Weak-Signal Phase Calibration Strategies for Large DSN Arrays

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Abstract—The NASA Deep Space Network (DSN) is studying arrays of large numbers of small, mass-produced radio antennas as a cost-effective way to increase downlink sensitivity and data rates for future missions. An important issue for the operation of large arrays is the accuracy with which signals from hundreds of small antennas can be combined. This is particularly true at Ka band (32 GHz) where atmospheric phase variations can be large and rapidly changing. A number of algorithms exist to correct the phases of signals from individual antennas in the case where a spacecraft signal provides a useful signal-to-noise ratio (SNR) on time scales shorter than the atmospheric coherence time. However, for very weak spacecraft signals it will be necessary to rely on background natural radio sources to maintain array phasing. Very weak signals could result from a spacecraft emergency or by design, such as direct-to-Earth data transmissions from distant planetary atmospheric or surface probes using only low gain antennas. This paper considers the parameter space where external real-time phase calibration will be necessary, and what this requires in terms of array configuration and signal processing. The inherent limitations of this technique are also discussed.

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1. Introduction

The Deep Space Network (DSN) is designing arrays composed of hundreds of mass-produced, small-diameter parabolic radio antennas which can increase the sensitivity available for downlink telemetry by providing a large increase in total collecting area ([1], [6]). This large array approach is a cost-effective way to increase downlink telemetry capabilities by 1-2 orders of magnitude over the current DSN. However, to take advantage of this increase in sensitivity it is necessary to calibrate the array (or a sub-array involving some fraction of the array antennas) and to maintain accurate calibration over a wide range of weather and spacecraft signal strength conditions.

The primary objective of array calibration is to allow the coherent addition of signals from all antennas in one or more sub-arrays. This involves the measurement and correction of delay and phase errors from multiple instrumental and propagation media sources, but the most significant error is expected to be differential path length variations caused by fluctuations in atmospheric water vapor along the lines of sight from different antennas. This effect is more severe at higher frequencies.

Other objectives of array calibration include accurate sky position measurements (astrometry), accurate estimates of received signal strength (radiometry), and real-time indicators of the over-all array performance.

2. Deep Space Network Large Arrays

The current plan for DSN arrays is to have approximately four hundred 12-meter diameter antennas in each of three longitude ranges, corresponding to the general longitudes of the existing three DSN sites in California, Australia, and Spain. Within each longitude range, the array antennas may be divided into two clusters separated by a few hundred km, or all array antennas may be located in a single cluster. Each cluster will have a compact configuration extending over an area of approximately one square km (see figure 1).

The main requirement of the array is to provide at least an order of magnitude increase in sensitivity for telemetry downlink compared to the current DSN antennas. Other DSN services such as spacecraft navigation (Δ-DOR, Doppler, and range measurements) will benefit from the increased sensitivity, but it is the need for much higher downlink data rates that is driving the array design.

In addition to improved performance (sensitivity), arrays of many small antennas have two important operational advan-
FIGURE 1. Artist concept of a small portion of a DSN array cluster of 12-meter antennas.

The first of these (geometric delay) can be determined through periodic observations of a number of radio sources with well-known positions. From the time variation in baseline phases it is possible to solve for the relative positions of the array antennas (e.g., section 12.2 in [15], [11], [17]). More precisely, one can solve for the vectors between the phase centers of the antennas. This calibration is needed only when the array geometry changes, as might occur due to mechanical work on an antenna or due to local ground motion.

The second set of calibration parameters (instrumental and signal transport delays from each antenna) can be calibrated through signals injected into the front end of the antennas. A well designed system should exhibit slowly varying instrumental delay and phase responses.

The third set of parameters (propagation delays) are the most important because they can vary by large amounts on short time scales. Both the interplanetary medium and the ionosphere produce delays that scale as wavelength squared, and thus they can be calibrated with simultaneous observations at widely spaced frequencies. Moreover, the operating frequencies of the DSN arrays will be X-band (8 GHz) and Ka-band (32 GHz). These frequencies are high enough that plasma effects will not be very important.

The troposphere is another matter. Significant fluctuations in atmospheric water vapor content, and consequently variations in the index of refraction and the line-of-sight delay, can occur on spatial scales as small as 100-200 meters. (For examples of atmospheric phase fluctuations at a relatively good observing site, see [3], [13], and [14].) Thus, antennas within a single array cluster could see very different tropospheric delays when looking in the same direction. For each antenna, these delays will change on time scales given by the spatial scale of water vapor fluctuations divided by the typical wind speed at the altitude of maximum water vapor (the lower few km of the atmosphere). This can be as short as a few seconds. Calibration of this changing delay is possible with precise water vapor radiometry [8], but for an array with many antennas it will be much more cost effective to use data from the array antennas for calibration instead of data from a separate and complex media calibration system.

For interferometry, it is the difference between antenna phases (the baseline phases) that are relevant. An array can be considered coherent if the rms baseline phase errors after calibration are much less than one radian. Note that shorter baselines will generally have higher coherence because the atmospheric phase variations will be more highly correlated and thus will cancel more completely. Because the phase error associated with a given delay error is proportional to frequency, the effects of imperfect troposphere delay calibration will be four times more severe at Ka band than at X band.

For a phase coherent array, the sensitivity of the combined output signal will be very nearly the sum of the sensitivity of

3. ARRAY CALIBRATION SCENARIOS

The basic parameters that need to be calibrated in order to make an array coherent (or "phased up") are

1) the geometry of the array antennas with respect to the direction of the spacecraft signal,

2) the total signal delay from each antenna to the signal combining point, and

3) the signal delay through the interstellar medium, the ionosphere, and troposphere.
all the individual antennas being used. In general, the coherence of an array will decrease with time until a new set of phase corrections is determined and applied. The coherence time is defined as the time until residual phase errors equal one radian.

The primary challenge of array calibration is to obtain accurate, time-varying antenna phase corrections continuously, or at least more frequently than the phase errors change significantly, during an observation. This can best be done by using the known properties of a received signal (for example, it is angularly unresolved and from a known direction) to determine antenna corrections that are most consistent with our a priori knowledge. The signal/noise ratio (SNR) of the received signal, usually from a distant spacecraft, determines the detailed calibration approach that must be used.

We will consider two cases: High spacecraft SNR and low spacecraft SNR. By low SNR, we mean that the spacecraft signal is sufficiently weak that real-time phase calibration using this signal alone is not possible.

**High Spacecraft SNR Case**

In the case of a spacecraft signal that is strong enough to be detected with at least unity SNR with a single array antenna in a time shorter than the atmospheric coherence time, we can use the spacecraft signal itself to determine and maintain array phasing. This is the best possible case, because the corrections are determined for exactly the line of sight we are interested in.

The maximum phase correction rate depends on the time required to obtain useful SNR on the spacecraft signal. If this time is short, the effects of the atmosphere can be almost perfectly removed and phase coherent observations can continue indefinitely. A number of algorithms have been developed for calibration in this regime (e.g., [9], [10], [16]). A well-known example is the iterative SUMPLE algorithm (see Chapter 8 in [12]).

An important practical advantage of these algorithms is that most do not require cross-correlation of signals for all array baselines. For an array with N antennas the number of baselines is N(N-1)/2, which implies a large hardware processor if N is large and cross-correlation coefficients for all baselines are needed.

**Low Spacecraft SNR Case**

There are many possible causes of low SNR:

- intrinsically weak signals (extremely distant or low-power spacecraft)
- spacecraft emergency (omnidirectional antenna)
- inaccurate knowledge of spacecraft position
- atmospheric attenuation at Ka band (rain)
- phased array needed at the start of a spacecraft track (no time to phase up on spacecraft signal before some critical event)

We must be able to deal with these situations. The way to do this is to use distant (background) compact radio sources to determine the array phase corrections, and then apply those corrections to the spacecraft signal beamforming process.

### 4. Phasing with Background Radio Sources

The best situation is when we have a strong enough background radio source for array phasing located within the primary beams of the array antennas. This gives us the same advantage as the high spacecraft SNR case, but requires a full cross-correlator to provide baseline phases for the background source. The antenna phase corrections derived from the baseline phases can be applied directly to the spacecraft signal beamforming. The spacecraft signal itself need not be detectable prior to the beamformer (coherent addition) output from the whole array. Section 5 considers the question of how strong a radio source is needed for this approach to work.

In most situations we will not have a sufficiently strong source in the primary beam area. Now we must use a subarray of antennas to observe a strong source a degree or two away, and interpolate the phase corrections from this subarray to the remaining antennas that are observing the spacecraft (see figure 2). The accuracy of the (spatial) phase interpolation will depend strongly on the typical distance between sub-array (calibration source) and main array (spacecraft tracking) antennas. Phase interpolation will be robust for antenna separations less than 100 meters, even at Ka band. This implies a compact cluster configuration (e.g., [7]). It also suggests that there will be a practical limit on the number of simultaneous sub-arrays.

This approach requires only a cross-correlator large enough to handle the calibration sub-array antennas, not the entire array.

### 5. Feasibility of Approach

The feasibility of any calibration strategy is determined mainly by the sensitivity of the array and the strength of the signal being used for calibration. Figure 3 illustrates the various regimes of array phase calibration, as a function of the sensitivity of the array (or sub-array) being used and the atmospheric phase stability. This is most relevant for Ka band, where atmosphere stability is expected to be the dominant source of phase errors.

The regime labeled "self-calibration using spacecraft telemetry" in figure 3 is the standard mode of array operation for high SNR spacecraft signals. The signal processing bandwidth needs to be only as wide as the spacecraft signal bandwidth.
The regime labeled "self-calibration using integrated spacecraft signal" uses cross-correlation of the spacecraft signal integrated for at least several seconds to determine phase corrections. The size of this regime depends only on the spacecraft signal strength (Jy), independent of bit rate or coding. It does not require a bandwidth larger than the spacecraft signal bandwidth.

The "in-beam phase referencing" regime is the next best option when the spacecraft signal is too weak to be used directly for array phasing. This regime requires high sensitivity to continuum radio sources to allow a (relatively weak) source within the primary antenna beams to be used. This translates into a requirement for a wide signal path and correlator bandwidth. The array phasing in this regime is independent of the spacecraft SNR.

Finally, "external phase referencing" requires a separate small sub-array of antennas to continuously observe a phase reference source that is outside of antenna primary beams, but within a couple of degrees of the spacecraft position. For angular separations this small, the lines of sight to the reference source and spacecraft pass through nearly the same region of the troposphere and thus have highly correlated atmospheric phase errors. In this regime the array phasing is independent of spacecraft SNR, but is more sensitive to atmospheric conditions. A wide-bandwidth correlator is also required, but only for the number of antennas in the sub-array observing the reference source.

Clearly, in the limit of very poor weather conditions (very short coherence times) the only viable options involve very strong spacecraft signals. Fortunately, such conditions are rare at good (high, dry) observing sites.

Based on the current plan for DSN arrays, we assume a densely-packed cluster of 200 parabolic antennas, each with a diameter of 12 meters, an aperture efficiency of 70%, and a total system temperature of 30K at X-band and 50K at Ka-band. The maximum IF bandwidth is 500 MHz for each of two orthogonal polarizations. A cross-correlator with a bandwidth of 500 MHz and multiple beamformers with bandwidths of 100 MHz each are expected to be available.

To see how this approach might work in practice, we first need to determine how strong a background radio source is needed for array calibration. The rms error in antenna gain measurements ($\sigma_{\text{gain}}$) for an array of $N$ identical antennas and a global least-squares solution is given by

\[
(\sigma_{\text{gain}})^2 = (\sigma^2/S^2)/(N - 2)
\]

for phase solutions only, and

\[
(\sigma_{\text{gain}})^2 = (\sigma^2/S^2)/(N - 3)
\]
for both amplitude and phase solutions, where \( \sigma = \) measurement error in Jy and \( S = \) reference source flux density in Jy [15]. The single-baseline rms phase error in radians is \( \sigma_{\phi} = 1/\text{SNR} \). Thus, for an array or sub-array of at least 200 antennas we will have an rms phase error of less than 6 degrees if the reference source produces an SNR of at least 0.7 on single baselines. An rms phase error of \( \sigma_{\phi} = \) 6 degrees for the array would produce a sensitivity loss of only

\[
e^{-\frac{(\sigma_{\phi})^2}{2}} = 0.005 = 0.02 \text{dB.}
\]

The rms thermal noise on a single baseline is given by [15]

\[
\frac{\sqrt{2} k_B T_{sys}}{A \sqrt{\Delta \nu \Delta \tau}}
\]

where \( k_B \) is Boltzmann's constant, \( T_{sys} \) is the total system temperature, \( A \) is the effective area (the total geometric collecting of both antennas times the aperture efficiency), \( \Delta \nu \) is the bandwidth, and \( \Delta \tau \) is the coherent integration time. We are ignoring small loss factors (of order unity) associated with digitizing and cross-correlation of the signals, and we assume that the incoming signals in unpolared.

Using our assumed values for system temperature, collecting area, efficiency, bandwidth, and an integration time of 10 seconds, we get a noise level of \( 5 \times 10^{-22} \) W m\(^2\) Hz\(^{-1}\), or 5 mJy. Thus, to get a single-baseline SNR of 0.7 in 10 seconds we need a source flux density of 3.5 mJy. By using both polarizations this can be reduced to 2.5 mJy.

What is the average angular separation between radio sources whose flux density is at least 2.5 mJy? At X-band the density of sources this strong is approximately 0.002 sources per square arcminute [4]. The average distance between sources is proportional to the inverse square root of the source density, and for the density given here the distance between sources is about 24 arcminutes. Thus, from a random position on the sky there will be a 2.5 mJy radio source about 12 arcminutes away. The half-power beam width of a 12-meter diameter antenna is about 10 arcminutes at X-band, so on average we will have less than one radio source strong enough for phase calibration within the primary beam. The conclusion is that we can calibrate the array at X-band using an in-beam background radio source some of the time, but not most of the time.

At Ka-band we can extrapolate source density from measurements made at higher frequencies and higher flux densities [2]. The average distance to a source scales as the \( 3/4 \) power of its flux density, but the scaling with frequency is less well known because different source populations dominate at lower frequencies (flat or steep spectrum synchrotron sources) and higher frequencies (inverted spectrum thermal sources). If we assume a flat spectrum on average between 32 GHz and 90 GHz, we find that the distance to a 2.5 mJy source is about 7 arcminutes. However, the antenna beamwidth is only about 2.5 arcminutes at Ka band, so in-beam calibration will be possible far less often than at X-band.

Recent work by Garrett, Wrobel, and Morganti [5] has suggested that array self-calibration using the combined response from a number of sub-mJy radio sources should be possible. This would allow in-beam calibration in any direction, although this approach places challenging demands on the time and frequency resolution of the cross-correlator. Despite the appeal of using such weak (and numerous) radio sources for calibration, the data processing requirements probably preclude this approach for real-time array calibration. Consequently we will not consider it further in this paper.

For external phase referencing we are in much better shape. The loss of sensitivity caused by using only a subset of array antennas to observe the calibration source is more than made up by the ability to find much stronger sources within 1-2 degrees of any position on the sky. Experience with the VLA at 22 and 43 GHz shows that phase fluctuations are highly correlated between antennas separated by up to 100 meters, and between lines of sight separated by up to a few degrees. These are exactly the conditions we need for external phase referencing. Sub-array based calibration will work for the full array as long as the typical distance between calibrating antennas is not much more than 100 meters and the calibrating antennas are located throughout the main array.

6. CONCLUSIONS

A large DSN array will often have a strong enough radio source for phase calibration within the primary antenna beams at X-band. This will allow all antennas to observe the spacecraft continuously and avoid interpolation errors. The full sensitivity of the array is available for detection of the spacecraft signal, no matter how weak it is.

However, in-beam calibration sources will not always be available at X-band, and will almost never be available at Ka-band — where there are fewer strong sources available and the primary beam area is 16 times smaller. In these situations the phase interpolation technique described here will allow array phasing in the spacecraft direction with some loss of sensitivity due to the need to have some antennas observing in a slightly different direction and to imperfect phase corrections.

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