ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

DYNAMIC ANALYSIS OF LARGE IN-SPACE DEPLOYABLE MEMBRANE ANTENNAS

Houfei Fang*[1], Bingen Yang[2], Hongli Ding[2], John Hah[3], Ubaldo Quijano[1], and John Huang[1]

[1] Jet Propulsion Laboratory, California Institute of Technology
Pasadena, California, USA
[2] Dept. of Aerospace and Mechanical Engineering, University of Southern California
Los Angeles, California, USA
[3] Hah Hah & Associates, Inc., Monterey Park, California, USA
hfang@jpl.nasa.gov (e-mail address of lead author)

Abstract

This paper presents a vibration analysis of an eight-meter diameter membrane reflectarray antenna, which is composed of a thin membrane and a deployable frame. This analysis process has two main steps. In the first step, a two-variable-parameter (2-VP) membrane model is developed to determine the in-plane stress distribution of the membrane due to pre-tensioning, which eventually yields the differential stiffness of the membrane. In the second step, the obtained differential stiffness is incorporated in a dynamic equation governing the transverse vibration of the membrane-frame assembly. This dynamic equation is then solved by a semi-analytical method, called the Distributed Transfer Function Method (DTFM), which produces the natural frequencies and mode shapes of the antenna. The combination of the 2-VP model and the DTFM provides an accurate prediction of the in-plane stress distribution and modes of vibration for the antenna.

1. INTRODUCTION

Deployable telecommunication reflectarray antennas are being developed at the Jet Propulsion Laboratory for future space missions. The major components of a membrane antenna are one or several layers of thin-membrane pre-tensioned by a space deployable frame. Advantages of using thin-film membranes in such an antenna design include ultra lightweight, small packaging volume, and large deployed aperture for operation. Inspired by recent successes in developing membrane antennas, which include several X-, Ku-, and Ka-band reflectarray antennas [1, 2], space science missions that will employ multiple-band reflectarray antennas with aperture sizes over 8 meters are being considered.

This paper discusses a vibration analysis process developed for analyzing an eight-meter diameter membrane antenna as shown in Figure 1. The membrane aperture of the antenna has very little out-of-plane bending stiffness. The out-of-plane stiffness of this membrane aperture comes from pre-tensioning. This stiffness, called the differential stiffness, is the function of the membrane stress distribution. Therefore, the dynamic analysis of a piece of membrane has two steps. The first step is static analysis to obtain the stress distribution due to pre-tensioning and the second step is modal analysis of the tensioned membrane with its differential stiffness incorporated.

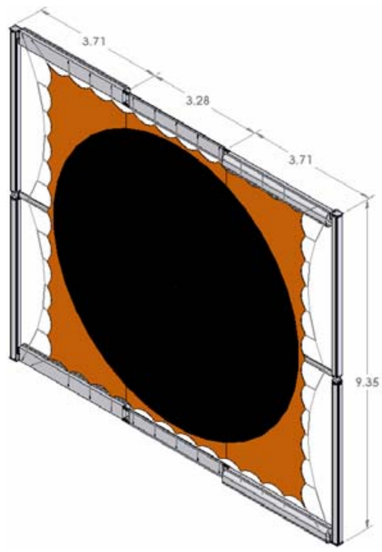

Figure 1 – Design drawing of the eight-meter diameter membrane antenna

A thin membrane, because of its little bending resistance, is easy to get wrinkled. This renders the stress distribution and dynamic behaviors of the membrane sensitive to loading and boundary conditions, which in turn imposes challenging problems in modeling and analysis. Using finite element method to analyze the stress distribution on a piece of thin-membrane is very problematical because wrinkles are usually introduced by uneven stress distribution. Wrinkles cause numerical instability and prevent successful numerical converging. In order to resolve this problem, an innovative analytical methodology, namely two-variable-parameter (2-VP) method, has been developed for calculating the differential stiffness on a pre-tensioned thin-membrane. Another analytical method called Distributed Transfer Function Method (DTFM) is then used to assemble the membrane with other antenna components and to perform the modal analysis.

2. MODELING AND VIBRATION ANALYSIS

The antenna, in consideration, is a deployable boom frame with a mounted thin-film membrane, as shown in Fig. 1. In modeling and analysis of such a structure, the boom frame is described by the DTFM [3, 4]; the membrane is described by 2-VP model [5-7] that systematically characterizes taut, wrinkled and slack states of the membrane. Assembly of the frame, as well as the membrane, leads to a dynamic equation and the solution of which gives the natural frequencies and mode shapes of the antenna structure.

2.1 Frame Members

The deployable frame is modeled as a space frame that is an assemblage of members that are rigidly connected at nodes. A frame member is a prismatic member sustaining bending, longitudinal and torsional deformations. The free vibration of such a member is governed by the differential equations

$$\begin{aligned} &\rho A\frac{\partial^2 u}{\partial t^2} - EA\frac{\partial^2 u}{\partial x^2} = 0, \quad \rho A\frac{\partial^2 v}{\partial t^2} + EI_z\frac{\partial^4 v}{\partial x^4} = 0 \\ &\rho A\frac{\partial^2 w}{\partial t^2} + EI_y\frac{\partial^4 w}{\partial y^4} = 0, \quad \rho J\frac{\partial^2 \theta}{\partial t^2} - GJ\frac{\partial^2 \theta}{\partial x^2} = 0 \end{aligned} \tag{1}$$

where u is the longitudinal displacement; v and w are the transverse displacements; θ is the rotation or twist; ρ, E, G and A are the linear density, Young's modules, shear modulus and cross-section area of the beam, respectively; J is the polar moment of inertia; I_y and I_x are moment of inertia of the y and z axes, respectively.

By the DTFM [4], the dynamic equilibrium equation of the boom member is derived as

$$\begin{pmatrix} p_i \\ p_j \end{pmatrix} = K_m(s)\begin{pmatrix} \alpha_i \\ \alpha_j \end{pmatrix} \tag{2}$$

where α_i and p_i are the vectors of nodal displacements and forces of the membrane at node i, $K_m(s)$ is the dynamic stiffness matrix, and s is the complex parameter that comes from Laplace transform with respect to time. For $s = 0$, $K_m(0)$ gives a stiffness matrix of the boom member. Unlike finite element modeling, $K_m(s)$ obtained herein is always of exact and closed form [3].

2.2 Membrane Model—Two Variable Parameter Method

In this study, the membrane of the deployable antenna is modeled as a thin plate with in-plane displacements u, v and transverse deflection w. It is assumed that the membrane does not resist negative in-plane principal stresses and that in vibration the in-plane membrane stresses remain unchanged [7]. By the assumptions, the plane deformation of the membrane satisfies the equilibrium equations

$$\frac{\partial}{\partial x}\sigma_x + \frac{\partial}{\partial y}\tau_{xy} = 0, \quad \frac{\partial}{\partial y}\sigma_y + \frac{\partial}{\partial x}\tau_{xy} = 0 \tag{3}$$

and the constraint conditions

$$\sigma_1 \geq 0, \quad \sigma_2 \geq 0 \tag{4}$$

where σ_1, σ_2 are the principal stresses of the membrane; the transverse vibration of the membrane is governed by

$$D\nabla^4 w + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + \rho \frac{\partial^2 w}{\partial t^2} = 0 \tag{5}$$

where ρ and D are the mass density (mass per unit area) and flexural rigidity of the plate, respectively; and N_x, N_y, N_{xy} are the internal forces of the membrane that are related to the membrane in-plane stresses by $N_x = \sigma_x h, \quad N_y = \sigma_y h, \quad N_{xy} = \sigma_{xy} h$.

Solution of the free vibration problem for a wrinkled membrane takes two steps:

(a) Solve the wrinkling problem of the membrane defined by Eq. (3) with conditions (4), for the in-plan stresses. This is done through the use of 2-VP membrane model [7], which describes the stress-strain relation of the membrane by

$$\{\sigma\} = \left[D(\lambda_1, \lambda_2)\right]\{\varepsilon\} \tag{6}$$

where constitutive matrix $\left[\tilde{D}(\lambda_1, \lambda_2)\right]$ is a function of nonnegative control parameters λ_1 and λ_2. As shown in [6], the control parameters are such that they can be adjusted to characterize three states of the membrane:

(S1) Taut state $\quad \lambda_1 = 0, \quad \lambda_2 = 0 \quad$ (7a)

(S2) Wrinkled state $\quad \lambda_1 = 0, \quad \lambda_2 > 0 \quad$ (7b)

(S3) Slack state $\quad \lambda_1 > 0, \quad \lambda_2 > 0 \quad$ (7c)

With Eq. (6), conditions (7) and a parametric variational principle [6, 7], the wrinkling problem is converted to a nonlinear complementarity problem (NCP) in optimization theory. Solution of the NCP via a smoothing Newton method yields the in-plane stresses of the membrane. One advantage of the 2-VP model and NCP in wrinkling analysis lies in that it guarantees convergent solutions, without the need of stress iteration. (Many numerical methods that depend on stress iteration often encounter numerical instability for thin membranes with wrinkled and slack regions.)

(b) Determine the transverse vibration of the membrane. With the membrane in-plane stresses determined in (a), the internal forces (N_x, N_y, N_{xy}) of the membrane are known. Thus, a finite element discretization of Eq. (5) yields

$$M_m \frac{d^2}{dt^2} W + \left(K_b + K_g\right) W = 0 \tag{8}$$

where W is a vector of nodal displacements by which the transverse displacement $w(x,t)$ of the membrane is interpolated, M_m is the effective mass matrix of the membrane, K_b is a bending stiffness matrix, and K_g is a differential (or geometric) stiffness matrix characterizing the internal forces N_x, N_y, N_{xy}. Solution of Eq. (8) gives the natural frequencies and mode shapes of the wrinkled membrane.

2.3 Assembly of Antenna Structure

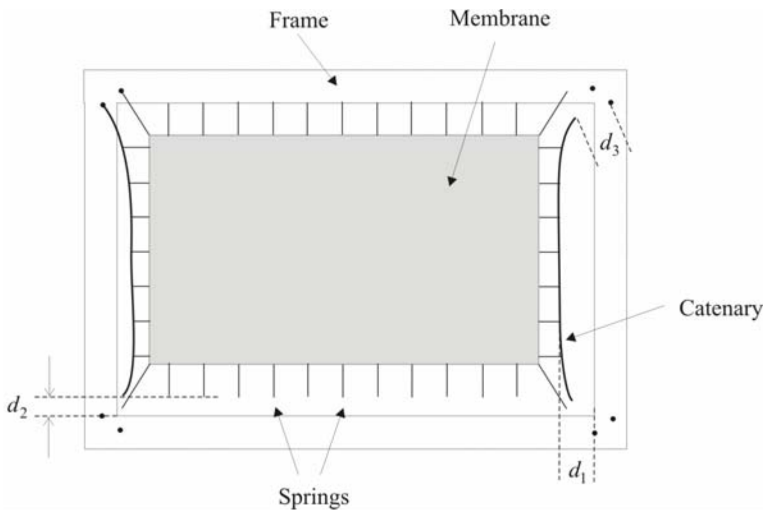

Figure 2 – Schematic of frame-membrane assembly

The membrane is mounted onto the frame by catenary and springs, as illustrated in Fig. 2, where d_1, d_2 and d_3 are clearances that set up tension forces in catenary and springs (for the membrane antenna considered in Fig. 1, more clearance parameters can be introduced). The clearance parameters are adjusted such that the membrane is evenly loaded along its boundary, and has desired normal stresses in its central area. The catenary cables, due to its high tension, can be viewed as elastic bars.

Assume that the membrane antenna in *static equilibrium* only experience in-plane deformations. As such, the equilibrium equations for the antenna are:

Membrane

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} u_{c1} \\ v_1 \end{Bmatrix} = \begin{Bmatrix} f_c \\ 0 \end{Bmatrix} \tag{9a}$$

Frame with catenary

$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{Bmatrix} u_{c2} \\ v_2 \end{Bmatrix} = \begin{Bmatrix} -f_c \\ 0 \end{Bmatrix} \tag{9b}$$

where A_{ij} are the membrane stiffness matrices that are obtained through use of the 2-VP mentioned in Section 2.1; B_{ij} are the stiffness matrices of the frame-catenary substructure that are obtained by the DTFM with $s = 0$; u_{c1} and u_{c2} are vectors of displacements that are involved in the coupling between the membrane and frame; and v_1 and v_2 are vectors of displacements that are not directly involved in the coupling. The f_c is a vector of elastic coupling forces, and is of the form

$$f_c = K_c \left(u_{c2} + d_c - u_{c1} \right) \tag{10}$$

where K_c is a matrix consisting of the coefficients of the springs that connect the membrane to the frame or catenary, and d_c is a vector of clearance parameters as illustrated in Fig. 2. Substituting Eq. (10) into Eqs. (9) gives

$$\begin{bmatrix} A_{11}+K_c & -K_c & A_{12} & 0 \\ -K_c & B_{11}+K_c & 0 & B_{12} \\ A_{21} & 0 & A_{22} & 0 \\ 0 & B_{21} & 0 & B_{22} \end{bmatrix} \begin{Bmatrix} u_{c1} \\ u_{c2} \\ v_1 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} K_c d_c \\ -K_c d_c \\ 0 \\ 0 \end{Bmatrix} \tag{11}$$

from which, the membrane in-plane deformation and the differential stiffness matrix K_g can be computed. As can be seen from Eq. (11), the clearances serve as external loads.

Now consider small vibration of the antenna from its equilibrium configuration. For the membrane, its transverse vibration is governed by Eq. (8); its in-plane vibration is described by

$$M_A \frac{d^2}{dt^2} \begin{Bmatrix} \Delta u_{c1} \\ \Delta v_1 \end{Bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} \Delta u_{c1} \\ \Delta v_1 \end{Bmatrix} = \begin{Bmatrix} q_c \\ 0 \end{Bmatrix} \tag{12}$$

and for the frame, its three-dimensional motion is governed by

$$M_B \frac{d^2}{dt^2} \begin{Bmatrix} u_{c2} \\ v_2 \\ w_2 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{Bmatrix} \Delta u_{c2} \\ \Delta v_2 \\ w_2 \end{Bmatrix} = \begin{Bmatrix} -q_c \\ 0 \\ 0 \end{Bmatrix} \tag{13}$$

Here Δu_{c1}, Δu_{c2}, Δv_1 and Δv_2 are perturbations of u_{c1}, u_{c2}, v_1 and v_2, respectively; w_2 is a vector of out-plane displacements of the frame; M_A and M_B are the inertia matrices; and q_c is a vector of coupling forces given by

$$q_c = K_c\left(\Delta u_{c2} - \Delta u_{c1}\right). \tag{14}$$

According to the above discussion, the free vibration of the membrane antenna takes the following two steps. First, solve Eq. (11) to determine the differential stiffness matrix K_g. Second, solve Eqs. (8), (12), (13) and (14) for the natural frequencies and mode shapes of the structure. Note that the transverse vibration and in-plane deformation of the membrane are not coupled. This is because the geometric nonlinearity of the membrane is neglected in the current linear vibration analysis. The three-dimensional motion of the frame, on the other hand, is coupled with the in-plane motion of the membrane.

3. NUMERICAL EXAMPLES

In order to verify the aforementioned membrane modelling method, a scaled engineering model of the reflectarray antenna membrane has been assembled to experimentally determine its natural frequencies. Fig. 3 shows the test set up. This model is a 1.5-meter by 1.5-meter square Liquid Crystal Polymer (LCP) membrane with copper coating on one side. The thickness of the LCP membrane is 51 micro-meters and the thickness of the copper coating is 18 micron-meter. The LCP has a Young's Modulus of 2.41 GPa and density of 1381 Kg/m^3 with a Poisson's Ratio of 0.33. The copper has a Young's Modulus of 117.2 GPa and density of 8912 Kg/m^3 with a Poisson's Ratio of 0.33. Catenaries are assembled to the four membrane edges to evenly tension the membrane [8]. Catenaries are tensioned by constant force springs, as shown in Fig. 3, to assure the membrane stress be kept at the design level without be deviated by environment, such as temperature changing.

Figure 3 – Test setup(left),close up at tensioning springs (right)

The frame was assembled using high-stiffness/light-weight BOSCH aluminium extrusions to separate the frame modes away from the membrane modes. A piece of membrane is very flimsy, non-contact excitation and measuring devices should be used to assure the pure membrane modes. This test used a speaker to excite the membrane acoustically and a non-contact laser vibrometer acquire the response modes and modal frequencies. Fig. 4 shows the first three mode shapes that were experimentally obtained. Table 1 gives modal frequencies obtained by test, Finite Element Analysis (FEA) and 2-VP method.

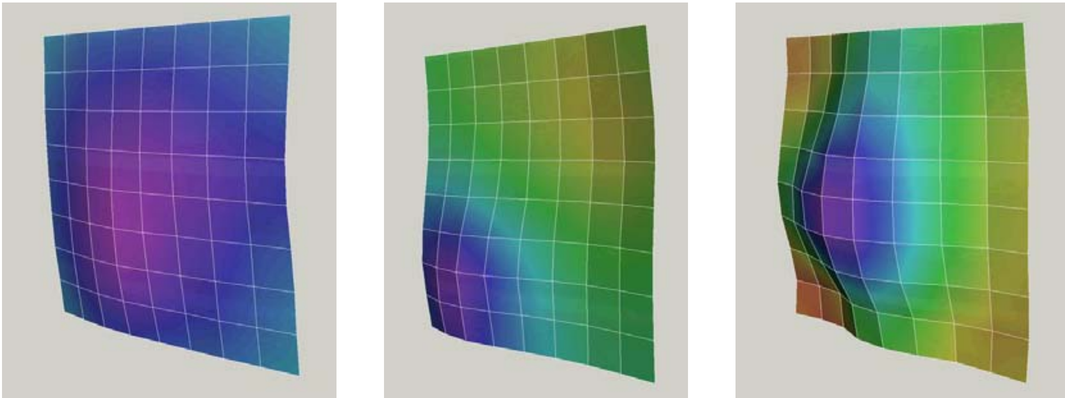

Figure 4 – First mode shape (left), second mode shape (middle), third mode shape (right)

Table 1 – model frequencies

Frequency	Test (Hz)	FEA (Hz)	2-VP (Hz)
f_1	2.44	2.43	2.24
f_2	4.75	4.49	4.21
f_3	7.44	--	7.04

Mode shapes acquired from 2-VP and FEA matched very well with tested mode shapes. Therefore, the 2-VP membrane analysis method has been verified to be suitable for analyzing membrane space structures.

4. CONCLUSIONS

Membrane reflectarrays and other membrane space structures are being developed for future space missions. Since membrane wrinkles while it is unevenly tensioned, analyzing membrane space structures remains to be a challenging problem. This paper presented the 2-VP method for membrane analysis. 2-VP method can systematically characterize taut, wrinkle, and slack statuses of the membrane. It guarantees convergent solutions without the need of stress iteration. This paper also discussed the DTFM for analyzing space deployable frame components and the assembly process to synthesize a complete membrane antenna system. Since it gives exact and closed-form solutions for the space deployable frame components, DTFM offers high computational efficiency and precision. A sub-scale membrane system of 1.5-m by 1.5-m has been assembled for experimental study and the 2-VP membrane analysis method has been verified by correlating its results with test results. As a conclusion of this study, 2-VP method and DTFM along with the assembly process are extremely suitable for being further developed to analyze future membrane space structures.

5. ACKNOLODGEMENTS

The authors wish to thank Drs. James Lesh, Faramaz Davarian, and Richard Lovick of JPL for their encouragement, guidance, and support. The authors also want to acknowledge Jose Perez, Kevin Knarr, Sam Szeto, and Julieta Morales for their contributions on both experimental and analytical aspects of this study.

The work described was performed at Jet Propulsion Laboratory, California Institute of Technology under a contract with the National Aeronautics and Space Administration.

REFERENCES

[1] Fang, H., Lou, M., Huang, J., Hsia, L., and Kerdanyan, G., “Inflatable Structure for A Three-Meter Reflectarray Antenna,” AIAA Journal of Spacecraft and Rockets, Vol. 41, No. 4, pp. 543-550, July-August 2004.

[2] Lou, M., and Fang, H., “Development of Inflatable Antenna structures”, Proceedings of the European Conference on Spacecraft Structures, Materials & Mechanical Testing, Toulouse, France, 11-13 December, 2002

[3] Yang, B., and Tan, C.A., 1992, “Transfer Functions of One-Dimensional Distributed Parameter Systems,” ASME Journal of Applied Mechanics, Vol. 59, No. 4, pp. 1009-1014.

[4] Yang, B., Ding, H., Fang, H., and Lou, M., 2005, “Modeling and Vibration Analysis of Deployable Space Boom Structures”, Proceedings of ASME 2005 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, September 24-28, Long Beach, California. Paper No. DETC2005-84519.

[5] Ding, H., Yang, B., Lou, M., and Fang, H., 2002, “New Numerical Method for Two-Dimensional Partially Wrinkled Membranes,” *AIAA Journal*, Vol. 41, No. 1, pp. 125-132.

[6] Ding, H., Yang, B., Lou, M., and Fang, H., 2002, “A Two-Viable Parameter Membrane Model for Wrinkling Analysis of Membrane Structures,” Proceedings of the AIAA Gossamer Spacecraft Forum, the 42nd AIAA/ASME/ASCE/ASC Structures, Structural Dynamics, and Materials Conference, Denver, Colorado, April 22-25, 2002.

[7] Yang, B., Ding, H., Fang, H., and Lou, M., 2005, “Wrinkling and Free Vibration of Catenary-Supported Thin Membranes”, Proceedings of ASME 2005 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, September 24-28, Long Beach, California. Paper No. DETC2005-85089.

[8] Fang, H., Lou, M., Hsia, L., Leung, P., "Catenary Systems for Membrane Structures," AIAA 2001-1342, 42nd AIAA/ASME/ASCE/AHS/ ASC Structures, Structural Dynamics, and Materials Conference, Seattle, WA, 16-19 April 2001.