Formation Flying for Distributed InSAR

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We consider two spacecraft flying in formation to create an interferometric synthetic aperture radar (InSAR). Several candidate orbits for such an InSAR formation have been previously determined based on radar performance and Keplerian orbital dynamics. However, without active control, disturbance-induced drift can degrade radar performance and (in the worst case) cause a collision. This study evaluates the feasibility of operating the InSAR spacecraft as a formation, that is, with inter-spacecraft sensing and control. We describe the candidate InSAR orbits, design formation guidance and control architectures and algorithms, and report the $\Delta v$ and control acceleration requirements for the candidate orbits for several tracking performance levels. As part of determining formation requirements, a formation guidance algorithm called Command Virtual Structure is introduced that can reduce $\Delta v$ requirements compared to standard Leader/Follower formation approaches. In addition, an orbit maintenance and formation reconfiguration guidance algorithm is developed for changing both relative orbits and the overall formation orbit.

I. Introduction

In distributed interferometric synthetic aperture radar (InSAR), at least two synthetic aperture radar spacecraft obtain additional information by differencing radar phase information from multiple viewpoints of the same target. InSAR can be used to generate digital elevation maps (e.g., Shuttle Radar Topography Mission) or measure surface velocities for geology and climatology (e.g., tectonic or glacial movement). Since arbitrary spacecraft motions in LEO are impractical, thrust-free orbits are designed in simplified dynamics models to generate desirable relative spacecraft motion (i.e., InSAR baselines). The resulting relative motion is referred to as a passive relative orbit (PRO).5,6

As part of an ongoing formation InSAR study, several candidate Keplerian PROs have been selected for a two-spacecraft InSAR. Since the minimum altitude for the InSAR formation is 500 km and the two InSAR spacecraft are essentially identical, the principal disturbance on the formation is the first zonal harmonic of Earth oblateness, $J_2$. The spacecraft must be actively controlled to prevent $J_2$-induced drift from degrading InSAR performance or, in the worst case, causing collision.

We consider formation flying InSAR, that is, explicitly controlling the relative spacecraft position by coupling the spacecraft through an automatic control law. This paper presents the results of a two-spacecraft, formation flying InSAR study. Our focus is on formation flying requirements and not on radar performance. Specifically, the study determines the $\Delta v$ and control acceleration requirements of several candidate InSAR PROs for several levels of performance. A sensitivity analysis with respect to control design parameters is also included. This analysis shows that the maximum acceleration required by the controller to maintain the formation can be tailored to the desired level.

As part of the formation InSAR study, a new formation guidance algorithm called Command Virtual Structure (CVS) has been developed. CVS, a type of Leader/Follower formation architecture,8 can reduce

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5Based on Ref. 7, a reasonable value of $C_D A/m$ (i.e., the inverse ballistic coefficient) for a SAR spacecraft is 0.0043 m$^2$/kg. In addition, the GRACE mission, which consists of two identical spacecraft separated by 220 km, has observed ballistic coefficient differences of 1%. For a 520 km altitude orbit, these facts result in a maximum differential drag acceleration of $2 \times 10^{-5}$ m/s$^2$. At the same altitude for spacecraft separated by 300 m, the maximum differential acceleration due to $J_2$ is $4 \times 10^{-6}$ m/s$^2$.
\( \Delta v \) requirements by up to an order of magnitude compared to standard Leader/Follower implementations. Additionally, a formation guidance algorithm has been developed for changing both the spacecraft's relative orbit and the overall formation orbit. This orbit maintenance/reconfiguration (OM/R) algorithm is an extension of standard formation linear programming approaches\(^9\) to more general boundary conditions.

Previous work in InSAR formations has focused on PRO design.\(^5\),\(^10\),\(^11\) In Refs. 7, 12 and 13, open loop \( \Delta v \) estimates are given to cancel differential aerodynamic drag and differential mean \( J_2 \). While there is considerable research in closed-loop formation control,\(^11\) the majority of LEO formation work has focused on showing stability or performance; not on determining \( \Delta v \) requirements as a function of performance. Several near-optimal orbit transfer methodologies have been developed for application in model predictive control laws. See Ref. 15 and references therein. However, sensor noise and actuation errors can limit model predictive control performance to tens of meters.\(^16\) Previous studies of \( \Delta v \) requirements for precision formation flying (i.e., better than 10 meter tracking performance) include Refs. 9, 17 and 18. These earlier studies considered a single PRO and a selection of control architectures. In addition to introducing the CVS and OM/R formation guidance algorithms, this paper extends these earlier studies by considering a spectrum of InSAR-relevant PROs for several levels of performance and control designs.

The paper is organized as follows. In Section II, four classes of InSAR-relevant PROs are reviewed briefly. A new PRO called the pseudo-frozen corkscrew is introduced. Additionally, we discuss the PRO parameterization methodology used in the formation guidance algorithms. Next, the overall control architecture and control algorithm design methodology are presented in Section III. Then two formation guidance algorithms, the new CVS architecture and a standard Leader/Follower architecture called Prescribed-\( \nu \), are developed in Section IV. In Section V, the simulation environment used to evaluate formation \( \Delta v \) and acceleration requirements is briefly described, including assumptions on sensor noise and disturbances. The main results of this paper, requirements for \( \Delta v \) and acceleration and the sensitivity of these requirements to control design parameters, are given in Section VI. Section VII presents the OM/R algorithm and representative \( \Delta v \) requirements for changing PROs. Additionally, the OM/R algorithm is validated against a Holmann transfer. Finally, we summarize and identify areas for future study.

In the sequel, we refer to the two InSAR spacecraft as \( SC_1 \) and \( SC_2 \). \( SC_1 \) is considered the Leader, but in implementation, the roles of the spacecraft would be switched to balance fuel. Regarding nomenclature, column vectors are given in bold and estimated values have a carat symbol (e.g., \( \hat{\rho} \) is the estimated relative position vector).

II. InSAR Passive Relative Orbits

In this section, the four classes of InSAR PROs are described. Relative motion is characterized in the rotating Local-Vertical, Local-Horizontal (LVLH) frame, which is defined as follows. The origin can be either a physical spacecraft or a prescribed location in space. The \( z \)-axis is along the position vector of the spacecraft (or prescribed point) with respect to the Earth, the \( x \)-axis is along the angular momentum vector, and the \( y \)-axis completes the triad. For a circular orbit, the \( y \)-axis is parallel to the velocity. The \( x \)-, \( y \)-, and \( z \)-axes are also referred to as radial, along-track, and cross-track. Unless otherwise noted, the orbit of \( SC_1 \) is the one for the LVLH frame.

The four classes of PROs that are being considered in the overall InSAR study are referred to as (i) cartwheel, (ii) corkscrew, (iii) pseudo-frozen corkscrew, and (iv) pendulum. Three of the classes are well-known,\(^19\) but the pseudo-frozen corkscrew (conceived by C.W. Chen) is new as far as the authors could determine. For each of these PRO classes, different altitudes and InSAR baseline lengths are considered. For all PROs, the two spacecraft have identical semi-major axes, as this is a necessary condition for periodic relative motion.\(^20\) Relations in this section were derived using Keplerian orbit geometry and projecting relative motion into InSAR baselines.

The cartwheel PRO consists of relative motion in the orbit plane. It is produced by placing two spacecraft on equally eccentric orbits with arguments of periapsis and initial true anomalies both \( 180^\circ \) apart. This phasing gives a maximum radial separation (i.e., baseline) of

\[
B_r = 2ae, \tag{1}
\]

where \( a \) and \( e \) are the semi-major axis and eccentricity of the two orbits. The inclinations and longitude of the ascending nodes are identical.

The corkscrew PRO begins with a cartwheel and then rotates the orbit of \( SC_2 \) about the periapsis vector
of $SC_1$’s orbit, producing a cross-track separation. The angle of rotation $\theta_p$ determines the cross-track baseline via

$$B_t = a(1 - e^2) \sin(\theta_p).$$

The semilatus rectum $p = a(1 - e^2)$, not the semi-minor axis $a\sqrt{1 - e^2}$, gives the maximum cross track baseline due to phasing within the two orbits. For a corkscrew PRO, both $B_t$ and $B_c$ must be specified. Depending on the argument of periapsis of $SC_1$, the periapsis rotation for a near-polar orbit produces a differential inclination, a differential longitude of the ascending node, or a combination thereof.

The pseudo-frozen corkscrew PRO partially compensates for $J_2$ and $J_3$-induced variations. First, for the desired semi-major axis and inclination, the frozen orbit argument of periapsis $\omega_f$ and eccentricity $e_f$ are calculated. See Ref. 21. The frozen values eliminate long period variations of eccentricity and argument of periapsis. The frozen eccentricity is given by

$$e_f = \frac{1}{2} \frac{J_3}{J_2} \frac{R_\oplus}{a} \sin i \sin \omega_f,$$

where $R_\oplus$ is the mean equatorial radius of the Earth, and recall $J_2$ and $J_3$ have opposite signs. Both $SC_1$ and $SC_2$ have an $\omega_f$ of 90°. Since the arguments of periapsis are identical, the spacecraft must have different eccentricities to obtain the radial baseline of the regular corkscrew. It can be shown that a differential eccentricity of

$$\Delta e = B_{e,f}/a$$

gives the desired horizontal baseline. Hence, $SC_1$ and $SC_2$ are given eccentricities of $e_f + \Delta e/2$ and $e_f - \Delta e/2$, respectively. Finally, to obtain the vertical baseline the orbits are again rotated about their common periapsis vector. However, Eq. (2) must be modified to

$$B_{e,f} = 2a(1 - e_f^2 - \Delta e^2) \sin(\theta_p).$$

The orbit of $SC_1$ and $SC_2$ are then rotated $\theta_p/2$ and $-\theta_p/2$ about their common periapsis vector.

The pendulum PRO is produced by rotating two identical orbits about their periapsis vector, giving a vertical baseline via Eq. (2). We use circular orbits, and define the argument of periapsis to be zero. To avoid collisions, the initial true anomalies of the orbits are phased to produce an along track separation of

$$B_{at} = a \sin(\nu_2),$$

where $\nu_2$ is the initial true anomaly of $SC_2$, and it is assumed that the initial true anomaly of $SC_1$ is $\nu_1 = 0$. For a pendulum PRO, both $B_{at}$ and $B_c$ must...

Examples of the PROs are shown in the LVLH frame of $SC_1$ in Figure 1. The curves represent the motion of $SC_2$ relative to $SC_1$. The relative motion in a frozen corkscrew differs from that in a regular corkscrew by less than a millimeter, and so the frozen corkscrew is not shown in Figure 1. Consider the cartwheel PRO, shown as a dashed line with circles. It is in the orbital plane of $SC_1$ and can be approximated by the standard, $2 \times 1$ elliptical “football” orbit of the Hill-Clothesby-Wiltshire (HCW) Equations. Note, however, that the cartwheel is designed in the nonlinear, Keplerian orbital dynamics model and not a simplified, linearized model. The pendulum PRO is the approximately vertical line with asterisks (there are centimeter-level oscillations in the $x$- and $y$-directions). The pendulum can be approximated by a pure $z$-oscillation in the HCW Equations. The true anomaly offset in this case results in a 100 m $y$-offset. Finally, the corkscrew can be approximated as a football orbit combined with the $z$-oscillation of a pendulum.

![Figure 1. PROs Considered in InSAR Study](image-url)
These InSAR PROs are based on Keplerian orbits, that is, based on the absolute motion of the two spacecraft. For formation control, we require a description of the relative spacecraft motion in a Cartesian LVLH frame. We parameterize the relative motion with a Fourier-like sum of sinusoidal basis functions, where the longest period is the orbital period of SC1. Specifically, the parameterization is

\[
x_f(v_1) = \left[ \begin{array}{c}
p_{f}(v_1) \\
\frac{\partial p_{f}(v_1)}{\partial v_1} 
\end{array} \right] = \left[ \begin{array}{c}
a_{b_0} + \sum_{k=1}^{N_p} \left( a_{1ck} \cos(kv_1) + a_{1sk} \sin(kv_1) \right) \\
\frac{a_{b_1}}{2} + \sum_{k=1}^{N_p} \left( a_{2ck} \cos(kv_1) + a_{2sk} \sin(kv_1) \right) \\
\vdots \\
\end{array} \right],
\]

where \( x_f = [\rho_f^T, (d\rho_f/dv_1)^T]^T \) is the fitted dynamic state vector of SC2 with respect to SC1 represented in the LVLH frame, \( v_1 \) is the true anomaly of SC1, \( \rho_f \) is the fitted relative position vector, \( a_{1ck} \) and \( a_{1sk} \) are constant coefficients, and \( N_p \) is the order of the basis function parameterization. Since \( \rho_f \) is a vector in the LVLH frame, its derivative is with respect to the LVLH frame.

In the formation guidance algorithm, the parameterization Eq. (7) is fit automatically. The specified Keplerian PRO is calculated for one orbital period, and the relative position and velocity of SC2 in the LVLH frame is stored at discrete points. Then, for a given \( N_p \), the coefficients \( a_{1ck}, a_{1sk}, a_{2ck}, \) etc. are solved for via least squares. The value for \( N_p \) is increased until the least squares residual is below a specified tolerance. Typically, \( N_p = 3 \) is sufficient to fit relative position to better than \( 1 \times 10^{-7} \) m and relative velocity to better than \( 1 \times 10^{-9} \) m/s for 2000 discrete points.

III. Formation Control Architecture and Algorithms

The Leader/Follower formation control architecture\(^8\) was selected for this study since it is robust and its stability properties are well understood.\(^14\) The Leader/Follower architecture is characterized by a hierarchical state dependency in spacecraft control laws. In contrast, Cyclic control architectures have a non-hierarchical control dependency. Even though Cyclic architectures can be more efficient, their stability is only understood in restricted cases.\(^24\)

Based on the Leader/Follower controller architecture, a control algorithm must be designed for SC2 to control itself relative to SC1. To limit the scope of this study, the formation control algorithm is designed using the steady-state Linear Quadratic Regulator (LQR). The control design model is a version of the HCW Equations augmented with the effects of \( J_2 \).\(^25\)

The cross-track effects of \( J_2 \) are nonlinear and so we only retain the in-orbit-plane (i.e., the radial/along-track plane) terms.

As a brief review, the components of an overall formation guidance and control system specialized to our scenario are shown in Figure 2. We consider a discrete time formulation as would be implemented onboard a spacecraft. Assume the spacecraft computer has a control cycle of length \( T_s \) that starts at discrete times indexed by \( k \) (i.e., \( T_s = t_{k+1} - t_k \)). Each time step, the formation guidance generates a desired position and velocity \( x_{d,k} = [\rho_{d,k}^T, \dot{\rho}_{d,k}^T]^T \) of SC2 with respect to SC1 in the LVLH frame, where the () indicates differentiation with respect to time. The estimated relative state vector \( \hat{x}_k \) is subtracted from the desired relative state to obtain the control tracking error \( e_k \). The control error is then passed to the Leader/Follower formation control algorithm that generates the relative acceleration command \( u_{2,k} \), where \( u_{1,k} \) is the control acceleration for SC1 in the \( k^{th} \) time step.

For LQR design, the \( J_2 \)- augmented HCW Equations must be discretized. Ignoring disturbances, the discretized equations of motion have the following form:

\[
x_{k+1} = Ax_k + Bu_k
\]

where \( A \) and \( B \) are appropriately-sized matrices, the state vector \( x_k \) consists of relative position and velocity of SC2 with respect to SC1 in the LVLH frame of SC1, and \( u_k = u_{2,k} \). Recall \( u_{1,k} \) is uniformly zero, except for orbit maintenance and possibly reconfiguration maneuvers.

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Two important aspects of the discretization process are (i) selecting the discrete time step size $T$, also referred to as the control sample period, and (ii) specifying how the control acceleration is applied in continuous time. Regarding the latter, the standard assumption is that $u_{2,k}$ is applied constantly from $t_k$ to $t_{k+1}$. This assumption does not necessarily give the desired performance. Instead, we assume that $u_{2,k}$ is applied constantly from $t_k$ to $t_k + T_s$, where $0 < T_s \leq T$. Both $T_s$ and $T$ are control design parameters and affect $A$ and $B$ in Eq. (8).

The LQR controller is optimal in that it minimizes the cost

$$J = \sum_{k=0}^{\infty} (x_k^T Q x_k + u_k^T R u_k)$$

subject to Eq. (8), where $Q \geq 0$ and $R > 0$ are weighting matrices. The steady-state LQR controller is given by $u_k = -G x_k$ where $G = (B^T S B + R)^{-1} B^T S A$, and $S$ is the solution to the algebraic Riccati equation

$$S = A^T [S - SB (B^T S B + R)^{-1} B^T S] A + Q.$$

We assume $R$ is the identity matrix and

$$Q = \begin{bmatrix}
\lambda_x & 0 & \cdots & 0 \\
0 & \lambda_y & 0 & \cdots \\
\cdots & 0 & \lambda_z & 0 & \cdots \\
0 & \lambda_x \lambda_y & 0 & \cdots \\
\cdots & 0 & \lambda_x \lambda_y & 0 \\
\cdots & 0 & \lambda_x \lambda_z
\end{bmatrix},$$

where $\lambda_x$, $\lambda_y$, and $\lambda_z$ weight the effect of position errors by axis, and $\lambda_v$ scales these position weights to velocity weights. Each of the four weights is a design parameter.

The LQR controller is a regulator, that is, it is the optimal controller for driving the state to the origin of phase space. However, we wish the state trajectory to track a PRO. To formulate the optimal linear quadratic tracker (LQT), the LQR cost Eq. (9) is modified to

$$J_T = \sum_{k=0}^{\infty} (x_k - x_{d,k})^T Q (x_k - x_{d,k}) + u_k^T R u_k.$$

In the special case when $x_{d,k}$ is a homogenous solution of Eq. (8), that is, when $x_{d,k+1} = A_T x_{d,k}$, the LQT controller reduces to the LQR controller. While not all the Keplerian PROs are not exact solutions to linearized models, and hence the LQR controller is sub-optimal, its performance is sufficient.

IV. Formation Guidance Algorithms

A formation guidance algorithm determines $x_{d,k} = [\rho_{d,k}^T, \rho_{d,k}^T]^T$, the desired relative position and velocity for $SC_2$. The Command Virtual Structure (CVS) guidance architecture developed here is an adaptation of a common Cyclic combined guidance/control architecture called Iterated Virtual Structure (IVS). In IVS, a formation template or “virtual structure” is fit to the spacecraft positions at the beginning of each time step. Then the spacecraft control to their fitted positions for that time step. The stability of an IVS control design is inferred from simulation. Aspects of IVS have been incorporated in other Cyclic algorithms, such as centroiding approaches, where the location but not the orientation of the template is fit. Here, IVS is modified to obtain a Leader/Follower-compliant guidance algorithm by fitting only the orientation of the formation template.

This study considers two formation guidance algorithms: CVS and Prescribed-$\nu$ Guidance. The latter may be considered the standard Leader/Follower approach. Prescribed-$\nu$ guidance commands $SC_2$ to track the exact PRO in both time and space via

$$\rho_{d,k} = \rho_f (\hat{\theta}_{1,k} - n 2\pi)$$

$$\rho_{d,k} = \frac{d\rho_1}{dt} \left[ \frac{d}{d\rho_1} \rho_f (\nu_1) \right]_{\rho_1 = \hat{\theta}_{1,k} - n 2\pi}$$

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where \( \hat{\theta}_{1,k} \) is the estimated argument of latitude of \( SC_1 \) at time step \( k \), \( n \) is the largest integer such that \( n2\pi \leq \hat{\theta}_{1,k} \), and the estimated true anomaly rate is given by

\[
\frac{d\hat{\nu}_1}{dt} = \frac{1}{\|\hat{\mathbf{r}}_{1,k}\|^2} \|\hat{\mathbf{r}}_{1,k} \times \hat{\mathbf{v}}_{1,k}\|, \tag{14}
\]

where \( \hat{\mathbf{r}}_{1,k} \) and \( \hat{\mathbf{v}}_{1,k} \) are the estimated position and velocity of \( SC_1 \) in an Earth-centered, reference frame at time step \( k \). Such information would be available from GPS. In the arguments of \( \mathbf{\rho}_f \) in Eq. (13), the true anomaly of \( SC_1 \) has been replaced with the estimated argument of latitude. This substitution is necessary to account for the \( J_2 \)-induced perturbation of the argument of perigee of \( SC_1 \). See Figure 3, which illustrates the need for the argument of latitude.

The CVS guidance algorithm (conceived by E.A. Murray) is based on the observation that the time history of the relative motion can vary temporally as long as the commanded relative motion follows the PRO path spatially. Hence, we set the desired relative position at \( t_k \) to

\[
\mathbf{\rho}_{d,k} = \mathbf{\rho}_f \left[ \alpha_k \left[ \hat{\theta}_{1,k} - n2\pi \right] \right] \tag{15a}
\]

\[
\hat{\mathbf{\rho}}_{d,k} = \frac{d}{dt} \left[ \mathbf{\rho}_f (\nu_1) \right] \bigg|_{\nu_1 = \alpha_k \left[ \hat{\theta}_{1,k} - n2\pi \right]} \tag{15b}
\]

for some time-varying \( \alpha_k \) that we select. If \( \alpha_k = 1 \) uniformly, then Eq. (15) reduces to Prescribed-\( \nu \) Guidance. To maintain phasing between InSAR baselines and orbital position, it is further desired that \( 1 - \delta \leq \alpha_k \leq 1 + \delta \) for some small \( \delta \).

The extra degree of freedom introduced by \( \alpha_k \) is used to reduce \( \Delta \nu \) requirements. Specifically, the formation guidance algorithm calculates the minimum weighted tracking error \( e_k^* \) via

\[
e_k^* = \min_{\tau \in [0, 2\pi]} \left\| \begin{bmatrix} \hat{\mathbf{\rho}}_k \\ \gamma \frac{d}{d\tau} \hat{\mathbf{\rho}}_k \\ \gamma \frac{d}{d\tau} \mathbf{\rho}_f (\tau) \end{bmatrix} - \begin{bmatrix} \mathbf{\rho}_k \\ \gamma \frac{d}{d\tau} \mathbf{\rho}_f (\tau) \end{bmatrix} \right\|, \tag{16}
\]

where the estimated inverse true anomaly rate \( d\hat{\nu}_1/d\nu_1 \) is obtained from Eq. (14), \( \hat{\mathbf{\rho}}_k \) and \( \hat{\mathbf{\rho}}_k \) are the estimated relative position and velocity of \( SC_2 \) in the LVLH frame, and \( \gamma \) is a velocity error weight. This weight is a design parameter, and should be tuned in concert with \( \lambda_v \). The parameter \( \alpha_k \) is solved for implicitly in Eq. (16).

The minimization in Eq. (16) can have multiple minima. We solve this problem by discretizing the interval \([0, 2\pi]\) and applying exhaustive search. Then a local optimization is performed with the minimum point as an initial condition.

Figure 4 illustrates the two guidance approaches and the difference between them for \( \gamma = 0 \). An example PRO is shown in the LVLH frame of \( SC_1 \), which is at the center of the ellipse. \( SC_2 \) travels along the elliptical PRO, and several time steps are indicated by values of \( \nu_1 \). These are the positions \( SC_2 \) would assume if there were no disturbances. However, with disturbances and without control \( SC_2 \) follows the
dashed line with the corresponding steps of \( \nu_1 \) shown. For Prescribed-\( \nu \) Guidance, the tracking error is the difference between the dashed and solid lines at the time steps as shown by solid arrows. For CVS with \( \gamma = 0 \), the tracking error is the minimum distance to the PRO.

V. Simulation Environment and Assumptions

A MATLAB-based formation flying simulation environment was developed to evaluate the two-spacecraft formation flying scenarios. The dynamics include forces due to a radially symmetric Earth and Earth oblateness as characterized by the first four zonal harmonics \( J_2 \) through \( J_6 \). The dynamics also model aerodynamic drag. However, to evaluate \( \Delta v \) requirements for maintaining PROs, sometimes several hundred orbits had to be averaged over. Since drag decreases the semi-major axes and the orbit maintenance/reconfiguration algorithm has not yet been fully integrated into the simulation environment, drag was omitted. As discussed in the Introduction, differential aerodynamic drag is three orders-of-magnitude smaller than differential \( J_2 \).

The simulation environment uses a fixed-step, 5th order Runge-Kutta integrator. The integrator time step was \( 1/100^\text{th} \) of \( SC_1 \)'s initial orbit (1 minute). Simulations were also performed with a time step 10 times smaller with no appreciable difference. The initial inclination for \( SC_1 \) was always 97.4°. The control sample period \( T_s \) was \( 1/100^\text{th} \) of \( SC_1 \)'s initial Keplerian orbit, and the control actuation time \( T_a \) was 1 s. These effects of varying these two control parameters are analyzed in the next section.

For sensor noise levels, we used carrier differential GPS (CDGPS) results from two- and three-spacecraft ground-based formation estimation testbeds\(^{29, 31} \) which have achieved relative sensing of 2 cm and 0.5 mm/s 1\( \sigma \) in relative position and velocity. For absolute knowledge, we assume GPS has real-time sensor noise of 1 m 1\( \sigma \).\(^ {32, 33} \) With accelerometers in addition to GPS, it was assumed absolute velocity knowledge was accurate to 0.1 m/s 1\( \sigma \). These values are mildly conservative since with the global differential correction system, real-time GPS position uncertainty can be reduced to tens of centimeters.\(^ {34} \) Actuator uncertainties included a thrust magnitude variation uniformly distributed in \( \pm 5\% \) of the requested value, attitude uncertainty of 1 arcmin (for transforming LVLH request forces to body frame for thrust allocation), and thruster misalignments of 15 arcsec.

However, sensor noise and actuator uncertainties were not included in the simulations for \( \Delta v \) and acceleration requirements. Differences in \( \Delta v \) requirements between these cases and their corresponding cases without sensor and actuator uncertainties are less than 1.5\%. Sensor noise and actuator uncertainties were included in the controller parameter sensitivity study since as the sample period \( T_s \) gets larger, sensor noise can become the principal limitation achievable tracking performance.\(^ {16} \)

VI. InSAR \( \Delta v \) Requirements

To determine formation \( \Delta v \) requirements, a spectrum of PRO classes, orbit altitudes, baselines, and controller error requirements were considered. The error requirements are specified as \((e_x, e_y, e_z)\), where \( e_i \) is the tracking error allowed in a given LVLH axis. For example, if \( e_z = 10 \) m, then the difference between the commanded z relative position and the actual can be \( \pm 10 \) m. Generally, the \( e_y \) requirement is larger than those for \( e_x \) and \( e_z \) since the InSAR formation is less sensitive to baseline offsets in the along-track direction. Table 1 shows the values considered in this study.

<table>
<thead>
<tr>
<th>PRO Class</th>
<th>Orbit Altitude(km) / InSAR Baseline(m)</th>
<th>Performance Requirements (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corkscrew, ( \omega_1 = 0^\circ ) and 90°</td>
<td>500 / 300</td>
<td>(10, 50, 10)</td>
</tr>
<tr>
<td>Pseudo-Frozen Corkscrew</td>
<td>500 / 1000</td>
<td>(25, 125, 25)</td>
</tr>
<tr>
<td>Cartwheel, ( \omega_1 = 0^\circ ) and 90°</td>
<td>1000 / 2000</td>
<td>(50, 250, 50)</td>
</tr>
<tr>
<td>Pendulum, ( \omega_1 = 0^\circ ) and 90°</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Characteristics of InSAR Formations to be Studied

Figures 5 and 6 show representative \( \Delta v \) requirements and tracking errors for formation control using CVS for a 500 km corkscrew with 90° initial argument of perigee and 300 m baselines. Additionally, Figure 5 shows the \( \Delta v \) requirement for controlling the same PRO using Prescribed-\( \nu \) formation guidance. The \( \Delta v \) requirements are shown as the average \( \Delta v \) required per orbit up to a given time. For example, the average
\( \Delta v \) per orbit at 1000 orbits in Figure 5 is the total \( \Delta v \) required over those 1000 orbits divided by 1000. For this PRO, CVS reduces the required \( \Delta v \) by 16%.

![Graph showing \( \Delta v \) requirements for a Corkscrew PRO with 500 km Altitude, 90° Initial Perigee, and 300 m Baseline for both Prescribed-\( v \) and CVS Formation Guidance.](image)

**Figure 5.** \( \Delta v \) Requirements for a Corkscrew PRO with 500 km Altitude, 90° Initial Perigee, and 300 m Baseline for Both Prescribed-\( v \) and CVS Formation Guidance.

The \( \Delta v \) requirements for the test cases of Table 1 using CVS formation guidance are shown in Table 2. The \( \Delta v \) requirements using Prescribed-\( v \) are still being determined, and initial results are shown in Table 3. The values for “w” are the initial argument of perigee.

These results show that operating a distributed InSAR mission as a formation is feasible: the majority of \( \Delta v \) requirements are less than 12 mm/s/orbit. Figure 7 can be used to interpret these \( \Delta v \) requirements. The figure shows the percentage of initial spacecraft wet mass that is fuel for a two-year mission with a semi-major axis of 7128 km (750 m altitude) as a function of average \( \Delta v \) per orbit for both cold gas thrusters (\( I_{sp} \) of 60 s) and electric thrusters (\( I_{sp} \) of 2500 s). For 12 mm/s/orbit, the fuel mass percentages are 18% and 0.3% for cold gas and electric thrusters, respectively.

VIA. Comparison of Prescribed-\( v \) and CVS Guidance

Figure 5 shows one comparison between the \( \Delta v \) required by a formation using CVS and a formation using Prescribed-\( v \) guidance algorithms. Considering Tables 2 and 3, CVS requires less \( \Delta v \) for cartwheel and pendulum PROs and for corkscrews with 90° initial perigee. Figure 8 shows a case where CVS uses 89% less \( \Delta v \), that is, nearly an order of magnitude improvement.

CVS achieves these savings by introducing a bias into the rate at which \( SC_2 \) travels around the PRO. Figure 9 shows the difference between the CVS command for the angular position of \( SC_2 \) in a corkscrew PRO and the true anomaly of \( SC_1, v_1 \). For Prescribed-\( v \), this difference is uniformly zero. When this difference reaches, for example, 90°, then a baseline that originally occurred at the equator now occurs at one of the poles. This secular drift, which is on the order of tenths of a degree per orbit, can be desirable for shifting baselines. If, however, the time at which a baseline occurs is to remain fixed, then the secular drift must be restricted. Also shown in Figure 9 is a version of constrained CVS in which the difference between the CVS command and \( v_1 \) is limited to 10°. As can be seen from the figure, after several days, constrained CVS is equivalent to Prescribed-\( v \) with a constant offset.

![Graph showing tracking error for a Corkscrew PRO with CVS, 500 km Altitude, 300 m Baseline, and (10, 25, 10) m Error Requirements.](image)

**Figure 6.** Tracking Error for a Corkscrew PRO with CVS, 500 km Altitude, 300 m Baseline, and (10, 25, 10) m Error Requirements.

![Graph showing fuel-to-wet mass for a two-year mission at 750 km Altitude.](image)

**Figure 7.** Fuel-to-Wet-Mass for Two Year Mission at 750 km Altitude.
VI.B. Acceleration Requirements

In addition to $\Delta v$ requirements, a formation control system must have realizable acceleration requirements. The force level of RCS thrusters can range from Newtons for cold gas to milli- and micro-Newton for electric thrusters. For a 1000 kg spacecraft, this translates into acceleration levels of $mm/s^2$ to $nm/s^2$. A large ion thruster, such as the UK-25, can produce 300 mN of thrust. Figure 10 shows the maximum acceleration for each of the CVS test cases and the acceleration limit for a large ion thruster. These acceleration levels are easily realizable with a cold gas RCS system, but are large for an electric thruster RCS system using $3 \sim 30 mN$ electric thrusters.

Recall that the CVS test cases used control design parameters $T_s = 1/100^{th}$ of an orbit and $T_a = 1.0 s$. $T_s$ is the control sample period, and $T_a$ is the duration that a commanded acceleration level is applied. A sensitivity analysis was performed to determine the impact of varying these parameters on formation $\Delta v$.
and acceleration requirements.

Tables 4 and 5 show the results for a corkscrew PRO using CVS with 10/50/10 m tracking error requirements. In these tables, the non-dimensional parameter \( \sigma = T_s/T_s \) replaces \( T_s \). If \( \sigma = 1 \), then \( SC_2 \) is always thrusting. From Table 4 we first conclude that the non-dimensionalized sample period, \( T_s/\sigma \) orbit period, and \( \sigma \) can be varied from nearly zero to nearly one with minimal impact on the \( \Delta v \) requirement. Furthermore, the longest control sample period that does not significantly affect \( \Delta v \) is 1/3\( r^d \) of an orbit. This period compares favorably with some model predictive control approaches that thrust every half orbit.\(^{16}\) Next, from Table 5 it is seen that for a 1000 kg spacecraft with constant thrusting, standard, milli-Newton-level electric thrusters can be used. If desired, thrusting could be done only half the time (\( \sigma = 0.5 \)) and still only require micro-Newton-level accelerations.

![Figure 10. Maximum Control Accelerations](image)

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Table 4. \( \Delta v \) Requirements in m/s for a Cartwheel PRO with 500 km Altitude, 90° Initial Perigee, 300 m Baseline, and CVS Formation Guidance as a Function of Control Design Parameters.

<table>
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</table>

Table 5. Acceleration Requirements in m/s\(^2\) for a Cartwheel PRO with 500 km Altitude, 90° Initial Perigee, 300 m Baseline, and CVS Formation Guidance as a Function of Control Design Parameters.

Having discussed formation guidance and control issues for maintaining a PRO, we now turn to formation algorithms for maintaining the absolute formation orbit and changing PROs.

VII. Orbit Maintenance/Reconfiguration Algorithm

VIIA. Problem Statement

The goal of the formation guidance OM/R algorithm is to plan fuel-optimal, acceleration-limited trajectories that maneuver two spacecraft in LEO from arbitrary initial conditions to final conditions specified by (i) the desired absolute orbit of the formation, and (ii) the desired PRO of \( SC_2 \) with respect to \( SC_1 \). This trajectory planning problem is a complex, nonlinear optimization problem that remains unsolved in its general form. Consistent with the scope of this study, we have made simplifying assumptions to obtain a near-optimal, computationally-efficient algorithm for evaluating formation fuel consumption. These assumptions are discussed subsequently.

Figure 11 shows the problem statement graphically. Let \( X(0) \) be the initial position on the desired formation reference orbit with respect to Earth’s center. This reference orbit is independent of the spacecraft states, has initial true anomaly \( \nu(0) \), and defines an LVLH frame. Subsequently, we allow \( \nu(0) \) to vary. The initial positions of \( SC_1 \) and \( SC_2 \) relative to the formation reference orbit in the LVLH frame are given by \( \rho_1(0) \) and \( \rho_2(0) \), respectively. The final condition of the formation reconfiguration or orbit maintenance maneuver consists of (i) \( SC_1 \) located on the formation reference orbit and (ii) \( SC_2 \) in the desired PRO.

![Figure 11. OM/R Problem Statement](image)
A key aspect of this problem is that the maneuver time is free. That is, as long as the maneuver is completed in some reasonable maximum amount of time, it can take half an orbit or three orbits. Figure 11 shows the final conditions for two different maneuver times, $t_{f,1}$ and $t_{f,2}$. Observe that the final PRO has the same shape for both maneuver times, but that the location of $SC_2$ within the PRO depends on the maneuver time. Another key aspect is that SAR spacecraft may not have a large “kick” motor for impulsive velocity changes. Hence, the OM/R algorithm must include the ability to plan low-thrust optimal trajectories. For example, assuming 5 N cold-gas thrusters are mounted on a 1000 kg spacecraft and two thrusters must fire to produce a pure force, translational accelerations are limited to 0.01 m/s².

VII.B. OM/R Algorithm Assumptions

To obtain a computationally-efficient, practical algorithm, we make the following assumptions. First, there are no environmental disturbances (i.e., no drag, solar pressure, or gravitational perturbations). The OM/R algorithm does partially account for $J_2$. However, for simplicity, we present the disturbance-free case. Second, each spacecraft has six thrusters that point in each of the cardinal directions of the IVLH frame. Each thruster can produce a maximum acceleration of $a_{thrust}$. Third, we are currently not considering collision avoidance. Our experience is that the planned trajectories of the two spacecraft rarely intersect. However, in future versions of this algorithm, collision avoidance can be included through either waypoints or starting the spacecraft along their trajectories at different times. See Ref. 35 for more on collision avoidance through phasing of maneuver start times. Finally, the spacecraft remain close enough to the formation reference orbit that linearized relative dynamics are accurate (i.e., tens of kilometers). If this last assumption is violated, the OM/R algorithm will still function, but it will be sub-optimal.

For orbit maintenance, the initial relative positions of the two spacecraft are close to the desired PRO, but the spacecraft have drifted away from their absolute formation orbit. The OM/R algorithm returns $SC_1$ to the origin of the IVLH frame and $SC_2$ to the desired PRO about $SC_1$.

A reconfiguration maneuver changes the PRO of $SC_2$ about $SC_1$. Since $SC_1$’s orbit is already close to the desired absolute orbit, only $SC_2$ maneuvers. To balance fuel consumption, a reconfiguration can also be used to switch the roles of $SC_1$ and $SC_2$.

VII.C. OM/R Solution

Our solution method is to apply linear programming to the HCW Equations augmented with the in-orbit-plane effects of $J_2$.

The eccentricities of the orbits for which the OM/R algorithm will be used are very small ($10^{-5}$), and so the HCW Equations are a valid approximation. Further, eccentricities of up to 0.2 can be included without restructuring the OM/R algorithm via time-explicit, eccentricity series-based linear relative dynamic models. The application of linear programming to relative path planning of spacecraft is not new. Our contributions are to generalize the boundary conditions and introduce an additional degree of freedom that further reduces fuel consumption.

Given a discrete, linear, time-invariant model of the form of Eq. (8), the state at time step $k$ can be written in terms of initial conditions $x_0$ (e.g., $p(0)$ and the associated velocity in Figure 11) and all previous thruster firings $u_i, i = 0, \ldots, k - 1$ as

$$x_k = G_k U_k + c_k$$  

$$G_k = [A^{k-1}B A^{k-2}B \cdots AB B]$$

$$c_k = B^k x_0$$

$$U_k = [u_1^T u_1^T \cdots u_{k-1}^T]^T$$

The trajectory optimization problem can now be formulated as a linear program. The goal is to have $x_f$ equal the desired final conditions $x_f$ while minimizing the fuel consumed. The fuel consumed is monotonically related to sum of the absolute values of the thruster accelerations. Finally, recall that the thrust must be...
limited to \( u_{\text{max}} \). The trajectory optimization problem can be stated in standardized form as

\[
\begin{align*}
\min_{\Lambda_n} & \quad [1 \cdots 1] \Lambda_n \\
\text{subject to} & \quad \begin{bmatrix} [G_n - G_n] \Lambda_n = x_f - c_n \\
I \\
-I \end{bmatrix} \Lambda_n \leq \begin{bmatrix} u_{\text{max}} \\
0 \end{bmatrix} \\
\Lambda_n &= \begin{bmatrix} \Lambda_n^+ \\
\Lambda_n^- \end{bmatrix}
\end{align*}
\]

(18)

where \( U_n = \Lambda_n^+ - \Lambda_n^- \). Since both \( \Lambda_n^+ \) and \( \Lambda_n^- \) are constrained to be positive, if \( u_k = -1 \text{ m/s}^2 \) for example, then the corresponding entry in \( \Lambda_n^+ \) would be 0 and in \( \Lambda_n^- \) the entry would be 1. While entries 1 and 2 would generate the same \( u_k \), these values are not minimal. Hence, minimizing the sum of the elements of \( \Lambda_n \) is equivalent to minimizing the sum of the absolute values of the thruster accelerations.

The orbit maintenance and reconfiguration problems have been reduced to a linear program for which standard numerical solution software exists. However, the OM/R algorithm must still optimize over the final time, which changes \( x_f \). Further, we also optimize over the initial formation reference point \( \nu(0) \), which changes the LVLH frame and hence \( x_o \). The maneuver final time is optimized by considering several discrete values. For shorter final times and smaller values of \( u_{\text{max}} \), the linear program becomes infeasible. This infeasibility indicates that the maneuver cannot be completed in the given time with the given acceleration limit.

To understand how the reference LVLH frame enters the optimization, observe that the trajectory optimization problem only requires a spacecraft state relative to an LVLH frame. The transformation of the inertial spacecraft states to LVLH-relative states introduces an additional degree of freedom to optimize over. Specifically, we are free to choose any LVLH frame on the formation reference orbit within given limits. Choosing different LVLH frames only affects the time of perigee passage for the final formation orbit. Recall that Earth oblateness will be perturbing the time of perigee passage, and so modifying this time slightly during an orbit maintenance maneuver is not a concern. Further, it is shown subsequently that the optimum modifications are typically less than 0.05° in true anomaly.

Consider Figure 12. The default frame is the LVLH frame of the reference orbit of \( SC_1 \), which is point \( O_o \) in Figure 12. Recall this point is assumed to be within tens of kilometers of the spacecraft for the linearized dynamics to be accurate. The possible reference frames are parameterized by a true anomaly offset \( \nu_{oa} \) with respect to the default reference frame. For example, a second reference frame is shown with origin \( O_o \). When changing the reference LVLH frame, the specification of the final position of \( SC_2 \) within the desired PRO must account for \( \nu_{oa} \). Therefore, the angle \( \theta \) specifying the final position of \( SC_2 \) in the desired PRO is \( \theta = \nu(0) + \nu_{oa} + \Delta \nu_{ml} \), where \( \Delta \nu_{ml} \) is the change in true anomaly over the maneuver time \( t_f \).

In summary, the OM/R algorithm considers ranges of maneuver times \( t_f \) and LVLH reference frame offsets \( \nu_{oa} \). For each combination of maneuver time and offset, the trajectory optimization linear program of Eq. (18) is solved. From the resulting family of trajectories, the overall fuel optimal is selected.

VIID. OM/R Algorithm Validation and Performance

The OM/R algorithm was verified by comparing to it a Hohmann transfer from a 499 km circular orbit to a 500 km circular orbit. To represent an impulsive transfer, \( u_{\text{max}} \) was set to 1 m/s². The Hohmann transfer time is 0.4995 of the period of the higher orbit, \( P_1 \). The default LVLH frame corresponded to the point in the higher orbit directly above \( SC_1 \)'s initial location on the lower orbit. Optimization over \( \nu_{oa} \) was performed. The resultant optimal \( \nu_{oa} \) was 0.03°. This offset is precisely the one needed so that the LVLH
frame, to which $SC_1$ is moving, ends at the correct point on the higher orbit. That is, with $\nu(0) = 0^\circ$, $\nu_{oa} = 0.03^\circ$, and $t_f = 0.4995 P_1$, we have $\nu(t_f) = 180^\circ$ as is required for a Hohmann transfer. See Figure 13. The Hohmann $\Delta v$ is 0.553 m/s and the OM/R algorithm calculated a trajectory with a $\Delta v$ of 0.555 m/s, that is, with 0.3% higher $\Delta v$. The Hohmann transfer and the impulsive OM/R transfer are shown in the reference LVLH frame in Figure 14.

Of more interest is how the OM/R algorithm performs with limited acceleration. Assume $u_{max} = 0.01 \text{ m/s}^2$, corresponding to an orbit transfer on RCS thrusters. The maneuver time is free. The OM/R algorithm calculated a trajectory taking 0.75 orbits, but with a $\Delta v$ only 0.1% greater than the globally optimal Hohmann $\Delta v$. This lower-thrust trajectory is also shown in Figure 14. The optimal $\nu_{oa}$ is 0.02°. Without optimizing over $\nu_{oa}$ the suboptimal trajectory required 6% more $\Delta v$. See Figure 15, which shows the optimal $\Delta v$ as a function of $\nu_{oa}$ for this lower-thrust case.

The OM/R algorithm was used to determine representative $\Delta v$ requirements for InSAR formation reconfigurations. Table 6 shows the $\Delta v$ requirements for transferring between PROs at a representative altitude of 500 km and baselines of 300 m. Reconfigurations can require from nearly 0.86 m/s down to 0.16 m/s, which are equivalent to approximately 15 days and 2.8 days of formation control, respectively.

![Figure 14. Hohmann, Impulsive OM/R, and Lower Thrust OM/R Trajectories in Reference LVLH Frames for 1 km Transfer Between Circular Orbits. Note that the reference frame for the Lower Thrust OM/R trajectory is different.](image)

![Figure 15. Optimal $\Delta v$ for Lower Thrust OM/R Trajectory as a Function of $\nu_{oa}$.](image)

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Table 6. $\Delta v$ Requirements in m/s for Reconfigurations at 500 km Altitude and 300 m Baselines
VIII. Summary and Future Work

To evaluate the feasibility of formation flying two InSAR spacecraft, formation guidance, control, reconfiguration, and orbit maintenance architectures and algorithms have been developed, including the new CVS formation guidance algorithm. CVS incorporates some of the fuel-efficiency of Cyclic formation control schemes while maintaining the stability guarantees of the Leader/Follower architecture. It can reduce Δv requirements by up to an order of magnitude over standard Leader/Follower guidance schemes, such as Prescribed-Δv. The principle underlying CVS is that baselines are allowed to vary temporally; the formation controls to the set of PRO baselines as opposed to a prescribed point in time and space. CVS achieves its Δv savings by introducing a secular drift in the phasing of a PRO. This drift can be used to shift phasing for InSAR observations.

Formation Δv and acceleration requirements were determined to maintain a spectrum of PROs. Generally, less than 12 mm/s/orbit of Δv was required, resulting in fuel mass percentages of better than 18% for cold gas thrusters and 0.5% for electric thrusters. It was further determined that by varying control design parameters, the maximum acceleration required for formation maintenance can be tailored from μm/s^2 to mm/s^2. Hence, for a 1000 kg spacecraft, milli-Newton-level electric thrusters can be used for formation control.

Finally, an orbit maintenance/reconfiguration (OM/R) algorithm was developed for both maintaining the absolute formation orbit and reconfiguring between passive relative orbits for InSAR observations. A key aspect of this algorithm is that it includes acceleration magnitude constraints so that such maneuvers can be performed using RCS thrusters, obviating the need for a kick motor. To achieve optimal trajectories, it is critical to optimize over the LVLH frame in which the trajectory generation is performed. By doing so, a lower-thrust transfer was obtained that required the same Δv as an impulsive Holmman transfer. Reconfigurations required Δv equivalent to a few days to a half-month of formation maintenance.

In our future work we intend to (i) incorporate OM/R algorithm into closed loop simulation environment and obtain mission Δv requirements with re-boosting, reconfigurations, oblateness, aerodynamic drag, solar radiation pressure, and sensor and actuator uncertainties, (ii) tailor the formation control laws to provide acceleration uncertainties on the order 10 mm/s^2 to allow for sub-millimeter precision orbit determination for radar data post-processing, and (iii) evaluate formation control with sensor noise levels corresponding to differential GPS as opposed to carrier differential GPS.

IX. Acknowledgments

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References


