Magnetometer based on the opto-electronic oscillator

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ABSTRACT
We theoretically propose and discuss properties of two schemes of an all-optical self-oscillating magnetometer based on an opto-electronic oscillator stabilized with an atomic vapor cell. Proof of the principle DC magnetic field measurements characterized with $2 \times 10^{-7}$ G sensitivity and $1 - 1000 \text{ mG}$ dynamic range in one of the schemes are demonstrated.

INTRODUCTION
Opto-Electronic Oscillator (OEO) is a photonic device that produces microwaves characterized with high spectral purity.\footnote{We propose to stabilize the oscillator with an atomic vapor cell to create a miniature active magnetometer (Fig. 1). The magnetometer is expected to generate a stable narrowband microwave signal with frequency proportional to the external magnetic field.} In a generic OEO a laser beam sent consequently to an amplitude modulator and a fast photodiode produces the electrical current that includes a component at the frequency, generally microwave, of the modulation. The electrical output of the photodetector is amplified, filtered, and fed back to the microwave input port of the modulator. The scheme starts to oscillate if the laser power/microwave amplification is large enough.

An OEO becomes a magnetometer if an atomic vapor cell with proper chemical content is inserted on the path of the modulated light. The cell transmits sidebands of modulated light only when the modulation frequency coincides with a half of the eigenfrequency of the
magnetic-field-sensitive hyperfine transition of the atoms. In other words, the atomic cell operates as a pass-band microwave filter. The OEO signal locks to the frequency of the filter. We here discuss two schemes of OEO magnetometers. The schemes are different in their optical parts. One scheme containing an electro-optical modulator and a CW monochromatic laser (Fig. 1a) was partially studied very recently, while the other, based on a directly modulated vertical cavity surface emitting laser (VCSEL) (Fig. 1b) is a novel one. We calculate the phase noise, stability, and estimate performance of the oscillators. Finally, we include the results of the proof of the principle measurements of the magnetic field with the devices.

The devices are self-oscillating and, obviously, differ from the passive EIT magnetometers. They are also different from other active magnetometers, where stability is achieved through the microwave field stored in a microwave resonator or an RF coil containing an atomic vapor cell. There is no need for a microwave cavity in the OEO because the microwave energy is carried as the sidebands of modulated light. This allows for minimizing the size and reducing the magnetometer power consumption without performance loss.

**ATOMIC VAPOR CELL**

To realize the pass-band optical filter sensitive to the magnetic field one can utilize alkali atoms, e.g. Rb$^{87}$, optical response of which can be explained using three level $\Lambda$ and two two-level configurations (Fig. 2). The $\Lambda$ system introduces the filter function we are interested in, while the two-level systems are responsible for the residual absorption of the sidebands of the modulated light. The carrier light is offresonant with the atomic transitions so we neglecting its influence on the filter properties of the cell. We discuss rubidium atomic cell in the following.

**Figure 2.** Schematic of interaction of $D1$ line of $Rb^{87}$ and the modulated light. The wavelength of the light is 795 nm and the modulation frequency is approximately 3.42 GHz. For the sake of simplicity we replace the exact level configuration with three-level and two-level systems frequency response of which could be evaluated analytically. $\Delta_D$ stands for the homogeneous width of the excited atomic state. We assume that $\Delta_D$ exceeds the inhomogeneous width.
Propagation of the sideband amplitudes \((E_+ \text{ and } E_-)\) is characterized by equations

\[
\begin{align*}
\frac{\partial}{\partial z} E_+ &= -\alpha(\omega_M) E_+, \\
\frac{\partial}{\partial z} E_- &= -\alpha(\omega_M) E_-,
\end{align*}
\]

where \(\omega_M\) is the modulation frequency and \(\alpha(\omega_M)\) and \(\beta(\omega_M)\) are coefficients that generally have a complicated form. However, for the particular case of strong enough optical sidebands they can be approximated by

\[
\begin{align*}
\alpha(\omega_M) &= \kappa \left[ \frac{\gamma_0 \Delta_D}{\Delta_D^2 + (\delta/2)^2} \right], \\
\beta(\omega_M) &= \kappa \left[ \frac{\gamma_0 \Delta_D}{\Delta_D^2 + (\delta/2)^2} \right],
\end{align*}
\]

where \(|\Omega|^2 = (\varphi/h)^2 |E_+|^2 + |E_-|^2\), \(\omega_0\) is the matrix element of transitions \(a \rightarrow b \text{ and } a \rightarrow c\), \(E_+\) and \(E_-\) are the amplitudes of the electric fields of the sidebands; \(\delta = 2\omega_M - \omega_{bc}\) is the two photon detuning, \(\omega_{bc} = \omega_0 + 2\pi \times 2nB\), \(a = 0.7 \text{ MHz/G}\), \(B\) is the value of the magnetic field, \(\omega_0 \approx 6.84 \text{ GHz}\) is the hyperfine splitting; \(\kappa = 3N\lambda^2/(8\pi)\), \(N\) is the concentration of the atoms, \(\lambda = 795 \text{ nm}\) is the wavelength of the carrier light; \(\gamma_0 = \gamma_2 = \gamma_a\) and \(\gamma\) are the natural decay rates of states \(a_1, a_2, \text{ and } a\), respectively; \(\gamma_{bc}\) is the decay rate of the low frequency coherence, \(\zeta\) is the scale parameter indicating the relative absorption due to the interaction of the light with the two and three level systems.

In practically interesting cases the contrast of the EIT resonance is small, and the solution of Eqs. (1) and (2) can be simplified. We introduce the filter function of the cell as

\[
\xi(\omega_M) = \alpha_F(L) \exp[i(\omega_{bc} - 2\omega_M)\tau_F(L)],
\]

so that \(E_+^{\text{out}} = \xi(\omega_M) E_+^{\text{in}}\) and \(E_-^{\text{out}} = \xi^*(\omega_M) E_-^{\text{in}}\). The absorptive and refractive coefficients of the filter function are

\[
\begin{align*}
\alpha_F(L) &= \exp \left[-\kappa L \frac{\gamma_0 \Delta_D}{\Delta_D^2 + (\delta/2)^2} \right], \\
\tau_F(L) &= \kappa \int_0^L \frac{\gamma [\alpha_F(z)^2 (|\Omega(z) = 0)|^2 - \delta^2] \, dz}{\alpha_F(z)^2 (|\Omega(z) = 0)|^2 - \delta^2 + \Delta_D \delta^2}.
\end{align*}
\]

**OSCILLATION CONDITIONS**

The oscillation conditions for the circuits shown in Fig. (1a) can be estimated using the calculation procedure presented in. The microwave power at the entrance of the modulator \((P_{\text{Min}})\) is determined by

\[
G \left( \frac{\alpha_F}{4} \right)^2 \rho R^2 P_{\text{sat}} \left[ 1 - \frac{P_{\text{Min}}}{P_{\text{sat}}} - \left( \frac{P_0}{P_{\text{sat}}} \right)^2 \right] = 1,
\]

where \(G\) is the gain coefficient of the microwave amplifier, \(R\) and \(P_{\text{sat}}\) are the responsivity and saturation power of the photodiode, \(P_0\) is the power of the laser light at the modulator.
entrance, \( P_{\text{sat}} \) is the saturation power of the modulator, \( \rho \) is the resistivity of the microwave transmission line.

The frequency of the oscillations is determined by

\[
(\omega_M - \omega_f) \tau_f + (2\omega_M - \omega_{bc}) \tau_F(L) = 2\pi m,
\]

where \( m \) is an integer number, \( \omega_f \) is an eigenfrequency of the optical-microwave loop that does not include the atomic cell (found from \( \omega_f \tau_f = 2\pi k \), where \( k \) is integer). Because generally \( \tau_F(L) \gg \tau_f \), the oscillation frequency is determined by the magnetic field sensitive frequency of the hyperfine splitting.

The amplitude oscillation condition (7) does not depend on the low contrast EIT effect. The oscillations occur anywhere where gain exceeds the absorption and, naturally, the system prefers to oscillate far away from the atomic transitions unless this is forbidden by condition (8). An additional microwave filter should be inserted to force the oscillations at the prescribed frequency. Otherwise, the system can be miniaturized such that the optical-microwave loop may not oscillate without the atomic cell.

**NOISE AND SENSITIVITY**

The sensitivity of the magnetometer is determined by the phase noise of the OEO. The phase noise is determined by phase transfer function of the oscillator. It is possible to show, that the phase transfer function of the oscillator stabilized with atomic cell is

\[
H(\delta\omega) = \left| 1 + \frac{1}{e^{\delta\omega [\tau_f + 2\tau_F(L, \delta_0)]} - 1} \right|^2
\]

where

\[
\delta\omega = \omega - \omega_{M0}, \quad \omega_{M0} = \frac{2\pi m}{\tau_f + 2\tau_F(L, \delta_0)} + \omega_{bc} \frac{\tau_F(L, \delta_0)}{\tau_f + 2\tau_F(L, \delta_0)} + \omega_f \frac{\tau_f}{\tau_f + 2\tau_F(L, \delta_0)}
\]

value \( \delta_0 = 2\omega_{M0} - \omega_{bc} \) corresponds to the two photon detuning at the oscillation frequency \( \omega_{M0} \). A typical frequency dependence of the phase transfer function is shown in Fig. 3. The phase noise spectrum of the oscillator is

\[
S_\phi = H(\delta\omega) \frac{FkT}{P_M},
\]

where \( P_M \) is the power of the generated microwave signal, \( F \) is the noise figure of the oscillator, \( k \) is the Boltzmann constant, and \( T \) is the temperature. The noise figure is primarily restricted by the shot noise of the optical part of the oscillator.

The sensitivity of the active magnetometer \( \Delta B(\tau) \) is determined by the uncertainty of the oscillation frequency measurements, which could be characterized by Allan variance \( \sigma_{\omega M/2\pi}(\tau) \). Allan variance and phase modulation noise are related by straightforward formula,

\[
\sigma_{\omega M/2\pi}^2(\tau) = \frac{32\pi^2}{\tau^2 \omega_M^2} \int_0^\infty \sin^4 \left( \frac{\omega M \tau}{2} \right) S_\phi \left( \frac{\omega M}{2\pi} \right) d\omega_M \frac{\omega M}{2\pi}, \quad \Delta B(\tau) \approx \frac{\sigma_{\omega M/2\pi}(\tau)}{a}
\]
Figure 3. Parameter $H(\delta \omega)$ calculated for $|\Omega| = 3 \times 10^6$ s$^{-1}$, $|\Omega|^2 \gg \gamma_c \Delta_D$, $\Delta_D = 10^9$ s$^{-1}$, $\tau_F(L, 0) \simeq \kappa L \gamma / |\Omega|^2 = 30$ $\mu$s, and $\tau_f = 3 \times 10^{-9}$ s. The curve does not reach the minimum possible value $H(\delta \omega)_{\text{min}} = 1$ because of the low contrast of the resonance. The lower is contrast, the higher is phase noise of the oscillator. For the particular calculation the local minimum is achieved for the maximum attainable phase shift $\delta \omega[\tau_f + 2\tau_F(L, 2\delta \omega + \delta_0)]_{\text{max}} \approx 0.28$

It is reasonable to expect that $\sigma_{\omega_M/2\pi}(1s)/(\omega_M/2\pi) = 10^{-11}$, as for usual EIT-based clocks, then $\Delta B(1s) = 5 \times 10^{-8}$ G.

EXPERIMENT

In our experiment the laser light was sent through a free-beam electro-optic phase modulator having resonance of $\sim 20$ MHz at microwave frequency $3.42$ GHz. The phase modulated light was transformed to the amplitude modulated light with a rotating half-wave plate and a polarizer. The light was directed to a centimeter-long cell contains pure $^{87}$Rb isotope at about $90^\circ$C. The cell was placed inside of a solenoid, which was used to create a magnetic field along the propagation direction of the optical beam. The solenoid and the cell were enclosed in a three-layer $\mu$-metal shield. The laser was tuned in between of the $5S_{1/2}, F=2 \rightarrow 5P_{1/2}, F=1$ and $5S_{1/2}, F=1 \rightarrow 5P_{1/2}, F=1$ transitions of $^{87}$Rb. Transition frequencies differ by $6.83468$ GHz at zero magnetic field, which is close to $2\omega_M$.

Light that passed through the cell was detected by a fast photodiode, the electric output of which was amplified and used to drive the modulator. A phase shifter was inserted into the circuit to achieve the desired microwave phase delay. As the result, the oscillation frequency $\omega_M$ is locked at the atomic hyperfine transition frequency for a suitable choice of the experimental parameters.

We have measured Allan variance of the oscillations and estimated sensitivity in the system locked either to $m = -1 \rightarrow m' = -1$ atomic microwave transition, setting the bias magnetic field at the level of approximately 1 G and recording the oscillation frequency as the bias
experienced steps varying from 1 mG to 100 mG (see Fig. 4a). The sensitivity was determined by the technical noises.

\[ G = 1.69 \times 10^{-2} \times 10^{-3} \times G \]

**Figure 4.** Proof of the principle demonstration of the OEO magnetometer. (a) The performance of the magnetometer with external EOM as per Fig. 1a. The trace illustrates the frequency deviations of the oscillator. The magnetic field bias abruptly changes by approximately 1 mG during the measurement. (b) The sensitivity of the measurement of the magnetic field estimated from the Allan variance calculated for the device shown in Fig. 1b.

Our preliminary measurements with direct modulation of the laser (VCSEL) instead of usage of the external modulator have shown poorer sensitivity (Fig. 4b). However, again, the performance was determined by purely technical reasons and improvement of the device performance by two-three orders of magnitude is feasible.

**CONCLUSION**

We have studied properties of a self-oscillating magnetometer based on an opto-electronic oscillator locked to a rubidium atomic cell. The oscillator is capable of generating stable microwave signals at a frequency determined by a magnetic field sensitive atomic transition.

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**REFERENCES**

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