Evolutionary Computing for Low-Thrust Navigation

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The development of new mission concepts requires efficient methodologies to analyze, design and simulate the concepts before implementation. New mission concepts are increasingly considering the use of ion thrusters for fuel-efficient navigation in deep space. This paper presents parallel, evolutionary computing methods to design trajectories of spacecraft propelled by ion thrusters and to assess the trade-off between delivered payload mass and required flight time. The developed methods utilize a distributed computing environment in order to speed up computation, and use evolutionary algorithms to find globally Pareto-optimal solutions. The methods are coupled with two main traditional trajectory design approaches, which are called direct and indirect. In the direct approach, thrust control is discretized in either arc time or arc length, and the resulting discrete thrust vectors are optimized. In the indirect approach, a thrust control problem is transformed into a costate control problem, and the initial values of the costate vector are optimized. The developed methods are applied to two problems: 1) an orbit transfer around the Earth and 2) a transfer between two distance retrograde orbits around Europa, the closest to Jupiter of the icy Galilean moons. The optimal solutions found with the present methods are comparable to other state-of-the-art trajectory optimizers and to analytical approximations for optimal transfers, while the required computational time is several orders of magnitude shorter than other optimizers thanks to an intelligent design of control vector discretization, advanced algorithmic parameterization, and parallel computing.

Nomenclature

a	=	semimajor axis
e	=	eccentricity
i	=	inclination
ω	=	argument of the periapsis
Ω	=	longitude of the ascending node
$\boldsymbol{u}$	=	control vector
$\boldsymbol{x}$	=	state vector
$\boldsymbol{\lambda}$	=	costate vector
t	=	time
dt	=	time step
f, p	=	generic functions
J	=	performance index
ψ	=	boundary condition

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I. Introduction

THIS paper addresses the problem of finding optimal orbit transfers for low-thrust spacecraft. A common goal for the optimization problem is to find the minimum-time, minimum-fuel, or Pareto-optimal trajectories, where the Pareto-optimality means that no other solutions are superior to them in terms of both flight time and fuel consumption. Particularly in an early mission design phase, where maximizing the deliverable payload mass is an equally attractive mission objective as minimizing time of flight, the Pareto optimal solutions that demonstrate the trades between flight time and deliverable payload mass are desired. In general, these optimization problems are difficult to solve due to not only the long transfer time and multi-revolutionary transfer but also the trade search for Pareto optimality.

Various methods have been used to solve this optimization problem. A majority of the work has utilized either direct or indirect methods.[1] The direct method approaches the problem by adjusting the control variables iteratively to reduce the performance index. The continuous control and state variables are often discretized, which results in a nonlinear programming. The indirect method, on the other hand, use the calculus of variations to transform the optimal control problem into a set of necessary conditions for a local optimum solution for the objective function. The transformation leads to a two-point boundary-value problem: state and costate variables at the initial time and at the final time.

For the nonlinear programming problem in the direct method and the two-point boundary-value problem in the indirect method, a gradient-based local optimization algorithm such as Newton's method and sequential quadratic programming algorithm has been a popular choice thanks to the maturity of the algorithm. However, the traditionally algorithm finds a locally optimal solution in the vicinity of the initial guess, and becomes unstable when the objective function is rugged and the function gradient is discontinuous. The trajectory optimization problem tends to have many local optimal solutions which makes difficult to find the globally optimal solution. Furthermore, the traditional optimization algorithm does not directly handle the multi-objective problem but converts it into a single scalar objective, so called weighting method. The resulting solution is highly sensitive to the weighting factors and is a single solution rather than a set of Pareto-optimal solutions. Therefore, the low-thrust orbit transfer problem calls for a more robust global and Pareto optimization algorithm.

This paper presents a global and Pareto optimization method to solve the nonlinear programming problem of the direct method and the two-point boundary-value problem of the indirect method for the low-thrust orbit transfer optimization. The present method consists of two global-search algorithms: a genetic algorithm and simulated annealing. Both algorithms do not require the objective function gradient and thus are likely to find a globally optimal solution in a rugged search domain, as opposed to the gradient-based algorithms. Additionally, the genetic algorithm takes advantage of the population-based search to directly solve the multi-objective optimization problem in a single run.

II. Methodologies

The following section defines the problem of the low-thrust trajectory optimization in a general way and describes the present method developed for the optimization problem.

A. Low-Thrust Trajectory Optimization Problem

Electric propulsion systems are one of the most efficient propulsion systems available for space missions. These systems use less propellant and thereby require a less massive spacecraft and a smaller launch vehicle. This propellant efficiency makes the electric propulsion system attractive for a budget-sensitive space program. The thrust provided by the electric propulsion system is relatively small, typically on the order of fractions of one Newton. Therefore, any significant maneuver of a spacecraft with the electric propulsion system requires continuous thrust over long periods of time. This makes the low-thrust trajectory optimization more challenging than the chemical-propulsion spacecraft trajectory optimization where a few impulsive maneuvers need to be optimized.

The equation of motions for a spacecraft is given by a set of ordinary differential equations, which include the effect of the engine thrust, gravitational sources, and inertial forces if non-inertial reference systems are used.

$$\dot{\boldsymbol{x}}(t) = f\left[\boldsymbol{x}(t), \boldsymbol{u}(t), t\right], \tag{1}$$

where $\boldsymbol{x}(t)$ is the state vector consisting of the position, velocity, and mass of the spacecraft, and $\boldsymbol{u}(t)$ is the control vector representing the thrust acceleration vector. The low-thrust orbit transfer optimization problem is to minimize the performance index J, subject to the equation of motions and the initial and final boundary conditions ψ's:

$$J = \int_{t_o}^{t_f} p\left[x(t), u(t), t\right] dt$$
$$\text{subject to} \tag{2}$$
$$\psi\left[x(t_o), u(t_o), t_o\right] = \psi_o \text{ and } \psi\left[x(t_f), u(t_f), t_f\right] = \psi_f.$$

Typically, the performance index J is given by the spacecraft final mass, the time of flight, or the linear combination of the two quantities. Instead of using a single performance index, the present method use both the final mass and the flight time as separate performance indices, and maximizes the spacecraft final mass and at the same time minimizes the time of flight. The resulting solutions are Pareto-optimal solutions that are considered equally good in terms of the vector objective.

B. Direct Method

In the direct method, the continuous control variables $u(t)$ are discretized in either arc time or arc length. The discrete control variables are directly adjusted and optimized to satisfy the boundary conditions and optimization criteria. There are various ways to discretize the control variables, and the choice of the discretization strategy can strongly affect the performance of the optimization. In general, a finer discretization leads to a better approximation to the original control problem, but requires a significantly more computation time. Several discretization strategies are implemented and tested to help guide an optimal strategy for a given problem.

C. Indirect Method

The indirect method transforms the optimal control problem into a two-point boundary-value problem: state and costate variables at the initial time and at the final time. The control vector is determined by the optimality necessary conditions resulting from the transformation. The two-point boundary-value problem as well as the minimization of the flight time and propellant mass formulates a constrained, multi-objective optimization problem.

D. Global Optimization Method

In order to solve the global optimization problems which appear in the nonlinear programming problem of the direct method and the two-point boundary-value problem of the indirect method, the present method uses a genetic algorithm and simulated annealing. The genetic algorithm is inspired by the natural selection and sexual reproduction process of living organisms, and the simulated annealing mimics the thermodynamic process of cooling molten metals. Both methods have mechanisms to escape from local minima in order to find a globally optimal solution. The global search mechanism is a reproduction operator in the case of the genetic algorithm and a Metropolis algorithm in the case of the simulated annealing.

E. Pareto Optimization Method

For the Pareto optimization, the genetic algorithm directly handles multiple objectives with nondominated sorting in a single run. The nondominated sorting uses the concepts of non-dominance and dominance to rank the population composed of candidate solutions. When comparing two solutions, a solution is termed dominated if the solution is inferior to the other solution in all objectives. Otherwise, the solution is termed nondominated. The nondominated sorting finds solutions that are nondominated in comparison with the rest of the candidate solutions in the population. The nondominated solutions constitute a first Pareto front and are assigned the best fitness value. The sorting continues with the dominated solutions (i.e., the complement of the nondominated solutions) by finding the next Pareto front and assigning a slightly worse fitness value. Since the nondominated sorting does not involve a weighting process of aggregating the multi-objectives into a single scalar objective function, the careful, educated guess of the weighting factors is not needed. The genetic algorithm accompanied by the nondominated sorting can generate Pareto-optimal solutions in a single synergetic optimization run.

F. Constraint Handling Method

The low-thrust trajectory optimization problems involve not only multiple objectives but also multiple constraints such as the boundary condition for a spacecraft final state to meet a given target state. Typically, the constraints are treated with a penalty function as a part of the fitness/energy function. The penalty function approach requires a weighting process when combining the penalty function and the objective function into a single scalar function. This approach is used for the simulated annealing application. A different approach named stochastic ranking is used to handle constraints in the genetic algorithm application. The stochastic ranking method strikes a balance between the objectives and the constraints in their contributions to the population ranking process by randomly choosing the ranking criterion between the two. A user-defined parameter determines how probable it is to choose one criterion versus the other.

G. Parallel Computing Method

The genetic algorithm uses a population-based search and thus is amenable to parallel computing. When the fitness evaluation is one of the computationally most expensive parts, the parallel computing becomes an ideal choice to reduce the computational time. The fitness evaluation of the candidate solutions in the population is distributed among several processors in the distributed memory system. The evaluation result is sent to the master processor on which the rest of the algorithmic process such as parent selection, offspring creation, and population replacement is executed. The fitness-value passing is the only message passing between the processors in the genetic algorithm run. As a result, the computational overhead due to the parallel computing is marginal.

III. Results

The present method is applied to two types of trajectory problems: 1) two-body orbit transfer problem and 2) restricted three-body orbit transfer problem. The optimization results are presented and compared with solutions found with other state-of-the-art optimizers in terms of solution quality and required computation time.

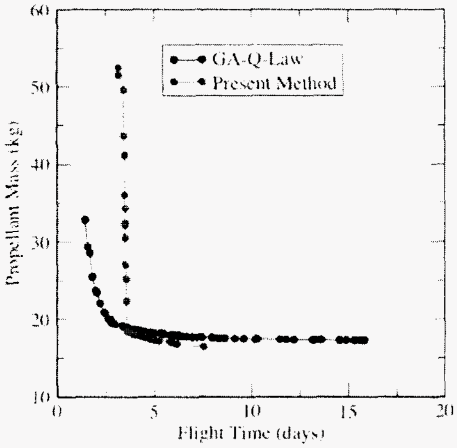

Figure 1. Pareto-optimal solutions found with the present method and GA-Q-Law for the low-thrust orbit transfer around the Earth.

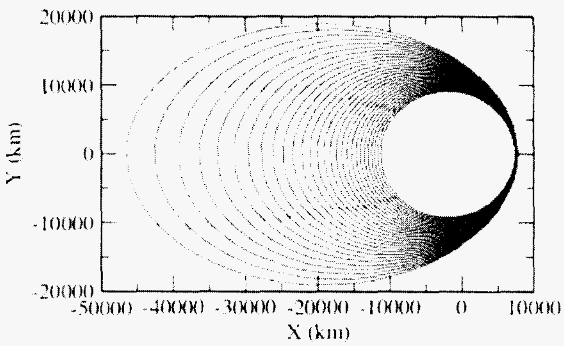

Figure 2. Minimum-fuel trajectory found with the present method for the low-thrust orbit transfer around the Earth.

A. Orbit Transfer around the Earth

As an example of two-body orbit transfer problems, a low-thrust orbit transfer around the Earth is considered. The Earth is the only gravitational body in this problem and is approximated as a point mass. The optimization problem is to find Pareto-optimal solutions for the transfer from a low-eccentricity small orbit to a high-eccentricity, coplanar, larger orbit. The initial orbit has the semimajor axis of 9,222.7 km and the eccentricity of 0.2, and the final orbit has the semimajor axis of 30,000 km and the eccentricity of 0.7. A relatively high thrust magnitude of 9.3 N is used for this transfer problem. The specific impulse of the thrust engine is set to 3100 s. The initial mass of the spacecraft is 300 kg.

Within the direct method, several discretization strategies for the approximation of the continuous control vectors are tested. One strategy is to discretize the control vectors into uniform-time thrust arcs. Another strategy is to discretize the control vectors into uniform-angle thrust arcs. Both strategies lead to an increasing number of discrete control vectors as the flight time increases, and become quickly impractical for multi-revolutionary transfers. To overcome the problem, a simplified but intelligent strategy is developed for the coplanar transfer. The strategy allocates two thrust arcs per revolution, and the arc is centered at each of the two apsides. The length of the thrust arc and the direction of the thrust vector are the resulting variables to optimize. Ideally, the thrust-arc length and the thrust-vector direction should vary from one revolution to another. However, the same length and vector is used for every revolution in order to reduce the number of independent variables to optimize. Although this strategy leads to only four independent parameters (two thrust-arc lengths and two thrust-vector angles), it captures energetically economic maneuvers, which occur near the apsides in the coplanar transfer.

Figure 1 shows the Pareto-optimal solutions found with the present method. The present solutions are compared with the solutions found with GA-Q-Law, which is an optimized heuristic control law based on a Lyapunov feedback control law named Q-law and a genetic algorithm. It has been demonstrated that the GA-Q-Law finds nearly Pareto-optimal solutions in a reasonable computation time. The present solutions are as good as the GA-Q-Law solutions for a long flight time case, while the present solutions are not as good as the GA-Q-Law solutions for a short flight time case. This comparison shows both the shortcoming and effectiveness of the simplified discretization strategy of the present method. It is expected that the present strategy would be inefficient for a short flight time case where a continuously varying control vector is essential to improve the fuel efficiency. Figure 2 shows the minimum-fuel trajectory among the Pareto-optimal solutions found with the present method.

B. Distant Retrograde Orbit (DRO) Transfer around Europa

As a restricted three-body orbit transfer problem, a DRO transfer around Europa, the closest to Jupiter of the icy Galilean moons, is considered. The Jupiter and Europa gravitational fields are included as point masses while the other moons' gravitational fields are excluded. The dynamics of the spacecraft is described in the rotating frame where Europa is at the center, the x-axis points along the Jupiter-Europa line, and the z-axis points along Europa's angular momentum vector with respect to Jupiter. The initial DRO is given by the position vector (0.07518, 0) and the velocity vector (0, -0.14992) in the x-y plane with the unit length of 67,0988 km and the unit time of 48831.6 seconds. Similarly, the final DRO has the position vector (0.03067, 0) and the velocity vector (0, -0.07274). The spacecraft is modeled with the specific impulse of 7365 s, the thrust magnitude of 4.983 mN, and the initial mass of 25,000 kg.

Both the direct and indirect methods are applied to this DRO transfer optimization. In the direct method, the same discretization strategy used for the orbit transfer around the Earth is chosen since it is found to capture the energetically efficient maneuvers of this DRO transfer as well. The two thrust angle (each at one of the apsides) and the two thrust arc lengths are optimized to meet the boundary condition as well as to maximize performance indices. The performance indices considered are the flight time and ΔV, which is the integration over time of the magnitude of the acceleration produced by the propulsion engine. In the indirect method, the initial values of the costate vector associated with a thrust angle, a thrust switching function, and their time derivatives are optimized to minimize the flight time and ΔV, while satisfying the final-state boundary condition given by the target DRO.

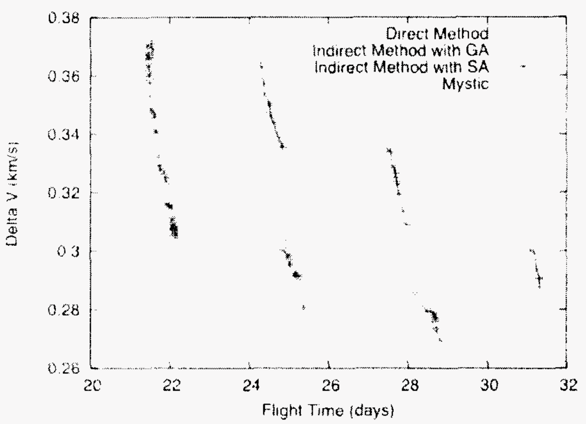

Figure 3. Pareto-optimal solutions found with the direct and indirect methods for the DRO transfer around Europa.

Figure 3 shows the Pareto-optimal solutions found with the direct and indirect methods. The present optimization results are compared with the solutions found with Mystic, which is a software package based on the static/dynamic control algorithm developed by Whiffen. The direct method with the two constant thrust arcs per revolution does not perform as well as the other optimization methods. However, the results of the indirect method are as good as Mystic solutions, which are considered to be nearly optimal. The comparison demonstrates that both the genetic algorithm and simulated annealing efficiently solve the two-point boundary-value problem and find Pareto-optimal solutions.

In terms of computation time, the present optimization of the indirect method requires a considerably smaller computational time than Mystic because the present method utilizes parallel and synergetic computing. With Intel 3.2 GHz Xeon processors, Mystic used about 300 minutes on one processor, while the present method used about 5 minutes in wall-clock time on 16 processors. Mystic required an educated initial guess to obtain the relatively short computation time of 300 minutes. Without the educated guess, the computation time for Mystic may increase by several orders of magnitude. In contrast, the present method efficiently runs without an expert's guidance. Another difference between the Mystic and the present computation is that several independents runs are required to obtain the Pareto-optimal solutions with Mystic while the present method generates the Pareto-optimal solutions in a single synergetic run.

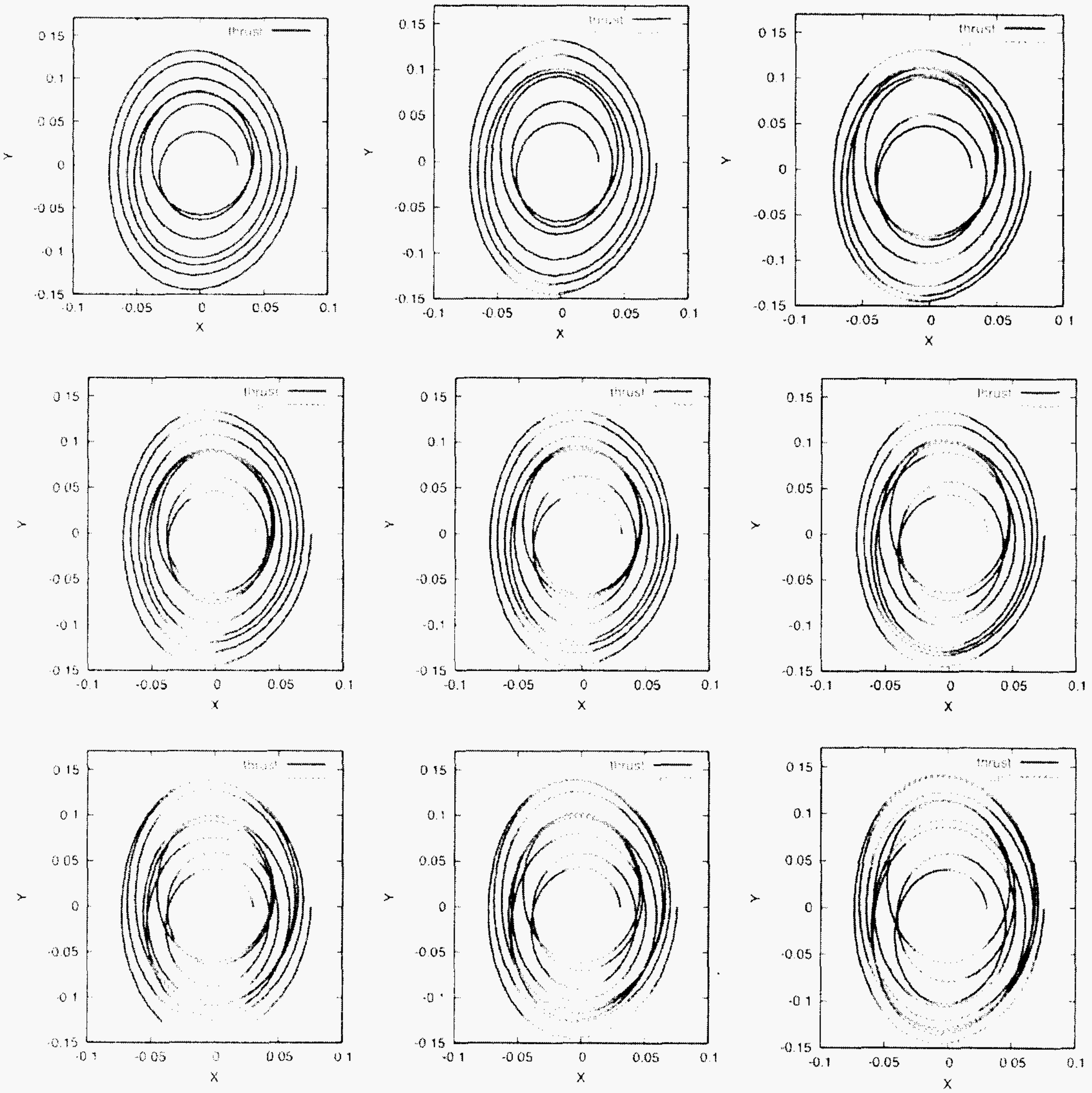

Figure 4. Pareto-optimal trajectories found with the indirect method optimization. From the top left panel to the bottom right, the flight time of the trajectory increases, and more and longer coast arcs appear.

Figure 4 shows the variations of the trajectory and control profile for Pareto-optimal solutions. The top row illustrates the trajectories with flight time of 21.4 days, 21.7 days, and 22.0 days from left to right. The middle row depicts the trajectories with flight time of 24.9 days, 25.0 days, and 25.1 days, and the bottom row represent the trajectories with flight time of 28.2 days, 28.4 days, and 28.7 days. The minimum-flight-time trajectory (the top-left figure) shows a continuous thrust arc (red line). As the flight time increases, several coast arcs (green dashed lines) are inserted around the y axis. This observation suggests that the present strategy taken for the control vector discretization in the direct method is a reasonable approximation, although it is not as general and efficient as the indirect method.

IV. Conclusions

We have developed a robust and efficient method for the global Pareto optimization of low-thrust orbit transfers by applying a genetic algorithm and simulated annealing to both the direct and indirect methods. The developed method is applied to solve a nonlinear programming problem in the direct method and a two-point boundary-value problem in the indirect method. The example problems demonstrate that this method finds nearly Pareto-optimal solutions of the constrained, multi-objective optimization problems within a reasonable computation time. The computational performance of the present method is improved by taking advantage of an intelligent design of control vector discretization, advanced algorithmic parameterization, and parallel computing. Future applications of the present method include the optimization of more complex two-body orbit transfers and restricted three-body orbit transfers which appear frequently in Earth and space missions.

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