EXTENDED ABSTRACT

# Coverage and Control of Constellations of Elliptical Inclined Frozen Lunar Orbits

Todd A. Ely
Guidance, Navigation, and Control Section
Jet Propulsion Laboratory
Mail Stop 301-125L
4800 Oak Grove Drive
Pasadena, CA 91109-8099
Phone: (818) 393-1744 FAX: (818) 393-6388
E-Mail: Todd.A.Ely@jpl.nasa.gov

Prepared for the
AAS/AIAA Astrodynamics Specialist Conference
Lake Tahoe, California, USA

August 7-11, 2005

## Introduction

A great deal of scientific interest exists regarding the permanently shadowed craters near the poles of the Moon where there may be frozen volatiles. These regions, particularly the Moon's South Pole, have been proposed for extensive robotic and human exploration. Unfortunately, they are typically not in view of Earth, and would require some form of communication relay to facilitate exploration via robotic and/or human missions. One solution for such a relay is a long-lived constellation of lunar telecommunication orbiters providing focused coverage at the pole of interest. Robust support requires this coverage to be continuous, redundant, and, in order to minimize costs, this constellation should consist of 3 satellites or fewer.

Prior results [1] have developed a methodology for selecting this constellation to provide persistent, stable coverage to either the North or South Pole with no requirement for stationkeeping under the influence of only gravitational perturbations. In the present study, coverage statistics are examined to determine the extent of coverage at lower latitudes. The sensitivity of this coverage in the presence of non-gravitational forces is determined, and control strategies are formulated that minimize any potential sensitivity to these accelerations.

## Elliptical Inclined Frozen Lunar Orbits

For the class of orbits being considered in this study the Earth produces the most significant perturbations. Simplified equations for the mean motion of the satellite orbits that are amenable to a qualitative analysis take the form,

$$\frac{de}{dt} = \frac{15}{8}\gamma\frac{n_{\mathrm{E}}^2}{n}e\sqrt{1-e^2}\sin^2 i^{\mathrm{op}}\sin 2\omega^{\mathrm{op}}\,, \tag{1}$$

$$\frac{d\omega^{\mathrm{op}}}{dt} = \frac{3}{8\sqrt{1-e^2}}\gamma\frac{n_{\mathrm{E}}^2}{n}\left[\left(5\cos^2 i^{\mathrm{op}} - 1 + e^2\right) + 5\left(1 - e^2 - \cos^2 i^{\mathrm{op}}\right)\cos 2\omega^{\mathrm{op}}\right]. \tag{2}$$

Where orbital elements with a superscript 'op' are reconciled in a frame at the Moon with its z-axis parallel to the normal of the Earth's apparent orbit around the Moon, the mass ratio is defined as $\gamma = \dfrac{m_{\mathrm{E}}}{m_{\mathrm{E}} + m_{\mathrm{M}}}$ with $m_{\mathrm{M}}$ the mass of the Moon and the mean motion of the Earth around the Moon is $n_{\mathrm{E}} = \sqrt{\dfrac{G(m_{\mathrm{E}} + m_{\mathrm{M}})}{a_{\mathrm{E}}^2}}$ and the mean motion of the spacecraft is $n \equiv \sqrt{\dfrac{Gm_{\mathrm{M}}}{a^3}}$. Solutions to Eqs. (1) and (2) that are of particular interest are the stable librations of $e - \omega^{\mathrm{op}}$ around the fixed point solutions where $e - \omega^{\mathrm{op}}$ remain constant (i.e., $\dfrac{de}{dt} = \dfrac{d\omega^{\mathrm{op}}}{dt} = 0$). Examination of the equations reveals the fixed point solutions when the following conditions are met,

$$\sin 2\omega^{\mathrm{op}} = 0\,, \tag{3}$$

and,

$$\left(5\cos^2 i^{\rm op} - 1 + e^2\right) + 5\left(1 - e^2 - \cos^2 i^{\rm op}\right)\cos 2\omega^{\rm op} = 0\,, \tag{4}$$

which leads to the following results,

$$\omega^{\rm op} = 90^\circ, 270^\circ \text{ and } e^2 + \frac{5}{3}\cos^2 i^{\rm op} = 1\,. \tag{5}$$

In addition to the particular fixed point solutions, Eqs. (1) and (2) are completely integrable with the following general solution,

$$\left(1 - e^2\right)\cos^2 i^{\rm op} \equiv \alpha \tag{6}$$

and,

$$e^2\left(1 - \frac{5}{2}\sin^2 i^{\rm op}\sin^2\omega^{\rm op}\right) \equiv \beta = \frac{5}{2}R + \frac{15}{8}\alpha\,. \tag{7}$$

where $\alpha$ is a constant of motion and is related to the z-component of the angular momentum and $\beta$ is a constant of motion related to the disturbing function $R$ (also a constant of motion) and $\alpha$ (c.f., the last term in Eq. (7)). Figure 1 illustrates some example trajectories resulting from solutions to Eqs. (6) and (7) in the $e - \omega^{\rm op}$ phase plane. Note that two types of trajectories for $e - \omega^{\rm op}$ motion are illustrated: closed librations and open circulations. The paired sets of numbers shown on figure are the initial eccentricity and inclination for each trajectory; all trajectories begin with an initial argument of periapsis value that is either 90° or 270°. The division between open or closed motions occurs when $\beta = 0$ which exists either for $e = 0$ or for $i^{\rm op} = \sin^{-1}(\sqrt{2/5} \approx 39.2^\circ$ (which is often called the 'critical inclination' in the 3rd body perturbed problem). A solution of particular interest that is shown in Figure 1 is the closed libration with,

$$(e_o, i_o^{\rm op}) = (0.6, 56.2^\circ)\ \&\ \Omega^{\rm op} = 0^\circ \quad \rightarrow \quad (e_o, i_o^{\rm ep}) = (0.6, 63^\circ)\ , \tag{8}$$

with $\alpha = 0.198414$ and $\beta =$ -0.26098. Using Eq. (5), the fixed point solution at $i_o^{\rm op} = 56.2^\circ$ is found to have an eccentricity of $e \sim 0.7$. In Figure 1 this value of eccentricity is at top of the loop formed by the solution in Eq. (8), and represents an upper bound to the largest value that this solution will have. The minimal value seen in the figure is the initial value of $e \sim 0.6$. The variation in inclination seen by this orbit can be ascertained from Eq. (6), and leads to $(i_{\rm min}^{\rm op}, i_{\rm max}^{\rm op}) = (51.4^\circ, 56.2^\circ)$. This range of values is useful for selecting a nominal semi-major axis value that satisfies the constellation coverage requirement to cover the selected pole and has a sufficient periapsis altitude. First, to ensure periapsis altitudes are above a minimal altitude the following constraint must be met,

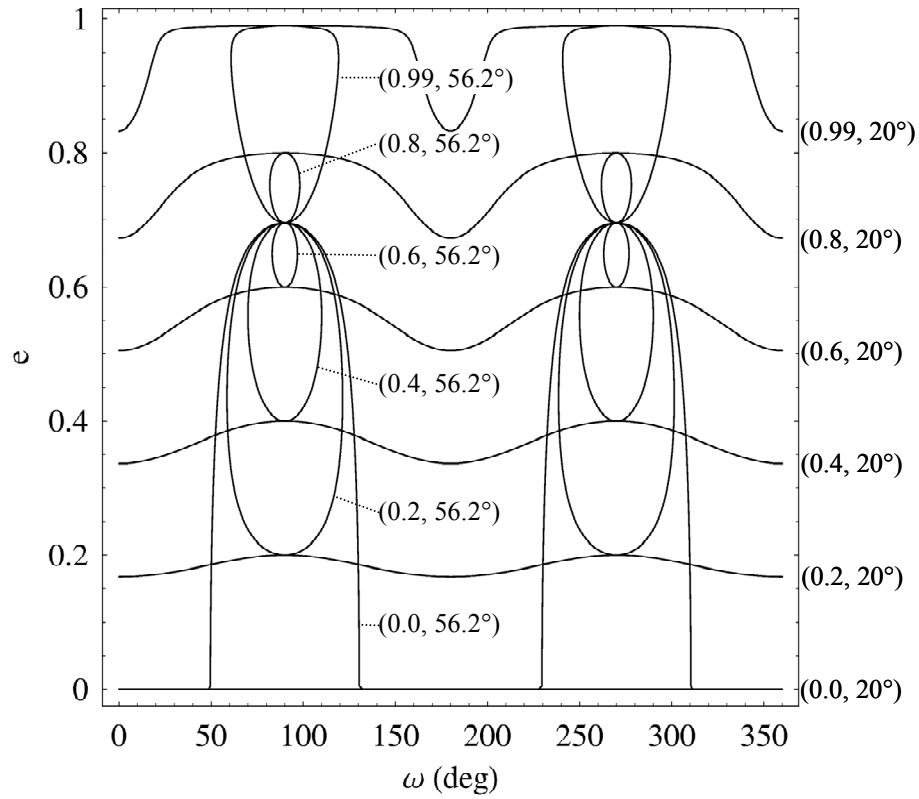


**Figure 1:** Trajectories in the $e - \omega^{\mathrm{op}}$ phase plane for selected initial values of eccentricity and inclination $(e_o, i_o^{\mathrm{op}})$. All trajectories have initial argument's of periapsis $\omega^{\mathrm{op}}$ = 90° or 270°. In general, trajectories with initial inclinations above 39.6° will librate, and below this value will circulate.

$$a(1 - e_{\max}) - R_{\mathrm{M}} \geq h_{\min} \quad \& \quad a(1 + e_{\max}) - R_{\mathrm{M}} \geq h_{\max} \tag{9}$$

where $R_{\mathrm{M}}$ is the mean radius of the Moon. A value for $h_{\min} = 225\ \mathrm{km}$ has been selected and leads to a semi-major axis value of $a \sim 6541.4$ km; furthermore, at apoapsis $h_{\max} = 9382\ \mathrm{km}$. To summarize, the preceding development yielded an initial orbit selection with the following initial values,

$$\{a, e, i^{\mathrm{op}}, \Omega^{\mathrm{op}}, \omega^{\mathrm{op}}\} = \{6541.4\ \mathrm{km},\ 0.6,\ 56.2°,\ 0°,\ 90°,\}. \tag{10}$$

This selection has been made using the analytical solution given by Eqs. (6) and (7) and results in a 'frozen' orbit in $e - \omega^{\mathrm{op}}$ phase space with an eccentricity variation of $\Delta e \sim +0.1$ and an inclination variation of $\Delta i^{\mathrm{op}} \sim -4.8°$.

Detailed numerical analysis [1] of the long term evolution of these orbits has shown that the coverage at South Pole is continuous and 2-fold with three satellites in the orbit plane defined by Eq. (10) that are each separated by 120° in mean anomaly. Consideration of conservative

perturbations has also shown that, given appropriate semi-major axes, this coverage remains 2-fold continuous for a period of at least 10 years.

## Current Study

In the current study, the impact on the aforementioned coverage properties from non-conservative perturbations (i.e., solar radiation pressure) is investigated. Preliminary results indicate that accommodations for solar radiation pressure can be made by making adjustments to the semi-major axis, and that the 2-fold coverage remains persistent at the poles. This initial result suggests that control requirements can still be minimized by appropriate orbit selection. This hypothesis will be investigated.

This constellation also provides coverage beyond the poles. The global coverage properties of the constellation will be explored as a function of the natural orbital evolution of the constellation. The relationship between the global coverage and the orbital evolution will be investigated.

## Acknowledgements

This work was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

## References

1. Ely, T. A., "Stable Constellations of Elliptical Inclined Lunar Orbits." Submitted for publication to The Journal of Astronautical Sciences, January 2005.