Analysis of Calibration Errors for Both Short and Long Stroke White Light Experiments

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ABSTRACT

The SIM PlanetQuest mission can provide microarcsecond (μas) accuracy for exoplanet searches and critical astrophysical research. SIM is the only mission which can measure angular wobbles caused by planets for determination of planetary masses. In order to reach μas accuracy the SIM instrument must be able to measure fringe parameters to the accuracy of picometers. It is necessary to investigate calibration techniques and to carefully analyze influences from ghost images in white light fringe measurements. This work will analyze focusing and tilt variations introduced by thermal changes in calibration processes. In particular the accuracy limits are presented for common short- and long-stroke experiments. A new, simple, practical calibration scheme is proposed and analyzed based on the SIM PlanetQuest's Micro-Arcsecond Metrology (MAM) testbed experiments.

Keywords: space interferometry, white light fringe, bias calibration

1. INTRODUCTION

It is a challenge task to search extra-solar planets (exoplanet). Although we have detected Doppler shifts of more than hundred exoplanets[1], and have "seen" the lights from a few of them[2], the key parameter of exoplanets, mass, has not been directly measured. Besides, most of exoplanets discovered so far are very hot and big. The Space Interferometry Mission (SIM PlanetQuest) is the only instrument which can provide directly measured and accurate mass of exoplanets, and is the only technique which can detect Earth-like habitable planets. In order to find the signatures of Earth-like planets outside of solar system, it is necessary to have angular measurements with accuracy of μas, which corresponds to the delay measurements of precision at the picometer (pm) level.

For this unprecedented μas astrometric accuracy in space interferometer it is necessary to test and check critical technology requirements. SIM PlanetQuest task force designed and built the MAM testbed for this purpose. The MAM has worked for about ten years, and provides numerous valuable data to evaluate various error sources and instrument performance. One of the key tests is the white light fringe measurements, including both short and long strokes experiments. Fringe measurements include four parameters, i.e. fringe intensity, visibility, wavelength and fringe phase (or optical path delays). The long stroke (about 20 μm) measurements are used for wavelength calibration, and short stroke (around 1 μm) data are applied for phase calibrations. In order to reach pm level of measurement precision both calibrations need to be carefully investigated for systematic biases. Since the maximum value of fringe visibility is 1, and the unresolved point source can be used for the calibration of fringe visibilities. However, for the calibration of wavelength and phase there is no true and absolutely correct value available. Random errors of wavelength and phase determination can be improved by increasing number of measurements. The systematic biases of fringe parameter determination are the main limit to the SIM mission. It is difficult to find systematic biases, and is more difficult to quantify such biases. This report will discuss a "color" algorithm for fringe determination first in Section 2. The difference between assumptions of monochromatic light and polychromatic light is studied for estimation of fringe parameters. The difficult and important issue on accuracy of wavelength and phase measurements is investigated in later sections. Through measurements of Full-Aperture Metrology laser signals, the issue of accuracy of wavelength determination is discussed. In particular, several error sources which affect systematic biases, including ghost images, focusing and beam shear variation, are investigated in detail. Finally, we conclude that we need to carry out further study for all possible sources of systematic biases, which can not be improved by more measurements. In particular, cyclic error and nonlinearity of path length modulation need to be investigated.

2. "Color" Algorithm of Fringe Parameter Determination

MAM testbed is a ground-based system of optical and electronic equipment for testing components, systems, and engineering concepts for SIM PlanetQust. It is crucial to develop accurate fringe determination algorithm in order to achieve OPD measurement precision to within tens of picometers. For a two-element interferometer coherence function of light from two arms, $\gamma(x)$, is given by the Fourier transform of the spectral responses of light source and optical system[3]:

$$\gamma(x) = \int S(\omega)O(\omega)D(\omega)e^{-2\pi j\omega x}d\omega \qquad (1)$$

where x is the optical delay, $S(\omega)$ is the stellar spectral density, $O(\omega)$ is the spectral response of optical components, including mirrors, beam spreader, AR coating etc., $D(\omega)$ is the detector's response[4]. Since the interferometer instrument has limited wavelength coverage ($\Delta\omega$), if we assume that optics and detector had constant response over $\Delta\omega$, the monochromatic light will have fringe pattern as follows:

$$F(x,\omega) = F_0(1 + V\cos(2\pi\omega x + \phi_0)), \qquad (2)$$

where F_0, V and ϕ_0 are fringe amplitude, visibility and phase offset, respectively. It is obvious that the fringe parameters determined by least square technique from the formula above is just the first order of approximation to the true fringes. In the real world star's thermal spectrum depends on surface temperature, chemical composition, age etc., and all of them have many absorption and/or emission lines, and continuum of spectra are quite different for different targets. Spectral responses of optical components in MAM and SIM are all wavelength dependent. So detected fringe is expressed as:

$$F(x,\overline{\omega}) = F_0(1 + E(x,\overline{\omega})\cos(2\pi\overline{\omega}x - \phi(x)), \qquad (3)$$

where $\overline{\omega}$ is the effective wave number, $\phi(x)$ is the phase modulation of white light fringe, and $E(x,\overline{\omega})$ ($= |\gamma(x)|$) is the envelop function, which depends on the delay and wavelength. Here effective wavelength can be calculated as

$$\overline{\lambda} = \frac{1}{\overline{\omega}} = \frac{\int_{\Delta\lambda} \lambda S(\lambda)O(\lambda)D(\lambda)d\lambda}{\int_{\Delta\lambda} S(\lambda)O(\lambda)D(\lambda)d\lambda} \qquad (4)$$

where $\Delta\lambda$ is wider than the boundaries of detector's pixels, and is determined by the diffraction effects of the instrument. In the case of white light fringe estimation the coherence length of the fringe is very short.

By using spectral responses of instrument Figure 1 demonstrates that difference between white light fringe (red thick dotted line) and a cosine approximation (green thin line) for the current MAM testbed. The envelop function is indicated by dash lines in that figure. The phase of the cosine fringe is consistent with modulated fringe only near the central peak. It is evident that phase modulation of white light fringe does not linearly change with moving away from central peak. In fact the phase modulation must be expressed as

$$\phi(x) = 2\pi\overline{\omega}(d_0 + \Delta_d + \Delta_m + \Delta_p) \qquad (5)$$

where d_0 is the geometric delay, Δ_d is the correction of delay dispersions, Δ_m is the correction of optical modeling residual, and Δ_p is the correction of nonlinear path length modulations. Since a prism spectrometer is used in MAM and SIM, dispersions change with wavelengths smoothly. So the smoothness of dispersions can be used as a criterion for judgment of measurement accuracies.

The "color" algorithm here is based on color star light, not on the assumption of monochromatic light. By using star spectra and model of instrument response the envelop function can be estimated accurately. The correction for dispersions, modeling residuals, and nonlinearities of path length modulations are carefully determined. The effective wavelengths for different pixels must be determined to the required precision of a few picometers[5].

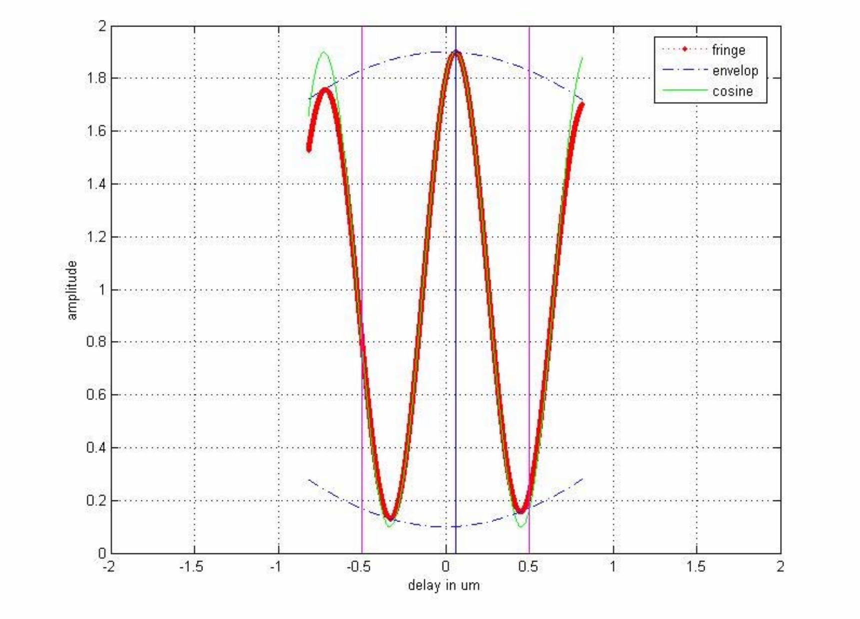

Figure 1 Comparisons between true fringe and cosine approximation

3. GHOST IMAGES IN THE WHITE LIGHT FRINGE

An interferometer includes various light sources and many kinds of optical components, such as mirrors, fibers, lenses, and beam splitters. It is inevitable that there are some unwanted scattering light, which produces ghost images on fringe detector. One scenario is as follows: two incident beams (A and B) transmit and reflect via beam splitter, and produce fringes on CCD. However, because of thickness and the coating on both sides of beam splitter, multiple reflections can bring ghost images to CCD. That ghost images bias the white light fringe parameters directly. One interesting experiment was made on MAM system. When the white light source on MAM was turned off, and the fringe signals were recorded continuously. We discovered that there exist white light fringes at 952 nm without the white light source[6], as shown in Figure 3. Also the FAM signal does exist on white light pixel 40. There is a long-pass filter in front of white light signals, and that filter suppose to block out any signal with wavelength short than 700 nm. Power spectral analysis in Figure 3 does show crosstalk over 33 pixels for FAM laser signals (659.5nm) on Pixels 6 and 7. More extensive experiments indicate that ghost WL signal comes from a green laser (532 nm) injected for alignment. It is thought that the fiber tip may produce a weak white light signal from strong green laser because all of the visible light is combined into one fiber.

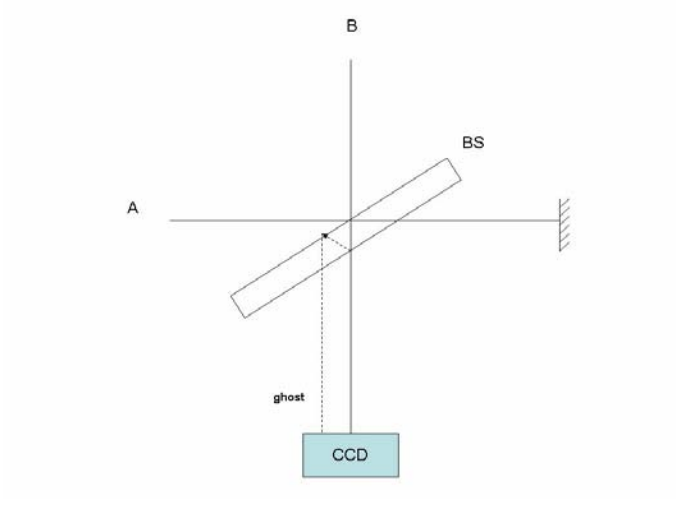

Figure 2 Ghost image produced by beam splitter

In order to check accuracy of wavelength determination we conducted a series of experiments on FAM's wavelength determination. FAM signal come from a 1319 nm NaYAG laser. It has extremely long coherence length, and is relatively easy for checking accuracy of wavelength measurements. FAM's signals are centered in the middle of pixel 6 and pixel 7 on MAM. Wavelengths determined from both pixels 6 and 7 are evaluated, and typical measurement stability is shown in Figure 4. As we can see from Figure 4, the Allan deviation of wavelength measurement stability is 76 pm at sampling time of 5.1 s. The blue straight line indicates the white light noise, which decreases with square-root of sampling time. The bump near 100s shows the existence of cyclic errors. The flat stability of 10 pm on sampling times greater than 700s demonstrates ultimate precision of wavelength determination for a single measurement.

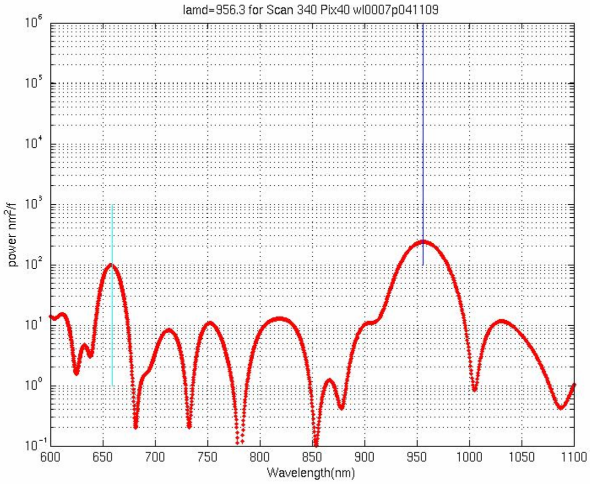

Figure 3 Ghost signals on Pixel 40 of MAM testbed

FAM's measured wavelengths for five runs are listed in Table 1. The average value of the above measurements is 659.586 ±0.002nm. By using commercial high resolution spectrometer the wavelength of FAM is determined as 1319.0852/2=659.543 nm[7]. So the difference is 43 pm. This systematic bias is 25 times more than the measurement precision of 2 pm. The important question is if this bias is true, or not?

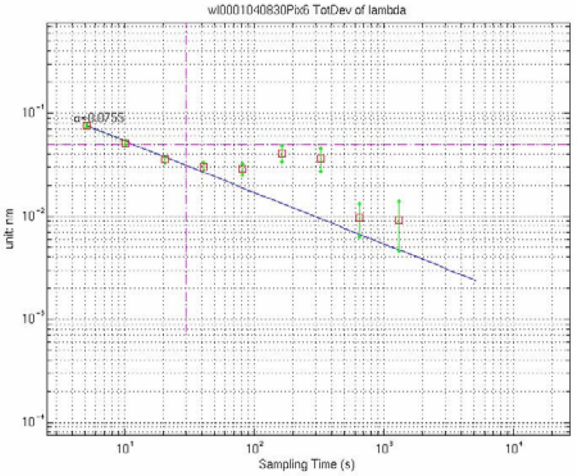

Figure 4 Stability of FAM wavelength determinations

In order to investigate the issue of wavelength bias we made special tests in MAM. In those tests the FAM and green laser signals are turned off, and the wavelengths on Pixel 6 and 7 are measured separately. The results are listed in Table 2. In the second column of each run the common corrections of FAM wavelength (-659.543 nm) are used. It is obvious that pixel 6 and 7 have had significant wavelength difference of 6.5 nm, and amplitude difference of 18%. By applying common correction of 659.543 nm the wavelength offsets are -2 nm and + 4.3 nm for pixels 6 and 7 respectively. We can not simply average the data from two pixels for FAM's wavelength.

The experiments above indicate that the fringes on Pixel 6 and 7 are combination of FAM and WL signals, and the wavelength corrections can be modelled as:

$$\varepsilon_i = \Delta_F * \Delta\lambda_i / \Delta, \qquad (6)$$

where Δ_F is the wavelength difference of FAM signals between Pixel 6 and 7, $\Delta\lambda_i$ is the difference between wavelength of pixel i and nominal wavelength of FAM signal, and Δ is the wavelength difference of white light signals between pixels 6 and 7. After adding corrections to the data in Table 1 estimated true wavelengths of FAM laser signal are presented in Table 3. From bias-corrected measurements the average wavelength of FAM signal is 659.556 ±

0.005 nm. Comparing with high resolution spectrometer's result of 659.543 nm, the difference now has been reduced to only 13 pm. From this example it is shown that measurement precision of FAM wavelength can reach 5 pm, and the original systematic offset is about 50 pm, and the accuracy of FAM wavelength determination can not be improved by averaging multiple runs from two pixels. Unless the physical reason from the influence of white light signals on those two pixels has been detected, the systematic bias can not be recognized. Difficult issues are how to find the physical reasons for biases, and how to model the systematic biases.

Table 1 Wavelength determination of FAM signals

wavelengths	Wl08p051114	Wl09p051114	Wl11p060124	Wl01p060215	Wl01p060228
Pixel 6	659.538±0.007	659.536±0.008	659.510±0.006	659.462±0.007	659.506±0.005
Pixel 7	659.630±0.007	659.632±0.009	659.650±0.007	659.781±0.007	659.612±0.005
Diff (nm)	0.092	0.096	0.140	0.319	0.106
Mean(nm)	659.584	659.584	659.580	659.622	659.559

Table 2 Wavelength determination on Pixels 6 and 7 without FAM signals

	Wl22p060215		Wl23p060215		Intensity
	measured	(-659.543)	measured	(-659.543)	
Pixel 6	657.495±0.061	-2.048 nm	657.288±0.071	-2.255 nm	340
Pixel 7	663.842±0.055	4.299 nm	663.778±0.053	4.235 nm	400
Diff	6.347 nm	6.347 nm	6.490 nm	6.490 nm	18%

Table 3 Wavelength determination of FAM signal with correction of WL signals

wavelengths	Wl08p051114	Wl09p051114	Wl11p060124	Wl01p060215	Wl01p060228
Pixel 6	659.565	659.567	659.557	659.568	659.541
Pixel 7	659.564	659.568	659.556	659.569	659.542
Diff (nm)	-0.001	0.001	-0.001.	0.001	0.001
Mean(nm)	659.564	659.567	659.557	659.568	659.541

4. FORCUSING AND TILT VARIATIONS FROM THERMAL CHANGES

Thermal stabilities of interferometric instrument play important roles for high accuracy measurement of fringe parameters. Wavelength dispersion and phase dispersion can be calibrated in high accuracy. However, optical elements and the whole system will change with environmental conditions. In particular the changes of focusing and tilt of the SIM spectrometer will directly affect the dispersion corrections. It is extremely important to find and to quantify such variations in focusing and tilt.

For MAM's prism spectrometer the linear dispersion is a function of index of refraction (n), focal length of imaging lens (F), and beam width (W):

$$\frac{dx}{d\lambda} = \frac{FL}{W}\frac{dn}{d\lambda} \tag{7}$$

where L is the base length. For fused silicon, index of refraction, n, is approximated by:

$$n^2 = 1 + \frac{A_1\lambda^2}{\lambda^2 - A_2} + \frac{B_1\lambda^2}{\lambda^2 - B_2} + \frac{C_1\lambda^2}{\lambda^2 - C_2} \tag{8}$$

where A_i, B_i, and C_i (i=1,2) are constants. By using the formulas above the theoretical wavelength dispersions are demonstrated in Figure 5. We can see that wave number differences among adjacent pixels are not constant, rather increase with decrease of wavelengths. Of course the intervals of wavelengths in different pixels may change as much twice as at two ends. On other hand, it is obvious that dispersion is a smooth function, i.e. wavelengths gradually change with pixels of CCD detector, with no wiggles, or jumps. Measured wavelengths, however, are complicated, and the wavelength of i-th pixel, λ_i, can be given by:

$$\lambda_i = \lambda_{i0} + \delta_{id} + \delta_{ic} + \delta_{if} \,, \tag{9}$$

where i = 1, … n, is the selected central part of n pixels, λ_{i0} is the nominal value of wavelength of i-th pixel, δ_{id} is the dispersion correction, δ_{ic} is the cyclic correction, and δ_{if} is fitting correction. Here δ_{id} come from theoretical model of fused silica prism spectrometer, and δ_{ic} and δ_{ic} can be obtained by retro mode measurements.

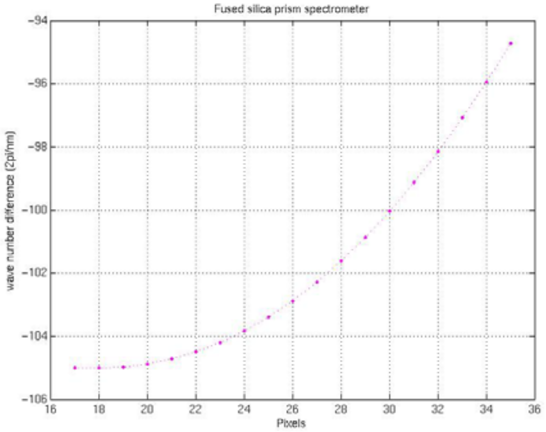

Figure 5 Wave number differences of fused silica prism spectrometer

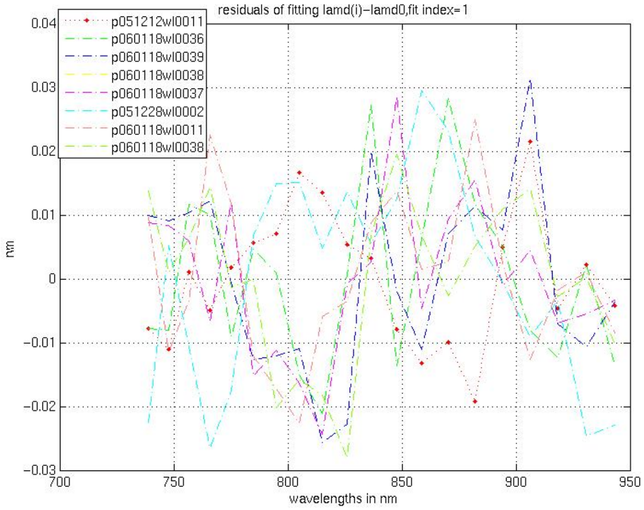

Figure 6 Residuals of wavelength determination

For the long stroke experiment, the stroke length of 23 µm is selected. There are several methods to analyze wavelengths of MAM spectrometer, including multi-parameter fitting, FFT, DFT, Pwelch, wavelet etc. Extensive experiments show that wiggles of wavelength determination in general is around a few hundred pm. That means the wavelength determination accuracy for a pixel with bandwidth of about 10 nm is only hundreds picometers, which is far away from required precision. By using models above it is necessary to make corrections for the raw wavelengths measurements. Dispersions of wavelengths on different days after correction are shown in Figure 6. It is noticed that the residuals of wavelength verses common nominal values demonstrate random fluctuations for different days and different wavelengths, and the rms value of wavelength fluctuations is about 20 pm. In fact that represents maximum accuracy limits of wavelength determination.

One more interesting test is the relative difference of wavelength dispersion. MAM spectrometer has 31 pixels available, which cover the range of wavelength from 670 to 970 nm. We can exclude a few pixels at the two ends, and make a fit to the dispersions. Relative wavelength dispersions are shown in Figure 7. We can see that relative dispersion differences are different on different days. In particular, out of two red vertical lines it is difficult to model the wavelength changes because there are some sudden changes which can not be repeated. Otherwise, it can be seen in Figure 7 that within the selected wavelength range those runs at the same day present close and parallel lines, which means no significant dispersion changes during a day. For some days, such as p051212, the dispersion has vertical shift of 0.5 nm, which indicates beam shear on detector. For some days the slops of dispersions have changed. That change of slope represents a focus change in the spectrometer. The MAM spectrometer has an imaging lens with focal length of 500 mm, and beam size of 45 mm. The changes of focusing are given by $f*\Delta_{slope}$, where f = 500 mm, typical relative changes of slope, $\Delta_{slope,}$ is 0.0004 in Fig. 7. So the focusing change is about 200 µm. In fact the depth of focus of MAM spectrometer is 390 µm. So focusing changes are within the depth of focus. However, such changes are not negligible for MAM's performance. Big slope change in Fig. 7 come from data on 11/14/2005 (p051114w0037). The reason is that the measurements are carried out before optical configuration changes.

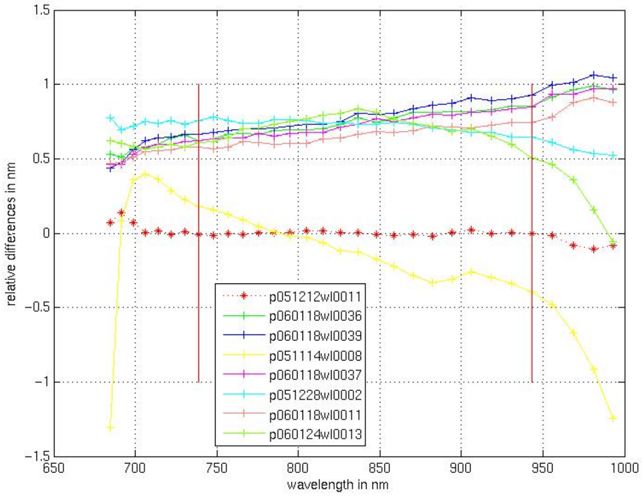

Figure 7 Relative wavelength dispersions

7. CONCLUSIONS

SIM PlanetQust is a high accuracy mission to search for Earth-like extra-solar planets and to conduct some unique and high resolution astrophysical researches. The required µas astrometric accuracy needs fringe parameter measurements of pm level precision. For both short and long stroke calibrations it is important to identify sources of systematic biases. Those biases are often mixed with measurement errors. Since measurement errors can be improved by increasing number of observations, the issue of accuracy is forgotten, and is neglected. It is difficult to recognize the systematic biases, and it is more difficult to model various biases. For the MAM system we discovered the ghost images by carefully analyzing long stroke calibration data. Even for the long coherence FAM signal it is easy to have tens of picometers offsets. It is easy to have hundreds of pm errors for white light channel's wavelengths. The results of this report indicate that wavelength determination of white light signal can have accuracy of 20 pm. For short stroke and phase measurements a "color" algorithm is presented. The key issue is to pay great attention to spectral responses of target, optics and detector. Significant biases of fringe parameters from assumption of monochromatic light have been pointed out. Besides, accuracy of effective wavelengths can not be neglected. By using long stroke experiments it has been shown that the focusing and tilt can be measured by using wavelength dispersion analysis. That technique is useful for the future examination of instrument thermal stabilities. We believe that there are other sources of systematic biases, such as cyclic error, nonlinear stroke length, among others. It is extremely critical to find those bias sources and to model their influences to the accuracy of fringe parameter measurements.

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