particular by the set-oriented approximation of invariant manifolds [2, 5]. The underlying idea of the advocated *target shooting* algorithm (which is reminescent of the multiple shooting techniques for solving boundary value problems, see e.g. [6]) is to search for *pseudo-orbits* in a simple model of the problem (i.e. the Circular Restricted Three Body Problem). Roughly speaking a pseudo-orbit is a finite sequence of short trajectories which connect a given initial point to a given end point while allowing for (small) discontinuities between the individual trajectories. These pseudo-orbits should provide a good initial guess for a direct method for the determination of the final trajectory (e.g. for the differential corrector as implemented in LTool) in the full model. We apply the algorithm within the context of the Genesis mission and show how to generate candidates for return trajectories without the undesired loop around the Earth.

2 Target shooting

We consider an autonomous differential equation of the form

$$\dot{x} = f(x), \quad f : X \to \mathbb{R}^n, \tag{1}$$

where X is a subset of $\mathbb{R}^n$, and denote by $\phi : X \times \mathbb{R} \to X$ the flow generated by (1).

DEFINITION 2.1 Let $\varepsilon = (\varepsilon_1, \dots, \varepsilon_\ell)$ be a vector of *jumps* $\varepsilon_j \in \mathbb{R}^n$ with $(\varepsilon_j)_i > 0$, $i = 1, \dots, n$, and let $t = (t_1, \dots, t_\ell) \in \mathbb{R}^\ell$ be a positive vector of integration times. An (ε, t)*-pseudo-orbit* of the flow ϕ is a sequence $\xi = (x_0, \dots, x_\ell)$ of points, $x_j \in \mathbb{R}^n$, such that

$$|\phi(x_{j-1}, t_j) - x_j| \leq \varepsilon_j, \quad j = 1, \dots, \ell, \tag{2}$$

where the inequality is meant to be read componentwise.

Our goal is to find some pseudo-orbit ξ which connects a given initial point y to a given end point z within a given accuracy ε. More precisely, for a given vector of jumps ε we are looking for integration times $t = (t_1, \dots, t_\ell)$ and an (ε, t)-pseudo-orbit $\xi = (x_0, \dots, x_\ell)$, such that

$$y = x_0, \quad z = x_\ell.$$

On a less ambitious level one would choose reasonable integration times $t_1, \dots, t_{\ell-1}$ as well as a maximum time for t_ℓ a priori and let only t_ℓ be determined automatically. This is the actual task which is eventually accomplished by the following algorithm.

ALGORITHM 2.2 (Target shooting)

`find_pseudo_orbit`$(x, \varepsilon, t, z, o)$
  `if length`(t)`=1`
    `if` $\exists\, s \leq t_1$ `such that` $|\phi(x, s) - z| \leq \varepsilon_1$
      `output` s `and` $[o, x]$ `and return`
  `choose points` $y_1, \dots, y_m$ `such that` $|\phi(x, t_1) - y_i| \leq \varepsilon_1$
  `for` $i = 1, \dots, m$
    `find_pseudo_orbit`$(y_i, \varepsilon', t', z, [o, x])$

Here $\varepsilon' = (\varepsilon_2, \ldots, \varepsilon_\ell)$ and $t' = (t_2, \ldots, t_\ell)$ when $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_\ell)$ and $t = (t_1, \ldots, t_\ell)$. For a list $o = [x_0, \ldots, x_k]$ of vectors $x_j \in \mathbb{R}^n$ we write $[o, x]$ for the list $[x_0, \ldots, x_k, x]$. The symbol [ ] denotes the empty list. A call of `find_pseudo_orbit`$(x, \varepsilon, t, z, [\,])$ eventually outputs pseudo-orbits connecting x to z.

REMARK 2.3 (i) Depending on the (initial) number of steps ℓ of the algorithm the number m of intermediate points in each step should not be chosen too large since the numercial effort increases exponentially in ℓ.

(ii) For the approximation of ϕ one would typically employ a numerical scheme with stepsize control. This stepsize control actually has to be amended in such a way that the neighborhood check in line 2 of Algorithm 2.2 can be performed.

(iii) The determination of a suitable pseudo-orbit can be viewed as an optimization problem. After all one is interested in minimizing the size and the number of jumps as a function of the pseudo-orbit and the integration times. An approach along these lines is currently under investigation.

3 Target Shooting for the Genesis Discovery Mission

In the context of the Genesis Discovery Mission we consider a halo orbit in the vicinity of the libration point L1 of the circular restricted three body problem

$$\begin{aligned} \dot{x} &= u, & \dot{u} &= 2v + x + c_1(x + \mu - 1) + c_2(x + \mu), \\ \dot{y} &= v, & \dot{v} &= -2u + y + (c_1 + c_2)y, \\ \dot{z} &= w, & \dot{w} &= (c_1 + c_2)z, \end{aligned}$$

where

$$c_1 = -\frac{\mu}{((x + \mu - 1)^2 + y^2 + z^2)^{\frac{3}{2}}}, \quad c_2 = -\frac{1 - \mu}{((x + \mu)^2 + y^2 + z^2)^{\frac{3}{2}}},$$

with the Sun and the Earth as the primaries (i.e. $\mu = 3.040423398444176 \cdot 10^{-6}$) and try to detect an energetically efficient trajectory back to Earth. We apply the target shooting method with end point

$$z = (0.99999695956708, 0, 0, 0, 0, 0)^T$$

(i.e. the center of the Earth) and a couple of initial points which approximately lie at a distance of 10^{-8} (≈ 1.5 km) of the halo orbit on its unstable manifold. We choose

$$\begin{aligned} \varepsilon &= ((0, 0, 0, 10^{-3}, 10^{-3}, 10^{-3})^T, (10^{-4}, 10^{-4}, 10^{-4}, 0.2, 0.2, 0.2)^T), \\ t &= (3.0, 4.0)^T, \end{aligned}$$

i.e. we allow for a Δv of approximately 30m/s after 3 time units ($\approx$ 174 days) and require the spacecraft to reach a box of radius 10^{-4} ($\approx$ 15.000 km) around the Earth after at most 4 time units ($\approx$ 232 days), while only loosely restricting the speed at the end point ($\approx$ 6000m/s). One of the computed pseudo-orbits is

$$\xi_1 = \left(\begin{pmatrix} 0.99166288413400 \\ -0.00374932610817 \\ -0.00096354805402 \\ -0.00162684773981 \\ -0.00651889317254 \\ 0.00356849174105 \end{pmatrix}, \begin{pmatrix} 1.00429768657337 \\ -0.00034866194692 \\ 0.00118222812383 \\ 0.01071479619234 \\ 0.02325360572805 \\ 0.00145685510564 \end{pmatrix}, \begin{pmatrix} 0.99989752216379 \\ -0.00003832672726 \\ 0.00002228673461 \\ 0.08815101759107 \\ -0.13143166406622 \\ 0.17335584104554 \end{pmatrix} \right)$$

cf. Figure 2 – this is actually an $(\varepsilon, (3.0, 1.33605681636518))$-pseudo-orbit. Observe

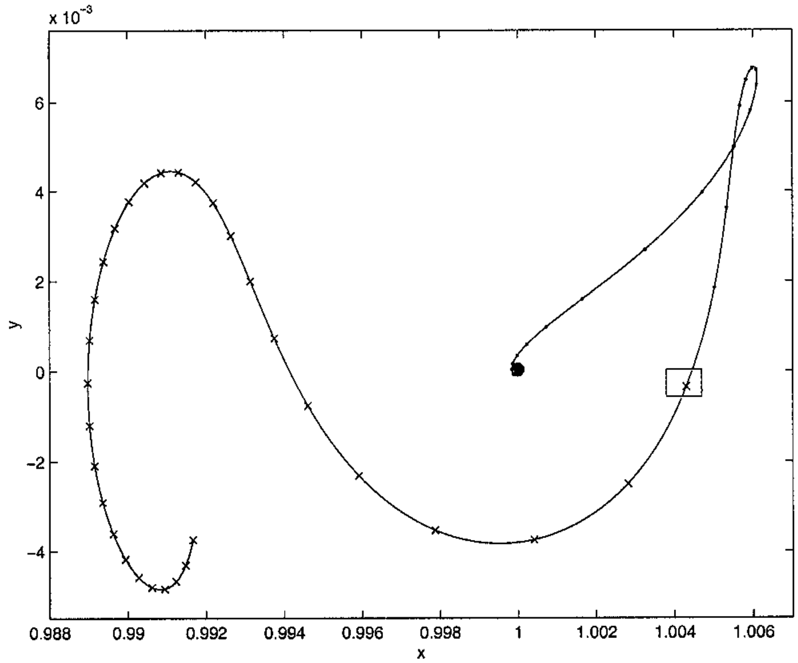

Figure 2: Sequence of two trajectories corresponding to the pseudo orbit ξ_1. The jump is marked with the box.

that these curves avoid the loop around the Earth as desired. We stress that such "no-loop trajectories" could not be found if one restricts the computations to trajectories lying entirely inside the unstable manifold of the halo orbit.

We finally use the pseudo-orbit ξ_1 as input for the differential corrector as implemented in LTool. The underlying model in this case is the restricted three body problem with the Sun and the Earth-Moon barycenter as the primaries. The corrected trajectory is shown in Figure 3.

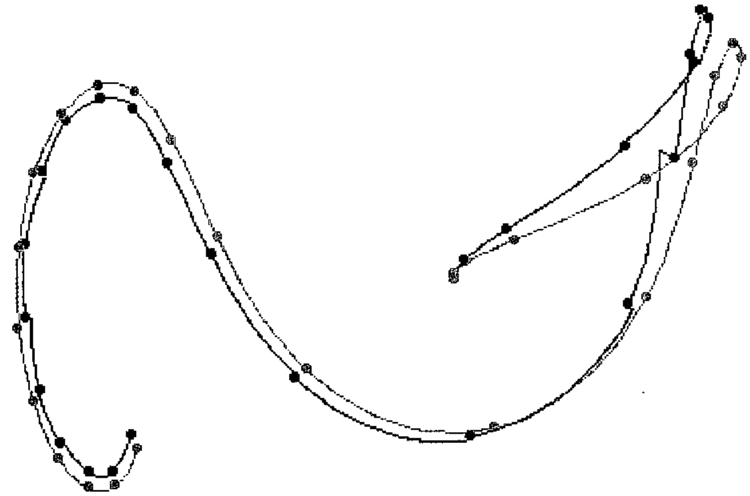

Figure 3: Pseudo-orbit (dark) and corrected trajectory in LTool.

A pseudo-orbit in the vicinity of the loop-trajectory

Note that the initial point of the pseudo-orbit ξ_1 (and also of the corrected trajectory) generated in the previous section (Figures 2 and 3) lie rather far away from the initial point of the return trajectory with loop (Figure 1). Our aim is now to find a pseudo-orbit connecting the initial point of the latter trajectory to the Earth. To this end we employ the target shooting algorithm with data

$$\begin{aligned} \epsilon &= (10^{-3}(0,0,0,1,1,1)^T, 10^{-3}(0,0,0,1,1,1)^T, (10^{-4}, 10^{-4}, 10^{-4}, 0.2, 0.2, 0.2)^T), \\ t &= (2.0, 2.0, 3.0)^T, \end{aligned}$$

i.e. this time we allow for two jumps after 2 and again 2 time units. The generated $(\varepsilon, (2.0, 2.0, 0.68312146298093113))$-pseudo-orbit together with the corresponding trajectories is shown in Figure 4. Finally we use the pseudo-orbit ξ_2 as input for the differential corrector in LTool, Figure 5 shows the resulting corrected trajectory.

4 Acknowledgements

This work was carried out in part at the Jet Propulsion Laboratory and California Institute of Technology under a contract with the National Aeronautics and Space Administration. Research of the authors is partly supported by the DAAD and the NSF.

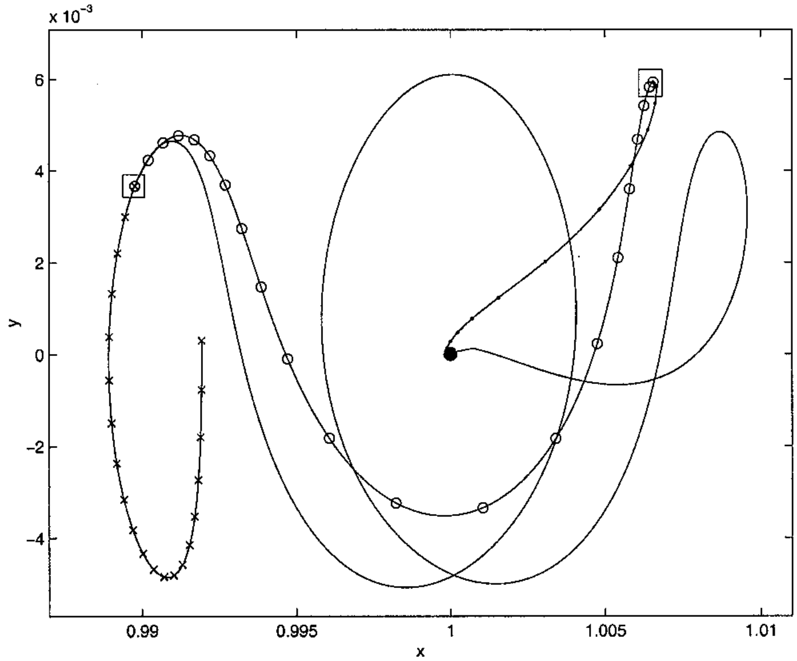

Figure 4: Initial return trajectory with loop (without markers) and sequence of three trajectories corresponding to the pseudo orbit ξ_2. The jumps are marked with boxes.

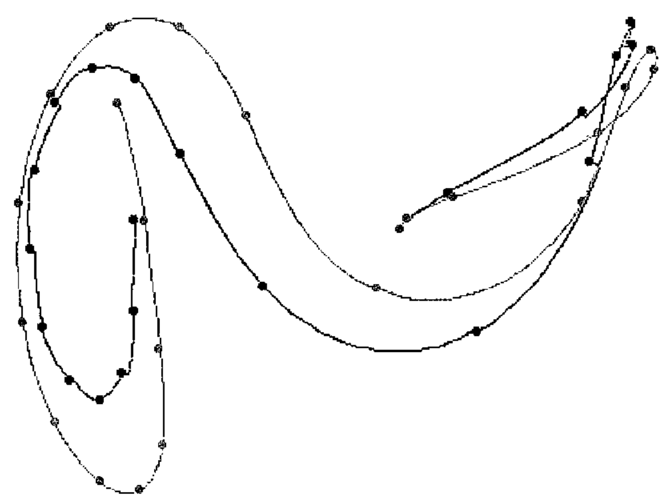

Figure 5: Pseudo-orbit ξ_2 (dark) and corrected trajectory in LTool.

References

[1] http://genesismission.jpl.nasa.gov.

[2] M. Dellnitz and A. Hohmann. The computation of unstable manifolds using subdivision and continuation. In H.W. Broer, S.A. van Gils, I. Hoveijn, and F. Takens, editors, *Nonlinear Dynamical Systems and Chaos*, pages 449–459. Birkhäuser, *PNLDE* 19, 1996.

[3] M. Dellnitz and A. Hohmann. A subdivision algorithm for the computation of unstable manifolds and global attractors. *Numerische Mathematik*, 75:293–317, 1997.

[4] M. Dellnitz and O. Junge. On the approximation of complicated dynamical behavior. *SIAM J. Numer. Anal.*, 36(2):491–515, 1999.

[5] O. Junge. *Mengenorientierte Methoden zur numerischen Analyse dynamischer Systeme.* PhD thesis, University of Paderborn, 1999.

[6] J. Stoer and R. Bulirsch. *Introduction to numerical analysis. Translated from the German by R. Bartels, W. Gautschi and C. Witzgall. 2. ed.*