Electron spin dephasing and decoherence by interaction with nuclear spins in self-assembled quantum dots

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Abstract

Electron spin dephasing and decoherence by its interaction with nuclear spins in self-assembled quantum dots are investigated in the framework of the empirical tight-binding model. Electron spin dephasing in an ensemble of dots is induced by the inhomogeneous precession frequencies of the electron among dots, while electron spin decoherence in a single dot arises from the inhomogeneous precession frequencies of nuclear spins in the dot. For $\mathrm{In_xGa_{1-x}As}$ self-assembled dots containing $\sim$ 30000 nuclei, the dephasing and decoherence times are predicted to be on the order of 100 ps and 1 μs.

I. INTRODUCTION

Quantum systems are predicted to provide new resources for more efficient computations and more secure communications.[1] One of the crucial requirements for the success of any form of quantum processing is to maintain the coherence of quantum systems.[2] The long electron spin decoherence time in a quantum dot suggests that the localized electron spin can be used as a qubit.[3, 4] The dominant mechanism for spin decoherence in bulk is related to spin-orbit coupling but it is strongly suppressed in a quantum dot due to the quantization of electron levels.[5] Spin decoherence due to the electron-hole exchange interaction can be avoided by removing holes in a dot. Spin decoherence via the interaction with phonons can be suppressed at low temperatures.[6] Therefore, the hyperfine interaction between electron and nuclear spins is expected to be a dominant mechanism for electron spin decoherence in a negatively charged quantum dot at low temperatures. In this paper, we investigate the spin decoherence time of a single dot and the spin dephasing time of an ensemble of dots due to the hyperfine interaction with nuclear spins.

II. HYPERFINE INTERACTION OF AN ELECTRON SPIN WITH NUCLEAR SPINS

The electron spin interaction with nuclear spins is described by the hyperfine Fermi contact interaction:

$$\hat{H}_{HF} = \frac{16\pi}{3}\mu_B\mu_N \sum_j g_j(\hat{\mathbf{S}} \cdot \hat{\mathbf{I}}_j)\delta(\mathbf{r} - \mathbf{R_j}), \tag{1}$$

where μ_B and μ_N are the Bohr magneton and the nuclear magneton, and g_j is the g factor of the jth nuclear spin. $\hat{\mathbf{S}}$ and $\hat{\mathbf{I}}_j$ are the spin operators of the electron and the jth nucleus, and $\mathbf{r}$ and $\mathbf{R_j}$ are the position vectors of the electron and the jth nucleus. Since the energy of the hyperfine interaction is much smaller than the energy spacing between quantized electron levels for quantum dots, the hyperfine Hamiltonian for a given electron level can be approximated with the first order perturbation theory:

$$\begin{aligned}\hat{H}_{HF} &= \frac{16\pi}{3}\mu_B\mu_N \sum_j g_j|\psi(\mathbf{R_j})|^2(\hat{\mathbf{S}} \cdot \hat{\mathbf{I}}_\mathbf{j}) \\ &= \sum_j A_j(\hat{\mathbf{S}} \cdot \hat{\mathbf{I}}_j), \end{aligned} \tag{2}$$

where $\psi(\mathbf{R})$ is the electron wave function, and A_j is the hyperfine coupling constant between the electron and the jth nuclear spin:

$$A_j = \frac{16\pi}{3}\mu_B\mu_N g_j \left|\psi(\mathbf{R_j})\right|^2 \tag{3}$$

The effective hyperfine magnetic field acting on the electron $\mathbf{B}_e$ and that acting on the jth nuclear spin $\mathbf{B}_j$ can be obtained from Eq. (2):

$$\mathbf{B}_e = \frac{1}{g_e\mu_B}\sum_j A_j \left\langle \hat{\mathbf{I}}_j \right\rangle_N, \tag{4}$$

$$\mathbf{B}_j = \frac{1}{g_j\mu_N} A_j \left\langle \hat{\mathbf{S}} \right\rangle_E, \tag{5}$$

where g_e is the g factor of the electron, and $\langle\cdots\rangle_N$ and $\langle\cdots\rangle_E$ are quantum mechanical averages over the nuclear wave function and the electron wave function, respectively. Furthermore, the Larmor frequencies of the electron precession (ω_e)and the jth nuclear precession (ω_j) in these fields are given by

$$\omega_e = \frac{g_e\mu_B}{\hbar}B_e = \frac{1}{\hbar}\left|\sum_j A_j \left\langle \hat{\mathbf{I}}_j \right\rangle_N\right|, \tag{6}$$

$$\omega_j = \frac{g_j\mu_N}{\hbar}B_j = \frac{1}{\hbar}A_j\left|\left\langle \hat{\mathbf{S}} \right\rangle_E\right| = \frac{A_j}{2\hbar}. \tag{7}$$

The electron precession frequency ω_e is much greater than the nuclear precession frequency ω_j since the electron interacts with a large number of nuclei while a nucleus interacts with the single electron. This means that the electron spin precession can be decoupled from the nuclear spin dynamics and can be described by the equation of motion of the electron spin in a fixed hyperfine magnetic field B_e during the time that is much smaller than the nuclear spin precession period $T_N = 1/\langle\omega_j\rangle$.

III. ENSEMBLE ELECTRON SPIN DEPHASING

In the time limit $t \ll T_N$ where the hyperfine magnetic field B_e can be approximated to be fixed, the motion of the electron spin in a single dot is coherent. However, the magnitude and direction of B_e are randomly distributed in an ensemble of dots. Hence the electron spin in each dot processes with a different frequency and this leads to ensemble spin dephasing. The dispersion of the electron spin precession frequency ω_e is given by

$$\Delta\omega_e = \sqrt{\langle\omega_e^2\rangle_{\text{ensemble}} - \langle\omega_e\rangle^2_{\text{ensemble}}}$$

$$= \frac{1}{\hbar}\sqrt{\sum_j A_j^2 I_j(I_j+1)}, \tag{8}$$

where $\langle\cdots\rangle_{\text{ensemble}}$ is an average over the ensemble of dots. The inhomogeneous broadening of the precession frequencies results in ensemble spin dephasing, of which the characteristic time is

$$T_2^* = 1/\Delta\omega_e. \tag{9}$$

IV. SINGLE ELECTRON SPIN DECOHERENCE

At times $t \sim T_N$, the time dependence of B_e becomes important and this leads to the single spin decoherence. In a single dot, nuclear spins process about the average electron spin direction at different rates given by Eq. (7). The inhomogeneous broadening of ω_j creates a time dependent B_e along the direction perpendicular to the average spin direction. As a result, the electron spin precesses about the slowly varying B_e and this leads to the spin decoherence. The characteristic time of the spin decoherence T_2 is determined by the dispersion of the nuclear spin precession frequency $\Delta\omega_n$:

$$\begin{aligned} T_2 &= 1/\Delta\omega_n & (10)\\ &= 1/\sqrt{\langle\omega_j^2\rangle - \langle\omega_j\rangle^2} & (11)\\ &= 2\hbar/\sqrt{\langle A_j^2\rangle - \langle A_j\rangle^2} & (12)\end{aligned}$$

To demonstrate the electron spin decoherence, we study the electron spin dynamics in a particular configuration, where the initial electron spin state is the eigenstate of $\hat{S}_z$ and the initial nuclear spin state is the eigenstate of $\hat{I}_{jz}$: $|\psi(0)\rangle = |\uparrow_e, \{I_{jz}\}\rangle$. Under the hyperfine Hamiltonian given by Eq. (2), the time evolution of $|\psi(t)\rangle$ can be described in the interaction picture by expanding it with the perturbation $\hat{V} = \sum_j A_j(\hat{S}^+\hat{I}_j^- + \hat{S}^-\hat{I}_j^+)/2$. Up to the first order perturbation, the electron and nuclear spin state is given by

$$|\psi(t)\rangle = |\uparrow_e, \{I_{jz}\}\rangle - \sum_j \frac{\omega_j}{\omega_z+\omega_j}\left(1 - e^{-i(\omega_z+\omega_j)t}\right) I_j^+ |\downarrow_e, \{I_{jz}\}\rangle, \tag{13}$$

where $\omega_z = \sum A_j I_{jz}/\hbar$. The electron spin decay can be evaluated by $\langle\hat{S}_z(t)\rangle$:

$$\begin{aligned}\langle\hat{S}_z(t)\rangle &= \left\langle\psi(t)\left|e^{it\hat{V}}\hat{S}_z e^{-it\hat{V}}\right|\psi(t)\right\rangle \\ &= \frac{1}{2}\left[1 - \sum_j \frac{2\omega_j^2}{(w_z+w_j)^2}\left(1-\cos(\omega_z+\omega_j)t\right)\langle\{I_{jz}\}|I_j^- I_j^+|\{I_{jz}\}\rangle\right]. \qquad (14)\end{aligned}$$

V. RESULTS AND CONCLUSIONS

The electron spin dephasing and decoherence times are calculated for InAs self-assembled dots containing about 30000 nuclei. The modeled dot is lens shaped with diameter 15 nm and height 5 nm. The InAs dot is embedded in GaAs matrix and is strongly strained due to the large lattice mismatch 7%. The strain profile is calculated with an atomistic valence force field model. The localized electron states are obtained using the empirical tight-binding model. For the calculation of the decoherence and dephasing times, the density of the electron state at the nuclear site is needed. The electron state obtained with the tight-binding model is a linear combination of atomic basis orbitals of which the real-space description is unknown. Hence, we empirically determine basis-orbital densities $|\phi_s(0)|^2$ and $|\phi_{s*}(0)|^2$ with experimental measurements using the Overhauser shift of the electron spin resonance.[7]

The electron spin dephasing time of an ensemble of the InAs dots is found to be on the order of 100 ps, which is similar to the measured T_2^*. The ensemble dephasing can be suppressed by applying a high external magnetic field. The electron spin decoherence time of the InAs single dot is found to be on the order of 1 μs. Figure 1 shows that the time evolution of the electron spin $\langle \hat{S}_z(t) \rangle$. The electron spin precesses with the period of 100 ns and decays exponentially with the decay time of 1 μs. The tight-binding calculations suggest that the variations of the dephasing and decoherence times among different electron states and different alloy composition ratios ($In_xGa_{1-x}As$) are minimal (about 10%).

Acknowledgments

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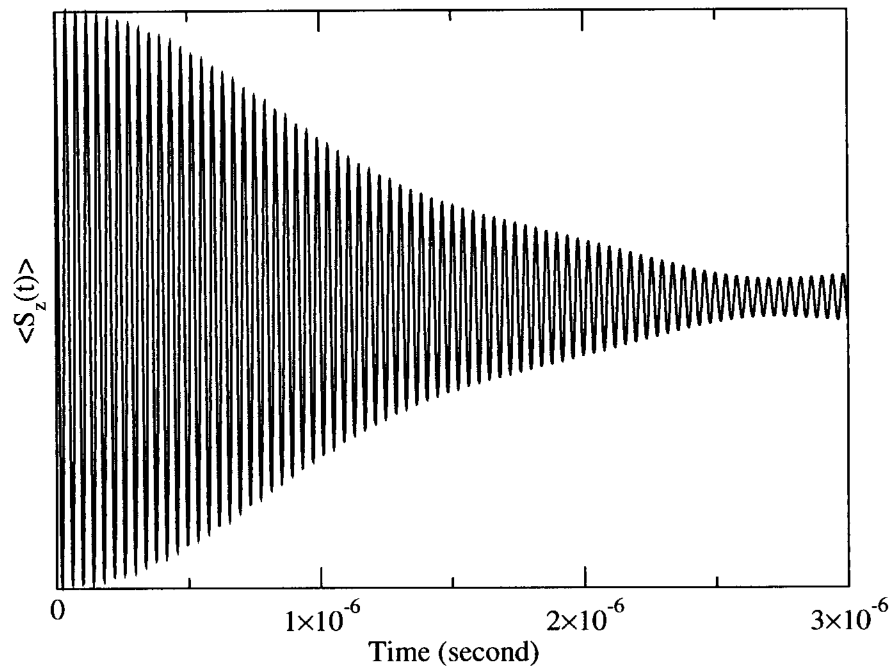

FIG. 1: Time evolution of electron spin $\langle \hat{S}_z \rangle$ under the hyperfine interaction with nuclear spins in an InAs self-assembled quantum dots containing ~30000 nuclei.

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