Black Hole Solar Systems

Extreme Mass Ratio Inspirals

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EMRIs as Black hole solar systems

\[ \frac{\mu}{M} \ll \frac{M}{M_{\odot}} \ll 10^2 \]

Mercury / Sun

\[ 10^{-7} \lesssim \frac{\mu}{M} \lesssim 10^{-3} \]

Jupiter / Sun

LISA Target

(\text{ignore spin of small body})

\[ 10^5 \lesssim \frac{M}{M_{\odot}} \lesssim 10^7 \]

light seconds \( \lesssim r \lesssim \) light minutes

\[ 1 \lesssim \frac{\mu}{M_{\odot}} \lesssim 10^2 \]
Data analysis

- Instantaneous strain is small compared to detector noise.
- Matched filtering for \(~\) year amplifies signal.
- Year-long matched filtering is computationally impractical (for EMRIs).
- Divide data into smaller segments - divide and conquer!
- Year 2013 computational budget allows segments of \(~\) few weeks.
- Gair et al: with this scheme, LISA can see \(~\)100’s to 1000’s of events out to \( z = 1 \).
Confusion
(from J. Gair’s talk, Capra meeting 2005)
Throwing the baby out with the bath water from Neil Cornish

$10^6 - 10^6 M_\odot$ black hole binary at $z = 1$
Throwing the baby out with the bath water from Neil Cornish

1,000 White Dwarf Binaries Later...

Some sort of “global fit” methods are needed

see Mock Data Challenge www.LISAscience.org

Can’t remove the Galactic “noise” to produce a “cleaned” data stream
Scales

\[ E \sim \mu \]

\[ \frac{dE}{dt} \sim \left( \frac{\mu}{M} \right)^2 \]

Light seconds \( \lesssim r \lesssim \) light minutes

\[ 1 \lesssim \frac{\mu}{M_{\odot}} \lesssim 10^2 \]
\[ 10^5 \lesssim \frac{M}{M_{\odot}} \lesssim 10^7 \]

\[ g_{\alpha\beta} = g_{\alpha\beta}^{\text{Kerr}} (M, a) + h_{\alpha\beta} \]

\[ h_{\alpha\beta} \sim \frac{\mu}{M} \]

\[ T_{\text{orbit}} \sim M \]

\[ T_{\text{radiation}} = \frac{E}{\frac{dE}{dt}} \sim \frac{M^2}{\mu} \]

\[ T_{\text{radiation}} \gg T_{\text{orbit}} \]
Inspiral: Adiabatic sequence of orbits

\[ T_{\text{radiation}} \gg T_{\text{orbit}} \]

\[ t = 0 \]

Kerr Geodesic
Inspiral: Adiabatic sequence of orbits

$T_{\text{radiation}} \gg T_{\text{orbit}}$

$t = T_{\text{radiation}}$

Kerr Geodesic
Inspiral: Adiabatic sequence of orbits

\[ T_{\text{radiation}} \gg T_{\text{orbit}} \]

\[ t = 2T_{\text{radiation}} \]
Inspiral: Adiabatic sequence of orbits

\[ T_{\text{radiation}} \gg T_{\text{orbit}} \]

\[ t = 3T_{\text{radiation}} \]
Expect any inclination and eccentricity

EMRI at $\sim (10 \text{ light days}) / 10^4$

|-10 light days-|
Orbit “shape” determined by three constants:

\[(r_{\text{min}}, r_{\text{max}}, \iota)\] or \[(e, p, \iota)\] or \[(E, L_z, Q)\] or...

Full description of orbit includes: \((t_0, r_0, \theta_0, \phi_0)\)
Orbit “shape” determined by three constants:

$$\left( r_{\text{min}}, r_{\text{max}}, \iota \right) \quad \text{or} \quad \left( e, p, \iota \right) \quad \text{or} \quad \left( E, L_z, Q \right) \quad \text{or}...$$

Full description of orbit includes: $$\left( t_0, r_0, \theta_0, \phi_0 \right)$$
Orbit “shape” determined by three constants:

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Full description of orbit includes: \[(t_0, r_0, \theta_0, \phi_0)\]
Orbit “shape” determined by three constants:

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Full description of orbit includes: \((t_0, r_0, \theta_0, \phi_0)\)
Generic Orbits

eccentricity = 1/3
inclination = 30 degrees
Generic Orbits

eccentricity = $1/3$
inclination = 30 degrees

eccentricity = $1/6$
inclination = 15 degrees
Orbit “shape” determined by three constants:

$$(r_{\text{min}}, r_{\text{max}}, \iota)$$ or $$(e, p, \iota)$$ or $$(E, L_z, Q)$$ or...

Full description of orbit includes: $$(t_0, r_0, \theta_0, \phi_0)$$
Orbit “shape” determined by three constants:

\( (r_{\text{min}}, r_{\text{max}}, \iota) \) or \( (e, p, \iota) \) or \( (E, L_z, Q) \) or...

Full description of orbit includes: \( (t_0, r_0, \theta_0, \phi_0) \)
Orbits in the frequency domain

\[ r(\lambda) = \sum_{n=-\infty}^{\infty} r_ne^{-in\gamma_r\lambda} \]

\[ \theta(\lambda) = \sum_{k=-\infty}^{\infty} \theta_ke^{-ik\gamma_\theta\lambda} \]

\[ \phi(\lambda) = \gamma_\phi\lambda + \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \phi_{kn}e^{-i(k\gamma_\theta+n\gamma_r)\lambda} \]

\[ t(\lambda) = \gamma_t\lambda + \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t_{kn}e^{-i(k\gamma_\theta+n\gamma_r)\lambda} \]

\[ \omega_{r,\theta,\phi} = \frac{\gamma_{r,\theta,\phi}}{\gamma_t} \]
Orbits in the frequency domain

\[ r(t) = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} r_{kn} e^{-i(k\omega_\theta + n\omega_r)t} \]

\[ \theta(t) = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \theta_{kn} e^{-i(k\omega_\theta + n\omega_r)t} \]

\[ \phi(t) = \omega_\phi t + \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \phi_{kn} e^{-i(k\omega_\theta + n\omega_r)t} \]

\[ \omega_{mkn} = m\omega_\phi + k\omega_\theta + n\omega_r \]
"spectral method" for geodesics
Waveforms

- Radiation from fixed geodesic - problem solved.
- For “inspiral as a sequence of geodesics” nearly solved at level needed to realize event rates predicted by Gair et al.
- Kludge waveforms: “quadrupole formula” for test particle on “inspiraling geodesic”.
- Teukolsky waveforms: first order (radiative) black hole perturbation theory.
- Capra waveforms: leading order radiative and conservative perturbation theory.
Kludge waveforms

- post-Newtonian (or better) approximation for fluxes

\[
\frac{dE}{dt} = \frac{dE}{dt}(E, L, Q) \implies E(t), L(t), Q(t)
\]

- Integrate “geodesic equation” (or better) for world line

\[
\frac{dx^\alpha}{d\tau} = u^\alpha[E(t), L_z(t), Q(t)] \implies x^\alpha(\tau)
\]

- Quadrupole formula (or better) for waveform

\[
h_{ij}(x^\alpha_{\text{field}}) = W_{ij}[x^\alpha_{\text{field}}, x^\alpha(\tau)]
\]
Teukolsky waveform snapshots

\[ h_+ - i h_\times = \frac{1}{r} \sum_{lmkn} \frac{Z_{lmkn}^H}{(-\omega_{mkn}^2)} S_{lmkn}(\theta) e^{im\phi} e^{-i(m\omega_\phi + k\omega_\theta + n\omega_\rho)t} \]

\[
\langle \frac{dE}{dt} \rangle = \sum_{lmkn} \frac{1}{4\pi \omega_{mkn}^2} \left( |Z_{lmkn}^H|^2 + \alpha_{lmkn} |Z_{lmkn}^\infty|^2 \right)
\]

\[
\langle \frac{dL_z}{dt} \rangle = (\text{similar}) \quad \omega_{mkn} = m\omega_\phi + k\omega_\theta + n\omega_\rho
\]

\[
\langle \frac{dQ}{dt} \rangle = (\text{similar}) \quad \text{see Sago et al gr-qc/0511151, or Drasco et al gr-qc/0505075}
\]
Teukolsky Waveforms

\[ h_+ - i h_\times = \frac{1}{r} \sum_{lmkn} \frac{Z^H_{lmkn}}{(-\omega^2_{mkn})} S_{lmkn}(\theta) e^{im\phi} e^{-i(m\omega_\phi+k\omega_\theta+n\omega_r)t} \]

\[ \left\langle \frac{dE}{dt} \right\rangle = \sum_{lmkn} \frac{1}{4\pi\omega^2_{mkn}} \left( |Z^H_{lmkn}|^2 + \alpha_{lmkn} |Z^\infty_{lmkn}|^2 \right) \]

\[ \left\langle \frac{dL_z}{dt} \right\rangle = \text{(similar)} \]

\[ \left\langle \frac{dQ}{dt} \right\rangle = \text{(similar)} \]

\[ Z^H,\infty : \text{spectral coefficients for solutions to the Teukolsky equation} \]

see Sago et al gr-qc/0511151, or Drasco et al gr-qc/0505075
\( a = 0.9M, \ e = 0.3, \ p = 12, \ i = 140 \text{ deg.}, \ \text{viewed from } \theta = 60 \text{ deg.} \)

\( a = 0.9M, \ e = 0.7, \ p = 6, \ i = 60 \text{ deg.}, \ \text{viewed from } \theta = 90 \text{ deg.} \)

\[ \frac{(r/M) h_+}{(t-r)/M} \]

\( M_\odot \approx 1 \text{ km } \approx 5 \mu s \)

Reminder:

\( M_\odot \approx 1 \text{ km } \approx 5 \mu s \)
$h_{\text{template}} = \cos[2(\phi_0 + \omega_0 t)]$

$h_{\text{true}} = \cos \left[ 2 \left( \phi_0 + \omega_0 t + \frac{1}{2} \dot{\omega}_0 t^2 \right) \right]$

$\omega_0 \sim \frac{1}{M}$ and $\dot{\omega}_0 \sim \frac{1}{M^2}$
$h_{\text{template}} = \cos[2(\phi_0 + \omega_0 t)]$

$h_{\text{true}} = \cos \left[ 2 \left( \phi_0 + \omega_0 t + \frac{1}{2} \dot{\omega}_0 t^2 \right) \right]$
Simple scaling argument
from Glampedakis and Babak (7 M < r < 20 M)

\[ \frac{M}{\mu} \frac{1}{2} \]

[numerical estimate]
from Glampedakis and Babak  (7 M < r < 20 M)

\[ T_{\text{snap}} \propto \sqrt{\frac{5\pi}{108}} \left( \frac{r}{M} \right)^{11/4} \left( \frac{M}{\mu} \right)^{1/2} \]

\[ (M/\mu)^{1/2} \]
While at KITP: new catalog of 2,000 snapshots

http://gmunu.mit.edu/sdrasco/snapshots

| black hole spin | = 0.8 M²

Power radiated to infinity / (μ/M)²
Power extracted from horizon / (μ/M)²

While at KITP: new catalog of 2,000 snapshots

http://gmunu.mit.edu/sdrasco/snapshots

| black hole spin | = 0.8 M²
While at KITP: new catalog of 2,000 snapshots
http://gmunu.mit.edu/sdprasco/snapshots

black hole spin \( I = 0.8 M^2 \)

\[
\frac{\text{radiation torque}}{(\mu^2/M)} = \frac{(\text{tidal torque})}{(\mu^2/M)}
\]
Summary

- Waveforms known well enough to detect some EMRIs today
- Soon, enough to realize Gair et al estimate of \( \sim 100\)'s to 1000's of detections to \( z = 1 \)
- Not yet enough to for precision parameter estimation of Barack and Cutler (mass and spin to \( 10^{-4} \))
- Some turning to the more exotic: non-Kerr background, gas interaction, third body, ...
- More status and refs: Drasco, gr-qc/0604115