MARS ORBITS WITH DAILY REPEATING GROUND TRACES

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ABSTRACT

This paper derives orbits at Mars with ground traces that repeat at the same times every solar day (sol). A relay orbiter in such an orbit would pass over in-situ probes at the same times every sol, ensuring consistent coverage and simplifying mission design and operations. 42 orbits in five classes are characterized:

- 14 circular equatorial prograde orbits
- 14 circular equatorial retrograde orbits
- 11 circular sun synchronous orbits
- 2 eccentric equatorial orbits
- 1 eccentric critically inclined orbit

The paper reports on the performance of a relay orbiter in some of the orbits.

INTRODUCTION

NASA has been building a Mars relay network to facilitate communications between Marscraft* and Earth and to provide navigation and timing services. This network currently includes UHF relays on two NASA orbiters – Mars Global Surveyor (MGS) and Mars Odyssey[1] – both in low orbits selected to optimize the collection of remote science rather than relay performance. The network will be augmented by UHF relays on the ESA Mars Express orbiter in 2004 and by the NASA Mars Reconnaissance Orbiter (MRO) in 2006.

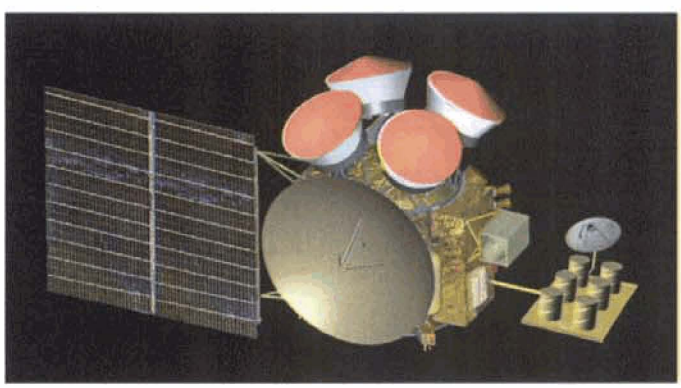

Figure 1 Artist Conception of Mars Telesat

NASA plans to send a telecommunications satellite to Mars in 2009. This Mars Telesat (Figure 1) will be placed into an orbit optimized for Marscraft relay support. While the Telesat may carry 4 CNES NetLanders to Mars and may also support a deep space

* A Marscraft is a lander, rover, aerobot or small orbiter in the vicinity of Mars.

optical communications experiment and some scientific instruments, its primary mission will be to relay communications to and from Marscraft. It thus will be placed into an orbit optimized for its relay communications mission.

Marscraft operators typically desire repeated contacts at the same time every day. This enables them to design their missions and operations based on an invariant pattern of relay contacts. To identify desirable Mars Telesat orbits, we have characterized orbits with ground traces that repeat every sol. This paper describes the Mars orbits we characterized and reports on the performance of the Mars Telesat operated in several of these orbits.

Similar collections of orbits with daily repeating ground traces could be easily assembled for other planetary bodies.

CIRCULAR EQUATORIAL ORBITS

An object in a circular equatorial orbit repeats the same ground trace every orbit. We wish to find orbits that have daily repeating ground tracks, i.e. which pass over the same locations at the same time each solar day. For this to happen, the orbiter must pass over the same spot on Mars that it passed over one sol earlier.

In order to pass over an object on Mars at the same time each sol, the time it takes for a circular prograde equatorial orbiter to complete Q periods P_Q (where Q, the Trace Repetition Parameter, is an integer), plus the amount of time it takes for the orbiter to cover the angular distance traversed by an object on Mars between a sidereal day and a solar day, must add up to a Mars sol. In equation form,

$$P_{sol} = QP_Q + \frac{P_Q\left(P_{sol} - P_{sid}\right)}{P_{sid}} \tag{1}$$

Rearranging,

$$P_Q = \frac{P_{sol}P_{sid}}{P_{sid}(Q-1)+P_{sol}} \tag{2}$$

For a retrograde orbit,

$$P_Q = \frac{P_{sol}P_{sid}}{P_{sid}(Q+1)-P_{sol}} \tag{3}$$

The Keplerian orbit period is equal to

$$P_K = 2\pi\sqrt{\frac{a^3}{\mu}} = \frac{2\pi}{n} \tag{4}$$

where

a = semi-major axis of the orbit,

μ = gravitational constant of the planet (GM), and

n = mean motion.

Rearranging equation 4,

$$a = \sqrt[3]{\frac{P_K^{\,2}\mu}{4\pi^2}} \tag{5}$$

Values for μ and other Mars constants are given in Table 1.

Table 1

MARS PLANETARY CONSTANTS

Gravitational constant μ	42,828.376645 km^3/s^2
Equatorial radius R_M	3,396.2 km
Constant of oblateness J_2	0.001958705252674
Sidereal rotation period P_{sid}	24.6230 hours
Solar rotation period P_{sol}	24.6598 hours
Orbit period around sun (Mars year)	668.6 sols
Mean rotation rate of Mars around the sun n_{sol}	1.059×10^{-7} rad/s

We can now find daily repeating ground track circular equatorial orbits by first computing orbit periods from equations 2 and 3 and then orbit radii from equation 5. Table 2 shows the basic characteristics of all such orbits at Mars.

Table 2

Circular equatorial orbits

	Prograde			Retrograde		
Q	Keplerian Period, Hours	Radius, km	Altitude, km	Keplerian Period, Hours	Radius, km	Altitude, km
1	24.6230	20,427.7	17,031.5	24.6967	20,468.5	17,072.3
2	12.3207	12,875.0	9,478.8	12.3391	12,887.9	9,491.7
3	8.2158	9,827.1	6,430.9	8.2240	9,833.7	6,437.5
4	6.1626	8,112.8	4,716.6	6.1673	8,116.8	4,720.6
5	4.9305	6,991.7	3,595.5	4.9334	6,994.5	3,598.3
6	4.1089	6,191.7	2,795.5	4.1110	6,193.8	2,797.6
7	3.5221	5,587.2	2,191.0	3.5236	5,588.8	2,192.6
8	3.0819	5,111.4	1,715.2	3.0831	5,112.6	1,716.4
9	2.7395	4,725.4	1,329.2	2.7404	4,726.5	1,330.3
10	2.4656	4,405.0	1,008.8	2.4663	4,405.8	1,009.6
11	2.2415	4,133.8	737.6	2.2421	4,134.6	738.4
12	2.0547	3,900.9	504.7	2.0552	3,901.5	505.3
13	1.8967	3,698.2	302.0	1.8971	3,698.8	302.6
14	1.7612	3,519.9	123.7	1.7616	3,520.4	124.2

These orbits are illustrated in Figure 2. The equatorial circular prograde orbit with Q=1 is an areostationary orbit, akin to geostationary orbits on Earth – an orbiter in this orbit will always be over the same location on the equator of Mars.

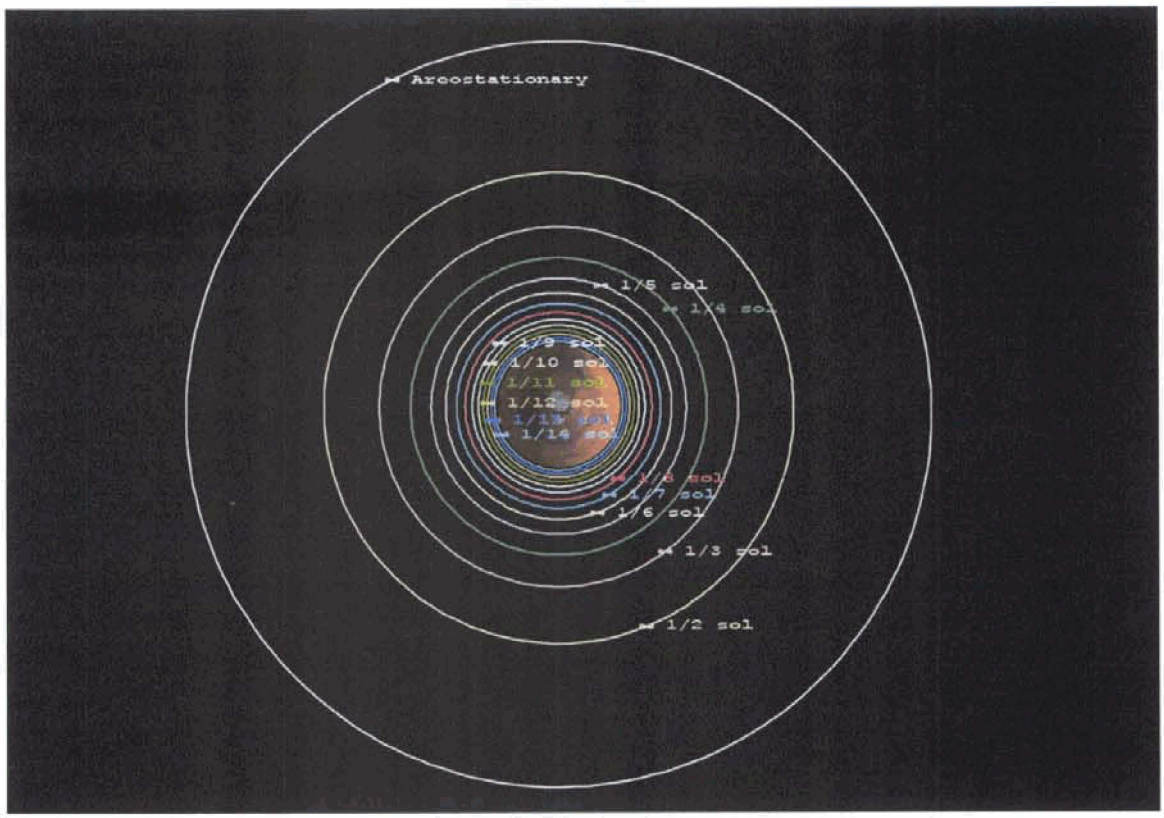

Figure 2 Equatorial Circular Daily Repeat Ground Track Orbits

CIRCULAR SUN SYNCHRONOUS ORBITS

For a circular inclined orbit to have a daily repeating ground trace, the node regression rate $\dot{\Omega}$ must be equal to the rotation rate n_{sol} of Mars about the sun. Such orbits are generally denoted sun synchronous because they always maintain the same orientation with respect to the sun.

Assuming that node regression at Mars is dominated by the oblateness of Mars, the node regression rate is given by

$$\dot{\Omega} = \frac{-3J_2\sqrt{\mu}\,R_{\mathrm{M}}{}^2\cos i}{2a^{7/2}(1-e^2)^2} \text{ rad/s,} \tag{7}$$

where

J_2 = constant of planet oblateness,

e = eccentricity of the orbit (0° for a circular orbit), and

i = orbit inclination.

Setting equation 7 equal to n_{sol} and solving for i, we have

$$i = \cos^{-1}\left(-\frac{2a^{7/2}n_{sol}}{3J_2\sqrt{\mu}R_M{}^2}\right) \tag{8}$$

We wish to calculate the inclination of each sun synchronous circular inclined orbit for which Q is an integer, i.e. all circular inclined orbits with daily repeating ground tracks. We need the nodal period P_Ω (the period between crossings of the equatorial plane) to be a submultiple of the solar rotation period of Mars, i.e.

$$P_\Omega = \frac{P_{sol}}{Q} \tag{9}$$

The nodal period of an orbit P_Ω is related to the Keplerian period P_K by[2]

$$P_\Omega = \frac{P_K}{1+\dfrac{3R_M{}^2J_2\left(4-5\sin^2 i+\sqrt{1-e^2}\left(2-3\sin^2 i\right)\right)}{4a^2(1-e^2)^2}} \tag{10}$$

For a circular orbit, $e = 0$ and we have

$$P_\Omega = \frac{P_K}{1+\dfrac{3R_M{}^2J_2\left(3-4\sin^2 i\right)}{2a^2}} \tag{11}$$

Equations 4, 8 and 11 can be solved iteratively to find a for each orbit with a nodal period equal to a submultiple of a sol (equation 9). The inclination of each orbit can then be computed from equation 8. Table 3 characterizes these orbits.

Table 3

CIRCULAR SUN SYNCHRONOUS ORBITS

Q	Nodal Period, Hours	Inclination	Radius, km	Altitude, km
4	6.165	136.683°	8118	4722
5	4.932	115.563°	6992	3596
6	4.110	106.362°	6190	2794
7	3.523	101.330°	5585	2188
8	3.082	98.266°	5108	1712
9	2.740	96.267°	4721	1325
10	2.466	94.894°	4400	1004
11	2.242	93.914°	4129	733
12	2.055	93.192°	3895	499
13	1.897	92.647°	3692	296
14	1.761	92.225°	3514	118

Figures 3 and 4 illustrate these orbits.

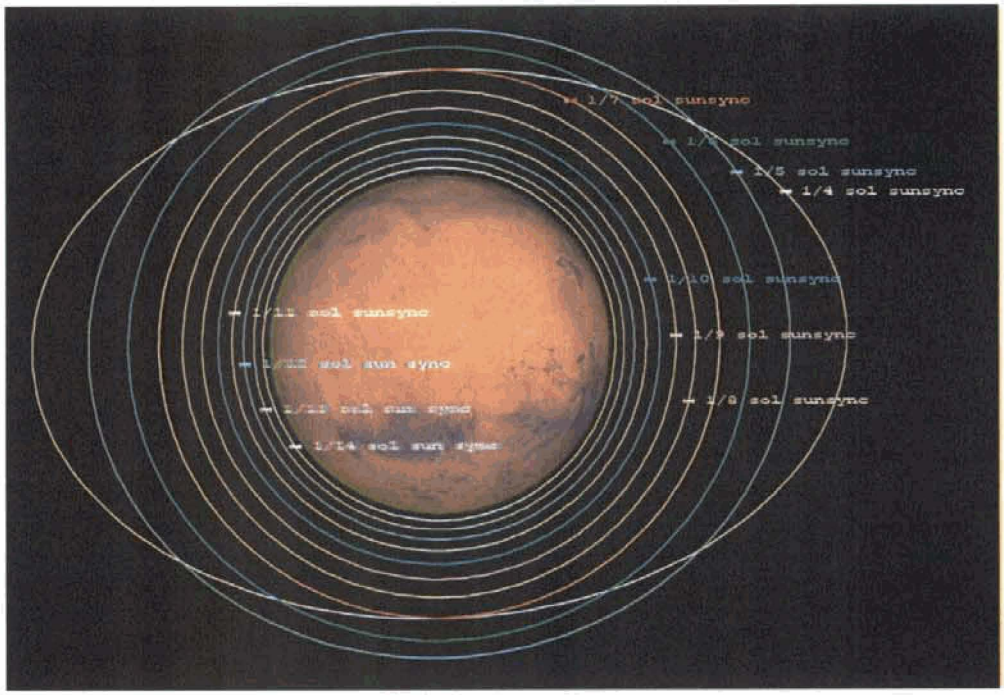

Figure 3 Circular Sun Synchronous Orbits Viewed Perpendicular to Nodes

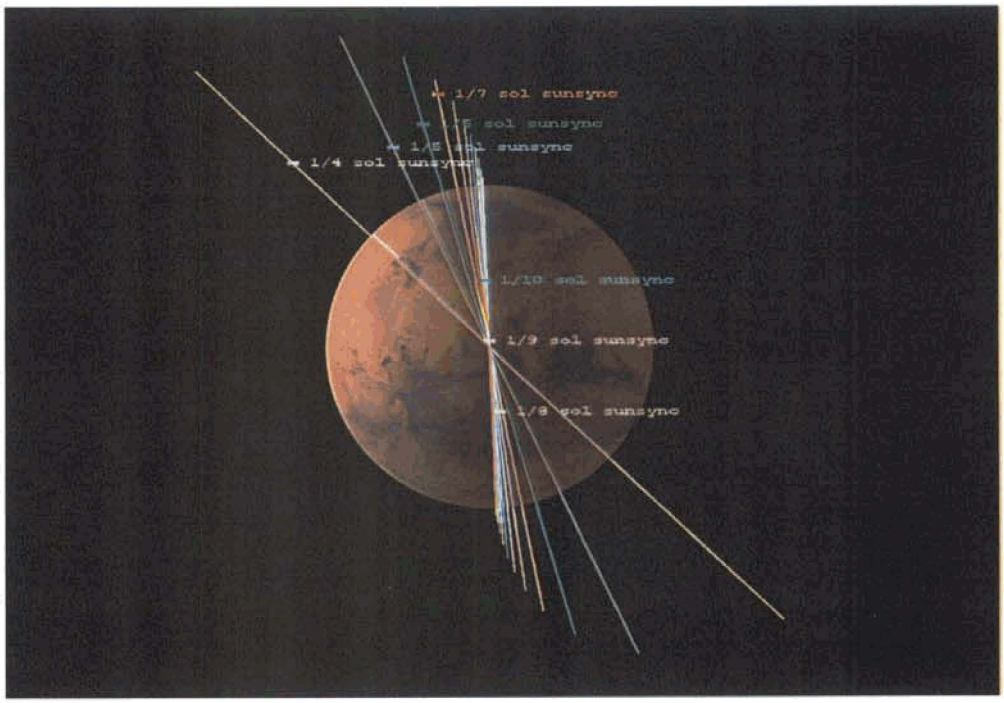

Figure 4 Circular Sun Synchronous Orbits Viewed Along Nodes

EQUATORIAL ECCENTRIC ORBITS

Assuming that rotation of the line of apsides of an eccentric orbit at Mars is dominated by the oblateness of Mars, the line of apsides rotates with respect to the line of nodes at a rate given by[3]

$$\varpi = \frac{3J_2\sqrt{\mu}R_M{}^2(5\cos^2 i - 1)}{4a^{7/2}(1-e^2)^2} \text{ rad/s.} \tag{12}$$

The rotation rate r of the line of apsides projected onto the equatorial plane is then

$$r = \varpi \cos i + \dot{\Omega} \text{ rad/s} \tag{13}$$

For a prograde equatorial orbit, we set $i = 0°$ and combine equations 7, 12 and 13 to get

$$r = \frac{3J_2\sqrt{\mu}R_M{}^2}{2a^{7/2}(1-e^2)^2}. \tag{14}$$

Setting $r = n_{sol}$ so that the line of apsides rotates in inertial space at a sun synchronous rate and solving for e, we have

$$e = \sqrt{1 - \sqrt{\frac{3J_2\sqrt{\mu}R_M{}^2}{2n_{sol}a^{7/2}}}} \tag{15}$$

For an equatorial orbiter, $i = 0$ and the nodal period is related to the Keplerian period by

$$P_\Omega = \frac{P_K}{1 + \frac{3R_M{}^2 J_2\left(2+\sqrt{1-e^2}\right)}{2a^2(1-e^2)^2}} \tag{16}$$

As for the sun synchronous circular orbits, 15 and 16 can be solved iteratively to find orbits with nodal periods equal to submultiples of a sol. There are just two such orbits, both prograde. Their characteristics are shown in Table 4. Turner[3] denotes such orbits at Earth Apogee at Constant time-of-day Equatorial (ACE) orbits; at Mars, we use the same acronym to denote Apoapsis at Constant time-of-day Equatorial.

Table 4

MARS ACE ORBIT CHARACTERISTICS

Orbit period, sols	½ Sol	⅓ Sol
Orbit period, hours	12.32 hours	8.21 hours
Semi Major Axis a	12,890 km	9,835 km
Eccentricity e	0.691	0.402
Periapsis Altitude	582 km	2,480 km
Apoapsis Altitude	18,400 km	10,400 km

CRITICALLY INCLINED ECCENTRIC

At the critical inclinations, i = 116.565° or 63.435°, $\dot{\omega} = 0$ and there is no apsidal rotation with respect to the line of nodes. Setting $\dot{\Omega} = n_{sol}$ and $\cos i = -\sqrt{1/5}$,† we can solve equation 7 for e:

$$e = \sqrt{1 - \sqrt[4]{\mu/5}\sqrt{\frac{3J_2 R_M{}^2}{a^{7/2} n_{sol}}}} \tag{17}$$

Setting $\sin^2 i = 4/5$ in equation 10, the nodal period is

$$P_\Omega = \frac{P_K}{1 - \frac{3R_M{}^2 J_2}{10a^2(1-e^2)^{3/2}}} \tag{18}$$

We can now find values of a which result in nodal periods that are submultiples of a sol as before through iteration. There is one such orbit at Mars (see Table 5). We denote this orbit Apoapsis at Constant time-of-day Critically Inclined, or ACCI. A similar orbit at Earth has been called in the past "triply synchronous."[4]

Table 5

MARS ACCI ORBIT CHARACTERISTICS

Nodal period in sols	¼ sol
Nodal period in hours	6.165 hours
Semi-major Axis a	8,114 km
Eccentricity e	0.464
Periapsis Altitude	953 km
Apoapsis Altitude	8,483 km

The Mars ACCI orbit and both Mars ACE orbits are illustrated in Figure 5.

† cos(i) must be negative for positive regression, so only 116.565° is viable.

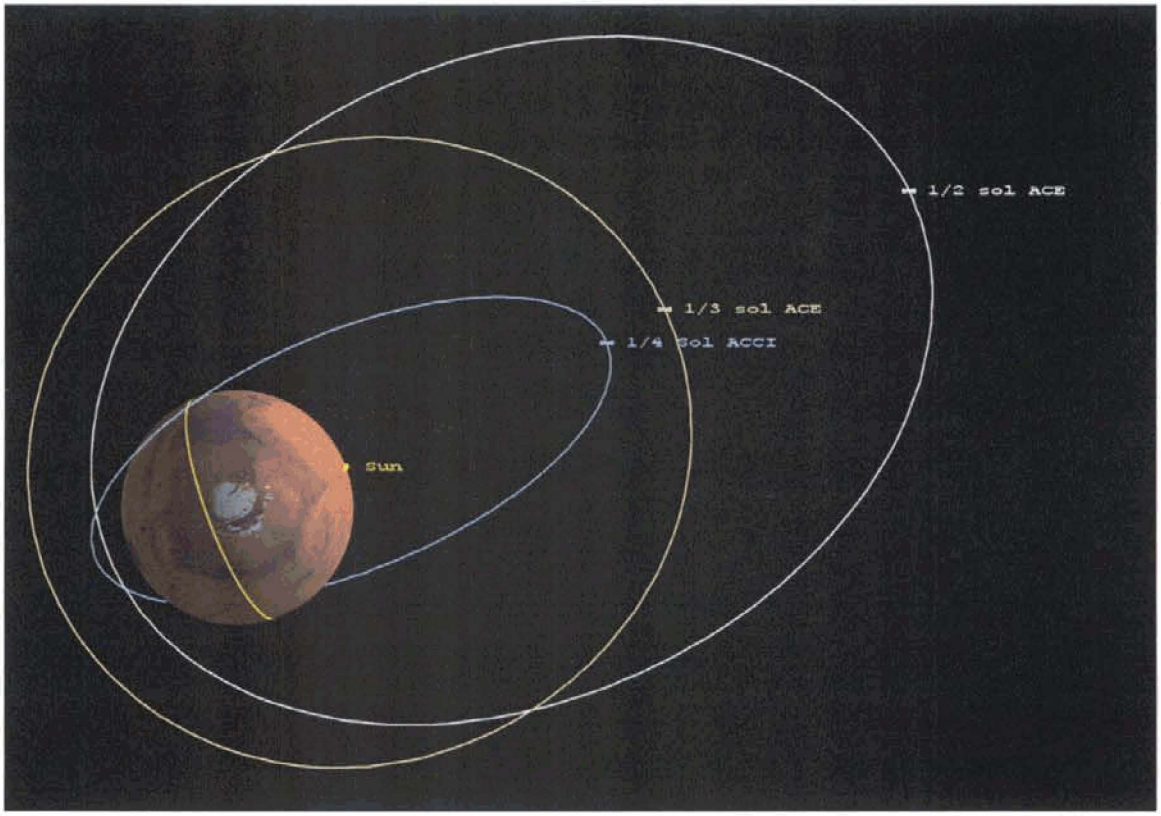

Figure 5 Elliptical Daily Repeat Ground Track Orbits

RELAY PERFORMANCE

Figure 6 compares the connectivity to Mars Science Laboratory (MSL) from various orbits. Note that connectivity through the Mars Telesat is typically much longer than through MRO, which is representative of the other low polar science orbiters now at Mars.

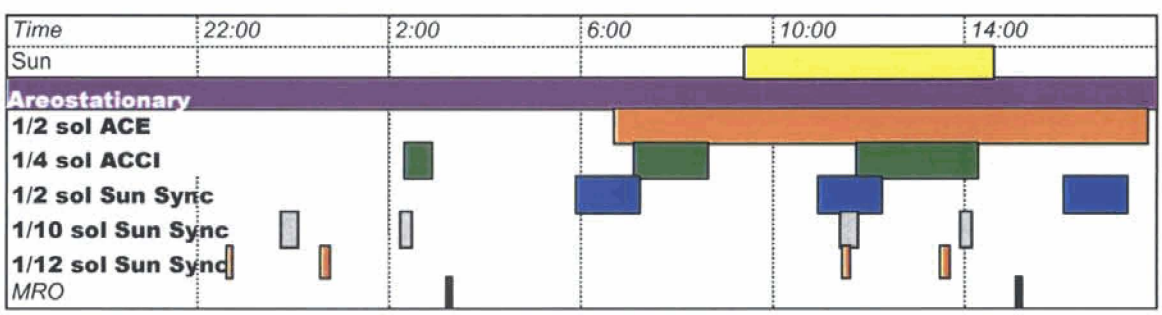

Figure 6. MSL Connectivity

The ½ sol ACE orbit has some interesting attributes that make it attractive for mid-latitude Mars rovers. Its highly eccentric orbit can be relatively easy to get into, and if appropriately set up, the orbiter can be in view every day for essentially all the time that

the rover is in sunlight. This could enable continuous rover monitoring and control capability.

Figure 7 shows the data volume that could be relayed from MSL through various orbiters. The Mars Telesat will increase the amount of data that can be relayed to Earth from MSL by more than an order of magnitude.

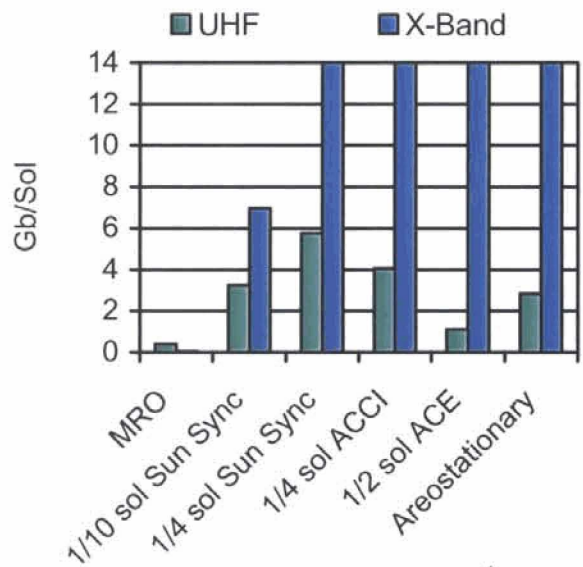

Figure 7 MSL Data Volume[1]

Figures 8 and 9 show mean pass lengths for links between landers and the ½ sol ACE and ¼ sol ACCI orbits, respectively. The tentative locations of the CNES NetLanders are shown as black crosses and MSL at its challenge location is shown as a black dot in Figure 8.

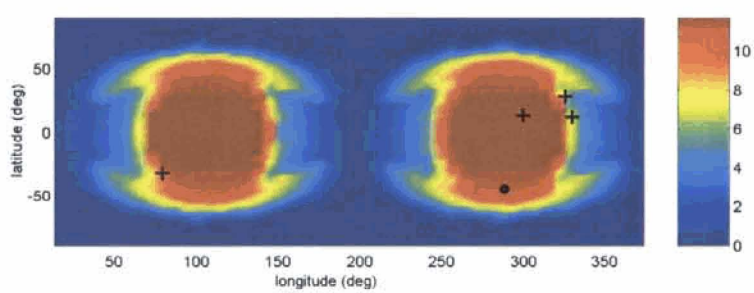

Figure 8 Mean Pass Length of ½ Sol ACE Orbit

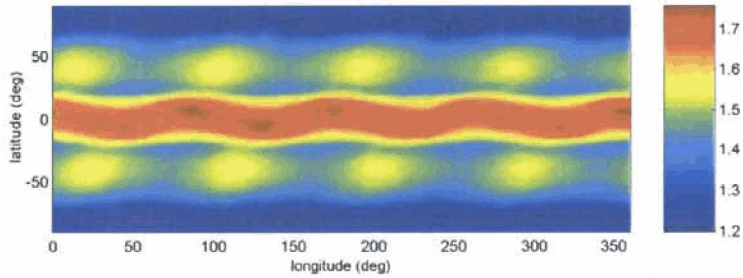

Figure 9 Mean Pass Length of ¼ Sol ACCI Orbit

While daily repeat ground track orbits are generally desirable for landers for operational reasons, they have some limitations that must be considered. Coverage tends to be granular in latitude and longitude – i.e. it is not uniform over all of the planet.

Because the orbit passes over the same areas each day, secular perturbations caused by lumps in the gravity field of Mars can build up over time. Exact repeat ground tracks are not important operationally for relay users, so it is likely this effect can be tolerated. Since the orbiter passes over landers with the same geometry each day, there is a reduction in the diversity of radiometric data that can be collected by the orbiter and a consequent reduction in the accuracy of radiometric-based navigation.

FUTURE WORK

MSL will be JPL's flagship mission of this decade and will be the dominant initial user of the Mars Telesat. The Mars Telesat and MSL teams at JPL are jointly constructing computer simulations of MSL operations using the Mars Telesat in various candidate orbits to characterize the science products that MSL could return through the Mars Telesat in each orbit and to help assess other operational implications of the candidate orbits. The Mars Telesat team is also simulating data returned from the CNES NetLanders through the Mars Telesat. The results of these simulations will be used to help select a final orbit for the Mars Telesat.

Mars Telesat mission designers are also determining the delta V required to place the Telesat into various candidate orbits. This depends on arrival geometry and many other factors.

The Mars Telesat will be just one part of the Mars network, albeit a key part. NASA requires a relay radio on every orbiter sent to Mars with a design life in orbit of more than one year. NASA science orbiters are generally put into low polar orbits, which provide excellent relay coverage in polar regions (up to 13 contacts per day per orbiter) but limited coverage of mid-latitude regions (1 contact per day or less). If put into an equatorial orbit, the Mars Telesat could fill in the gaps left by the other orbiters. On the other hand, the Mars Telesat is to be a long-life orbiter and might be the only orbiter available to support important polar missions, so it may require global coverage – precluding an equatorial orbit.

Final selection of the Mars Telesat orbit will await the results of the simulation studies, assessments of the longevity of the other Mars orbiters, and consideration of how the Mars Telesat can best fit into the mix of future orbiter and lander missions to Mars.

ACKNOWLEDGEMENTS

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[3] Turner, Andrew E., New Non-Geosynchronous Orbits for Communications Satellites to Off-Load Daily Peaks in Geostationary Traffic, AAS/AIAA Astrodynamics Specialist Conference, Kalispell, Montana, August 10-13, 1987.

[4] Bachtell, Edward E., et. al., Satellite Voice Broadcast System Study, Martin Marietta Aerospace, NASA Contract NAS3-24233, November, 1985.