Thermal Equilibrium Between Radiation and Matter: A Lead to the Maxwell-Boltzmann and Planck Distributions

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5 September, 2003

(Note: formulas are planned to be better formatted and explained with some expansion)
Thermal Equilibrium Between Radiation and Matter

Introduction

- Milestones: 1901: Planck’s constant and blackbody radiation law
  1916: Einstein rederived the blackbody radiation law

- Einstein's assumptions are adopted with two exceptions
- Seeking the conditions for: the thermal equilibrium of matter
  - the spectral distribution of radiation

Results:

- The existence of the constants of Boltzmann and Planck
- The Maxwell-Boltzmann distribution
- Planck’s law of blackbody radiation
- Bohr’s frequency condition
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Wien's law

- Wien's displacement law
- Scaling and dimensional considerations
- \( k \) and \( h \) are dimensioned as the constants of Boltzmann and Planck, but currently with undetermined values

\[
u_{\nu} = \nu^3 \ W( \nu / T)\]

\[
u_{\nu} = (8 \pi \nu^3 / c^3) \ h \ W(\beta h \nu)\]

\[
\beta = 1 / \theta \\
\theta = kT = \text{thermodynamic temperature}
\]
Thermal Equilibrium Between Radiation and Matter

Thermal equilibrium

- Thermal balance between radiation and matter

\[ f_2 \, B_{12} \, (u_{\nu} + a) = f_1 \, B_{12} \, u_{\nu} \, (f \text{ with a hat}) \]

\[ K_{2}/(b \, f(\beta' \, E_1)/f \, \beta' \, E_2 - 1) = W(\beta \, h \, \nu) \]
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Thermal equilibrium

- Thermal balance between radiation and matter

\[ f_{-2} B_{-12} (u_{nu} + a) = f_{-1} B_{-12} u_{nu} \ (f \text{ with a hat}) \]

\[ K_{-2}/( b f( beta' E_{-1})/f beta' E_{-2} - 1) = W(beta h nu) \]

\[ beta' = 1/theta \]
\[ theta' = \text{statistical temperature} \]
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Final Equation and its Solution

- g is function of W

\[
\ln f((\beta' E_1/\beta h \nu) \beta h \nu) - \ln f((\beta' E_2/\beta h \nu) \beta h \nu) = \ln g(\beta h \nu)
\]

\[
f(\beta' E) = \exp(- (\beta' E)^n) \\
\beta = K_2 \beta' \\
E_2^n - E_1^n = (K_1 h \nu)^n \\
W(\beta h \nu) = K_1 (b \exp(K_2 \beta' h \nu)^n - 1)
\]
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Classical Limit

- classical limit: \( \beta' (E_2 - E_1) \ll 1 \)
- \( \sigma(1) = 1 \)

\[
b^{-1} - b(K_3 \beta' h \nu) = K_1 \beta' h \nu / \sigma(n)
\]
Thermal Equilibrium Between Radiation and Matter

References


