

General Theory of Relativity: Will it survive the next decade?

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Abstract

The nature of gravity is fundamental to our understanding of our own solar system, the galaxy and the structure and evolution of the Universe. Einstein's general theory of relativity is the standard model that is used for almost ninety years to describe gravitational phenomena on these various scales. We review the foundations of general relativity, discuss the recent progress in the tests of relativistic gravity, and present motivations for high-accuracy gravitational experiments in space. We also summarize the science objectives and technology needs for the laboratory experiments in space with laboratory being the entire solar system. We discuss the advances in our understanding of fundamental physics anticipated in the near future and evaluate discovery potential for the recently proposed gravitational experiments.

1 Introduction

To understand the Universe in its vast and complex splendor seems a daunting task, yet human curiosity and wonder over centuries and civilizations have always led humankind to seek answers to some of the most compelling questions of all – How did the Universe come to be? What is it made of? What forces rule its behavior? Why is it the way it is? What will ultimately become of it? With its prominent influence on natural phenomena at every distance scale, gravitation plays a pivotal role in this intellectual quest.

Gravity was known to humans long before the present-day picture of four fundamental interactions was formed. The nature of gravity is fundamental to our understanding of our solar system, the galaxy and the structure and evolution of the Universe. It was Newton who first understood that gravity not only dictates the fall of apples and all bodies on Earth, but also planetary motion in our solar system and the sun itself are governed by the same physical principles. On the larger scales the effects of gravity are even more pronounced, guiding the evolution of the galaxies, galactic clusters and ultimately determining the fate of the Universe. Presently the Einstein's general theory of relativity is a key to the understanding a wide range of phenomena, spanning from the dynamics of compact astrophysical objects such as neutron stars and black holes, to cosmology, where the Universe itself is the object of study. Its striking predictions include gravitational lensing and waves, and only black holes have not yet been directly confirmed.

The significance that general relativity (GR) plays for our understanding of nature, makes the theory a focus of series of experimental efforts performed with ever increasing accuracy. However, even after more than ninety years since general relativity was born, Einstein's theory has survived every test. Such longevity does not mean that it is absolutely correct, but serves to motivate more precise tests to determine the level of accuracy at which it is violated. This motivates various precision tests of gravity both in laboratories and in space; as a result, we have witnessed an impressive progress in this area over the last two decades. However, there are a number of reasons to question the validity of this theory, both theoretical and experimental.

On the theoretical front, the problems arise from several directions, most dealing with the strong gravitational field regime; this includes the appearance of spacetime singularities and the inability to describe the physics

of very strong gravitational fields using the standard of classical description. A way out of this difficulty would be attained through gravity quantization. However, despite the success of modern gauge field theories in describing the electromagnetic, weak, and strong interactions, it is still not understood how gravity should be described at the quantum level. Our two foundational theories of nature, quantum mechanics and GR, are not compatible with each other. In theories that attempt to include gravity, new long-range forces can arise in addition to the Newtonian inverse-square law. Even at the classical level, and assuming the Equivalence Principle, Einstein's theory does not provide the most general way to establish the spacetime metric. Regardless of whether the cosmological constant should be included, there are also important reasons to consider additional fields, especially scalar fields. Although the latter naturally appear in these modern theories, their inclusion predicts a non-Einsteinian behavior of gravitating systems. These deviations from GR lead to a violation of the Equivalence Principle, a foundation of general relativity, modification of large-scale gravitational phenomena, and cast doubt upon the constancy of the fundamental "constants." These predictions motivate new searches for very small deviations of relativistic gravity from GR and provide a new theoretical paradigm and guidance for further gravity experiments.

Meanwhile, on the experimental front, recent cosmological observations has forced us to accept the fact that our current understanding of the origin and evolution of the Universe is at best incomplete, and possibly wrong. It turned out that, to our surprise, most of the energy content of the Universe resides in presently unknown dark matter and dark energy that may permeate much, if not all of spacetime. If so, then this dark matter may be accessible to laboratory experimentation. It is likely that the underlying physics that resolve the discord between quantum mechanics and GR will also shed light on cosmological questions addressing the origin and ultimate destiny of the Universe. Recent progress in the development of vastly superior measurement technology placed fundamental physics in a unique position to successfully address these vital questions. Moreover, because of the ever increasing practical significance of the general theory of relativity (i.e. its use in spacecraft navigation, time transfer, clock synchronization, etalons of time, weight and length, etc.) this fundamental theory must be tested to increasing accuracy.

This paper is organized as follows: Section 2 discusses the foundations of the general theory of relativity and reviews the results of the recent experiments designed to test the foundations of this theory. Section 3 presents motivations for extending the theoretical model of gravity provided by GR; it presents a model arising from string theory, discusses the scalar-tensor theories of gravity, and also highlights phenomenological implications of these proposals. This section also reviews the motivations and the search for new interactions of nature and discusses the hypothesis of gravitational shielding. Section 4 addresses the astrophysical and cosmological phenomena that led to some recent proposals that modify gravity on large scales; it discusses some of these proposals and reviews their experimental implications. Section 5 discusses future missions and experiments aiming to expand our knowledge of gravity. Finally, conclusions and an outlook are presented.

2 Testing Foundations of General Relativity

General relativity began its empirical success in 1915, by explaining the anomalous perihelion precession of Mercury's orbit, using no adjustable theoretical parameters. Shortly thereafter, Eddington's 1919 observations of stellar lines-of-sight during a solar eclipse confirmed the doubling of the deflection angles predicted by the Einstein's theory, as compared to Newtonian-like and Equivalence Principle arguments; this made the theory an instant success. From these beginnings, GR has been extensively tested in the solar system, successfully accounting for all data gathered to date. Thus, microwave ranging to the Viking Lander on Mars yielded accuracy ~ 0.2 in the tests of GR [1, 2, 3]. Spacecraft and planetary radar observations reached an accuracy of ~ 0.15 [4]. The astrometric observations of quasars on the solar background performed with Very-Long Baseline Interferometry improved the accuracy of the tests of gravity to ~ 0.045 [5, 6, 7]. Lunar laser ranging ~ 0.011 verification of GR via precision measurements of the lunar orbit [8, 9, 10, 11, 12, 13, 14]. Finally, the recent experiments with the Cassini spacecraft improved the accuracy of the tests to ~ 0.0023 [15]. As a result, GR became the standard theory of gravity when astrometry and spacecraft navigation are concerned.

To date, GR is also in agreement with the data collected from the binary millisecond pulsars. In fact,

recently a considerable interest has been shown in the physical processes occurring in the strong gravitational field regime with relativistic pulsars providing a promising possibility to test gravity in this qualitatively different dynamical environment. The general theoretical framework for pulsar tests of strong-field gravity was introduced in [16]; the observational data for the initial tests were obtained with PSR1534 [17]. An analysis of strong-field gravitational tests and their theoretical justification was presented in [18, 19, 20]. The recent analysis of the pulsar data tested GR to ~ 0.04 at a 3σ confidence level [21].

In this Section we present the framework used to plan and analyze the data in a weak-field and slow motion approximation which is appropriate to describe dynamical conditions in the solar system.

2.1 Metric Theories of Gravity and PPN Formalism

Within the accuracy of modern experiments, the weak-field and slow motion approximation provides a useful starting point for testing the predictions of different metric theories of gravity in the solar system. Following Fock [22, 23] and Chandrasekhar [24], a matter distribution in this approximation is often represented by the perfect fluid model with the density of energy-momentum tensor \hat{T}^{mn} as given below:

$$\hat{T}^{mn} = \sqrt{-g} \left(\left[\rho_0(1 + \Pi) + p \right] u^m u^n - p g^{mn} \right), \quad (1)$$

where ρ_0 is the mass density of the ideal fluid in coordinates of the co-moving frame of reference, $u^k = dz^k/ds$ are the components of invariant four-velocity of a fluid element, and $p(\rho)$ is the isentropic pressure connected with ρ by an equation of state. The quantity $\rho\Pi$ is the density of internal energy of an ideal fluid. The definition of Π results from the first law of thermodynamics, through the equation $u^n (\Pi_{;n} + p(1/\hat{\rho})_{;n}) = 0$, where the subscript $;n$ denotes a covariant derivative, $\hat{\rho} = \sqrt{-g}\rho_0 u^0$ is the conserved mass density (see further details in Refs. [23, 24, 25, 26]). Given the energy-momentum tensor, one finds the solutions of the gravitational field equations for a particular theory of gravity.¹

Metric theories of gravity have a special position among all the other possible theoretical models. The reason is that, independently of the many different principles at their foundations, the gravitational field in these theories affects the matter directly through the metric tensor g_{mn} , which is determined from the field equations. As a result, in contrast to Newtonian gravity, this tensor expresses the properties of a particular gravitational theory and carries information about the gravitational field of the bodies.

Generalizing on a phenomenological parameterization of the gravitational metric tensor field, which Eddington originally developed for a special case, a method called the parameterized post-Newtonian (PPN) metric has been developed [32, 33, 34]. This method represents the gravity tensor's potentials for slowly moving bodies and weak inter-body gravity, and is valid for a broad class of metric theories, including GR as a unique case. The several parameters in the PPN metric expansion vary from theory to theory, and they are individually associated with various symmetries and invariance properties of the underlying theory. Gravity experiments can be analyzed in terms of the PPN metric, and an ensemble of experiments will determine the unique value for these parameters, and hence the metric field itself.

As we know it today, observationally, GR is the most successful theory so far as solar system experiments are concerned (see e.g. [35] for an updated review). The implications of GR for solar system gravitational phenomena are best addressed via the PPN formalism for which the metric tensor of the general Riemannian spacetime is generated by some given distribution of matter in the form of an ideal fluid, given by Eq. (1). It is represented by a sum of gravitational potentials with arbitrary coefficients, the PPN parameters. If, for simplicity, one assumes that Lorentz invariance, local position invariance and total momentum conservation

¹A powerful approach developing a weak-field approximation for GR was presented in Refs. [27, 28, 29]. It combines an elegant ‘‘Maxwell-like’’ treatise of the spacetime metric in both the *global* and *local* reference frames with the Blanchet-Damour multipole formalism [30]. This approach is applicable for an arbitrary energy-stress tensor and is suitable for addressing problems of strong field regime. Application of this method to a general N-body problem in a weak-field and slow motion approximation was developed in Ref. [31].

hold, the metric tensor in four dimensions in the so-called PPN-gauge² as

$$\begin{aligned} g_{00} &= -1 + 2U - 2\beta U^2 + 2(\gamma + 1)\Phi_1 + 2\left[(3\gamma + 1 - 2\beta)\Phi_2 + \Phi_3 + 3\gamma\Phi_4\right] + \mathcal{O}(c^{-5}), \\ g_{0i} &= -\frac{1}{2}(4\gamma + 3)V_i - \frac{1}{2}W_i + \mathcal{O}(c^{-5}), \quad g_{ij} = \delta_{ij}(1 + 2\gamma U) + \mathcal{O}(c^{-4}). \end{aligned} \quad (2)$$

The order of magnitude of the various terms is determined according to the rules $U \sim v^2 \sim \Pi \sim p/\rho \sim \epsilon$, $v^i \sim |d/dt|/|d/dx| \sim \epsilon^{1/2}$. The parameter γ represents the measure of the curvature of the spacetime created by the unit rest mass; the parameter β is the measure of the non-linearity of the law of superposition of the gravitational fields in a theory of gravity or the measure of the metricity. The generalized gravitational potentials, proportional to U^2 , result from integrating the energy-stress density, Eq. (1), are given by

$$U(\mathbf{x}, t) = \int d^3\mathbf{x}' \frac{\rho_0(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|}, \quad V^\alpha(\mathbf{x}, t) = - \int d^3\mathbf{x}' \frac{\rho_0(\mathbf{x}', t)v^\alpha(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|}, \quad (3)$$

$$W^i(\mathbf{x}, t) = \int d^3\mathbf{x}' \rho_0(\mathbf{x}', t)v_j(\mathbf{x}', t) \frac{(x^j - x'^j)(x^i - x'^i)}{|\mathbf{x} - \mathbf{x}'|^3}, \quad (4)$$

$$\Phi_1(\mathbf{x}, t) = - \int d^3\mathbf{x}' \frac{\rho_0(\mathbf{x}', t)v^2(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|}, \quad \Phi_2(\mathbf{x}', t) = \int d^3\mathbf{x}' \frac{\rho_0(\mathbf{x}', t)U(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|}, \quad (5)$$

$$\Phi_3(\mathbf{x}, t) = \int d^3\mathbf{x}' \frac{\rho_0(\mathbf{x}', t)\Pi(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|} d^3z'^\nu, \quad \Phi_4(\mathbf{x}, t) = \int d^3\mathbf{x}' \frac{p(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|}. \quad (6)$$

In the complete PPN framework, a particular metric theory of gravity in the PPN formalism with a specific coordinate gauge might be fully characterized by means of ten PPN parameters [26, 36]. Thus, besides the parameters γ, β , there other eight parameters $\alpha_1, \alpha_2, \alpha_3, \zeta, \zeta_1, \zeta_2, \zeta_3, \zeta_4$. The formalism uniquely prescribes the values of these parameters for each particular theory under study. In the standard PPN gauge [26] these parameters have clear physical meaning, each quantifying a particular symmetry, conservation law or fundamental tenant of the structure of spacetime. Thus, in addition to the parameters γ and β discussed above, the group of parameters $\alpha_1, \alpha_2, \alpha_3$ specify the violation of Lorentz invariance (or the presence of the privileged reference frame), the parameter ζ quantifies the violation of the local position invariance, and, finally, the parameters $\zeta_1, \zeta_2, \zeta_3, \zeta_4$ reflect the violation of the law of total momentum conservation for a closed gravitating system. Note that GR, when analyzed in standard PPN gauge, gives: $\gamma = \beta = 1$ and all the other eight parameters vanish. The Brans-Dicke theory [37] is the best known of the alternative theories of gravity. It contains, besides the metric tensor, a scalar field and an arbitrary coupling constant ω , which yields the two PPN parameter values, $\beta = 1$, $\gamma = (1 + \omega)/(2 + \omega)$, where ω is an unknown dimensionless parameter of this theory. More general scalar tensor theories (see Section 3.2) yield values of β different from one [38].

The main properties of the PPN metric tensor given by Eqs. (3)-(6) are well established and widely in use in modern astronomical practice [39, 40, 36, 25, 41, 26]. For practical purposes one uses this metric to generate the equations of motion for the bodies of interest. These equations are then used to produce numerical codes in relativistic orbit determination formalisms for planets and satellites [40, 41, 36] as well as for analyzing the gravitational experiments in the solar system [26, 42].

In what follows, we discuss the foundations of general theory of relativity together with our current empirical knowledge on their validity. We take the standard approach to GR according to which the theory is supported by the following basic tenants:

- 1). Equivalence Principle (EP), which states that freely falling bodies do have the same acceleration in the same gravitational field independent on their compositions, which is also known as the principle of universality of the free fall (discussed in Section 2.2);
- 2). Local Lorentz invariance (LLI), which suggests that clock rates are independent on the clock's velocities (discussed in Section 2.3);

²Note the geometrical units $= c = G = 1$ are used throughout, as is the metric signature convention $(- + + +)$.

- 3). Local position invariance (LPI), which postulates that clocks rates are also independent on their spacetime positions (discussed in Section 2.4).

2.2 The Equivalence Principle (EP)

Since Newton, the question about the equality of inertial and passive gravitational masses has risen in almost every theory of gravitation. Thus, almost one hundred years ago Einstein postulated that not only mechanical laws of motion, but also all non-gravitational laws should behave in freely falling frames as if gravity was absent. It is this principle that predicts identical accelerations of compositionally different objects in the same gravitational field, and also allows gravity to be viewed as a geometrical property of spacetime—leading to the general relativistic interpretation of gravitation.

Below we shall discuss two different “flavors” of the Equivalence Principle, the weak and the strong forms of the EP that are currently tested in various experiments performed with laboratory tests masses and with bodies of astronomical sizes.

2.2.1 The Weak Equivalence Principle (WEP)

The weak form of the EP (the WEP) states that the gravitational properties of strong and electro-weak interactions obey the EP. In this case the relevant test-body differences are their fractional nuclear-binding differences, their neutron-to-proton ratios, their atomic charges, *etc.*. Furthermore, the equality of gravitational and inertial masses implies that different neutral massive test bodies will have the same free fall acceleration in an external gravitational field, and therefore in freely falling inertial frames the external gravitational field appears only in the form of a tidal interaction [43]. Apart from these tidal corrections, freely falling bodies behave as if external gravity is absent [44].

According to GR, the light rays propagating near a gravitating body are achronically scattered by the curvature of the spacetime generated by the body’s gravity field. The entire trajectory of the light ray is bent towards the body by an angle depending on the strength of the body’s gravity. In the solar system, the sun’s gravity field produces the largest effect, deflecting the light by as much as $1.75'' \cdot (R_{\odot}/b)$, where R_{\odot} is the solar radius and b is the impact parameter. The Eddington’s 1919 experiment confirmed the fact that photons obey the laws of free fall in a gravitational field as predicted by GR. The original accuracy was only 10% which was recently improved to 0.0023% by a solar conjunction experiment performed with the Cassini spacecraft [15].

The Pound-Rebka experiment, performed in 1960, further verified effects of gravity on light by testing the universality of gravity-induced frequency shift, $\Delta\nu$, that follows from the WEP:

$$\frac{\Delta\nu}{\nu} = \frac{gh}{c^2} = (2.57 \pm 0.26) \times 10^{-15}, \quad (7)$$

where g is the acceleration of gravity and h the height of fall [45].

The WEP can be scrutinized by studying the free fall of antiprotons and antihydrogen, even though the experimental obstacles are considerable; the subject has been extensively reviewed in Ref. [46]. This would help investigating to what extent does gravity respect the fundamental CPT symmetry of local quantum field theories, namely if antiparticles fall as particles in a gravitational field. As we shall see later, CPT symmetry may be spontaneously broken in some string/M-theory vacua’s; some implications of this will also be mentioned in the context of the validity Local Lorentz invariance. The ATHENA (ApparaTus for High precision Experiments on Neutral Antimatter) and the ATRAP collaborations at CERN have developed techniques to deal with the difficulties of storing antiprotons and creating an antihydrogen atom (see Refs. [47, 48] for recent accounts), but no gravitational has been performed so far. On the other hand, the former CPLEAR Collaboration has reported on a test of the WEP involving neutral kaons [49], with limits of 6.5, 4.3 and 1.8×10^{-9} respectively for scalar, vector and tensor potentials originating from the sun with a range much greater than 1 AU acting on kaons and antikaons. Despite their relevance, these results say nothing about new forces that couple to the

baryon number, and therefore are at best complementary to further tests yet to be performed with antiprotons and antihydrogen atoms.

Most extensions to GR are metric in nature, that is, they assume that the WEP is valid. However, as emphasized by [50, 51], almost all extensions to the standard model of particle physics generically predict new forces that would show up as apparent violations of the EP; this occurs specially in theories containing macroscopic-range quantum fields and thus predicting quantum exchange forces that generically violate the WEP, as they couple to generalized “charges”, rather than to mass/energy as does gravity [52].

In a laboratory, precise tests of the EP can be made by comparing the free fall accelerations, a_1 and a_2 , of different test bodies. When the bodies are at the same distance from the source of the gravity, the expression for the equivalence principle takes the elegant form

$$\frac{\Delta a}{a} = \frac{2(a_1 - a_2)}{a_1 + a_2} = \left(\frac{M_G}{M_I}\right)_1 - \left(\frac{M_G}{M_I}\right)_2 = \Delta \left(\frac{M_G}{M_I}\right), \quad (8)$$

where M_G and M_I are the gravitational and inertial masses of each body. The sensitivity of the EP test is determined by the precision of the differential acceleration measurement divided by the degree to which the test bodies differ (*e.g.* composition).

The WEP has been subject to various laboratory tests throughout the years. In 1975, Collela, Overhauser and Werner [53] showed with their interferometric experiment that a neutron beam split by a silicon crystal traveling through distinct gravitational paths interferes as predicted by the laws of quantum mechanics, with a gravitational potential given by Newtonian gravity, thus enabling an impressive verification of the WEP applied to an elementary hadron. Present-day technology has achieved impressive limits for the interferometry of atoms rising against gravity, of order 3×10^{-8} [54].

Various experiments have been performed to measure the ratios of gravitational to inertial masses of bodies. Recent experiments on bodies of laboratory dimensions verify the WEP to a fractional precision $\Delta(M_G/M_I) \lesssim 10^{-11}$ by [55], to $\lesssim 10^{-12}$ by [56, 57] and more recently to a precision of $\lesssim 1.4 \times 10^{-13}$ [58]. The accuracy of these experiments is sufficiently high to confirm that the strong, weak, and electromagnetic interactions each contribute equally to the passive gravitational and inertial masses of the laboratory bodies. A review of the most recent laboratory tests of gravity can be found in Ref. [59].

Quite recently, Nesvizhevsky and collaborators have reported evidence for the existence of gravitational bound states of neutrons [60]; the experiment was, at least conceptually, put forward long ago, in 1978 [61]. Subsequent steps towards the final experiment are described in Ref. [62]. This consists in allowing ultracold neutrons from a source at the Institute Laue-Langevin reactor in Grenoble to fall towards a horizontal mirror under the influence of the Earth’s gravitational field. This potential confines the motion of the neutrons, which do not move continuously vertically, but rather jump from one height to another as predicted by quantum mechanics. It is reported that the minimum measurable energy is of 1.4×10^{-12} eV, corresponding to a vertical velocity of 1.7 cm/s. A more intense beam and an enclosure mirrored on all sides could lead to an energy resolution down to 10^{-18} eV.

We remark that this experiment opens fascinating perspectives, both for testing non-commutative versions of quantum mechanics, as well as the connection of this theory with gravity [63]. It also enables a new criteria for the understanding the conditions for distinguishing quantum from classical behavior in function of the size of an observed system [64].

This impressive evidence of the WEP for laboratory bodies is incomplete for astronomical body scales. The experiments searching for WEP violations are conducted in laboratory environments that utilize test masses with negligible amounts of gravitational self-energy and therefore a large scale experiment is needed to test the postulated equality of gravitational self-energy contributions to the inertial and passive gravitational masses of the bodies [32]. Recent analysis of the lunar laser ranging data demonstrated that no composition-dependent acceleration effects [65] are present.

Once the self-gravity of the test bodies is non-negligible (currently with bodies of astronomical sizes only), the corresponding experiment will be testing the ultimate version of the EP – the strong equivalence principle,

that is discussed below.

2.2.2 The Strong Equivalence Principle (SEP)

In its strong form the EP is extended to cover the gravitational properties resulting from gravitational energy itself. In other words, it is an assumption about the way that gravity begets gravity, i.e. about the non-linear property of gravitation. Although GR assumes that the SEP is exact, alternate metric theories of gravity such as those involving scalar fields, and other extensions of gravity theory, typically violate the SEP [32, 66, 8, 67]. For the SEP case, the relevant test body differences are the fractional contributions to their masses by gravitational self-energy. Because of the extreme weakness of gravity, SEP test bodies that differ significantly must have astronomical sizes. Currently, the Earth-Moon-Sun system provides the best solar system arena for testing the SEP.

A wide class of metric theories of gravity are described by the parameterized post-Newtonian formalism [66, 33, 34], which allows one to describe within a common framework the motion of celestial bodies in external gravitational fields. Over the last 35 years, the PPN formalism has become a useful framework for testing the SEP for extended bodies. To facilitate investigation of a possible violation of the SEP, in that formalism the ratio between gravitational and inertial masses, M_G/M_I , is expressed [32, 66] as

$$\left[\frac{M_G}{M_I} \right]_{\text{SEP}} = 1 + \eta \left(\frac{\Omega}{Mc^2} \right), \quad (9)$$

where M is the mass of a body, Ω is the body's (negative) gravitational self-energy, Mc^2 is its total mass-energy, and η is a dimensionless constant for SEP violation [32, 66, 8]. Any SEP violation is quantified by the parameter η : in fully-conservative, Lorentz-invariant theories of gravity [26, 68] the SEP parameter is related to the PPN parameters by $\eta = 4\beta - \gamma - 3$. In GR $\gamma = \beta = 1$, so that $\eta = 0$.

The self energy of a body B is given by

$$\left(\frac{\Omega}{Mc^2} \right)_B = -\frac{G}{2M_B c^2} \int_B d^3\mathbf{x} d^3\mathbf{y} \frac{\rho_B(\mathbf{x})\rho_B(\mathbf{y})}{|\mathbf{x} - \mathbf{y}|}. \quad (10)$$

For a sphere with a radius R and uniform density, $\Omega/Mc^2 = -3GM/5Rc^2 = -3v_E^2/10c^2$, where v_E is the escape velocity. Accurate evaluation for solar system bodies requires numerical integration of the expression of Eq. (10). Evaluating the standard solar model [69] results in $(\Omega/Mc^2)_\odot \sim -3.52 \times 10^{-6}$. Because gravitational self-energy is proportional to M^2 and also because of the extreme weakness of gravity, the typical values for the ratio (Ω/Mc^2) are $\sim 10^{-25}$ for bodies of laboratory sizes. Therefore, the experimental accuracy of a part in 10^{13} [58] which is so useful for the WEP is not sufficient to test on how gravitational self-energy contributes to the inertial and gravitational masses of small bodies. To test the SEP one must consider planetary-sized extended bodies, where the ratio Eq. (10) is considerably higher.

Nordtvedt [32, 8, 70] suggested several solar system experiments for testing the SEP. One of these was the lunar test. Another, a search for the SEP effect in the motion of the Trojan asteroids, was carried out by [71, 72]. Interplanetary spacecraft tests have been considered by [44] and discussed [73]. An experiment employing existing binary pulsar data has been proposed [74]. It was pointed out that binary pulsars may provide an excellent possibility for testing the SEP in the new regime of strong self-gravity [18, 19], however the corresponding tests have yet to reach competitive accuracy [75, 76].

To date, the Earth-Moon-Sun system has provided the most accurate test of the SEP; recent analysis of LLR data test the EP to a high precision, yielding $\Delta(M_G/M_I)_{\text{EP}} = (-1.0 \pm 1.4) \times 10^{-13}$ [14]. This result corresponds to a test of the SEP of $\Delta(M_G/M_I)_{\text{SEP}} = (-2.0 \pm 2.0) \times 10^{-13}$ with the SEP violation parameter $\eta = 4\beta - \gamma - 3$ found to be $\eta = (4.4 \pm 4.5) \times 10^{-4}$. Using the recent Cassini result for the PPN parameter γ , PPN parameter β is determined at the level of $\beta - 1 = (1.2 \pm 1.1) \times 10^{-4}$ (see more details in [14]).

2.3 Local Lorentz Invariance (LLI)

Invariance under Lorentz transformations states that the laws of physics are independent of the frame velocity; this is an underlying symmetry of all current physical theories. However, some evidence recently found in the context of string field theory indicates that this symmetry can be spontaneously broken. Naturally, the experimental verification of this breaking poses a significant challenge. It has already been pointed out that astrophysical observations of distant sources of gamma radiation could hint what is the nature of gravity-induced wave dispersion in vacuum [77, 78] and therefore points towards physics beyond the Standard Model of Particles and Fields (hereafter – Standard Model). Limits on Lorentz symmetry violation based on the observations of high-energy cosmic rays with energies beyond 5×10^{19} eV, the so-called Greisen-Zatsepin-Kuzmin (GKZ) cut-off [79], have also been discussed [80, 81, 82, 83].

A putative violation of Lorentz symmetry has been a repeated object of interest in the literature. A physical description of the effect of our velocity with respect to a presumably preferred frame of reference relies on a constant background cosmological vector field, as suggested in [84]. Based on the behavior of the renormalization group β -function of non-abelian gauge theories, it has also been argued that Lorentz invariance could be just a low-energy symmetry [85].

Lorentz symmetry breaking due to non-trivial solutions of string field theory was first discussed in Ref. [86]. These arise from the string field theory of open strings and may have implications for low-energy physics. For instance, assuming that the contribution of Lorentz-violating interactions to the vacuum energy is about half of the critical density implies that feeble tensor-mediated interactions in the range of $\sim 10^{-4}$ m should exist [87]. Furthermore, Lorentz violation may induce the breaking of conformal symmetry; this, together with inflation may explain the origin of the primordial magnetic fields required to explain the observed galactic magnetic field [88]. Also, violations of Lorentz invariance may imply in a breaking of the fundamental CPT symmetry of local quantum field theories [89]. Quite remarkably, this can be experimentally verified in neutral-meson [90] experiments,³ Penning-trap measurements [92] and hydrogen-antihydrogen spectroscopy [93]. This spontaneous breaking of CPT symmetry allows for an explanation of the baryon asymmetry of the Universe: in the early Universe, after the breaking of the Lorentz and CPT symmetries, tensor-fermion interactions in the low-energy limit of string field theories give rise to a chemical potential that creates in equilibrium a baryon-antibaryon asymmetry in the presence of baryon number violating interactions [94].

Limits on the violation of Lorentz symmetry are available from laser interferometric versions of the Michelson-Morley experiment, by comparing the velocity of light, c and the maximum attainable velocity of massive particles, c_i , up to $\delta \equiv |c^2/c_i^2 - 1| < 10^{-9}$ [95]. More accurate tests can be performed via the Hughes-Drever experiment [96, 97], where one searches for a time dependence of the quadrupole splitting of nuclear Zeeman levels along Earth's orbit. This technique achieves an impressive limit of $\delta < 3 \times 10^{-22}$ [98]. A recent reassessment of these results reveals that more stringent bounds can be reached, up to 8 orders of magnitude higher [99]. The parameterized post-Newtonian parameter α_3 can be used to set astrophysical limits on the violation of momentum conservation and the existence of a preferred reference frame. This parameter, which vanishes identically in GR can be accurately determined from the pulse period of pulsars and millisecond pulsars [35, 100]. The most recent results yields a limit on the PPN parameter α_3 of $|\alpha_3| < 2.2 \times 10^{-20}$ [101].

After the cosmic microwave background radiation (CMBR) has been discovered, an analysis of the interaction between the most energetic cosmic-ray particles and the microwave photons was mandatory. As it turns out, the propagation of the ultra-high-energy nucleons is limited by inelastic collisions with photons of the CMBR, preventing nucleons with energies above 5×10^{19} eV from reaching Earth from further than 50–100 Mpc. This is the already mentioned GZK cut-off [79]. However, events where the estimated energy of the cosmic primaries is beyond the GZK cut-off were observed by different collaborations [102, 103, 104, 105]. It has been suggested [80, 81] (see also [82]) that slight violations of Lorentz invariance would cause energy-dependent effects that would suppress otherwise inevitable processes such as the resonant scattering reaction, $p + \gamma_{2.73K} \rightarrow \Delta_{1232}$. The study of the kinematics of this process produces a quite stringent constraint on the validity of Lorentz invariance, $\delta < 1.7 \times 10^{-25}$ [83, 106].

³These CPT violating effects are unrelated with those due to possible non-linearities in quantum mechanics, presumably arising from quantum gravity and already investigated by the CPLEAR Collaboration [91].

Quite recently, the High Resolution Fly’s Eye collaboration suggested that the gathered data show that the GZK cutoff holds for their span of observations [107]. Confirmation of this result is of great importance, and the coming into operation of the Auger collaboration [108] in the near future will undoubtedly provide further insight into this fundamental question. It is also worth mentioning that the breaking of Lorentz invariance can occur in the context of non-commutative field theories [109], even though this symmetry may hold (at least) at first non-trivial order in perturbation theory of the non-commutative parameter [110]. Actually, the idea that the non-commutative parameter may be a Lorentz tensor has been considered in some field theory models [111]. Also, Lorentz symmetry can be broken in loop quantum gravity [112], quantum gravity inspired spacetime foam scenarios [113] or via the spacetime variation of fundamental coupling constants [114]. For a fairly complete review about Lorentz violation at high-energies the reader is directed to Ref. [115]. Note that a gravity model where LLI is spontaneously broken was proposed in Ref. [116, 117] and solutions were discussed in Ref. [118].

2.3.1 Spontaneous Violation of Lorentz Invariance

The impact of a spontaneous violation of Lorentz invariance on theories of gravity is of great interest. In this context, the breaking of Lorentz invariance may be implemented, for instance, by allowing a vector field to roll to its vacuum expectation value. This mechanism requires that the potential that rules the dynamics of the vector field possesses a minimum, in the way similar to the Higgs mechanism [116]. This, so-called, “bumblebee” vector thus acquires an explicit (four-dimensional) orientation, and Lorentz symmetry is spontaneously broken. The action of the bumblebee model is written as

$$S = \int d^4x \left[\frac{\sqrt{-g}}{2} (R + \xi B^\mu B^\nu R_{\mu\nu}) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - V(B^\mu B_\mu \pm b^2) \right], \quad (11)$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, ξ and b^2 are a real coupling constant and a free real positive constant, respectively. The potential V driving Lorentz and/or CPT violation is supposed to have a minimum at $B^\mu B_\mu \pm b^2 = 0$, $V'(b_\mu b^\mu) = 0$. Since one assumes that the bumblebee field B_μ is frozen at its vacuum expectation value, the particular form of the potential driving its dynamics is irrelevant. The scale of b_μ , which controls the symmetry breaking, must be derived from a fundamental theory, such as string theory or from a low-energy extension to the Standard Model; hence one expects either b of order of the Planck mass, $M_P = 1.2 \times 10^{19}$ GeV, or of order of the electroweak breaking scale, $M_{EW} \simeq 10^2$ GeV.

Previously, efforts to quantify an hypothetical breaking of Lorentz invariance were primarily directed towards the phenomenology of such spontaneous Lorentz symmetry breaking in particle physics. Only recently its implications for gravity have been more thoroughly explored [116, 117]. In that work, the framework for the LSB gravity model is set up, developing the action and using the *vielbein* formalism. A later study [118] considered consequences of such a scenario, assuming three plausible cases: i) the bumblebee field acquires a purely radial vacuum expectation value, ii) a mixed radial and temporal vacuum expectation value and iii) a mixed axial and temporal vacuum expectation value.

In the first case, an exact black hole solution was found, exhibiting a deviation from the inverse square law such that the gravitational potential of a point mass at rest depends on the radial coordinate as r^{-1+p} where p is a parameter related to the fundamental physics underlying the breaking of Lorentz invariance. This solution has a removable singularity at a horizon of radius $r_s = (2Mr_0^{-p})^{1/(1-p)}$, slightly perturbed with respect to the usual Schwarzschild radius $r_{s0} = 2M$, which protects a real singularity at $r = 0$. Known deviations from Kepler’s law yield $p \leq 2 \times 10^{-9}$.

In the second case, no exact solutions was discovered, although a perturbative method allowed for the characterization of the Lorentz symmetry breaking in terms of the PPN parameters $\beta \approx 1 - (K + K_r)/M$ and $\gamma \approx 1 - (K + 2K_r)/M$, directly proportional to the strength of the induced effect, given by K and $K_r \sim K$, where K and K_r are integration constants arising from the perturbative treatment of the timelike spontaneous LSB superimposed on the vacuum Schwarzschild metric. An analogy with Rosen’s bimetric theory yields the parameter $\gamma \simeq (A + B)d$, where d is the distance to the central body and A and B rule the temporal and radial components of the vector field vacuum expectation value.

Table 1: The accuracy of determination of the PPN parameters γ and β [14, 35, 42].

PPN parameter	Experiment	Result
$\gamma - 1$	Cassini 2003 spacecraft radio-tracking	2.3×10^{-5}
	Observations of quasars with Astrometric VLBI	3×10^{-4}
$\beta - 1$	Helioseismology bound on perihelion shift	3×10^{-3}
	LLR test of the SEP, assumed: $\eta = 4\beta - \gamma - 3$ and the Cassini result for PPN γ	1.1×10^{-4}

In the third case, a temporal/axial vacuum expectation value for the bumblebee vector field clearly breaks isotropy, thus forbidding a standard PPN analysis. However, for the case of the breaking of Lorentz invariance occurring in the x_1 direction, similar direction-dependent PPN-like parameters were defined as $\gamma_1 \simeq b^2 \cos^2 \theta/2$ and $\gamma_2 \simeq a^2 b^2 \cos^2 \theta/4$, where a and b are proportional respectively to the temporal and axial components of the vacuum expectation value acquired by the bumblebee. This enables a crude estimative of the measurable PPN parameter γ , yielding $\gamma \approx b^2(1 - e^2)/4$, where e is the orbit's eccentricity. A comparison with experiments concerning the anisotropy of inertia yields $|b| \leq 2.4 \times 10^{-11}$ [98].

2.4 Local Position Invariance (LPI)

Given that both the WEP and LLI postulates have been tested with great accuracy, experiments concerning the universality of the gravitational red-shift measure the level to which the LPI holds. Therefore, violations of the LPI would imply that the rate of a free falling clock would be different when compared with a standard one, for instance on the Earth's surface. The accuracy to which the LPI holds as an invariance of Nature can be parameterized through $\Delta\nu/\nu = (1 + \mu)U/c^2$. From the already mentioned Pound-Rebka experiment (cf. Eq. (7)) one can infer that $\mu \simeq 10^{-2}$; the most accurate verification of the LPI was performed by Vessot and collaborators, by comparing hydrogen-maser frequencies on Earth and on a rocket flying to altitude of 10^4 km [119]. The resulting bound is $|\mu| < 2 \times 10^{-4}$. Recently, an one order of magnitude improvement was attained, thus establishing the most stringent result on the LPI so far [120], $|\mu| < 2.1 \times 10^{-5}$.

2.5 Summary of Solar System Tests of Relativistic Gravity

Although, these available experimental data fit quite well with GR, while allowing for the existence of putative extensions of this successful theory, provided any new effects are small at the post-Newtonian scale [26]. We shall here discuss the available phenomenological constraints for alternative theories of gravity.

Lunar Laser Ranging (LLR) investigates the SEP by looking for a displacement of the lunar orbit along the direction to the sun. The equivalence principle can be split into two parts: the WEP tests the sensitivity to composition and the SEP checks the dependence on mass. There are laboratory investigations of the WEP which are about as accurate as LLR [65, 58]. LLR is the dominant test of the SEP with the most accurate testing of the EP at the level of $\Delta(M_G/M_I)_{EP} = (-1.0 \pm 1.4) \times 10^{-13}$ [14]. This result corresponds to a test of the SEP of $\Delta(M_G/M_I)_{SEP} = (-2.0 \pm 2.0) \times 10^{-13}$ with the SEP violation parameter $\eta = 4\beta - \gamma - 3$ found to be $\eta = (4.4 \pm 4.5) \times 10^{-4}$. Using the recent Cassini result for the PPN parameter γ , PPN parameter β is determined at the level of $\beta - 1 = (1.2 \pm 1.1) \times 10^{-4}$ (see Figure 1).

LLR data yielded the strongest limits to date on variability of the gravitational constant (the way gravity is affected by the expansion of the Universe), the best measurement of the de Sitter precession rate, and is relied upon to generate accurate astronomical ephemerides. The possibility of a time variation of the gravitational constant, G , was first considered by Dirac in 1938 on the basis of his large number hypothesis, and later developed by Brans and Dicke in their theory of gravitation (for more details consult [26, 68]). Variation might be related to the expansion of the Universe, in which case $\dot{G}/G = \sigma H_0$, where H_0 is the Hubble constant, and

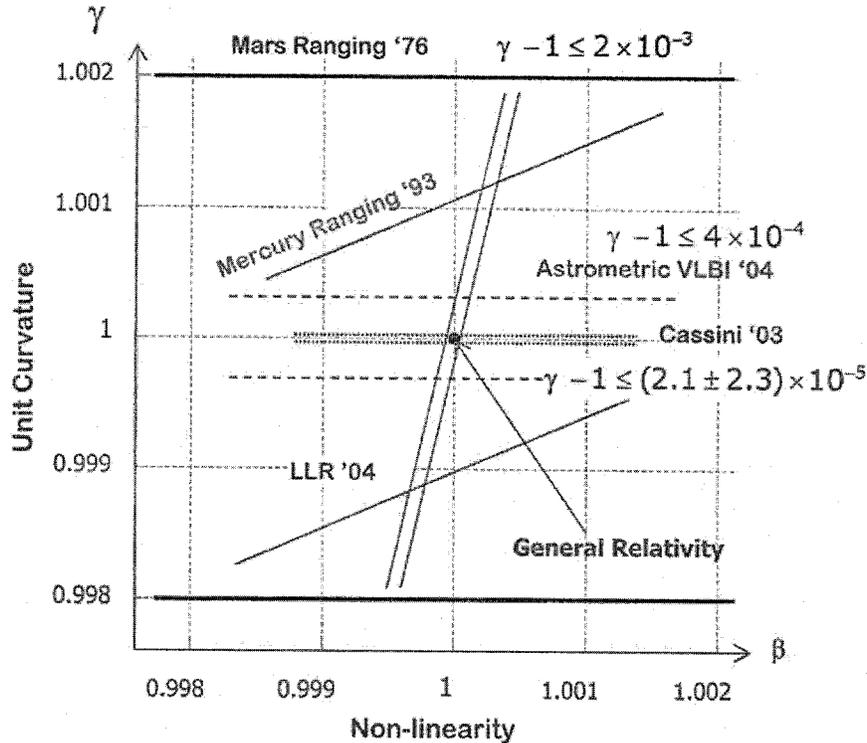


Figure 1: The progress in determining the PPN parameters γ and β for the last 30 years (adopted from [121]).

σ is a dimensionless parameter whose value depends on both the gravitational constant and the cosmological model considered. Revival of interest in Brans-Dicke-like theories, with a variable G , was partially motivated by the appearance of string theories where G is considered to be a dynamical quantity [122].

In Brans-Dicke theory, as well as in more general scalar-tensor theories, the gravitational coupling depends on the cosmic time. Observational bounds arising from the timing of the binary pulsar PSR1913+16 yield quite restrictive bounds [123] of $\dot{G}/G = (1.0 \pm 2.3) \times 10^{-11} \text{ yr}^{-1}$, with a magnitude similar to the cosmological bounds available [124, 125, 126] (see Ref. [127] and references therein for a discussion on a connection with the accelerated expansion of the Universe). Varying- G solar models [128] and measurements of masses and ages of neutron stars yield even more stringent limits [129] of $\dot{G}/G = (-0.6 \pm 2.0) \times 10^{-12} \text{ yr}^{-1}$.

The most stringent limit on a change of G comes from LLR, which is one of the important gravitational physics result that LLR provides. GR does not predict a changing G , but some other theories do, thus testing for this effect is important. As we have seen, the most accurate limit published is the current LLR test, yielding $\dot{G}/G = (4 \pm 9) \times 10^{-13} \text{ yr}^{-1}$ [14]. The \dot{G}/G uncertainty is 83 times smaller than the inverse age of the Universe, $t_0 = 13.4 \text{ Gyr}$ with the value for Hubble constant $H_0 = 72 \text{ km/sec/Mpc}$ from the WMAP data [130]. The uncertainty for \dot{G}/G is improving rapidly because its sensitivity depends on the square of the data span. This fact puts LLR, with its more than 35 years of history, in a clear advantage as opposed to other experiments.

LLR has also provided the only accurate determination of the geodetic precession. Ref. [14] reports a test of geodetic precession, which expressed as a relative deviation from GR, is $K_{gp} = -0.0019 \pm 0.0064$. The GP-B satellite should provide improved accuracy over this value, if that mission is successfully completed. LLR also has the capability of determining PPN β and γ directly from the point-mass orbit perturbations. A future possibility is detection of the solar J_2 from LLR data combined with the planetary ranging data. Also possible are dark matter tests, looking for any departure from the inverse square law of gravity, and checking for a variation of the speed of light. The accurate LLR data has been able to quickly eliminate several suggested alterations of physical laws. The precisely measured lunar motion is a reality that any proposed laws of attraction and motion must satisfy.

3 Search for New Physics Beyond General Relativity

The nature of gravity is fundamental to the understanding of the solar system and the large scale structure of the Universe. This importance motivates various precision tests of gravity both in laboratories and in space. To date, the experimental evidence for gravitational physics is in agreement with GR; however, there are a number of reasons to question the validity of this theory. Despite the success of modern gauge field theories in describing the electromagnetic, weak, and strong interactions, it is still not understood how gravity should be described at the quantum level. In theories that attempt to include gravity, new long-range forces can arise in addition to the Newtonian inverse-square law. Even at the purely classical level, and assuming the validity of the Equivalence Principle, Einstein's theory does not provide the most general way to describe the space-time dynamics, as there are reasons to consider additional fields and, in particular, scalar fields.

Although scalar fields naturally appear in the modern theories, their inclusion predicts a non-Einsteinian behavior of gravitating systems. These deviations from GR lead to a violation of the EP, modification of large-scale gravitational phenomena, and imply that the constancy of the “constants” must be reconsidered. These predictions motivate searches for small deviations of relativistic gravity from GR and provide a theoretical paradigm and constructive guidance for further gravity experiments. As a result, this theoretical progress has motivated high precision tests of relativistic gravity and especially those searching for a possible violation of the Equivalence Principle. Moreover, because of the ever increasing practical significance of the general theory of relativity (i.e. its use in spacecraft navigation, time transfer, clock synchronization, standards of time, weight and length, *etc.*) this fundamental theory must be tested to increasing accuracy.

An understanding of gravity at a quantum level will allow one to ascertain whether the gravitational “constant” is a running coupling constant like those of other fundamental interactions of Nature. String/M-theory [131] hints a negative answer to this question, given the non-renormalization theorems of Supersymmetry, a symmetry at the core of the underlying principle of string/M-theory and brane models, [132, 133]. However, 1-loop higher-derivative quantum gravity models may permit a running gravitational coupling, as these models are asymptotically free, a striking property [134]. In the absence of a screening mechanism for gravity, asymptotic freedom may imply that quantum gravitational corrections take effect on macroscopic and even cosmological scales, which of course has some bearing on the dark matter problem [135] and, in particular, on the subject of the large scale structure of the Universe [136, 137] (see, however, [124]). Either way, it seems plausible to assume that quantum gravity effects manifest themselves only on cosmological scales.

In this Section we review the arguments for high-accuracy experiments motivated by the ideas suggested by proposals of quantization of gravity.

3.1 String/M-Theory

String theory is currently referred to as string/M-theory, given the unification of the existing string theories that is achieved in the context M-theory. Nowadays, it is widely viewed as the most promising scheme to make GR compatible with quantum mechanics (see [131] for an extensive presentation). The closed string theory has a spectrum that contains as zero mass eigenstates the graviton, g_{MN} , the dilaton, Φ , and an antisymmetric second-order tensor, B_{MN} . There are various ways in which to extract the physics of our four-dimensional world, and a major difficulty lies in finding a natural mechanism that fixes the value of the dilaton field, since it does not acquire a potential at any order in string perturbation theory.

Damour and Polyakov [52] have studied a possible a mechanism to circumvent the above difficulty by suggesting string loop-contributions, which are counted by dilaton interactions, instead of a potential. After dropping the antisymmetric second-order tensor and introducing fermions, $\hat{\psi}$, Yang-Mills fields, \hat{A}^μ , with field strength $\hat{F}_{\mu\nu}$, in a spacetime described by the metric $\hat{g}_{\mu\nu}$, the relevant effective low-energy four-dimensional action is

$$S = \int_M d^4x \sqrt{-\hat{g}} B(\Phi) \left[\frac{1}{\alpha'} [\hat{R} + 4\hat{\nabla}_\mu \hat{\nabla}^\mu \Phi - 4(\hat{\nabla}\Phi)^2] - \frac{k}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} - \bar{\hat{\psi}} \gamma^\mu \hat{D}_\mu \hat{\psi} + \dots \right], \quad (12)$$

where

$$B(\Phi) = e^{-2\Phi} + c_0 + c_1 e^{2\Phi} + c_2 e^{4\Phi} + \dots, \quad (13)$$

α' is the inverse of the string tension and k is a gauge group constant; the constants c_0, c_1, \dots , etc., can, in principle, be determined via computation.

In order to recover Einsteinian gravity, a conformal transformation must be performed with $g_{\mu\nu} = B(\Phi) \hat{g}_{\mu\nu}$, leading to an action where the coupling constants and masses are functions of the rescaled dilaton, ϕ :

$$S = \int_M d^4x \sqrt{-g} \left[\frac{1}{4q} R - \frac{1}{2q} (\nabla\phi)^2 - \frac{k}{4} B(\phi) F_{\mu\nu} F^{\mu\nu} - \bar{\psi} \gamma^\mu D_\mu \psi + \dots \right], \quad (14)$$

from which follows that $4q = 16\pi G = \frac{1}{4}\alpha'$ and the coupling constants and masses are now dilaton-dependent, through $g^{-2} = kB(\phi)$ and $m_A = m_A(B(\phi))$. Damour and Polyakov proposed the minimal coupling principle (MCP), stating that the dilaton is dynamically driven towards a local minimum of all masses (corresponding to a local maximum of $B(\phi)$). Due to the MCP, the dependence of the masses on the dilaton implies that particles fall differently in a gravitational field, and hence are in violation of the WEP. Although, in the solar system conditions, the effect is rather small being of the order of $\Delta a/a \simeq 10^{-18}$, application of already available technology can potentially test prediction. Verifying this prediction is an interesting prospect, as it would present a distinct experimental signature of string/M-theory. We have no doubts that the experimental search for violations of the WEP, as well as of the fundamental Lorentz and CPT symmetries, present important windows of opportunity to string physics and should be vigorously pursued.

Recent analysis of a potential scalar field's evolution scenario based on action (14) discovered that the present agreement between GR and experiment might be naturally compatible with the existence of a scalar contribution to gravity. In particular, Damour and Nordtvedt [38] (see also [52] for non-metric versions of this mechanism together with [138] for the recent summary of a dilaton-runaway scenario) have found that a scalar-tensor theory of gravity may contain a "built-in" cosmological attractor mechanism toward GR. These scenarios assume that the scalar coupling parameter $\frac{1}{2}(1-\gamma)$ was of order one in the early Universe (say, before inflation), and show that it then evolves to be close to, but not exactly equal to, zero at the present time.

The Eddington parameter γ , whose value in general relativity is unity, is perhaps the most fundamental PPN parameter, in that $\frac{1}{2}(1-\gamma)$ is a measure, for example, of the fractional strength of the scalar gravity interaction in scalar-tensor theories of gravity [18, 19]. Within perturbation theory for such theories, all other PPN parameters to all relativistic orders collapse to their general relativistic values in proportion to $\frac{1}{2}(1-\gamma)$. Under some assumptions (see *e.g.* [38]) one can even estimate what is the likely order of magnitude of the left-over coupling strength at present time which, depending on the total mass density of the Universe, can be given as $1-\gamma \sim 7.3 \times 10^{-7} (H_0/\Omega_0^3)^{1/2}$, where Ω_0 is the ratio of the current density to the closure density and H_0 is the Hubble constant in units of 100 km/sec/Mpc . Compared to the cosmological constant, these scalar field models are consistent with the supernovae observations for a lower matter density, $\Omega_0 \sim 0.2$, and a higher age, $(H_0 t_0) \approx 1$. If this is indeed the case, the level $(1-\gamma) \sim 10^{-6} - 10^{-7}$ would be the lower bound for the present value of the PPN parameter γ [38]. This is why measuring the parameter γ to accuracy of one part in a billion, as suggested for the LATOR mission [121], is important.

3.2 Scalar-Tensor Theories of Gravity

In many alternative theories of gravity, the gravitational coupling strength exhibits a dependence on a field of some sort; in scalar-tensor theories, this is a scalar field φ . A general action for these theories can be written

as

$$S = \frac{c^3}{4\pi G} \int d^4x \sqrt{-g} \left[\frac{1}{4} f(\varphi) R - \frac{1}{2} g(\varphi) \partial_\mu \varphi \partial^\mu \varphi + V(\varphi) \right] + \sum_i q_i(\varphi) \mathcal{L}_i, \quad (15)$$

where $f(\varphi)$, $g(\varphi)$, $V(\varphi)$ are generic functions, $q_i(\varphi)$ are coupling functions and \mathcal{L}_i is the Lagrangian density of the matter fields; it is worth mentioning that the graviton-dilaton system in string/M-theory can be viewed as one of such scalar-tensor theories of gravity. An emblematic proposal is the well-known Brans-Dicke theory [37] corresponds to the specific choice

$$f(\varphi) = \varphi, \quad g(\varphi) = \frac{\omega}{\varphi}, \quad (16)$$

and a vanishing potential $V(\varphi)$. Notice that in the Brans-Dicke theory the kinetic energy term of the field φ is non-canonical, and the latter has a dimension of energy squared. In this theory, the constant ω marks observational deviations from GR, which is recovered in the limit $\omega \rightarrow \infty$. We point out that, in the context of the Brans-Dicke theory, one can operationally introduce the Mach's Principle which, we recall, states that the inertia of bodies is due to their interaction with the matter distribution in the Universe. Indeed, in this theory the gravitational coupling is proportional to φ^{-1} , which depends on the energy-momentum tensor of matter through the field equations. Observational bounds require that $|\omega| \gtrsim 500$ [5, 2], and even higher values $|\omega| \gtrsim 40000$ are reported in [35]. In the so-called *induced gravity models* [139], the functions of the fields are initially given by $f(\varphi) = \varphi^2$ and $g(\varphi) = 1/2$, and the potential $V(\varphi)$ allows for a spontaneous symmetry breaking, so that the field φ acquires a non-vanishing vacuum expectation value, $f(\langle 0|\varphi|0\rangle) = \langle 0|\varphi^2|0\rangle = M_P^2 = G^{-1}$. Naturally the cosmological constant is given by the interplay of the value $V(\langle 0|\varphi|0\rangle)$ and all other contributions to the vacuum energy.

Therefore, it is clear that in this setup Newton's constant arises from dynamical or symmetry-breaking considerations. It is mesmerizing to conjecture that the φ field could be locally altered: this would require the coupling of this field with other fields, in order to locally modify its value. This feature can be found in some adjusting mechanisms devised as a solution of the cosmological constant problem (see *e.g.* [140] for a list of references). However, Weinberg [140] has shown that these mechanisms are actually unsuitable for this purpose, although they nevertheless contain interesting multi-field dynamics. Recent speculations suggesting that extra dimensions in braneworld scenarios may be rather large [141, 142] bring forth gravitational effects at the much lower scale set by M_5 , the 5-dimensional Planck mass. Phenomenologically, the existence of extra dimensions should manifest itself through a contribution to Newton's law on small scales, $r \lesssim 10^{-4} m$, as discussed next in Section 3.3.

3.3 Search for New Interactions of Nature

The existence of new fundamental forces beyond the already known four fundamental interactions, if confirmed, will have several implications and bring important insights into the physics beyond the Standard Model. A great interest on the subject was sparked after the 1986 claim of evidence for an intermediate range interaction with sub-gravitational strength [143], both theoretical (see [46] for a review) as well as experimental, giving rise to a wave of new setups, as well as repetitions of "classical" ones using state of the art technology.

In its simplest versions, a putative new interaction or a fifth force would arise from the exchange of a light boson coupled to matter with a strength comparable to gravity. Planck-scale physics could give origin to such an interaction in a variety of ways, thus yielding a Yukawa-type modification in the interaction energy between point-like masses. This new interaction can be derived, for instance, from extended supergravity theories after dimensional reduction [46, 144], compactification of 5-dimensional generalized Kaluza-Klein theories including gauge interactions at higher dimensions [145], and also from string/M-theory. In general, the interaction energy,

$V(r)$, between two point masses m_1 and m_2 can be expressed in terms of the gravitational interaction as⁴

$$V(r) = -\frac{G_\infty m_1 m_2}{r} (1 + \alpha e^{-r/\lambda}), \quad (17)$$

where $r = |\mathbf{r}_2 - \mathbf{r}_1|$ is the distance between the masses, G_∞ is the gravitational coupling for $r \rightarrow \infty$ and α and λ are respectively the strength and range of the new interaction. Naturally, G_∞ has to be identified with Newton's gravitational constant and the gravitational coupling becomes dependent on r . Indeed, the force associated with Eq. (17) is given by:

$$\mathbf{F}(\mathbf{r}) = -\nabla V(\mathbf{r}) = -\frac{G(r)m_1 m_2}{r^2} \hat{\mathbf{r}}, \quad (18)$$

where

$$G(r) = G_\infty [1 + \alpha (1 + r/\lambda) e^{-r/\lambda}]. \quad (19)$$

The suggestion of existence of a new interaction arose from assuming that the coupling α is not an universal constant, but instead a parameter depending on the chemical composition of the test masses [146]. This comes about if one considers that the new bosonic field couples to the baryon number $B = Z + N$, which is the sum of protons and neutrons. Hence the new interaction between masses with baryon numbers B_1 and B_2 can be expressed through a new fundamental constant, f , as:

$$V(r) = -f^2 \frac{B_1 B_2}{r} e^{-r/\lambda}, \quad (20)$$

such that the constant α can be written as

$$\alpha = -\sigma \left(\frac{B_1}{\mu_1} \right) \left(\frac{B_2}{\mu_2} \right), \quad (21)$$

with $\sigma = f^2/G_\infty m_H^2$ and $\mu_{1,2} = m_{1,2}/m_H$, m_H being the hydrogen mass.

The above equations imply that in a Galileo-type experiment a difference in acceleration exists between the masses m_1 and m_2 , given by

$$\mathbf{a}_{12} = \sigma \left(\frac{B}{\mu} \right)_\oplus \left[\left(\frac{B_1}{\mu_1} \right) - \left(\frac{B_2}{\mu_2} \right) \right] \mathbf{g}, \quad (22)$$

where \mathbf{g} is the field strength of the Earth's gravitational field.

Several experiments (see, for instance, Refs. [143, 46] for a list of the most relevant) studied the parameters of a new interaction based on the idea of a composition-dependence differential acceleration, as described in Eq. (22), and other composition-independent effect⁵. The current experimental status is essentially compatible with the predictions of Newtonian gravity, in both composition-independent or -dependent setups. The bounds on parameters α and λ are summarized below (Figure 2):

- Laboratory experiments devised to measure deviations from the inverse-square law are most sensitive in the range $10^{-2} \text{ m} \lesssim \lambda \lesssim 1 \text{ m}$, constraining α to be smaller than about 10^{-4} ;
- Gravimetric experiments sensitive in the range of $10 \text{ m} \lesssim \lambda \lesssim 10^3 \text{ m}$ indicate that $\alpha \lesssim 10^{-3}$;
- Satellite tests probe the ranges of about $10^5 \text{ m} \lesssim \lambda \lesssim 10^7 \text{ m}$ suggest that $\alpha \lesssim 10^{-7}$;
- Analysis of the effects of the inclusion of scalar fields into the stellar structure yields a bound in the range $10^8 \text{ m} \lesssim \lambda \lesssim 10^{10} \text{ m}$, limiting α to be smaller than approximately 10^{-2} [148].

The latter bound, although modest, is derived from a simple computation of the stellar equilibrium configuration in the polytropic gas approximation, when an extra force due to a Yukawa potential is taken into account on the hydrostatic equilibrium equation.

⁴We use here the units $c = 1$.

⁵For instance, neutron interferometry has been suggested to investigate a possible new force that couples to neutron number [147].

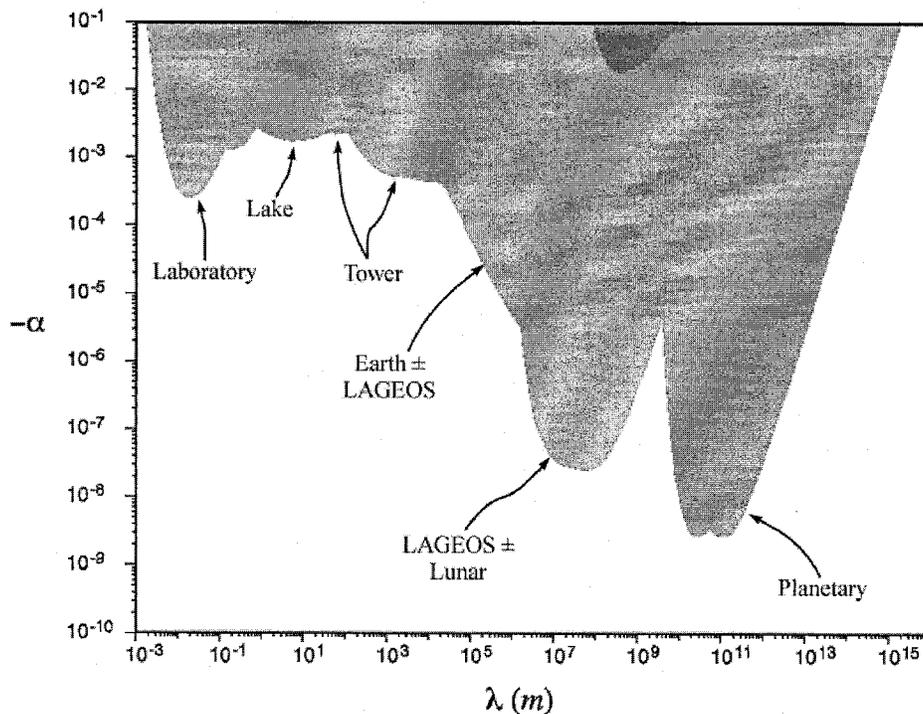


Figure 2: Experimentally excluded regions for the range and strength of possible new forces, as shown in Ref. [148].

Remarkably, α is so far essentially unconstrained for $\lambda < 10^{-3}$ m and $\lambda > 10^{13}$ m. The former range is particularly attractive as a testing ground for new interactions, since forces with sub-millimetric range seems to be favored from scalar interactions in supersymmetric theories [149]; this is also the case in the recently proposed theories of TeV scale quantum gravity, which stem from the hypothesis that extra dimensions are not necessarily of Planck size [141, 142]. The range $\lambda < 10^{-3}$ m also arises if one assumes that scalar [150] or tensor interactions associated with Lorentz symmetry breaking in string theories [87] account for the vacuum energy up to half of the critical density. Putative corrections to Newton's law at millimeter range could have relevant implications, especially taking into account that, in certain models of extra dimensions, these corrections can be as important as the usual Newtonian gravity [142, 151]. From the experimental side, this range has recently been available to experimental verification; state of the art experiments rule out extra dimensions over length scales down to 0.2 mm [152].

3.4 Gravity Shielding - the Majorana Effect

The possibility that matter can shield gravity is not predicted by modern theories of gravity, but it is a recurrent idea and it would cause a violation of the equivalence principle test. In fact, the topic has been more recently reviewed in [125] renewing the legitimacy of this controversial proposal; consequently, a brief discussion is given in this subsection.

The idea of gravity shielding goes back at least as far as to Majorana's 1920 paper [153]. Since then a number of proposals and studies has been put forward and performed to test the possible absorption of the gravitational force between two bodies when screened from each other by a medium other than vacuum. This effect is a clear gravitational analog of the magnetic permeability of materials, and Majorana [153] suggested the introduction of a screening or extinction coefficient, h , in order to measure the shielding of the gravitational force between masses m_1 and m_2 induced by a material with density $\rho(r)$; such an effect can be modeled as

$$F' = \frac{G m_1 m_2}{r^2} \exp \left[-h \int \rho(r) dr \right]. \quad (23)$$

which clearly depends on the amount of mass between attracting mass elements and a universal constant h . Naturally, one expects h to be quite small.

Several attempts to derive this parameter from general principles have been made. Majorana gave a closed form expression for a sphere's gravitational to inertial mass ratio. For weak shielding a simpler expression is given by the linear expansion of the exponential term, $M_G/M_I \approx 1 - hfR\bar{\rho}$, where f is a numerical factor, $\bar{\rho}$ is the mean density, and R is the sphere's radius. For a homogeneous sphere Majorana and Russell give $f = 3/4$. For a radial density distribution of the form $\rho(r) = \rho(0)(1 - r^2/R^2)^n$ Russell derives $f = (2n + 3)^2/(12n + 12)$. Russell [154] realized that the large masses of the Earth, Moon and planets made the observations of the orbits of these bodies and the asteroid Eros a good test of such a possibility. He made a rough estimate that the equivalence principle was satisfied to a few parts per million, which was much smaller than a numerical prediction based on Majorana's estimate for h . If mass shields gravity, then large bodies such as, for instance, the Moon and Earth will partly shield their own gravitational attraction. The observable ratio of gravitational mass to inertial mass would not be independent of mass, which would violate the equivalence principle.

Eckhardt [155] showed that LLR can be used to set the limit $h \leq 1.0 \times 10^{-22} \text{ m}^2 \text{ kg}^{-1}$, six orders of magnitude smaller than the geophysical constraint. In [155], an LLR test of the equivalence principle was used to set a modern limit on gravity shielding. That result is updated as follows: the uniform density approximation is sufficient for the Moon and $fR\bar{\rho} = 4.4 \times 10^9 \text{ kg m}^{-2}$. For the Earth we use $n \approx 0.8$ with Russell's expression to get $fR\bar{\rho} = 3.4 \times 10^{10} \text{ kg m}^{-2}$. Using the difference $-3.0 \times 10^9 \text{ g/cm}^2 h$ along with the LLR EP solution for the difference in gravitational to inertial mass ratios gives $h = (3 \pm 5) \times 10^{-24} \text{ m}^2 \text{ kg}^{-1}$ [156]. The value is not significant compared to the uncertainty. To give a sense of scale to the uncertainty, for the gravitational attraction to be diminished by 1/2 would require a column of matter with the density of water stretching at least half way from the solar system to the center of the galaxy. The LLR equivalence principle tests give no evidence that mass shields gravity and the limits are very strong.

For completeness, let us mention that Weber [157] argued that a quasi-static shielding could be predicted from a analysis of relativistic tidal phenomena, concluding that such effect should be extremely small. Finally, the most stringent laboratory limit on the gravitational shielding constant had been obtained during the recent measurement of Newton's constant, resulting in $h \leq 4.3 \times 10^{-15} \text{ m}^2 \text{ kg}^{-1}$ [158].

4 The "Dark Side" of Modern Physics

To a worldwide notice, recent cosmological observations dealt us a challenging puzzle forcing us to accept the fact that our current understanding of the origin and evolution of the Universe is incomplete. Surprisingly, it turns out that most of the energy content of the Universe is in the form of the presently unknown dark matter and dark energy that may likely permeate all of spacetime. It is possible that the underlying physics that resolve the discord between quantum mechanics and GR will also shed light on cosmological questions addressing the origin and ultimate destiny of the Universe.

In this Section we shall consider mechanisms that involve new physics beyond GR to explain the puzzling behavior observed at galactic and cosmological scales.

4.1 Cold Dark Matter

The relative importance of the gravitational interaction increases as one considers large scales, and it is at the largest scales where the observed gravitational phenomena do not agree with our expectations. Thus, based on the motion of the peripheral galaxies of the Coma cluster of galaxies, in 1933 Fritz Zwicky found a discrepancy between the value inferred from the total number of galaxies and brightness of the cluster.

Specifically, this estimates of the total amount of mass in the cluster revealed the need for about 400 times more mass than expected. This led Zwicky conclude that there is another form of matter in the cluster which, although unaccounted, contains most of the mass responsible for the gravitational stability of the cluster. This non-luminous matter became known as the “dark matter”. The dark matter hypothesis was further supported by related problems, namely the differential rotation of our galaxy, as first discussed by Oort in 1927, and the flatness of galactic rotation curves [159].

The most common approach to these problems is to assume the presence of unseen forms of energy that bring into agreement the observed phenomena with GR. The standard scenario to explain the dynamics of galaxies consists in the introduction of an extra weakly interacting massive particle, the so-called Cold Dark Matter (CDM), that clusters at the scales of galaxies and provides the required gravitational pull to hold them together. The explanation of the observed acceleration of the expansion of the Universe requires however the introduction of a more exotic form of energy, not necessarily associated with any form of matter but associated with the existence of space-time itself – vacuum energy.

Although CDM can be regarded as a natural possibility given our knowledge of elementary particle theory, the existence of a non-vanishing but very small vacuum energy remains an unsolved puzzle for our high-energy understanding of physics. However, the CDM hypothesis finds problems when one begin to look at the details of the observations. Increasingly precise simulations of galaxy formation and evolution, although relatively successful in broad terms, show well-known features that seem at odds with their real counterparts, the most prominent of which might be the “cuspy core” problem and the over-abundance of substructure seen in the simulations (see, for instance, [160]).

At the same time the CDM hypothesis is required to explain the correlations of the relative abundances of dark and luminous matter that seem to hold in a very diverse set of astrophysical objects [161]. These correlations are exemplified in the Tully-Fisher law [162] and can be interpreted as pointing to an underlying acceleration scale, $a_0 \simeq 10^{-10} \text{ m s}^{-2}$, below which the Newtonian potential changes and gravity becomes stronger. This is the basic idea of MOND (MODified Newtonian Dynamics), a successful phenomenological modification of Newton’s potential proposed in 1983 [163] whose predictions for the rotation curves of spiral galaxies have been realized with increasing accuracy as the quality of the data has improved [164]. Interestingly, the critical acceleration required by the data is of order $a_0 \sim cH_0$ where H_0 is today’s Hubble constant and c the speed of light (that we will set to 1 from now on). The problem with this idea is that MOND is just a modification of Newton’s potential so it is inadequate in any situation in which relativistic effects are important. Efforts have been made to obtain MONDian phenomenology in a relativistic generally covariant theory by including other fields in the action with suitable couplings to the spacetime metric [165].

On the other hand, in what concerns the CDM model one can state that if all matter is purely baryonic, early structure formation does not occur, as its temperature and pressure could not account for the latter. The presence of cold (i.e., non-relativistic) dark matter allows for gravitational collapse and thus solves this issue. Another hint of the existence of exotic dark matter lies in the observation of gravitational lensing, which may be interpreted as due to the presence of undetected clouds of non-luminous matter between the emitting light source and us, which bends the light path due to its mass. This could also be the cause for the discrepancies in the measured Lyman-alpha forest, the spectra of absorption lines of distant galaxies and quasars. The most likely candidates to account for the dark matter include a linear combination of neutral supersymmetrical particles, the neutralinos (see *e.g.* [166]), axions [167], self-interacting scalar particles [168], *etc.*

On a broader sense, one can say that these models do not address in a unified way the Dark Energy (discussed in Section 4.2) and Dark Matter problems, while a common origin is suggested by the observed coincidence between the critical acceleration scale and the Dark Energy density. This unification feature is found in the so-called generalized Chaplygin gas [169] (see Section 4.2 below)

4.1.1 Modified Gravity as an Alternative to Dark Matter

There are two types of effects in the dark-matter-inspired gravity theories that are responsible for the infrared modification. First, there is an extra scalar excitation of the spacetime metric besides the massless graviton.

The mass of this scalar field is of the order of the Hubble scale in vacuum, but its mass depends crucially on the background over which it propagates. This dependence is such that this excitation becomes more massive near the source, and the extra degree of freedom decouples at short distances in the spacetime of a spherically symmetric mass. This feature makes this excitation to behave in a way that remind of the chameleon field of [170, 171, 172], however, quite often this “chameleon” field is just a component of the spacetime metric coupled to the curvature.

There is also another effect in these theories – the Planck mass that controls the coupling strength of the massless graviton also undergoes a rescaling or “running” with the distance to the sources (or the background curvature). This phenomenon, although a purely classical one in our theory, is reminiscent of the quantum renormalization group running of couplings. So one might wonder if MONDian type actions could be an effective classical description of strong renormalization effects in the infrared that might appear in GR [134, 173], as happens in QCD. A phenomenological approach to structure formation based on these effects has been attempted in Ref. [136]. Other implications, such as lensing, cosmic virial theorem and nucleosynthesis, were analyzed in Refs. [124, 174, 175]. Additionally, these models offer a phenomenology that seems well suited to describe an infrared strongly coupled phase of gravity and especially at high energies/curvatures when one may use the GR action or its linearization being a good approximation; however, when one approaches low energies/curvatures one finds a non-perturbative regime. At even lower energies/curvatures perturbation theory is again applicable, but the relevant theory is of scalar-tensor type in a de Sitter space.

Clearly there are many modifications of the proposed class of actions that would offer a similar phenomenology, such that gravity would be modified below a characteristic acceleration scale of the order of the one required in MOND. Many of these theories also offer the unique possibility of being tested not only through astrophysical observations, but also through well-controlled laboratory experiments where the outcome of an experiment is correlated with parameters that can be determined by means of cosmological and astrophysical measurements.

4.2 Dark Energy as Modern Cosmological Puzzle

In 1998 Perlmutter and collaborators [176] and Riess and collaborators [177] have gathered data of the magnitude-redshift relation of Type Ia supernovae with redshifts $z \geq 0.35$ and concluded that it strongly suggest that we live in an accelerating Universe, with a low matter density with about one third of the of the energy content of the Universe. Currently there are about 250 supernovae data points which confirm this interpretation. Dark energy is assumed to be a smooth distribution of non-luminous energy uniformly distributed over the Universe so to account for the extra dimming of the light of far away Type Ia supernovae, standard candles for cosmological purposes. If there is a real physical field responsible for Dark Energy, it may be phenomenologically described in terms of an energy density ρ and pressure p , related instantaneously by the equation-of-state parameter $w = p/\rho$. Furthermore, covariant energy conservation would then imply that ρ dilutes as $a^{-3(1+w)}$, with a being the scale factor. Note that $p = w\rho$ is not necessarily the actual equation of state of the Dark Energy fluid, meaning that perturbations may not generally obey $\delta p = w\delta\rho$; however, if one were to have such an equation of state, one can define the speed of sound by $c_s^2 = \partial p/\partial\rho$. The implications of this phenomenology would make much more sense in the context of theories proposed to provide the required microscopic description.

4.2.1 Cosmological Constant and Dark Energy

One of the leading explanations for the accelerated expansion of the Universe is the presence of a non-zero cosmological constant. As can be seen from Einstein’s equation, the cosmological term can be viewed not as a geometric prior to the spacetime continuum, but instead interpreted as a energy-momentum tensor proportional to the metric, thus enabling the search for the fundamental physics mechanism behind its value and, possibly, its evolution with cosmic time. An outstanding question in today’s physics lies in the discrepancy between the observed value for Λ and the prediction arising from quantum field theory, which yields a vacuum energy density about 120 orders of magnitude larger than the former. To match the observed value, requires a yet unknown cancelation mechanism to circumvent the fine tuning of 120 decimal places necessary to account for

the observations. This is so as observations require the cosmological constant to be of order of the critical density $\rho_c = 3H_0^2/8\pi G \simeq 10^{-29} \text{ g cm}^{-3}$:

$$\rho_V \equiv \frac{\Lambda}{8\pi G} \simeq 10^{-29} \text{ g cm}^{-3} \simeq 10^{-12} \text{ eV}^4, \quad (24)$$

while the natural number to expect from a quantum gravity theory is $M_P^4 \simeq 10^{76} \text{ GeV}^4$.

Besides the cosmological constant, a slow-varying vacuum energy⁶ of some scalar field, usually referred to as “quintessence” [179], or an exotic fluid like the generalized Chaplygin gas [169] are among other the most discussed candidates to account for this dominating contribution for the energy density. It is worth mentioning that the latter possibility allows for a scenario where dark energy and dark matter are unified.

We mention that the presented bounds result from a variety of sources, of which the most significant are the CMBR, high- z supernovae redshifts and galaxy cluster abundances. These joint constraints establish that the amount of dark energy, dark matter and baryons are, in terms of the critical density, $\Omega_\Lambda \simeq 0.73$, $\Omega_{\text{DM}} = 0.23$ and $\Omega_{\text{Baryons}} = 0.04$, respectively [180].

Current observational constraints imply that the evolution of Dark Energy is entirely consistent with $w = -1$, characteristic of a cosmological constant (Λ). The cosmological constant was the first, and remains the simplest, theoretical solution to the Dark Energy observations. The well-known “cosmological constant problem” – why is the vacuum energy so much smaller than we expect from effective-field-theory considerations? – remains, of course, unsolved.

Recently an alternative mechanism to explain Λ has arisen out of string theory. It was previously widely perceived that string theory would continue in the path of QED and QCD wherein the theoretical picture contained few parameters and a uniquely defined ground state. However recent developments have yielded a theoretical horizon in distinct opposition to this, with a “landscape” of possible vacua generated during the compactification of 11 dimensions down to 3 [181]. Given the complexity of the landscape, anthropic arguments have been put forward to determine whether one vacuum is preferred over another. It is possible that further development of the statistics of the vacua distribution, and characterization of any distinctive observational signatures, such as predictions for the other fundamental coupling constants, might help to distinguish preferred vacua and extend beyond the current vacua counting approach.

Although Dark Energy is the most obvious and popular possibility to the recently observed acceleration of the Universe, other competing ideas have been investigated, and among them is modifications of gravity on cosmological scales. Indeed, as we discussed earlier, GR is well tested in the solar system, in measurements of the period of the binary pulsar, and in the early Universe, via primordial nucleosynthesis. None of these tests, however, probes the ultra-large length scales and low curvatures characteristic of the Hubble radius today. Therefore, one can potentially think that gravity is modified in the very far infrared allowing the Universe to accelerate at late times.

In this section we will discuss some of the gravity modification proposals suggested to provide a description of the observed acceleration of the Universe.

4.2.2 Modified Gravity as an Alternative to Dark Energy

A straightforward possibility is to modify the usual Einstein-Hilbert action by adding terms that are blow up as the scalar curvature goes to zero [182, 183]. Recently, models involving inverse powers of the curvature have been proposed as an alternative to Dark Energy [183, 184]. In these models one generically has more propagating degrees of freedom in the gravitational sector than the two contained in the massless graviton in GR. The simplest models of this kind add inverse powers of the scalar curvature to the action ($\Delta\mathcal{L} \propto 1/R^n$), thereby introducing a new scalar excitation in the spectrum. For the values of the parameters required to explain

⁶For earlier suggestions see Refs. [178].

the acceleration of the Universe this scalar field is almost massless in vacuum and one might worry about the presence of a new force contradicting solar system experiments. However, it can be shown that models that involve inverse powers of other invariant, in particular those that diverge for $r \rightarrow 0$ in the Schwarzschild solution, generically recover an acceptable weak field limit at short distances from sources by means of a screening or shielding of the extra degrees of freedom at short distances [185]. Such theories can lead to late-time acceleration, but unfortunately typically lead to one of two problems. Either they are in conflict with tests of GR in the solar system, due to the existence of additional dynamical degrees of freedom [186], or they contain ghost-like degrees of freedom that seem difficult to reconcile with fundamental theories. The search is ongoing for versions of this idea that are consistent with experiment.

A more dramatic approach would be to imagine that we live on a brane embedded in a large extra dimension. Although such theories can lead to perfectly conventional gravity on large scales, it is also possible to choose the dynamics in such a way that new effects show up exclusively in the far infrared. An example is the Dvali-Gabadadze-Porrati (DGP) braneworld model, in which the strength of gravity in the bulk is substantially less than that on the brane [187]. Such theories can naturally lead to late-time acceleration [188, 189], but may have difficulties with strong-coupling issues [190]. Furthermore, the DGP model does not properly account for the supernova data, as does its generalization, the Dvali-Turner model, and also other *ad hoc* modifications of the Friedmann equation, the so-called Cardassian model [191]. Most interestingly, however, DGP gravity and other modifications of GR hold out the possibility of having interesting and testable predictions that distinguish them from models of dynamical Dark Energy. One outcome of this work is that the physics of the accelerating Universe may be deeply tied to the properties of gravity on relatively short scales, from millimeters to astronomical units.

4.2.3 Scalar field Models as Candidate for Dark Energy

One of the simplest candidates for dynamical Dark Energy is a scalar field, φ , with an extremely low-mass and an effective potential, $V(\varphi)$, as shown by Eq. (15) [127]. If the field is rolling slowly, its persistent potential energy is responsible for creating the late epoch of inflation we observe today. For the models that include only inverse powers of the curvature, besides the Einstein-Hilbert term, it is however possible that in regions where the curvature is large the scalar has naturally a large mass and this could make the dynamics to be similar to those of GR [192]. At the same time, the scalar curvature, while being larger than its mean cosmological value, is still very small in the solar system (to satisfy the available results of gravitational tests). Although a rigorous quantitative analysis of the predictions of these models for the tests in the solar system is still noticeably missing in the literature, it is not clear whether these models may be regarded as a viable alternative to Dark Energy.

Effective scalar fields are prevalent in supersymmetric field theories and string/M-theory. For example, string theory predicts that the vacuum expectation value of a scalar field, the dilaton, determines the relationship between the gauge and gravitational couplings. A general, low energy effective action for the massless modes of the dilaton can be cast as a scalar-tensor theory as Eq. (15) with a vanishing potential, where $f(\varphi)$, $g(\varphi)$ and $q_i(\varphi)$ are the dilatonic couplings to gravity, the scalar kinetic term and gauge and matter fields respectively, encoding the effects of loop effects and potentially non-perturbative corrections.

A string-scale cosmological constant or exponential dilaton potential in the string frame translates into an exponential potential in the Einstein frame. Such quintessence potentials [193, 194] can have scaling [195], and tracking [196] properties that allow the scalar field energy density to evolve alongside the other matter constituents. A problematic feature of scaling potentials [195] is that they do not lead to accelerative expansion, since the energy density simply scales with that of matter. Alternatively, certain potentials can predict a Dark Energy density which alternately dominates the Universe and decays away; in such models, the acceleration of the Universe is transient [197, 198, 199]. Collectively, quintessence potentials predict that the density of the Dark Energy dynamically evolve in time, in contrast to the cosmological constant. Similar to a cosmological constant, however, the scalar field is expected to have no significant density perturbations within the causal horizon, so that they contribute little to the evolution of the clustering of matter in large-scale structure [200].

In addition to couplings to ordinary matter, the quintessence field may have nontrivial couplings to dark matter [201, 202]. Non perturbative string-loop effects do not lead to universal couplings, with the possibility

that the dilaton decouples more slowly from dark matter than it does from gravity and fermions. This coupling can provide a mechanism to generate acceleration, with a scaling potential, while also being consistent with Equivalence Principle tests. It can also explain why acceleration is occurring only recently, through being triggered by the non-minimal coupling to the cold dark matter, rather than a feature in the effective potential [203, 204]. Such couplings can not only generate acceleration, but also modify structure formation through the coupling to CDM density fluctuations [205], in contrast to minimally coupled quintessence models. Dynamical observables, sensitive to the evolution in matter perturbations as well as the expansion of the Universe, such as the matter power spectrum as measured by large scale surveys, and weak lensing convergence spectra, could distinguish non-minimal couplings from theories with minimal effect on clustering. The interaction between dark energy and dark matter is, of course, present in the generalized Chaplygin gas model, as in this proposal the fluid has a dual behavior.

It should be noted that for the run-away dilaton scenario presented in Section 3.1, comparison with the minimally coupled scalar field action,

$$S_\phi = \frac{c^3}{4\pi G} \int d^4x \sqrt{-g} \left[\frac{1}{4} R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right], \quad (25)$$

reveals that the negative scalar kinetic term leads to an action equivalent to a “ghost” in quantum field theory, and is referred to as “phantom energy” in the cosmological context [206]. Such a scalar field model could in theory generate acceleration by the field evolving *up* the potential toward the maximum. Phantom fields are plagued by catastrophic UV instabilities, as particle excitations have a negative mass [207, 208]; the fact that their energy is unbounded from below allows vacuum decay through the production of high energy real particles and negative energy ghosts that will be in contradiction with the constraints on ultra-high energy cosmic rays [209].

Such runaway behavior can potentially be avoided by the introduction of higher-order kinetic terms in the action. One implementation of this idea is “ghost condensation” [210]. Here, the scalar field has a negative kinetic energy near $\dot{\phi} = 0$, but the quantum instabilities are stabilized by the addition of higher-order corrections to the scalar field Lagrangian of the form $(\partial_\mu \phi \partial^\mu \phi)^2$. The “ghost” energy is then bounded from below, and stable evolution of the dilaton occurs with $w \geq -1$ [211]. The gradient $\partial_\mu \phi$ is non-vanishing in the vacuum, violating Lorentz invariance, and may have interesting consequences in cosmology and in laboratory experiments.

In proposing the scalar field as physical and requiring it to replace CDM and DE, one has to also calculate how the scalar field density fluctuations evolve, in order to compare them with density power spectra from large-scale structure surveys. This is true for the broader set of phenomenological models including the generalized Born-Infeld action, associated to the generalized Chaplygin gas model [169]. Despite being consistent with kinematical observations, it has been pointed that they are disfavored in comparison to the Λ CDM scenario [212, 213], even though solutions have been proposed [214].

5 Gravitational Physics and Experiments in Space

Recent progress in observational astronomy, astrophysics, and cosmology has raised important questions related to gravity and other fundamental laws of Nature. There are two approaches to physics research in space: one can detect and study signals from remote astrophysical objects, while the other relies on a carefully designed experiment. Although the two methods are complementary, the latter has the advantage of utilizing a well-understood and controlled laboratory environment in the solar system. Newly available technologies in conjunction with existing space capabilities offer unique opportunities to take full advantage of the variable gravity potentials, large heliocentric distances, and high velocity and acceleration regimes that are present in the solar system. As a result, solar system experiments can significantly advance our knowledge of fundamental physics and are capable of providing the missing links connecting quarks to the cosmos.

In this section we will discuss theoretical motivation of and innovative ideas for the advanced gravitational

space experiments.

5.1 Testable Implications of Recent Theoretical Proposals

The theories that were discussed in the previous section offer a diverse range of characteristic experimental predictions differing from those of GR that would allow their falsification. The most obvious tests would come from the comparison of the predictions of the theory to astrophysical and cosmological observations where the dynamics are dominated by very small gravitational fields. As a result, one might expect that these mechanisms would lead to small effects in the motion of the bodies in the solar system, short- and long-scale modifications of Newton’s law, as well as astrophysical phenomena.

Below we will discuss these possible tests and estimate the sizes of the expected effects.

5.1.1 Testing Newton’s Law at Short Distances

It was observed that many recent theories predict observable experimental signatures in experiments testing Newton’s law at short distances. For instance in the case of MOND-inspired theories discussed in Section 4.1.1, there may be an extra scalar excitation of the spacetime metric besides the massless graviton. Thus, in the effective gravitational theory applicable to the terrestrial conditions, besides the massless spin two graviton, one would also have an extra scalar field with gravitational couplings and with a small mass. A peculiar feature of such a local effective theory on a Schwarzschild background is that there will be a preferred direction that will be reflected in an anisotropy of the force that this scalar excitation will mediate. For an experiment conducted in the terrestrial conditions one expects short ranges modifications of Newton’s law at distances of ~ 0.1 mm, regime that is close to that already being explored in some laboratory experiments [215, 216].

For an experiment on an Earth-orbiting platform, one explores another interesting regime for which the solar mass and the Sun-Earth distance are the dominant factors in estimating the size of the effects. In this case the range of interest is $\sim 10^4$ m. However in measuring the gravitational field of an object one has to measure this field at a distance that is larger than the critical distance for which the self-shielding of the extra scalar excitation induced by the object itself is enough to switch off the modification. This means that, for an experiment in the inner solar system, we could only see significant modifications in the gravitational field of objects whose characteristic distance is smaller than 10^4 m, thus limiting the mass of the body to be below $\sim 10^9$ kg. As an example, one can place an object with mass of 10^3 kg placed on a heliocentric orbit at ~ 1 AU distance. For this situation, one may expect modifications of the body’s gravitational field at distances within the range of $\sim 10 - 10^4$ m. Note that at shorter distances the scalar effectively decouples because of the self-gravitational effect of the test object; also, at longer distances the mass induced by solar gravitational field effectively decouples the scalar.

5.1.2 Solar System Tests of Relativistic Gravity

Although many effects expected by gravity modification models are suppressed within the solar system, there are measurable effects induced by some long-distance modifications of gravity (notably the DGP model [187]). For instance, in the case of the precession of the planetary perihelion in the solar system, the anomalous perihelion advance, $\Delta\phi$, induced by a small correction, δV_N , to Newton’s potential, V_N , is given in radians per revolution [217] by

$$\Delta\phi \simeq \pi r \frac{d}{dr} \left(r^2 \frac{d}{dr} \left(\frac{\delta V_N}{r V_N} \right) \right). \quad (26)$$

The most reliable data regarding the planetary perihelion advances come from the inner planets of the solar system [218], where most of the corrections are negligible. However, with its excellent 2-cm-level range accuracy [14], LLR offers an interesting possibility to test for these new effects. Evaluating the expected magnitude of

the effect to the Earth-Moon system, one predicts an anomalous shift of $\Delta\phi \sim 10^{-12}$, to be compared with the achieved accuracy of 2.4×10^{-11} [217]. Therefore, the theories of gravity modification result in an intriguing possibility of discovering new physics, if one focuses on achieving higher precision in modern astrometrical measurements; this accuracy increase is within the reach and should be attempted in the near future.

The quintessence models discussed in Section 4.2.3 offer the possibility of observable couplings to ordinary matter, makes these models especially attractive for the tests even on the scales of the solar system. Even if we restrict attention to non-renormalizable couplings suppressed by the Planck scale, tests from fifth-force experiments and time-dependence of the fine-structure constant imply that such interactions must be several orders of magnitude less than expected [219]. Further improvement of existing limits on violations of the Equivalence Principle in terrestrial experiments and also in space would also provide important constraints on dark-energy models.

Another interesting experimental possibility is provided by the “chameleon” effect [170, 172]. Thus, by coupling to the baryon energy density, the scalar field value can vary across space from solar system to cosmological scales. Though the small variation of the coupling on Earth satisfies the existing terrestrial experimental bounds, future gravitational experiments in space such as measurements of variations in the gravitational constant or test of Equivalence Principle, may provide critical information for the theory.

There is also a possibility that the dynamics of the quintessence field evolves to a point of minimal coupling to matter. In [52] it was shown that ϕ could be attracted towards a value $\phi_m(x)$ during the matter dominated era that decoupled the dilaton from matter. For universal coupling, $f(\varphi) = g(\varphi) = q_i(\varphi)$ (see Eq. 15), this would motivate for improving the accuracy of the equivalence principle and other tests of GR. Ref. [220] suggested that with a large number of non-self-interacting matter species, the coupling constants are determined by the quantum corrections of the matter species, and ϕ would evolve as a run-away dilaton with asymptotic value $\phi_m \rightarrow \infty$. More recently, in Refs. [138] the quantity $\frac{1}{2}(1 - \gamma)$ has been estimated, within the framework compatible with string theory and modern cosmology, which basically confirms the previous result [38]. This recent analysis discusses a scenario where a composition-independent coupling of dilaton to hadronic matter produces detectable deviations from GR in high-accuracy light deflection experiments in the solar system. This work assumes only some general property of the coupling functions and then only assume that $(1 - \gamma)$ is of order of one at the beginning of the controllably classical part of inflation. It was shown in [138] that one can relate the present value of $\frac{1}{2}(1 - \gamma)$ to the cosmological density fluctuations; the level of the expected deviations from GR is $\sim 0.5 \times 10^{-7}$ [138]. Note that these predictions are based on the work in scalar-tensor extensions of gravity which are consistent with, and indeed often part of, present cosmological models provide a strong motivation for improvement of the accuracy of gravitational tests in the solar system.

5.1.3 Observations on Astrophysical and Cosmological Scales

The new theories also suggest an interesting observable effects on astrophysical and cosmological observations (see for instance [221]). In this respect, one can make unambiguous predictions for the rotation curves of spiral galaxies with the mass-to-light ratio being the only free parameter. Specifically, it has been argued that a skew-symmetric field with a suitable potential could account for galaxy and cluster rotation curves [222]. One can even choose an appropriate potential that would then give rise to flat rotation curves that obey the Tully-Fisher law [162]. But also other aspects of the observations of galactic dynamics can be used to constrain a MOND-like modification of Newton’s potential (see [223]). And notice also that our theory violates the strong equivalence principle, as expected for any relativistic theory for MOND [163], since locally physics will intrinsically depend on the background gravitational field. This will be the case if the background curvature dominates the curvature induced by the local system, similarly to the “external field effect” in MOND.

At larger scales, where one can use the equivalence with a scalar-tensor theory more reliably, one can then compare the theory against the observations of gravitational lensing in clusters, the growth of large scale structure and the fluctuations of the CMBR. In fact, it has been pointed out that if GR was modified at large distances, an inconsistency between the allowed regions of parameter space would allow for Dark Energy models verification when comparing the bounds on these parameters obtained from CMBR, and large scale structure

[224]. This means that although some cosmological observables, like the expansion history of the Universe, can be indistinguishable in modified gravity and Dark Energy models, this degeneracy is broken when considering other cosmological observations and in particular the growth of large scale structure and the Integrated Sachs-Wolfe effect (ISW) have been shown to be good discriminators for models in which GR is modified [225]. It has been recently pointed out that the fact that in the DGP model the effective Newton's constant increases at late times as the background curvature diminishes, causes a suppression of the ISW that brings the theory into better agreement with the CMBR data than the Λ CDM model [226].

5.2 New Experiments and Missions

Theoretical motivations presented above have stimulated development of several highly-accurate space experiments. Below we will briefly discuss science objectives and experimental design for several advanced experiments, namely MICROSCOPE, STEP, and HYPER missions, APOLLO LLR facility, and the LATOR mission.

5.2.1 MICROSCOPE, STEP, and HYPER Missions

Ground experiments designed to verify the validity of the WEP are limited by unavoidable microseismic activity of Earth, while the stability of space experiments offers an improvement in the precision of current tests by a factor of 10^6 . Most probably, the first test of the WEP in space will be carried out by the MICROSCOPE (MICROSatellite a trainee Compensee pour l'Observation du Principe d'Equivalence) mission led by CNES and ESA. The drag-free MICROSCOPE satellite, transporting two pairs of test masses, will be launched into a sun-synchronous orbit at 600 km altitude. The differential displacements between each test masses will of a pair be measured by capacitive sensors at room-temperature, with an expected precision of one part in 10^{15} .

The more ambitious joint ESA/NASA STEP (Satellite Test of the Equivalence Principle) mission which is proposed to be launched in the near future into a circular, sun-synchronous orbit with altitude of 600 km. The drag-free STEP spacecraft will carry four pairs of test masses stored in a dewar of superfluid He at a 2 K temperature. Differential displacements between the test masses of a pair will be measured by SQUID (Superconducting QUantum Interference Device) sensors, testing the WEP with an expected precision of $\Delta a/a \sim 10^{18}$.

Another quite interesting test of the WEP involves atomic interferometry: high-precision gravimetric measurements can be taken via the interferometry of free-falling caesium atoms, and such a concept has already yielded a precision of 7 parts per 10^9 [227]. This can only be dramatically improved in space, through a mission like HYPER (HYPER-precision cold atom interferometry in space). ESA's HYPER spacecraft would be in a sun-synchronous circular orbit at 700 km altitude. Two atomic Sagnac units are to be accommodated in the spacecraft, comprising four cold atom interferometers able to measure rotations and accelerations along two orthogonal planes. By comparing the rates of fall of caesium and rubidium atoms, the resolution of the atom interferometers of the HYPER experiment could, in principle, test the WEP with a precision of one part in 10^{15} or 10^{16} [228].

It is worth mentioning that proposals have been advanced to test the WEP by comparing the rate of fall of protons and antiprotons in a cryogenic vacuum facility that will be available at the ISS [229]. The concept behind this Weak Equivalence Antimatter eXperiment (WEAX) consists of confining antiprotons for a few weeks in a Penning trap, in a geometry such that gravity would produce a perturbation on the motion of the antiprotons. The expected precision of the experiment is of one part in 10^6 , three orders of magnitude better than for a ground experiment.

It is clear that testing the WEP in space requires pushing current technology to the limit; even though no significant violations of this principle are expected, any anomaly would provide significant insight into new and fundamental physical theories. The broad perspectives and the potential impact of testing fundamental physics in space were discussed in Ref. [230].

5.2.2 APOLLO – a *mm*-class LLR Facility

The Apache Point Observatory Lunar Laser-ranging Operation is a new LLR effort designed to achieve millimeter range precision and corresponding order-of-magnitude gains in measurements of fundamental physics parameters. The APOLLO project design and leadership responsibilities are shared between the University of California at San Diego and the University of Washington. In addition to the modeling aspects related to this new LLR facility, a brief description of APOLLO and associated expectations is provided here for reference. A more complete description can be found in [231].

The principal technologies implemented by APOLLO include a robust Nd:YAG laser with 100 ps pulse width, a GPS-slaved 50 MHz frequency standard and clock, a 25 ps-resolution time interval counter, and an integrated avalanche photo-diode (APD) array. The APD array, developed at Lincoln Labs, is a new technology that will allow multiple simultaneous photons to be individually time-tagged, and provide two-dimensional spatial information for real-time acquisition and tracking capabilities.

The overwhelming advantage APOLLO has over current LLR operations is a 3.5 m astronomical quality telescope at a good site. The site in the Sacramento Mountains of southern New Mexico offers high altitude (2780 m) and very good atmospheric “seeing” and image quality, with a median image resolution of 1.1 arcseconds. Both the image sharpness and large aperture enable the APOLLO instrument to deliver more photons onto the lunar retroreflector and receive more of the photons returning from the reflectors, respectively. Compared to current operations that receive, on average, fewer than 0.01 photons per pulse, APOLLO should be well into the multi-photon regime, with perhaps 5-10 return photons per pulse. With this signal rate, APOLLO will be efficient at finding and tracking the lunar return, yielding hundreds of times more photons in an observation than current operations deliver. In addition to the significant reduction in statistical error ($\sim \sqrt{N}$ reduction), the high signal rate will allow assessment and elimination of systematic errors in a way not currently possible.

The new LLR capabilities offered by the newly developed APOLLO instrument offer a unique opportunity to improve accuracy of a number of fundamental physics tests. The APOLLO project will push LLR into the regime of millimetric-range precision which translates to an order-of-magnitude improvement in the determination of fundamental physics parameters. For the Earth and Moon orbiting the Sun, the scale of relativistic effects is set by the ratio $(GM/rc^2) \sim v^2/c^2 \sim 10^{-8}$. Relativistic effects are small compared to Newtonian effects. The APOLLO’s 1 mm range accuracy corresponds to 3×10^{-12} of the Earth-Moon distance. The resulting LLR tests of gravitational physics would improve by an order of magnitude: the Equivalence Principle would give uncertainties approaching 10^{-14} , tests of GR effects would be $< 0.1\%$, and estimates of the relative change in the gravitational constant would be 0.1% of the inverse age of the Universe. This last number is impressive considering that the expansion rate of the Universe is approximately one part in 10^{10} per year.

Therefore, the gain in our ability to conduct even more precise tests of fundamental physics is enormous, thus this new instrument stimulates development of better and more accurate models for the LLR data analysis at a mm-level [232].

5.2.3 The LATOR Mission

The recently proposed Laser Astrometric Test Of Relativity (LATOR) [121, 233, 234, 235] is an experiment designed to test the metric nature of gravitation – a fundamental postulate of Einstein’s theory of general relativity. By using a combination of independent time-series of highly accurate gravitational deflection of light in the immediate proximity to the sun, along with measurements of the Shapiro time delay on interplanetary scales (to a precision respectively better than 10^{-13} radians and 1 cm), LATOR will significantly improve our knowledge of relativistic gravity. The primary mission objective is to i) measure the key post-Newtonian Eddington parameter γ with accuracy of a part in 10^9 . The quantity $(1 - \gamma)$ is a direct measure for presence of a new interaction in gravitational theory, and, in its search, LATOR goes a factor 30,000 beyond the present best result, Cassini’s 2003 test. Other mission objectives include: ii) first measurement of gravity’s non-linear effects on light to $\sim 0.01\%$ accuracy; including both the traditional Eddington β parameter via gravity effect on light to $\sim 0.01\%$ accuracy and also the spatial metric’s 2-nd order potential contribution δ (never measured before);

iii) direct measurement of the solar quadrupole moment J_2 (currently unavailable) to accuracy of a part in 200 of its expected size; iv) direct measurement of the “frame-dragging” effect on light due to the sun’s rotational gravitomagnetic field, to 0.1% accuracy. LATOR’s primary measurement pushes to unprecedented accuracy the search for cosmologically relevant scalar-tensor theories of gravity by looking for a remnant scalar field in today’s solar system. The key element of LATOR is a geometric redundancy provided by the laser ranging and long-baseline optical interferometry.

As a result, LATOR will be able to test the metric nature of the Einstein’s general theory of relativity in the most intense gravitational environment available in the solar system – the extreme proximity to the sun. It will also test alternative theories of gravity and cosmology, notably scalar-tensor theories, by searching for cosmological remnants of scalar field in the solar system. LATOR will lead to very robust advances in the tests of fundamental physics: this mission could discover a violation or extension of GR, or reveal the presence of an additional long range interaction in the physical law. There are no analogs to the LATOR experiment; it is unique and is a natural culmination of solar system gravity experiments [121].

LATOR mission is the 21st century version of Michelson-Morley-type experiment searching for a cosmologically evolved scalar field in the solar system. In spite of the previous space missions exploiting radio waves for tracking the spacecraft, this mission manifests an actual breakthrough in the relativistic gravity experiments as it allows to take full advantage of the optical techniques that recently became available. LATOR has a number of advantages over techniques that use radio waves to measure gravitational light deflection. Thus, optical technologies allow low bandwidth telecommunications with the LATOR spacecraft. The use of the monochromatic light enables the observation of the spacecraft at the limb of the sun. The use of narrowband filters, coronagraph optics and heterodyne detection will suppress background light to a level where the solar background is no longer the dominant noise source. The short wavelength allows much more efficient links with smaller apertures, thereby eliminating the need for a deployable antenna. Finally, the use of the ISS enables the test above the Earth’s atmosphere – the major source of astrometric noise for any ground based interferometer. This fact justifies LATOR as a space mission. LATOR is envisaged as a partnership between European and US institutions and with clear areas of responsibility between the space agencies: NASA provides the deep space mission components, while optical infrastructure on the ISS would be an ESA contribution.

Conclusions

General theory of relativity is one of the most elegant theories of physics; it is also one of the most empirically verified. Thus, almost ninety years of testing have also proved that GR has so far successfully accounted for all encountered phenomena and experiments in the solar system and with binary pulsars. However, despite that there are predictions of the theory that require still confirmation and detailed analysis, most notably the direct detection of gravitational waves. However, there are new motivations to test the theory to even a higher precisions that already led to a number of experimental proposals to advance the knowledge of fundamental laws of physics.

Recent progress in observational astronomy, astrophysics, and cosmology has raised important questions related to gravity and other fundamental laws of Nature. There are two approaches to physics research in space: one can detect and study signals from remote astrophysical objects, while the other relies on a carefully designed experiment. Although the two methods are complementary, the latter has the advantage of utilizing a well-understood and controlled laboratory environment in the solar system.

Newly available technologies in conjunction with existing space capabilities offer unique opportunities to take full advantage of the variable gravity potentials, large heliocentric distances, and high velocity and acceleration regimes that are present in the solar system. A common feature of precision gravity experiments is that they must operate in the noise free environment needed to achieve the ever increasing accuracy. These requirements are supported by the progress in the technologies, critical for space exploration, namely the highly-stable, high-powered, and space-qualified lasers, highly-accurate frequency standards, and the drag-free technologies. This progress advances both the science and technology for the laboratory experiments in space with laboratory

being the entire solar system. As a result, solar system experiments can significantly advance our knowledge of fundamental physics and are capable of providing the missing links connecting quarks to the cosmos.

Concluding, it is our hope that the recent progress will lead to establishing a more encompassing theory to describe all physical interactions in an unified fashion that harmonizes the spacetime description of GR with quantum mechanics. This unified theory is needed to address many of the standing difficulties we face in theoretical physics: Are singularities an unavoidable property of spacetime? What is the origin of our Universe? How to circumvent the cosmological constant problem and achieve a successful period of inflation and save our Universe from an embarrassing set of initial conditions? The answer to these questions is, of course, closely related to the nature of gravity. It is an exciting prospect to think that experiments carried out in space will be the first to provide the essential insights on the brave new world of the new theories to come.

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