

Ensemble Weight Enumerators for Protograph LDPC Codes

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Abstract— Recently LDPC codes with projected graph, or protograph structures have been proposed. In this paper, finite length ensemble weight enumerators for LDPC codes with protograph structures are obtained. Asymptotic results are derived as the block size goes to infinity. In particular we are interested in obtaining ensemble average weight enumerators for protograph LDPC codes which have minimum distance that grows linearly with block size. As with irregular ensembles, linear minimum distance property is sensitive to the proportion of degree-2 variable nodes. In this paper the derived results on ensemble weight enumerators show that linear minimum distance condition on degree distribution of unstructured irregular LDPC codes is a sufficient but not a necessary condition for protograph LDPC codes.

I. INTRODUCTION

Low-density parity-check (LDPC) codes were proposed by Gallager [1] in 1962. Ensemble weight enumerators for unstructured irregular LDPC codes and turbo like codes have been reported in [12], [13], [14], [15], [16], [17], [18], [19], [21]. Recently a flurry of work has been conducted on the design of LDPC codes with imposed sub-structures, starting with the introduction of multi-edge type codes in [5] and [6]. Protograph based LDPC codes are a subclass of multi-edge LDPC codes. In [11] a different method for computation of asymptotic weight enumerators for LDPC codes with protograph structure has been proposed. The method in this paper starts with computation of ensemble weight enumerators for finite block size LDPC with a protograph structure. The results then are extended to asymptotic cases as the block size goes to infinity.

For high-speed decoding, it is advantageous for an LDPC code to be constructed from a projected graph [4], or protograph [3]. A protograph is a Tanner graph with a relatively small number of nodes. A protograph $G = (V, C, E)$ consists of a set of variable nodes V , a set of check nodes C , and a set of edges E . Each edge $e \in E$ connects a variable node $v_e \in V$ to a check node $c_e \in C$. Parallel edges are permitted, so the mapping $e \rightarrow (v_e, c_e) \in V \times C$ is not necessarily 1:1. As a simple example, we consider the protograph shown in Fig. 1. This graph consists of 3 variable nodes and 2 check nodes, connected by 5 edges. In this example we have 5 edge types i.e. each edge in the base protograph represents an edge

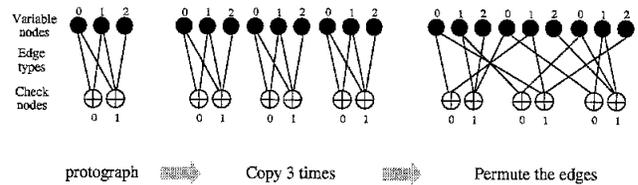


Fig. 1. Copy and Permute operation for a protograph to generate larger graphs

type. For multi-edge LDPC codes, a group of edges (number of edges in each group can be different) represents an edge type. For unstructured irregular LDPC codes, there is only one edge type. Having the base protograph, we can obtain a larger graph by a “copy-and-permute” operation as shown in Fig. 1. This operation consists of first making N copies of the protograph, and then permuting the endpoints of each edge type among the N variable and N check nodes connected to the set of N edges copied from the same edge type in the protograph. The derived or lifted graph is the graph of a code N times as large as the code corresponding to the protograph, with the same rate and the same distribution of variable and check node degrees.

As follows in the protograph representation in the figures, those variable nodes say m nodes that are connected to the channel (transmitted nodes) will be shown as dark filled circles. Those variable node that are not connected to the channel (punctured nodes or not transmitted nodes) will be depicted by a blank circle. The check nodes will be depicted by circles with a plus sign inside. The code rate for the protograph is $R_c = \frac{n_v - n_c}{m}$ provided that the parity check matrix of the derived or lifted graph is full rank.

II. ENSEMBLE WEIGHT ENUMERATOR FOR A PROTOGRAPH BASED LDPC

Consider a protograph based LDPC code as shown in Fig. 2 (for base protograph ignore the interleavers) with n_v variable and n_c check nodes.

Let q_{v_i} represent the degree of variable node v_i , q_{c_i} represent the degree of check node c_i and let $|E|$ represent the total number of edges types. Suppose we lift (copy and permute) this protograph by a factor of N . This is equivalent

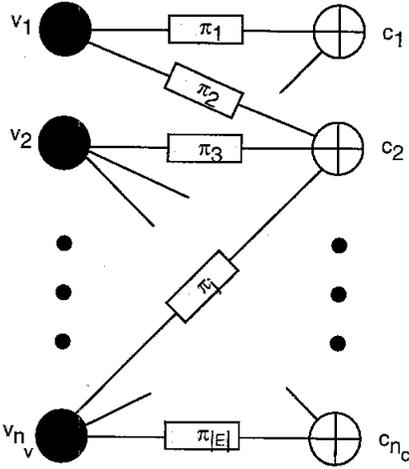


Fig. 2. An LDPC with protograph structure

to using random interleavers each of size N per each edge of the base protograph as shown in Fig. 2. Assign weight d_i to variable node v_i for $i = 1, \dots, n_v$. Using the results for serial concatenation with interleaved codes [9] and for turbo-like codes [2] we can compute the ensemble weight enumerators for protograph LDPC codes. Consider each variable and check node in the protograph as a constituent code. These constituent codes are connected through edges in the protograph where we assume each edge has a uniform interleaver [8] of size N . Each variable node v_i can be assumed as a constituent code with one input of weight d_i and q_{v_i} outputs. The input-output weight coefficient for node v_i is $\binom{N}{d_i} \delta_{d_i, w_{i,1}} \dots \delta_{d_i, w_{i, q_{v_i}}}$, where Kronecker Delta is defined as $\delta_{x,y} = 1$ if $x = y$, and zero otherwise. Each check node c_i can be assumed as constituent code with q_{c_i} inputs and no output (or a fictitious output with weight zero). Let $A_{w_i}^{c_i}$ be the input weight enumerator for the check node c_i that satisfies the check node constraint. The $A_{w_i}^{c_i}$ represents the number of sequences with input weight $w_i = (w_{i,1}, w_{i,2}, \dots, w_{i, q_{c_i}})$. Let A_d represent number of sequences each with weight vector $d = (d_1, d_2, \dots, d_{n_v})$ that is applied to the variable nodes and satisfy the protograph constraints. Then ensemble partial weight enumerators for the protograph based LDPC can be computed as:

$$A_d = \sum_{\substack{w_{m,u} \\ m=1, \dots, n_v \\ u=1, \dots, q_{v_m}}} \frac{\prod_{k=1}^{n_v} \left[\binom{N}{d_k} \delta_{d_k, w_{k,1}} \dots \delta_{d_k, w_{k, q_{v_k}}} \right] \prod_{i=1}^{n_c} A_{w_i}^{c_i}}{\prod_{s=1}^{n_v} \prod_{r=1}^{q_{v_s}} \binom{N}{w_{s,r}}} \quad (1)$$

which reduces to

$$A_d = \frac{\prod_{i=1}^{n_c} A_{d_i}^{c_i}}{\prod_{i=1}^{n_v} \binom{N}{d_i}^{q_{v_i}-1}} \quad (2)$$

where $d_i = (d_{i,1}, d_{i,2}, \dots, d_{i, q_{c_i}})$. Partial weights $d_{i,k}$ s corresponds to edge connections of a check node c_i to the set

of variable nodes $\{v_{i_1}, v_{i_2}, \dots, v_{i_{q_{c_i}}}\}$. Let S_d be the set of all possible partial weights of m variable nodes that are transmitted through the channel, say with indices l_1, l_2, \dots, l_m such that $d_{l_1} + d_{l_2} + \dots + d_{l_m} = d$. Let S_p be the set of all possible partial weights of the remaining variable nodes in the protograph, namely the punctured variable nodes. Then the ensemble weight enumerator for the protograph LDPC code can be written as:

$$A_d = \sum_{\{d_k\} \in S_d} \sum_{\{d_j\} \in S_p} A_d \quad (3)$$

A. Computation of A_w^c for a check c with degree 3

Define $s = \frac{w_1 + w_2 + w_3}{2}$. If $w_1 + w_2 + w_3$ is even and $\max\{w_1, w_2, w_3\} \leq s \leq N$, then

$$A_{w_1, w_2, w_3} = \binom{N}{s} \frac{s!}{(s-w_1)!(s-w_2)!(s-w_3)!} \quad (4)$$

otherwise $A_{w_1, w_2, w_3} = 0$.

Proof: We use the following theorem.

Multinomial theorem:

$$\left(\sum_{i=1}^k a_i \right)^n = \sum_{\substack{n_1, n_2, \dots, n_k \geq 0 \\ n_1 + n_2 + \dots + n_k = n}} C(n; n_1, n_2, \dots, n_k) \prod_{i=1}^k a_i^{n_i} \quad (5)$$

where

$$C(n; n_1, n_2, \dots, n_k) = \binom{n}{n_1 \ n_2 \ \dots \ n_k} = \frac{n!}{n_1! n_2! \dots n_k!} \quad (6)$$

Multidimensional z-transform for A_{w_1, w_2, w_3}^c can be written as:

$$\begin{aligned} A^c(W_1, W_2, W_3) &= (1 + W_1 W_2 + W_1 W_3 + W_2 W_3)^N \\ &= \sum_{\substack{n_1, n_2, n_3, n_4 \geq 0 \\ n_1 + n_2 + n_3 + n_4 = N}} C(N; n_1, n_2, n_3, n_4) \\ &\quad \times 1^{n_1} (W_1 W_2)^{n_2} (W_1 W_3)^{n_3} (W_2 W_3)^{n_4} \\ &= \sum_{\substack{n_1, n_2, n_3, n_4 \geq 0 \\ n_1 + n_2 + n_3 + n_4 = N}} C(N; n_1, n_2, n_3, n_4) \\ &\quad \times W_1^{n_2 + n_3} W_2^{n_2 + n_4} W_3^{n_3 + n_4} \end{aligned} \quad (7)$$

To obtain A_{w_1, w_2, w_3}^c we need to set $n_2 + n_3 = w_1$, $n_2 + n_4 = w_2$, and $n_3 + n_4 = w_3$. Solving this set of equations we get $n_1 = N - s$, $n_2 = s - w_3$, $n_3 = s - w_2$, and $n_4 = s - w_1$, where $s = \frac{w_1 + w_2 + w_3}{2}$, provided that $w_1 + w_2 + w_3$ is even. Since $n_i \geq 0$ this implies that $\max\{w_1, w_2, w_3\} \leq s \leq N$ Q.E.D.

B. Computation of A_w^c for a check c with degree higher than 3

The partial weight enumerators for checks with degree higher than 3 can be obtained from the result for a check with

degree 3 by concatenation. For example A_{w_1, w_2, w_3, w_4}^c can be obtained as

$$A_{w_1, w_2, w_3, w_4}^c = \sum_{l=1}^N \frac{A_{w_1, w_2, l} A_{w_3, w_4, l}}{\binom{N}{l}} \quad (8)$$

The weight enumerators for higher degree checks can be obtained in a similar way.

III. ASYMPTOTIC ENSEMBLE WEIGHT ENUMERATORS

We express the normalized logarithmic asymptotic weight distribution of a code as $r(\delta) = \frac{\ln(A_d)}{n}$ where d is Hamming distance, $\delta = \frac{d}{n}$, and A_d is the ensemble weight distribution. One application of having $r(\delta)$ is to obtain an upper bound on minimum E_b/N_o threshold when maximum likelihood decoding is used. The bound has been derived in [7]. This bound is the tightest closed-form threshold on minimum E_b/N_o that can be written as

$$\left(\frac{E_b}{N_o}\right)_{min} \leq \frac{1}{R_c} \max_{\delta} (1 - e^{-2r(\delta)}) \frac{1 - \delta}{2\delta} \quad (9)$$

Let γ_{ml} represent the upper bound in (9) in dB. We use this maximum likelihood threshold in the examples to compare with the iterative decoding threshold for infinite block size which will be denoted by γ_{iter} in dB.

If the first zero crossing of $r(\delta)$ (i.e. $r(\delta_{min}) = 0$ for $\delta_{min} > 0$) exists, then it will indicate that the non-zero normalized minimum distance of the code grows linearly with the block size, i.e. $d_{min} = \delta_{min} \times n$.

However to make the expressions simple, we first define $\tilde{r}(\delta) = \frac{\ln(A_d)}{N}$ where $\delta = \frac{d}{N}$ as $N \rightarrow \infty$. Then $r(\delta)$ can be obtained as $r(\delta) = \frac{1}{m} \tilde{r}(m\delta)$, where m represents the number of variable nodes in the protograph that are connected to the channel and $n = mN$.

$$\tilde{r}(\delta) = \max_{\{\delta_k\} \in \mathcal{S}_\delta} \max_{\{\delta_j\} \in \mathcal{S}_\pi} \lim_{N \rightarrow \infty} \sum_{i=1}^{n_c} \frac{\ln A_{d_i}^{c_i}}{N} - \sum_{i=1}^{n_v} \frac{(q_{v_i} - 1) \ln \binom{N}{d_i}}{N} \quad (10)$$

Define $a^{c_i}(\delta_i) \triangleq \lim_{N \rightarrow \infty} \frac{\ln A_{d_i}^{c_i}}{N}$. Vector δ_i represents the normalized version of the vector d_i , where each component is normalized by N . Then

$$\tilde{r}(\delta) = \max_{\{\delta_k\} \in \mathcal{S}_\delta} \max_{\{\delta_j\} \in \mathcal{S}_\pi} \sum_{i=1}^{n_c} a^{c_i}(\delta_i) - \sum_{i=1}^{n_v} (q_{v_i} - 1) H(\delta_i) \quad (11)$$

where $H(x) = -(1-x) \ln(1-x) - x \ln x$ is the entropy function. The sets \mathcal{S}_δ and \mathcal{S}_π are normalized version of the sets \mathcal{S}_d and \mathcal{S}_p i.e. each component of the latter sets is divided by N as $N \rightarrow \infty$.

Let $\sigma = (\epsilon_1 + \epsilon_2 + \epsilon_3)/2$ such that $\max\{\epsilon_1, \epsilon_2, \epsilon_3\} \leq \sigma \leq 1$, and define

$$H_3(x_1, x_2, x_3) = -\left(1 - \sum_{i=1}^3 x_i\right) \ln\left(1 - \sum_{i=1}^3 x_i\right) - \sum_{i=1}^3 x_i \ln x_i \quad (12)$$

then

$$a^c(\epsilon_1, \epsilon_2, \epsilon_3) = H_3((\sigma - \epsilon_1), (\sigma - \epsilon_2), (\sigma - \epsilon_3)) \quad (13)$$

$$a^c(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4) = \max_{\lambda} \{a^c(\epsilon_1, \epsilon_2, \lambda) + a^c(\epsilon_3, \epsilon_4, \lambda) - H(\lambda)\} \quad (14)$$

where

$$\max\{|\epsilon_1 - \epsilon_2|, |\epsilon_3 - \epsilon_4|\} \leq \lambda \leq \min\{(\epsilon_1 + \epsilon_2), (\epsilon_3 + \epsilon_4), (2 - (\epsilon_1 + \epsilon_2)), (2 - (\epsilon_3 + \epsilon_4))\} \quad (15)$$

The asymptotic weight enumerators for higher degree checks can be obtained in a similar way.

A. Example 1

Consider a (3,6) regular LDPC code with a protograph as shown in Fig. 3(a).

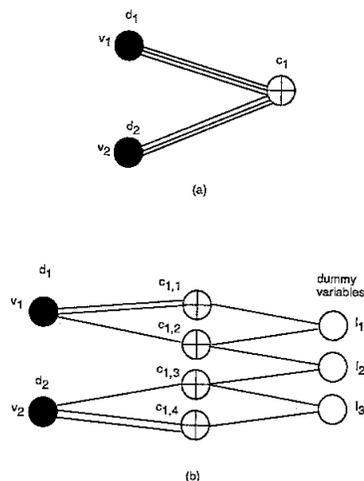


Fig. 3. (a) Protograph of a (3,6) regular LDPC code, (b) equivalent protograph for ensemble weight enumeration.

Using the result in (2), it is easy to show that

$$A_{d_1, d_2}^{c_1} = \frac{A_{d_1, d_1, d_1, d_2, d_2, d_2}^{c_1}}{\binom{N}{d_1}^2 \binom{N}{d_2}^2} \quad (16)$$

Using the results in previous and this section we can compute $A_{d_1, d_1, d_1, d_2, d_2, d_2}^{c_1}$ using (4), or we can use an equivalent protograph as shown in Fig. 3(b) with all degree 3 check nodes

to compute the ensemble weight enumerators. Using either approach we get

$$A_{d_1, d_2} = \sum_{l_1, l_2, l_3} \frac{A_{d_1, d_1, l_1}^{c_{1,1}} A_{d_1, l_1, l_2}^{c_{1,2}} A_{d_2, l_2, l_3}^{c_{1,3}} A_{d_2, d_2, l_3}^{c_{1,4}}}{\binom{N}{d_1}^2 \binom{N}{d_2}^2 \binom{N}{l_1} \binom{N}{l_2} \binom{N}{l_3}} \quad (17)$$

The asymptotic weight enumerators can be obtained as

$$\begin{aligned} \tilde{r}(\delta_1, \delta_2) = & \max_{\lambda_1, \lambda_2, \lambda_3} \{a^{c_{1,1}}(\delta_1, \delta_1, \lambda_1) + a^{c_{1,2}}(\delta_1, \lambda_1, \lambda_2) \\ & + a^{c_{1,3}}(\delta_2, \lambda_2, \lambda_3) + a^{c_{1,4}}(\delta_2, \delta_2, \lambda_3) \\ & - 2H(\delta_1) - 2H(\delta_2) \\ & - H(\lambda_1) - H(\lambda_2) - H(\lambda_3)\} \end{aligned} \quad (18)$$

$$\tilde{r}(\tilde{\delta}) = \max_{\delta_1} \tilde{r}(\delta_1, \tilde{\delta} - \delta_1) \quad (19)$$

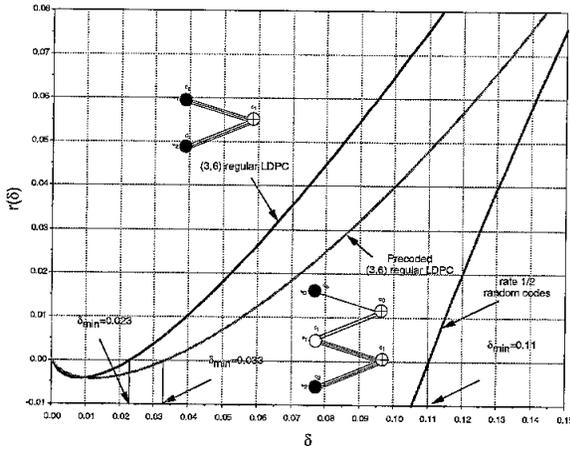


Fig. 4. Asymptotic normalized weight distributions and zero crossings for (3,6) regular LDPC, precoded (3,6) LDPC, and rate 1/2 random codes

IV. PRECODED LDPC CODES

LDPC codes with protograph structure can be precoded with an accumulator for better performance. Precoding places a degree 1 variable node between a constraint node and a higher degree variable (forming an accumulator) which is then optionally erased. Precoding often lowers the iterative decoding threshold of a given protograph without altering its rate [10]. Through examples we show that precoding also improves $r(\delta)$ and δ_{min} .

Next we investigate the weight enumerators for precoded LDPC codes. Consider an LDPC code with a protograph having n_v variable nodes that are connected to the channel. Let $A_{d_1, d_2, \dots, d_{n_v}}$ be the ensemble weight enumerators for partial weights $\{d_1, d_2, \dots, d_{n_v}\}$ for this protograph. Without loss of generality suppose we precode the variable node v_1 with an accumulator as shown in the Fig. 5. The node is disconnected from the channel (punctured) and it is connected to the output of an accumulator. The input of accumulator is

connected to the channel. Define the ensemble weight enumerators for the precoded version of the original protograph by $A_{d_0, d_1, d_2, \dots, d_{n_v}}^{pre}$, where d_0 is the partial weight of the variable node v_0 corresponding to the input of the accumulator.

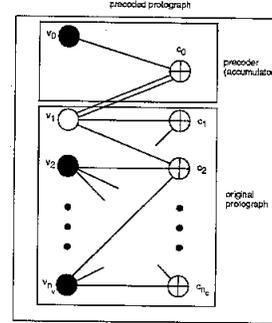


Fig. 5. Protograph of the precoded version of an LDPC code using an accumulator

Using the result in (2), it is easy to show that

$$A_{d_0, d_1, d_2, \dots, d_{n_v}}^{pre} = A_{d_1, d_2, \dots, d_{n_v}} \frac{A_{d_0, d_1, d_1}^{c_0}}{\binom{N}{d_1}^2} \quad (20)$$

However

$$\frac{A_{d_0, d_1, d_1}^{c_0}}{\binom{N}{d_1}^2} = \frac{\binom{N-d_1}{\frac{d_0}{2}} \binom{d_1}{\frac{d_0}{2}}}{\binom{N}{d_1}} \quad (21)$$

Note that $d_1 \geq \frac{d_0}{2}$, and d_0 is even. When $d_1 = \frac{d_0}{2}$ we can show that the right hand side of (21) is less than 1. For $d_1 > \frac{d_0}{2}$ we can upperbound the right hand side of (21) by

$$\frac{A_{d_0, d_1, d_1}^{c_0}}{\binom{N}{d_1}^2} \leq \theta(d_0, d_1) N^{-(d_1 - \frac{d_0}{2})} \quad (22)$$

where $\theta(d_0, d_1)$ does not depend on N . Thus the precoding introduces a so-called *interleaving gain*. Also for fixed d_0 and d_1 , there exist an N_1 such that for $N > N_1$ we have $\theta(d_0, d_1) N^{-(d_1 - \frac{d_0}{2})} < 1$. Thus

$$A_{d_0, d_1, d_2, \dots, d_{n_v}}^{pre} < A_{d_1, d_2, \dots, d_{n_v}} \quad (23)$$

The asymptotic result can be written as

$$\tilde{r}^{pre}(\delta_0, \delta_1, \delta_2, \dots, \delta_{n_v}) < \tilde{r}(\delta_1, \delta_2, \dots, \delta_{n_v}) \quad (24)$$

A. Example 2

Consider the (3,6) regular LDPC code in the example 1. This code has $\delta_{min}=0.023$, $\gamma_{ml} = 0.79$ dB, and $\gamma_{iter} = 1.1$ dB. However if we precode the (3,6) LDPC code with an accumulator, then we obtain $\delta_{min}=0.033$, $\gamma_{ml} = 0.31$ dB, and $\gamma_{iter} = 0.87$ dB. The asymptotic normalized weight distributions and zero crossing that specify the δ_{min} for the precoded (3,6) LDPC in this example, are shown in Figures 4.

B. Example 3

Consider Repeat Jagged Accumulate (RJA) and its precoded version Accumulate Repeat Jagged Accumulate (ARJA) LDPC codes [20]. The RJA code has $\delta_{min}=0.013$, $\gamma_{ml} = 0.83$ dB, and $\gamma_{iter} = 1.0$ dB. However if we precode the RJA LDPC code with an accumulator namely ARJA, then we obtain $\delta_{min}=0.015$, $\gamma_{ml} = 0.35$ dB, and $\gamma_{iter} = 0.628$ dB. The asymptotic normalized weight distributions and zero crossing for the RJA and ARJA LDPC codes are shown in Fig. 6.

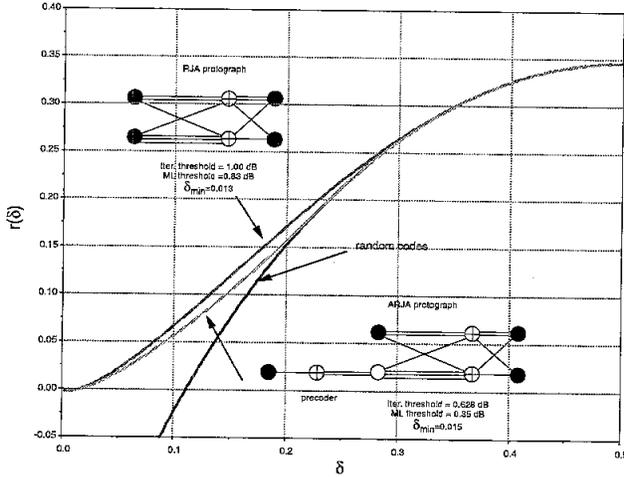


Fig. 6. Asymptotic normalized weight distributions for RJA LDPC code, precoded RJA (ARJA) LDPC code, and rate 1/2 random codes

C. Example 4

If we increase the degree 2 variable nodes to 2/3 of the inner checks in ARJA LDPC code we can further reduce the iterative decoding threshold at the expense of lowering the linearity coefficient that represent the growth of minimum distance with respect to n . An example of such code is shown in Fig. 7. Note that this code does not satisfy the relation $\lambda'(0)\rho'(1) < 1$ [14], where $\lambda(x)$, and $\rho(x)$ are the degree distributions for variable and constraint nodes. However, we note that for a protograph this condition is only a sufficient, but not a necessary, condition for minimum distance growing with n . Specifically, the ensemble asymptotic minimum distance over block size for this protograph is a small, but positive number $\delta_{min} = 0.004$.

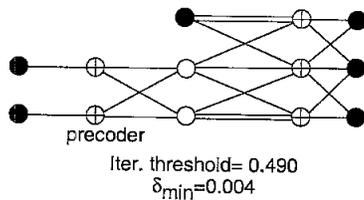


Fig. 7. Protograph of rate 1/2 ARJA with 5-checks (2/3 of inner checks are degree 2). The asymptotic minimum distance over block size for this code is $\delta_{min} = 0.004$

ACKNOWLEDGMENT

This research was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

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