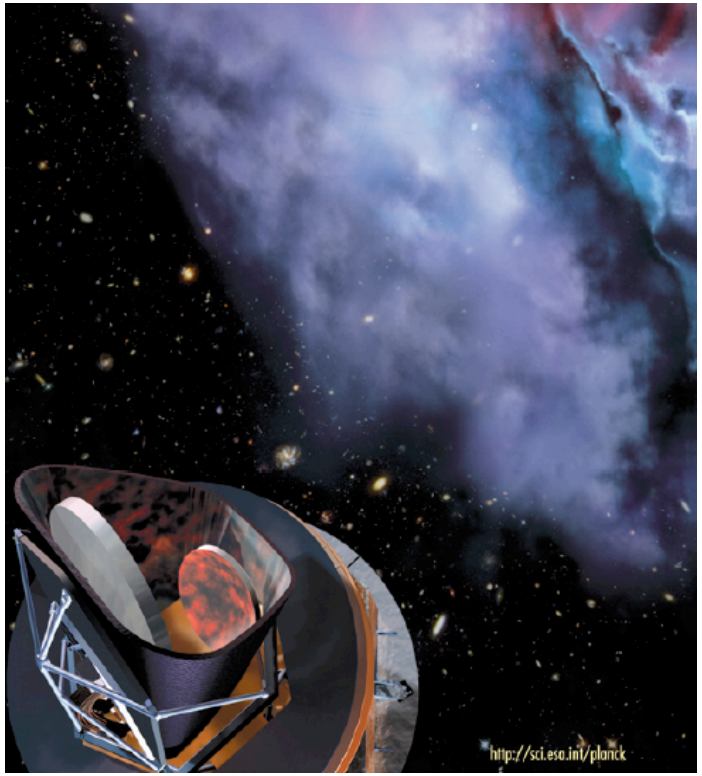
Monte Carlo Algorithms for a Bayesian Analysis of the Cosmic Microwave Background



J. Jewell(JPL)

And the Gibbs Sampling Team -

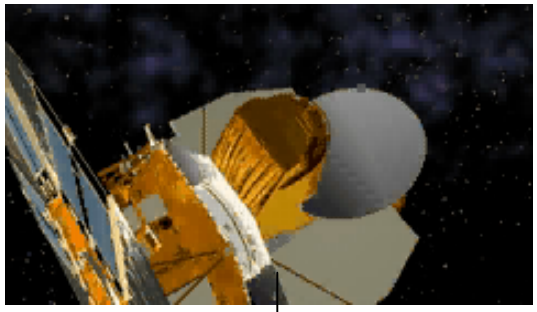
H.K. Eriksen(U. of Oslo), I.J.O'Dwyer (JPL), B.D. Wandelt (UIUC), D. Larson (UIUC), G. Huey (UIUC)

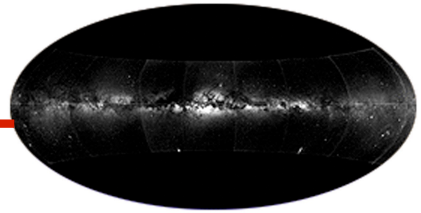
Overview…

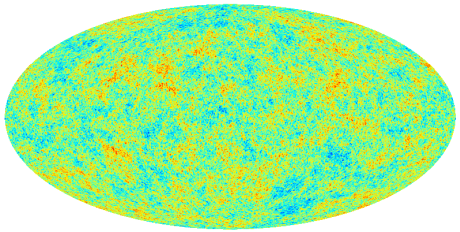
- Review of Bayesian approach to CMB analysis -
- Numerical implementation with Gibbs sampling
- Summary of application to WMAP I (currently running on WMAP 3…)
- Work in progress - generalizations to polarization, foregrounds, asymmetric beams, 1/f noise

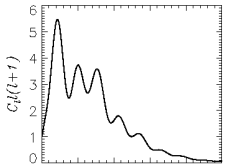
Simulation and Inference…

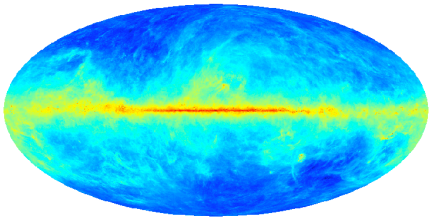
$$p(\Omega,\theta,s,f,d) = p(d \mid f,s)\,p(f \mid \theta)\,p(s \mid \Omega)\,p(\theta,\Omega)$$



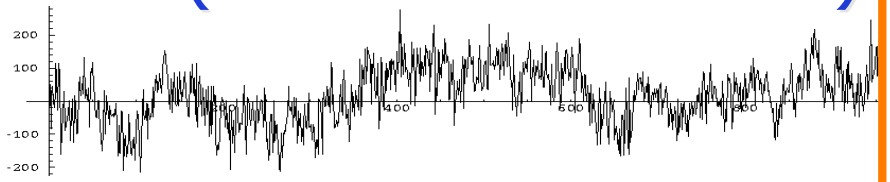








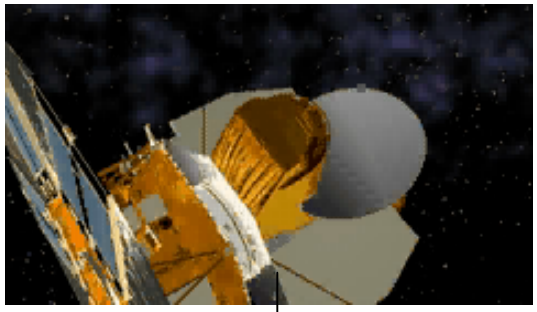
TOD (Time-ordered Data)

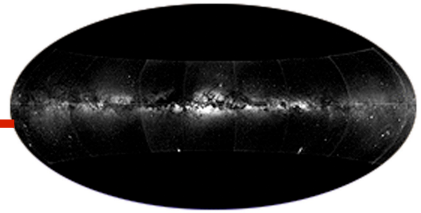


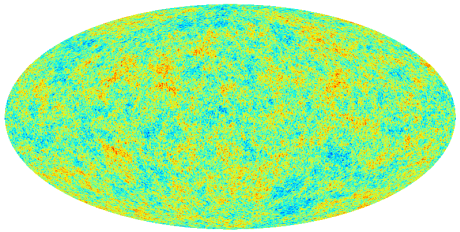
$$p(\Omega,\theta \mid d) = p(\theta,\Omega)\int d(s,f)\; p(d \mid f,s)\,p(f \mid \theta)\,p(s \mid \Omega)$$

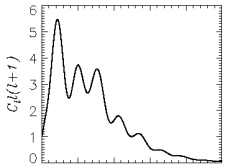
Gibbs Sampling…

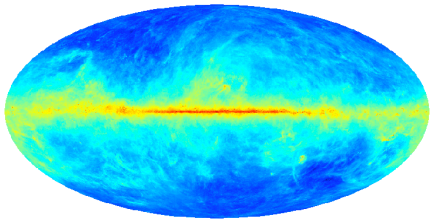
$$p(\Omega,\theta,s,f,d) = p(d \mid f,s)\,p(f \mid \theta)\,p(s \mid \Omega)\,p(\theta,\Omega)$$



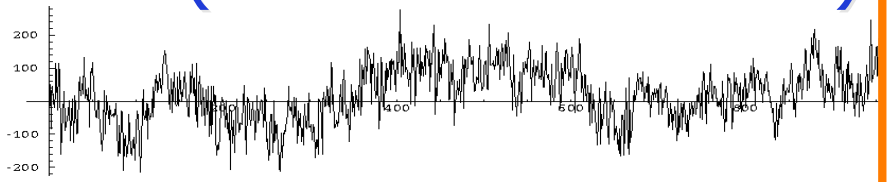








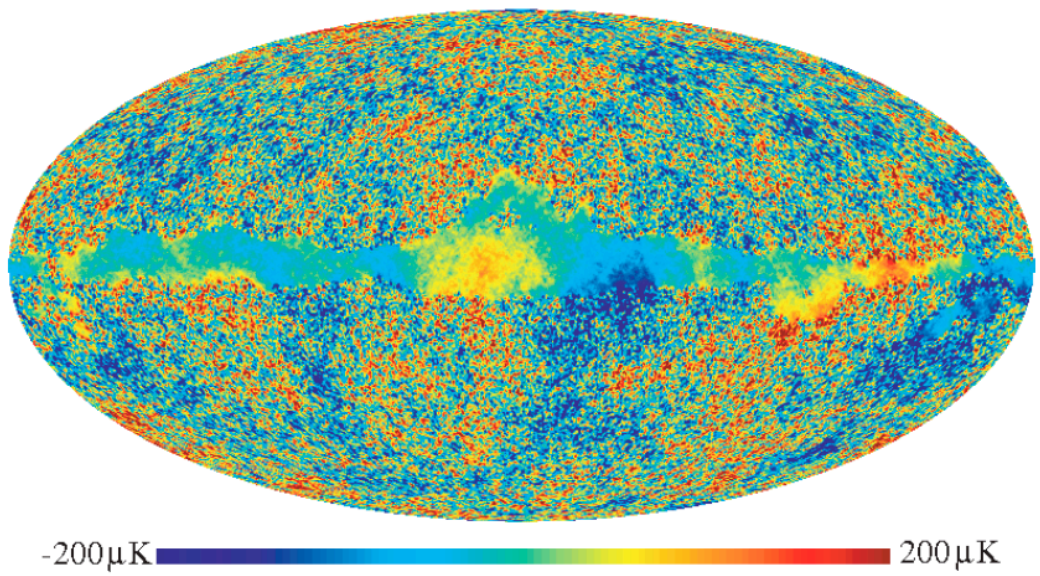
TOD (Time-ordered Data)



$$p(\Omega \mid d) = \int d\Omega' \left[\int ds\; p(\Omega \mid s,d)\, p(s \mid \Omega',d) \right] p(\Omega' \mid d)$$

$$p(\Omega \mid d) = \int ds\; p(\Omega \mid s)\, p(s \mid \Omega', d)\, p(\Omega', s \mid d)$$

$$\boxed{[I + C^{+1/2} A^T N_{tt'}^{-1} A C^{+1/2}](C^{-1/2}\hat{s}) = C^{+1/2} A^T N_{tt'}^{-1} d}$$

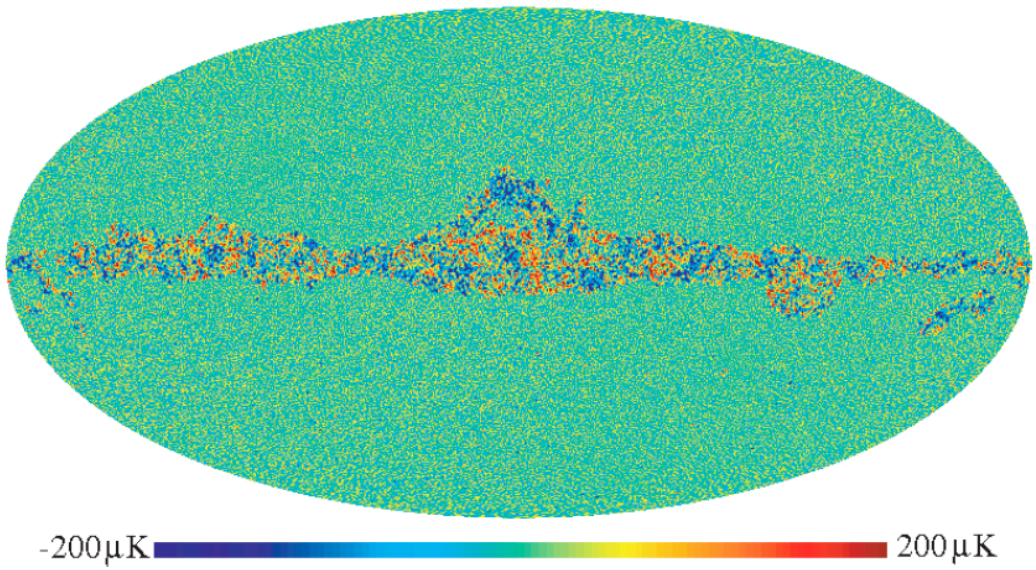


Mean Field map (given power spectrum estimate and data)

Eriksen, et al, ApJS,155, 227, Dec 2004

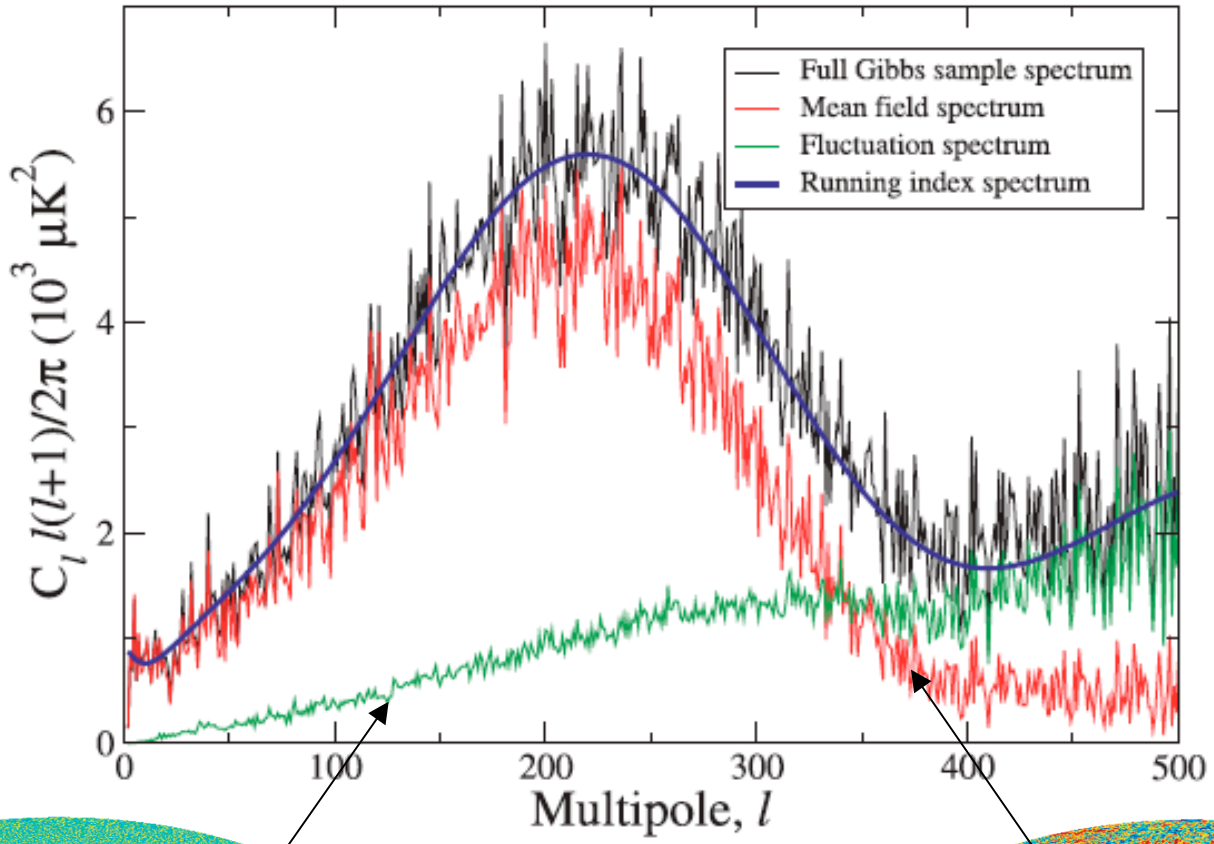
$$p(\Omega \mid d) = \int ds \; p(\Omega \mid s)\, p(s \mid \Omega', d)\, p(\Omega', s \mid d)$$

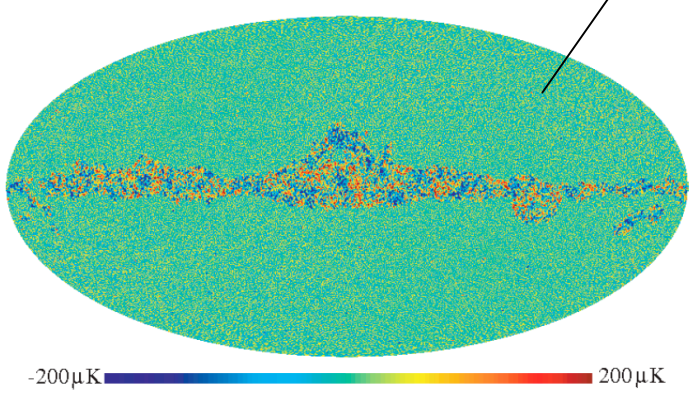
$$\boxed{[I + C^{+1/2} A^T N_t^{-1} A C^{+1/2}]\,(C^{-1/2}\xi) = \omega + C^{+1/2} A^T N_t^{-1/2} \tau}$$

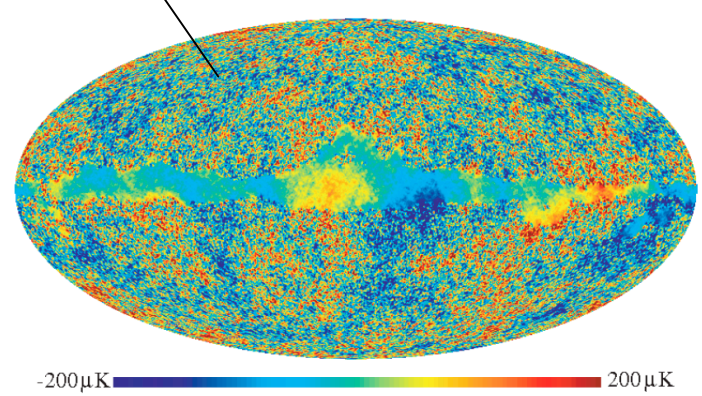


Fluctuation map (given power spectrum estimate)

Eriksen, et al, ApJS,155, 227, Dec 2004





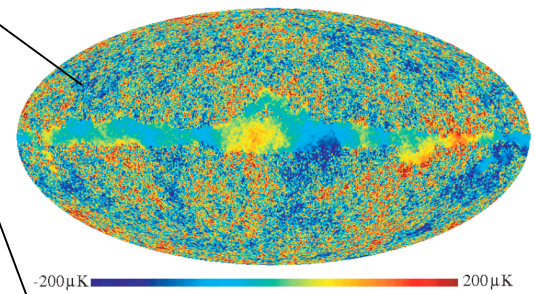


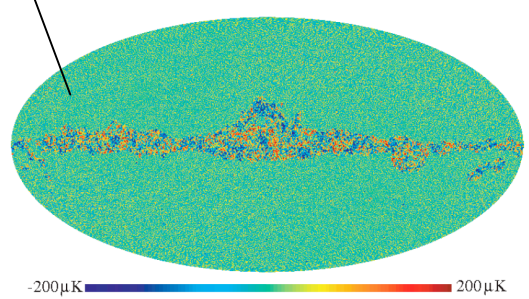
Eriksen, et al, ApJS,155, 227, Dec 2004

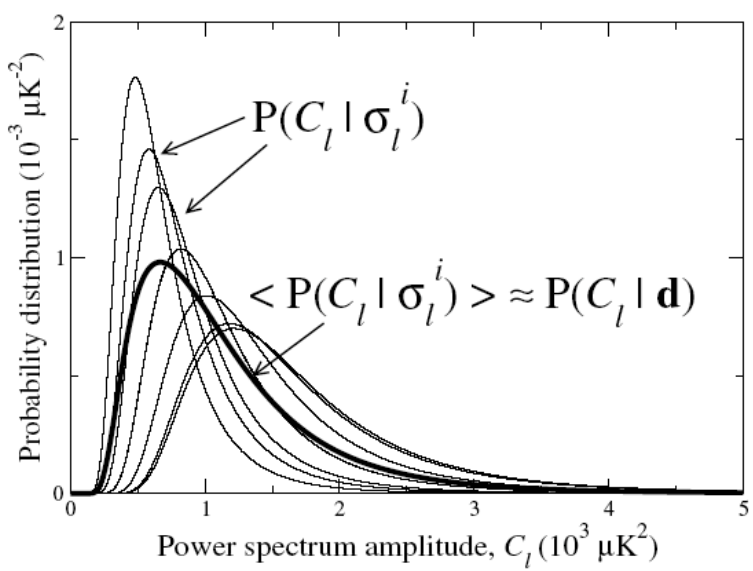
Sampling the Spectrum Given All Sky Maps…

$$p(\Omega \mid d) = \int ds\; p(\Omega \mid s)\, p(s \mid \Omega', d)\, p(\Omega', s \mid d)$$

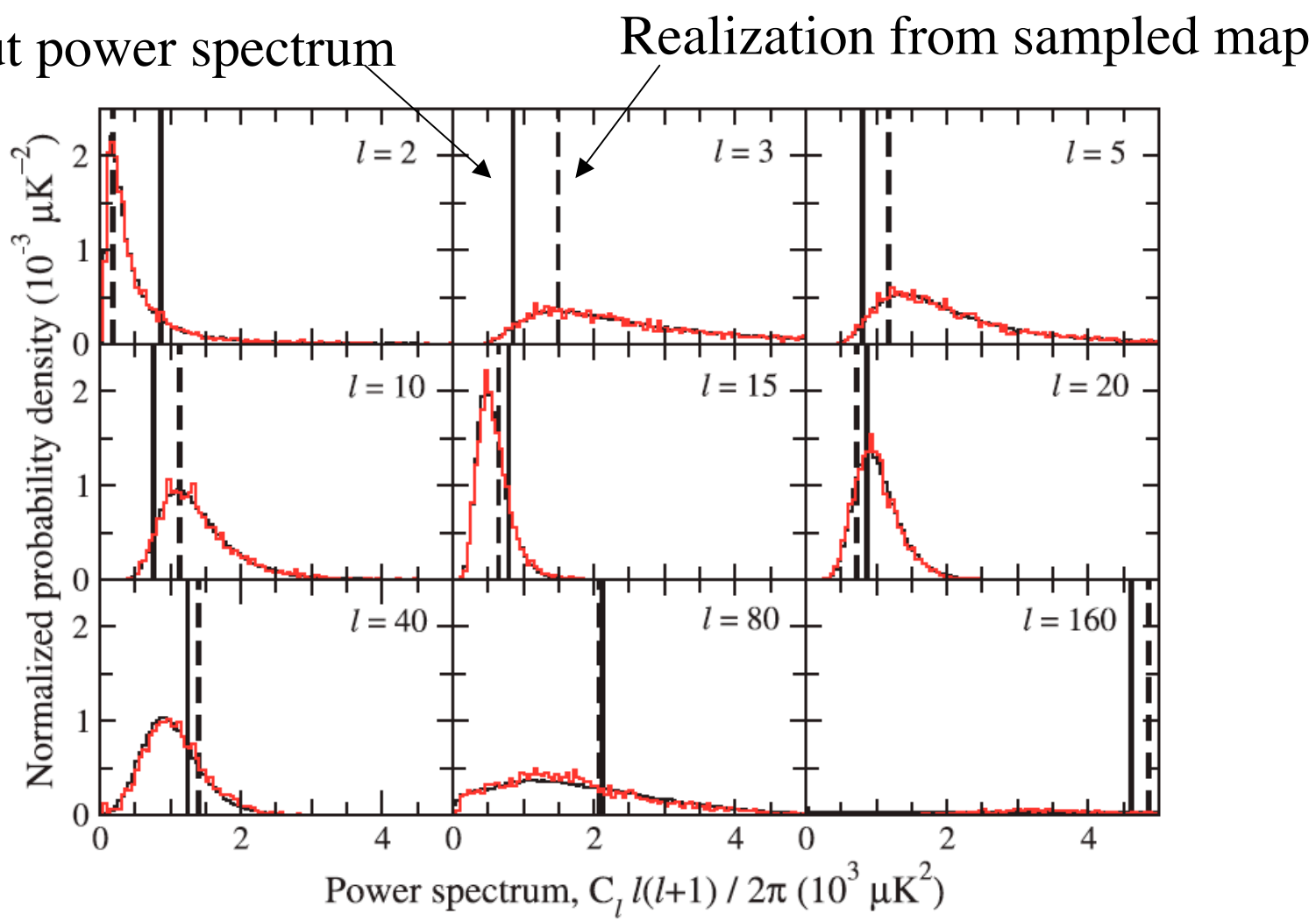
$$p(\Omega \mid s) \propto p(\Omega) \prod_{lm} \frac{e^{-\left|\langle lm \mid s \rangle\right|^2 / 2C_l(\Omega)}}{\sqrt{2\pi}\, C_l^{1/2}(\Omega)}$$







Simulations - Marginal Densities for Power at Selected Multipoles

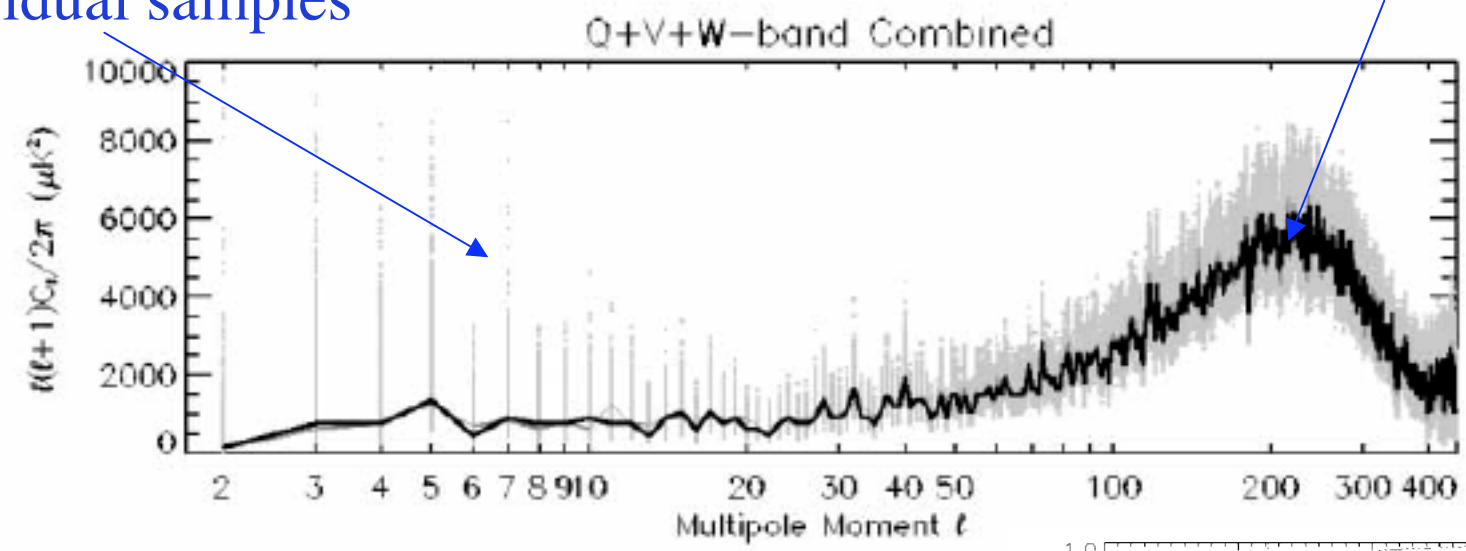


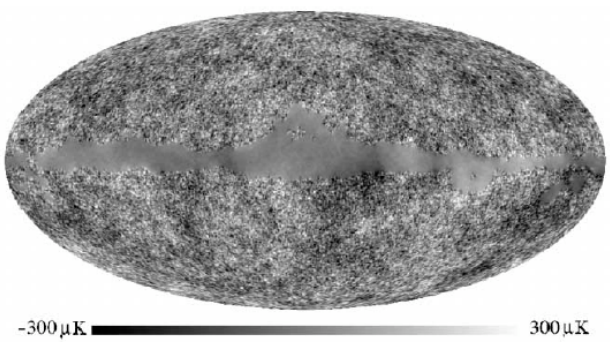
Eriksen, et al, ApJS,155, 227, Dec 2004

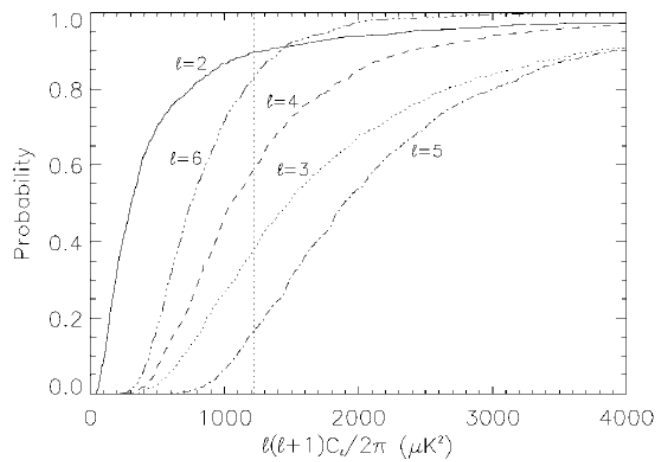
Gibbs Sampling Applied to WMAP I

Individual samples

Posterior mode







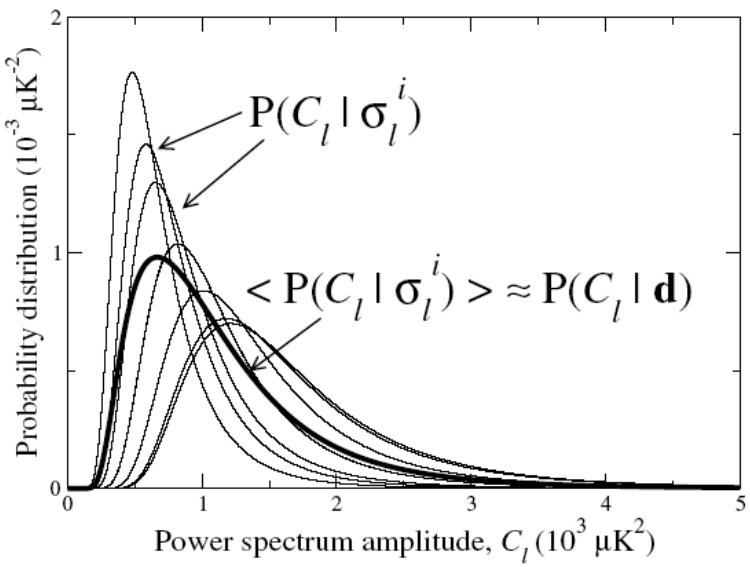
I.J. O'Dwyer et al, ApJ,617,L99,Dec. 2004

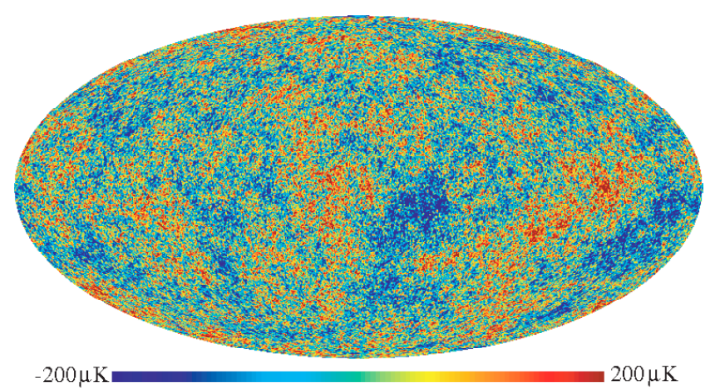
Blackwell-Rao Estimator For Parameter Posterior…

$$p(\Omega \mid d) \approx \int ds\; p(\Omega \mid s)\left[\int dC_l\; p(s \mid d, C_l)\, p(C_l \mid d)\right]$$

$$p(\Omega \mid s) \propto p(\Omega) \prod_{lm} \frac{e^{-\sigma_l / 2C_l(\Omega)}}{\sqrt{2\pi}\, C_l^{1/2}(\Omega)}$$

Sampled Maps from Gibbs chain, marginalized Over power spectra…



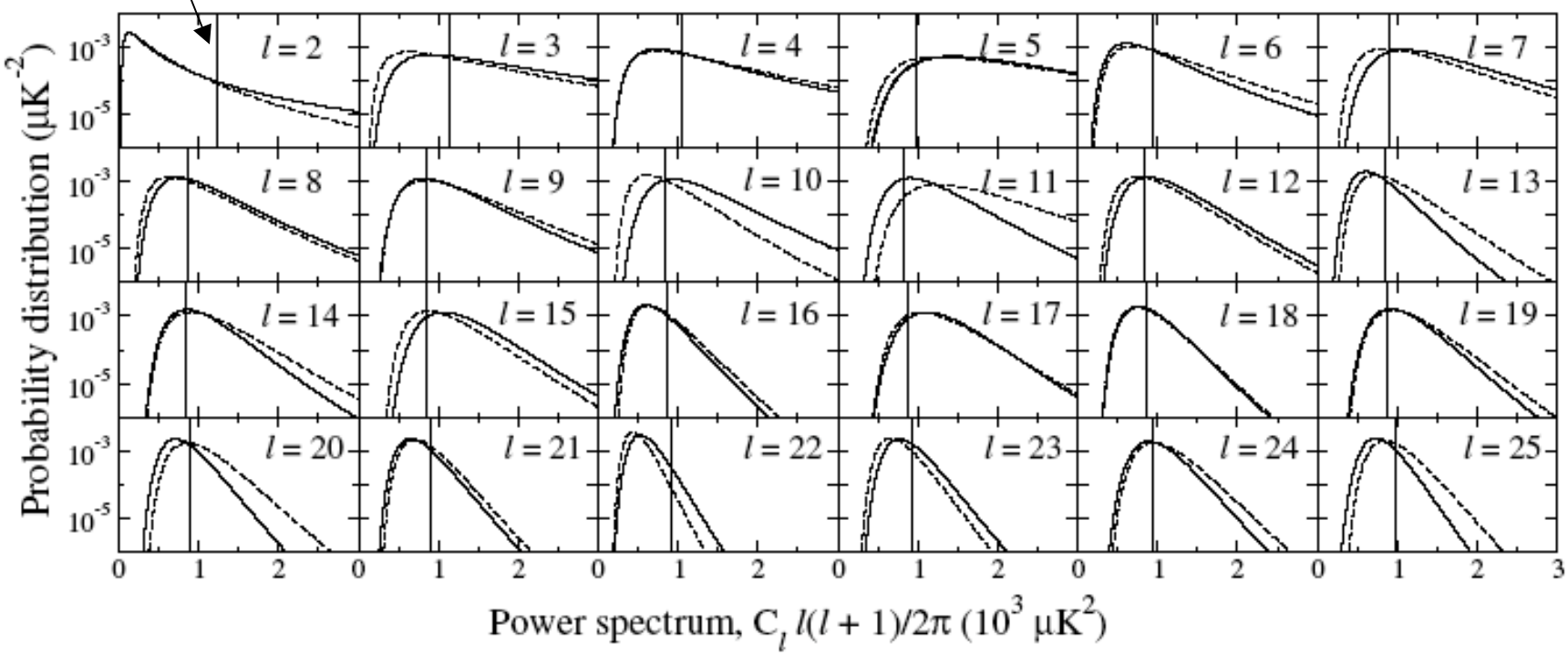


M. Chu et al, Phys.Rev. D, 71,2005

Blackwell-Rao Estimator of the Full Posterior

WMAP — Gibbs sampled BR estimate (solid curves)

Best-fit — WMAP analytic estimate (dashed curves)



M. Chu et al, Phys.Rev. D, 71,2005

Effect on Cosmological Parameters - WMAP I

	$dn_s/d\ln k$ fixed to zero			$dn_s/d\ln k$ free to vary		
Parameter	WMAP $P(C_\ell \mid \mathbf{d})$	Hybrid	Difference/σ	WMAP $P(C_\ell \mid \mathbf{d})$	Hybrid	Difference/σ
n_s	0.97 ± 0.03	0.97 ± 0.03	0.0	0.880 ± 0.048	0.903 ± 0.047	0.4
τ	$0.132^{+0.097}_{-0.048}$	$0.140^{+0.080}_{-0.053}$	0.1	0.202 ± 0.065	0.208 ± 0.063	0.1
A	0.80 ± 0.10	0.79 ± 0.10	0.1	0.91 ± 0.11	0.90 ± 0.11	0.1
ω_b	0.023 ± 0.001	0.023 ± 0.001	0.0	0.0215 ± 0.0013	0.0219 ± 0.0012	0.3
ω_m	0.136 ± 0.014	0.132 ± 0.013	0.3	$0.140 \pm .015$	0.134 ± 0.014	0.4
h	0.72 ± 0.05	0.73 ± 0.05	0.2	0.682 ± 0.054	0.708 ± 0.054	0.5
$dn_s/d\ln k$	$\cdots$	$\cdots$	$\cdots$	-0.079 ± 0.031	-0.063 ± 0.031	0.5

M. Chu et al, Phys.Rev. D, 71,2005

Extensions…

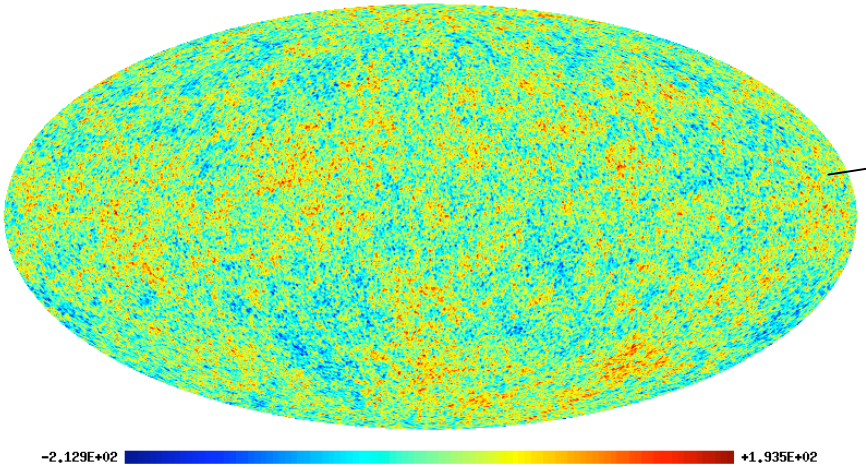
- Polarization
- Foregrounds
- Asymmetric Beams
- 1/f noise

Asymmetric Beams - Planck "Nominal" Scan

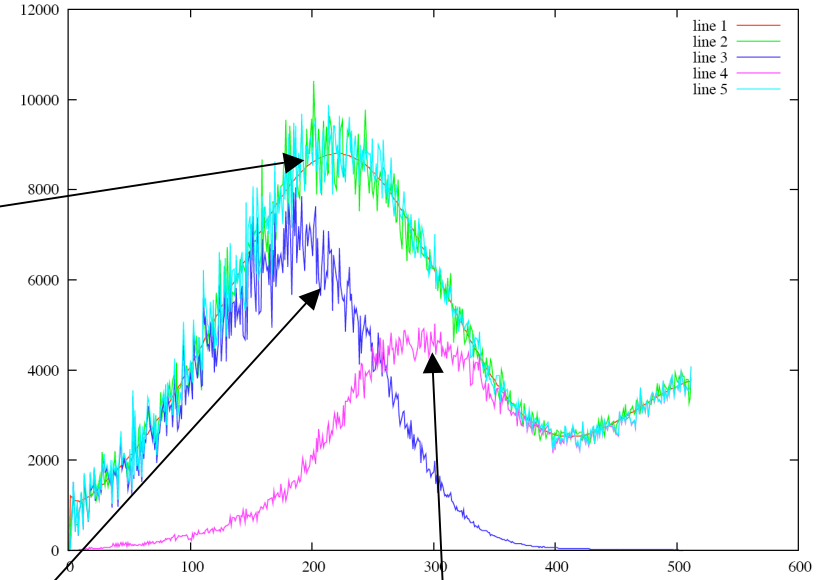
$$[I + C^{+1/2} A^T N^{-1} A C^{+1/2}]\,(C^{-1/2}\xi) = C^{1/2} A^T N^{-1} d + \omega_1 + C^{+1/2} A^T N^{-1} \omega_2$$

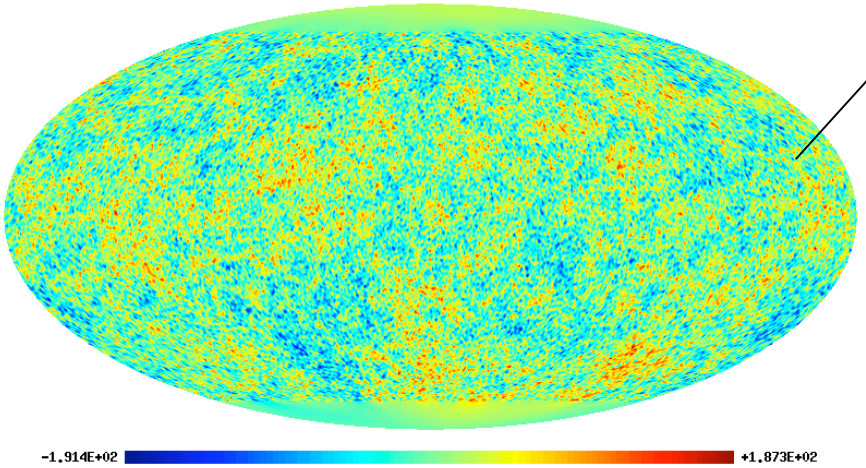
- Nominal Scan (no precession) with no galactic plane - a good case study for including other effects…
- Full "map-making" matrix is block diagonal
- Exact inversion in O(L^4), and multiplication in O(L^3)!
- Allows numerical study of many other effects, including asymmetric beams, 1/f noise,…

Nominal Scan Gibbs Map-making: Elliptical Beam (E=2)

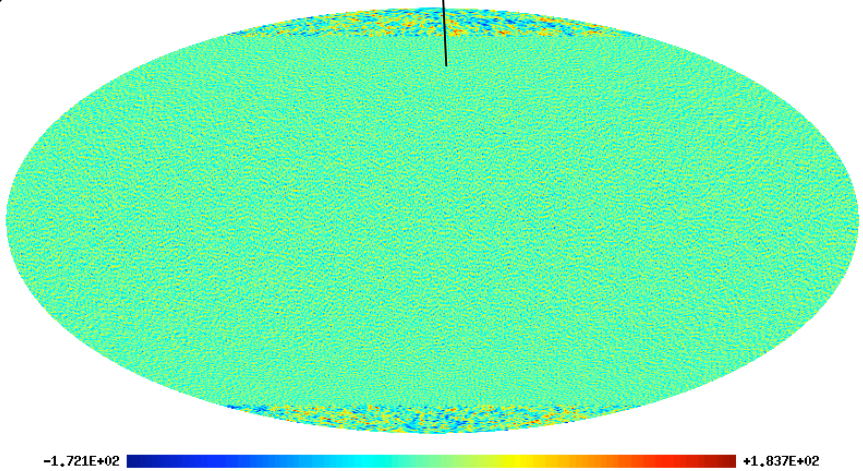


Total map





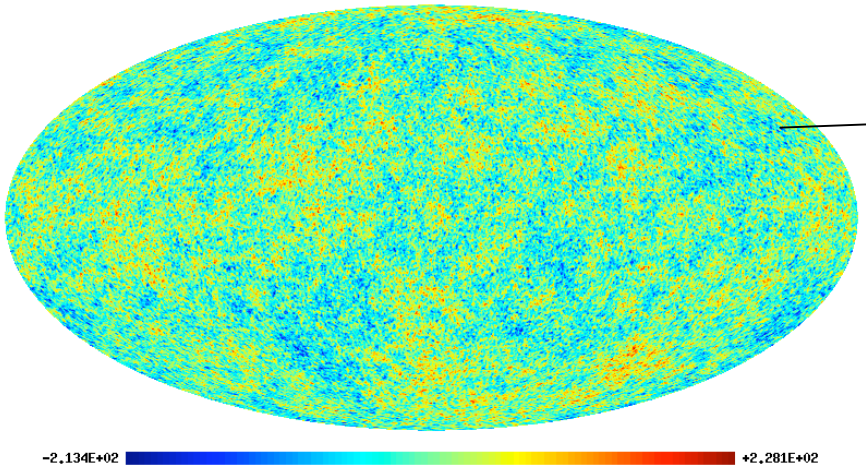
Mean-field map

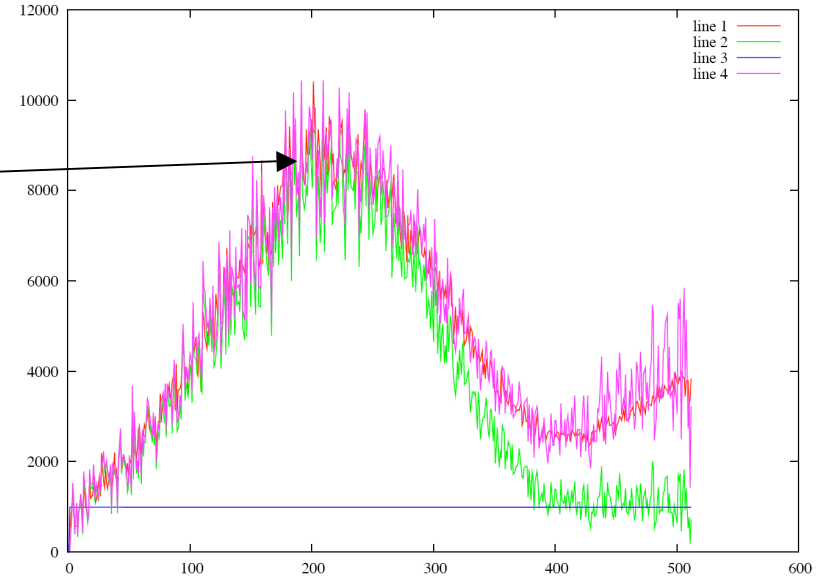


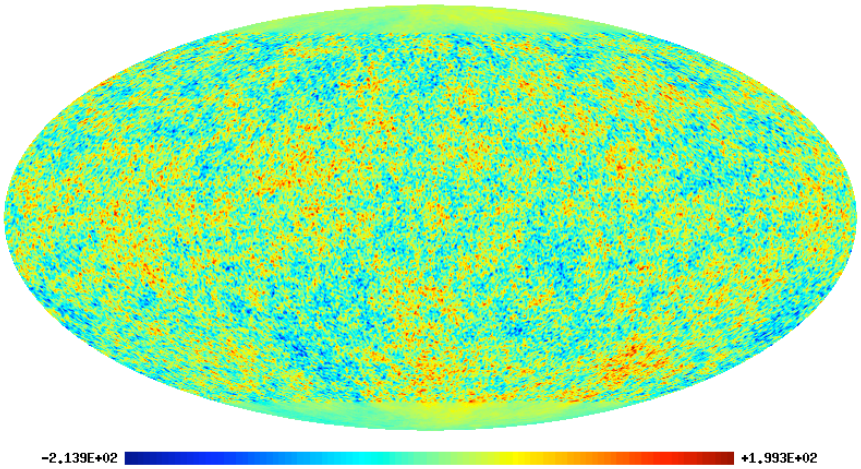
Fluctuation map

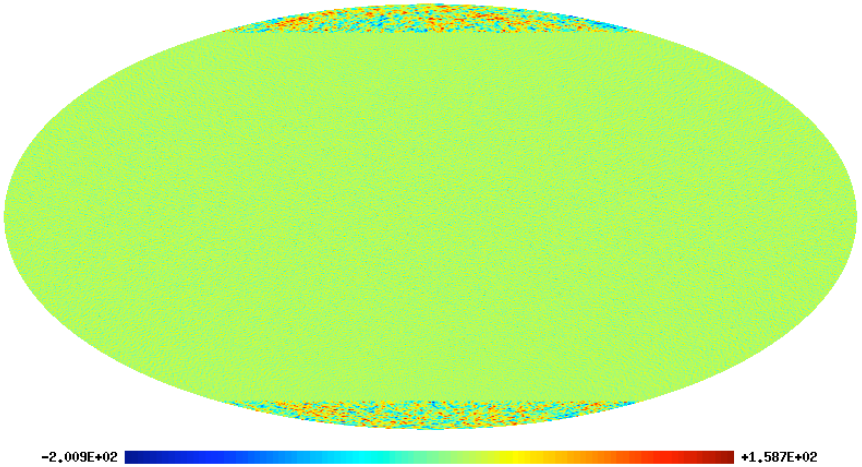
Maps in 0.5 sec with 128 processors!!

Expectation Maximization Power Spectrum: Elliptical Beam (E=2)

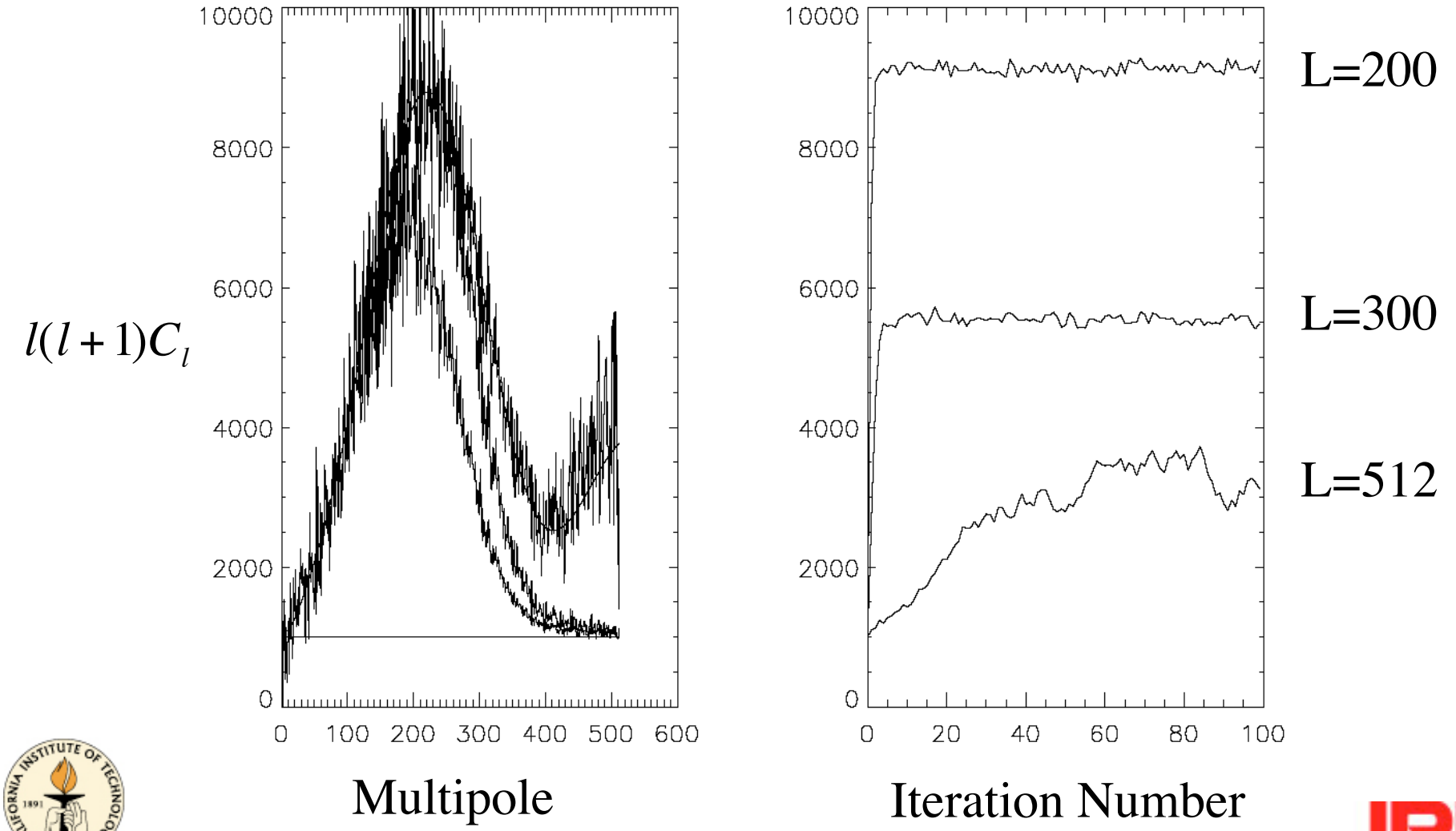








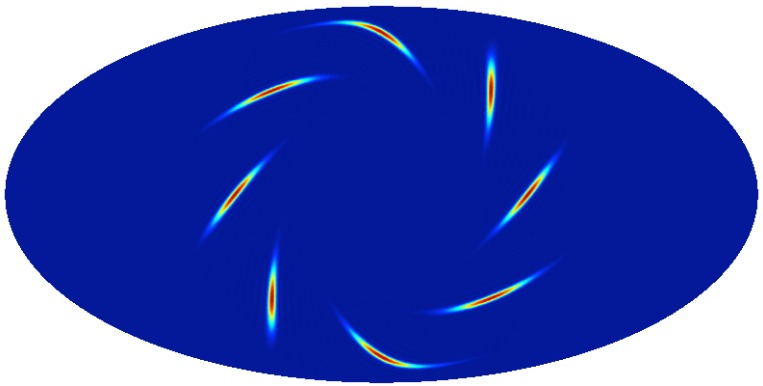
EM Convergence Rate - Elliptical Beam (E=2)

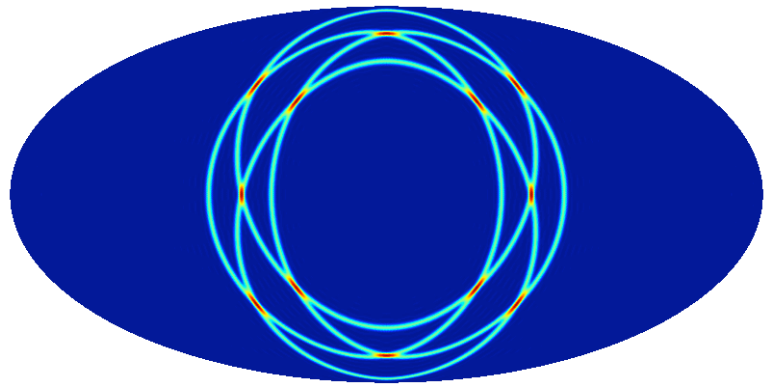


Beyond the "Nominal Scan" - All Spin Axis Pointings

$$[I + C^{+1/2} A^T N^{-1} A C^{+1/2}]\,(C^{-1/2}\xi) = C^{1/2} A^T N^{-1} d + \omega_1 + C^{+1/2} A^T N^{-1/2} \omega_2$$

Involves convolution and transpose…





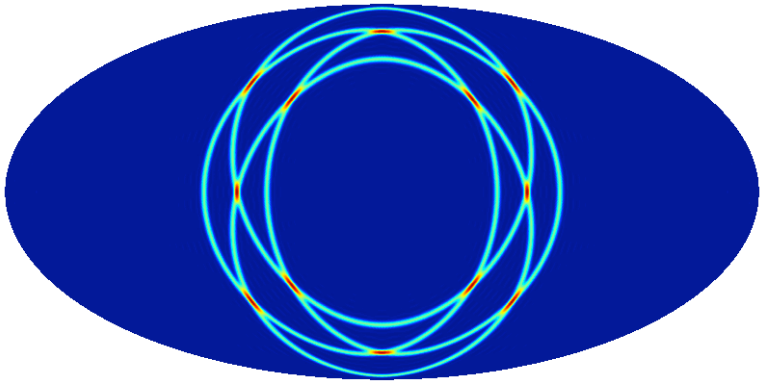
$$T(\varphi_P, \varphi_S, \varphi) = \sum_{l,j,k,n} s_{lj} e^{ij\varphi_P} d^l_{jk}(\pi/2) e^{ik\varphi_S} d^l_{kn}(\theta_P) e^{in\varphi} B_{\ln}$$

Offset Beam Precomputed -

$$B_{\ln} = \sum_m d^l_{nm}(\theta_S) e^{im\gamma} b_{lm}$$

Parallelization Scheme

$$T_{kn}(\varphi_P) = \sum_{l,j} s_{lj} e^{ij\varphi_P} d^l_{jk}(\pi/2) d^l_{kn}(\theta_P) B_{\ln}$$



$$T(\varphi_P, \varphi_S, \varphi) = \sum_{kn} e^{ik\varphi_S} e^{in\varphi} T_{kn}(\varphi_P)$$

- Processors divided into N groups of M processors
- Precession axis pointings (in azimuth) distributed over the N groups.
- O(L^3) per precession axis pointing - distributed over M processors (sum over “pyramid” in L distributed in load balanced fashion, followed by parallel 2D FFT).

O(L^4) Performance on the JPL Dell Xeon Cluster

$$[I + C^{+1/2} A^T N^{-1} A C^{+1/2}]\,(C^{-1/2}\xi) = C^{1/2} A^T N^{-1} d + \omega_1 + C^{+1/2} A^T N^{-1/2} \omega_2$$

Lmax	Procs.	Init (min)	Mult by A (min)	Mult by A^T (min)	I+S/N (min)	Memory/Proc (GB)
128	128	0.3	0.03	0.03	0.07	0.012
256	128	1.5	0.5	0.5	1	0.09
512	256	19	4	4	8	0.7

Summary:

- The challenge of CMB analysis – from complicated time streams, want to learn something about the universe
- MCMC algorithms (Gibbs Sampling as a special case) provide the means for a numerical implementation of a Bayesian approach to CMB analysis
- Bayesian approach in three steps:
  1. Write down the simulation "pipeline" which gives the probability density for "everything",
  2. Condition on the data and write down the integral for the posterior density
  3. Sample with MCMC!
- Current and future extensions – polarization, foregrounds, asymmetric beams, 1/f noise
- Stay tuned for WMAP3 results!