The "Brick Wall" Radio Loss Approximation and the Performance of Strong Channel Codes for Deep Space Applications at High Data Rates*

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Abstract

In order to evaluate performance of strong channel codes in presence of imperfect carrier phase tracking for residual carrier BPSK modulation in this paper an approximate "brick wall" model is developed which is independent of the channel code type for high data rates. It is shown that this approximation is reasonably accurate (less than 0.7dB for low FERs for (1784,1/6) code and less than 0.35dB for low FERs for (8920,1/6) code). Based on the approximation's accuracy, it is concluded that the effects of imperfect carrier tracking are more or less independent of the channel code type for strong channel codes. Therefore, the advantage that one strong channel code has over another with perfect carrier tracking translates to nearly the same advantage under imperfect carrier tracking conditions. This will allow the link designers to incorporate projected channel code performance of strong channel codes into their design tables without worrying about their behavior in the face of imperfect carrier phase tracking.

I. Introduction

In recent years turbo codes have been introduced which have a performance very close to the Shannon limit under ideal tracking conditions [1]. Given their ideal performance, these codes are strong candidates for deep space missions. However, there are still questions about whether or not under typical residual carrier tracking conditions used by such missions [2], the radio losses due to imperfect tracking of the carrier offset the advantage that these codes offer. In this paper we attempt to address this issue by developing a code independent approximation for radio losses.

This approximation shows that if the channel code is strong enough that its frame error rate (FER) performance could be approximated by a "brick wall" curve, then the radio losses that it suffers under typical tracking conditions at very high data rates is independent of the code. Thus any advantage a strong channel code offers over another strong channel code under ideal tracking conditions translates to almost the same advantage under typical residual carrier tracking conditions at high data rates. In Section II we present the theoretical background for the "brick wall" approximation of radio losses. In Section III we apply this approximation to two turbo codes ((8920,1/6) and (1784,1/6)) to demonstrate its accuracy. In Section IV we briefly discuss why such an approximation is a useful tool. In Section V we present our conclusions.

II. Theoretical Background

Consider a very powerful error correcting channel code such as the Turbo Codes or Concatenated Codes. Each of these codes has the characteristic that when its input bit signal to noise ratio, $\frac{E_b}{N_0}$, is higher than a set threshold, $\left(\frac{E_b}{N_0}\right)_{thresh}$, it can be decoded almost always perfectly. On the other hand if

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the bit signal to noise ratio is less than $\left(\frac{E_b}{N_0}\right)_{thresh}$, the code almost always fails to decode. This characteristic is called a "brick wall" characteristic because the frame error rate curve for these codes resemble a brick wall with the drop occurring at $\left(\frac{E_b}{N_0}\right)_{thresh}$. In other words, the frame error rate function, f_{FER}, of the code could be approximated by an indicator function (see Figure 1), i.e.:

$$f_{FER}\left(\frac{E_b}{N_0}\right) \approx \begin{cases} 1 & \frac{E_b}{N_0} < \left(\frac{E_b}{N_0}\right)_{thresh} \\ 0 & \frac{E_b}{N_0} \geq \left(\frac{E_b}{N_0}\right)_{thresh} \end{cases} \quad (1)$$

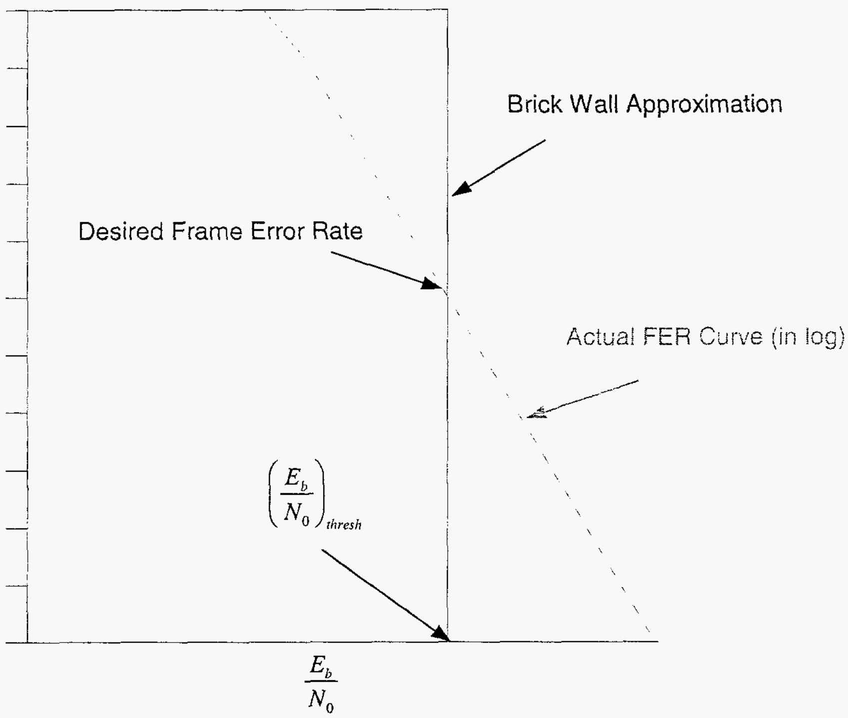

Figure 1: Idealized Frame Error Rate "Brick Wall" Curve

In residual carrier communications received $\frac{E_b}{N_0}$ is affected by the carrier tracking phase error. Let θ be the carrier tracking phase error, then the received energy per bit, E_b, is reduced by a factor of $\cos^2\theta$. In the worst case scenario all the bits in a frame are affected by the same tracking phase error. This is called the Ultra High Rate Model because the frame rate is as high as the carrier phase update rate. Under this condition, if the $\frac{E_b}{N_0}$ into the receiver is $\left(\frac{E_b}{N_0}\right)_r$ then the $\frac{E_b}{N_0}$ into the decoder for the frame would be:

$$\left(\frac{E_b}{N_0}\right)_d = \cos^2\theta \cdot \left(\frac{E_b}{N_0}\right)_r \quad (2)$$

Therefore, a frame fails to decode if

$$
\begin{aligned}
\left(\frac{E_b}{N_0}\right)_d &< \left(\frac{E_b}{N_0}\right)_{thresh} \\
\cos^2\theta \cdot \left(\frac{E_b}{N_0}\right)_r &< \left(\frac{E_b}{N_0}\right)_{thresh} \\
\cos^2\theta &< \frac{\left(\frac{E_b}{N_0}\right)_{thresh}}{\left(\frac{E_b}{N_0}\right)_r}
\end{aligned}
\tag{3}
$$

Let $p_\Theta(\theta)$ be the probability density function of the tracking phase error. Then the frame error probability function is given by:

$$
\begin{aligned}
\Pr\left\{\cos^2\Theta < \frac{\left(\frac{E_b}{N_0}\right)_{thresh}}{\left(\frac{E_b}{N_0}\right)_r}\right\} &= \Pr\left\{-\sqrt{\frac{\left(\frac{E_b}{N_0}\right)_{thresh}}{\left(\frac{E_b}{N_0}\right)_r}} < \cos\Theta < \sqrt{\frac{\left(\frac{E_b}{N_0}\right)_{thresh}}{\left(\frac{E_b}{N_0}\right)_r}}\right\} \\
&= \Pr\left\{\text{-Arccos}\left(-\sqrt{\frac{\left(\frac{E_b}{N_0}\right)_{thresh}}{\left(\frac{E_b}{N_0}\right)_r}}\right) < \Theta < \text{-Arccos}\left(\sqrt{\frac{\left(\frac{E_b}{N_0}\right)_{thresh}}{\left(\frac{E_b}{N_0}\right)_r}}\right)\right\} \\
&+ \Pr\left\{\text{Arccos}\left(\sqrt{\frac{\left(\frac{E_b}{N_0}\right)_{thresh}}{\left(\frac{E_b}{N_0}\right)_r}}\right) < \Theta < \text{Arccos}\left(-\sqrt{\frac{\left(\frac{E_b}{N_0}\right)_{thresh}}{\left(\frac{E_b}{N_0}\right)_r}}\right)\right\} \\
&= \int_{\text{-Arccos}\left(-\sqrt{\frac{(E_b/N_0)_{thresh}}{(E_b/N_0)_r}}\right)}^{\text{-Arc cos}\left(\sqrt{\frac{(E_b/N_0)_{thresh}}{(E_b/N_0)_r}}\right)} p_\Theta(\theta)d\theta + \int_{\text{Arccos}\left(\sqrt{\frac{(E_b/N_0)_{thresh}}{(E_b/N_0)_r}}\right)}^{\text{Arc cos}\left(-\sqrt{\frac{(E_b/N_0)_{thresh}}{(E_b/N_0)_r}}\right)} p_\Theta(\theta)d\theta
\end{aligned}
\tag{4}
$$

If we want to achieve a frame error rate of ε then we have to solve for $\left(\frac{E_b}{N_0}\right)_r$ value that lets the quantity calculated by Equation 4 equal to ε. The radio loss is then defined as

$$
L_{radio} = \frac{\left(\frac{E_b}{N_0}\right)_r}{\left(\frac{E_b}{N_0}\right)_{thresh}} \tag{5}
$$

Substituting 5 in 4 we obtain:

$$\Pr\{\text{Frame Error}\} = \int_{\mathrm{Arccos}(\sqrt{1/L_{radio}})}^{\mathrm{Arccos}(-\sqrt{1/L_{radio}})} p_\Theta(\theta)d\theta + \int_{-\mathrm{Arccos}(-\sqrt{1/L_{radio}})}^{-\mathrm{Arccos}(\sqrt{1/L_{radio}})} p_\Theta(\theta)d\theta \qquad (6)$$

Therefore, if a code displays the idealized "brick wall" behavior depicted in Figure 1 then its radio loss would be independent of $\left(\frac{E_b}{N_0}\right)_{thresh}$ for the Ultra High Rate Model.

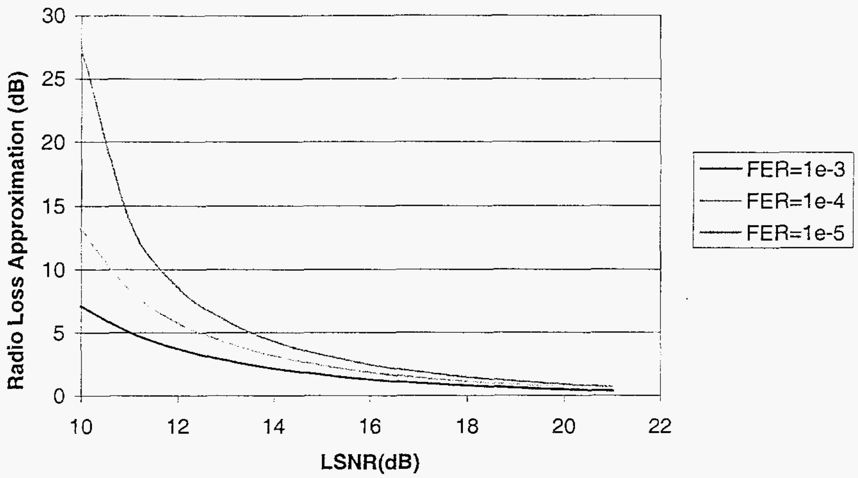

Figure 2: Radio Loss Approximation vs. PLL Loop SNR

If the carrier is tracked with a phased lock loop (PLL) then $p_\Theta(\theta)$ is a Tikhonov density function [2][3][4] given by:

$$p_\Theta(\theta) = \begin{cases} \dfrac{\exp\left(\dfrac{\cos(\theta)}{\sigma_\Theta^2}\right)}{2\cdot\pi\cdot I_0(\sigma_\Theta^{-2})} & -\pi \le \theta < \pi \\ 0 & Otherwise \end{cases} \qquad (7)$$

where σ_Θ^2 is the variance of the phase error. σ_Θ^2 is equal to the inverse of the carrier power to noise power within the bandwidth of the PLL filter (otherwise known as the loop signal to noise ratio).

Depending on the loop bandwidth, the carrier signal to noise ratio, $\frac{P_c}{N_0}$, and the stability of the spacecraft oscillator, the loop signal to noise ratio varies. It is desirable to express the radio losses only as a function of the loop signal to noise ratio (we define loop signal to noise ratio as the inverse of the variance of the Tekhinov distribution, σ_Θ^2, expressed in dBs) since different combinations of loop bandwidths, $\frac{P_c}{N_0}$ values and oscillator types could result in the same loop signal to noise ratio. For the purpose of this paper we have selected desirable frame error rates of 10^{-3}, 10^{-4} and 10^{-5} for the loop signal to noise ratio of 10dB to 21dB. The results of this analysis are shown in Figure 2. In addition, in order to show the validity of the theoretical method used for calculating a code's radio loss, we also consider FER values of 6.10^{-5} and 3.10^{-4} for which simulation results are available for (8920,1/6) and (1784,1/6) turbo codes.

As we can see in Figure 2, the lower the frame error rate requirement the higher are the losses for the same loop signal to noise ratio. Furthermore, we observe that the lower the loop signal to noise ratio, the higher are the losses. For example for 10dB loop SNR, the radio loss for the desirable frame error rate of 10^{-5} is 27.35dB. For 20dB loop SNR this value is reduced to 0.89dB.

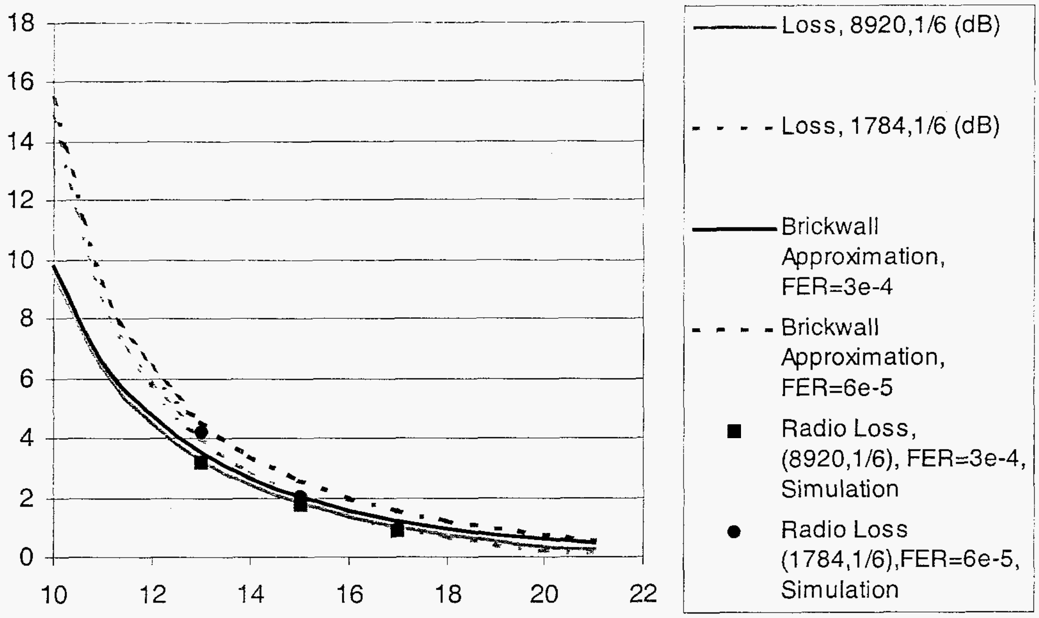

Figure 3: Radio Losses for (8920,1/6) and (1784,1/6) Turbo Codes, Analytical, Approximation and Simulation

III. Application of Brick Wall Approximations to Turbo Codes

In order to demonstrate the validity of the "brick wall" approximation of radio losses we have selected two turbo codes: (8920,1/6) and (1784,1/6). The performance of these codes have been extensively analyzed and they have been shown to have a frame error rate curve that could be very well approximated by a negative exponential function of the form

$$f_{FER}\left(\frac{E_b}{N_0}\right)=\begin{cases} 1 & \alpha_0/\alpha_1 \geq \frac{E_b}{N_0} \\ \exp\left(\alpha_0-\alpha_1\cdot\frac{E_b}{N_0}\right) & otherwise \end{cases} \quad (8).$$

For (8920,1/6) code $\alpha_0 = 129.2476$ and $\alpha_1 = 140.7286$. For (1784,1/6) code $\alpha_0 = 61.72809$ and $\alpha_1 = 66.21811$ [5].

Using the standard analytical approach for calculating high rate radio losses as outlined in [2], we have obtained the radio losses for these two codes using the FER function in Equation (8) for FER values of 10^{-3}, 10^{-4} and 10^{-5}. In addition, since simulation results are available for FER of 3.10^{-4} for (8920,1/6) code and FER of 6.10^{-5} for (1784,1/6) code, we use these FER rates to compare the accuracy of the calculation method for radio losses offered in [2] and actual radio losses.

Figure 3 presents the radio losses for (8920,1/6) code at an FER of 3.10^{-4} and for (1784,1/6) code for an FER of 6.10^{-5}, the "brick wall" approximation of those losses at those error rates and the simulation results for at these error rates for each code. As we can see from Figure 3, the simulation results match the theoretical radio losses for the two codes almost perfectly. In addition, the difference between the approximation and the actual radio losses are quite small. This difference is depicted in Figure 4.

As we can see from Figure 4, the difference between the approximation and the theoretical radio losses is less than 0.6dB. Furthermore, the approximation is more accurate for (8920,1/6) code than it is for (1784,1/6) code. The reason for this is twofold. First of all, (8920,1/6) has a steeper FER curve than the (1784,1/6) code, that is, the error rate decreases more rapidly for the (8920,1/6) code as the $\frac{E_b}{N_0}$ increases. This means that the "brick wall" approximation is more accurate for (8920,1/6) code than it is for (1784,1/6) code. Furthermore, the error rate that is approximated for the (1784,1/6) code is lower than that for (8920,1/6) code. To illustrate these points further, we look at Figures 5 and 6.

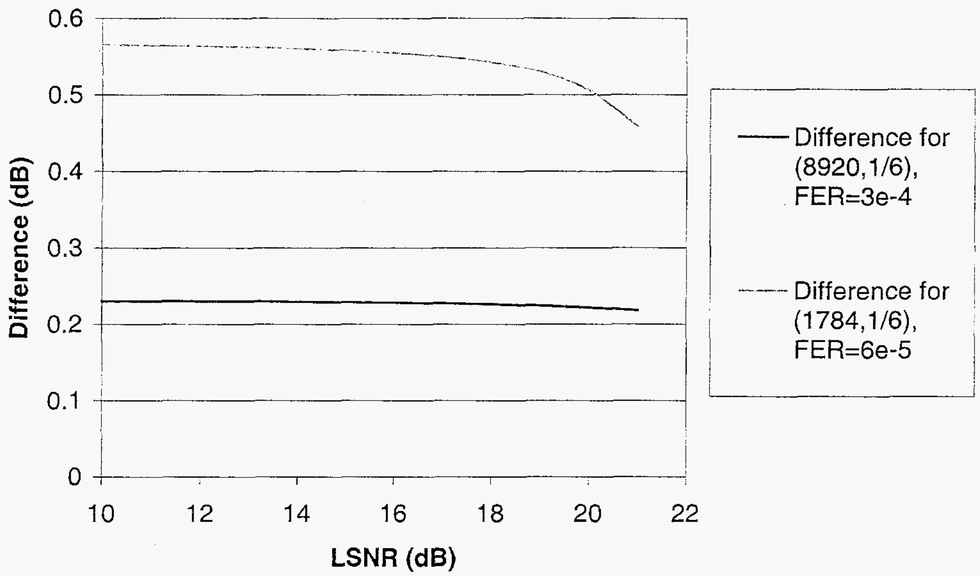

Figure 4. The Difference Between the "Brick Wall" Approximation and the Theoretical Radio Losses for (8920,1/6) at FER of 3.10^{-4} and (1784,1/6) at FER of 6.10^{-5}.

Figure 5 indicates the difference between the theoretical radio loss for (8920,1/6) code and the "brick wall" approximation for FER values of 10^{-3}, 10^{-4} and 10^{-5}. Figure 6 indicates the same for (1784,1/6) code. As we can see from these figures, for the same FER value, the difference between the approximation and the theoretical radio loss is lower for (8920,1/6) code than it is for (1784,1/6) code. Furthermore, for each code the difference increases as the FER value decreases. As mentioned before, the reason that the approximation is more accurate for (8920,1/6) code is due to the fact that (8920,1/6) code has a steeper FER curve.

The reason that the approximation becomes less accurate as the FER decreases is that the difference between the actual FER curve and the "brick wall" approximation becomes more tangible for lower FER values. Finally, note that difference between the approximation and the theoretical radio losses remain relatively constant over the range of loop SNR values considered and it decreases slightly as the loop SNR increases. The reason for this slight decrease is that as the loop SNR becomes very large, the loop starts to track perfectly and therefore, producing zero radio loss. This is true both for the actual code and the "brick wall" approximation. Therefore, as the loop SNR decreases, the difference between the actual radio loss and the "brick wall" approximation decreases. Finally, note that this difference is always less than 0.7dB for the codes under consideration.

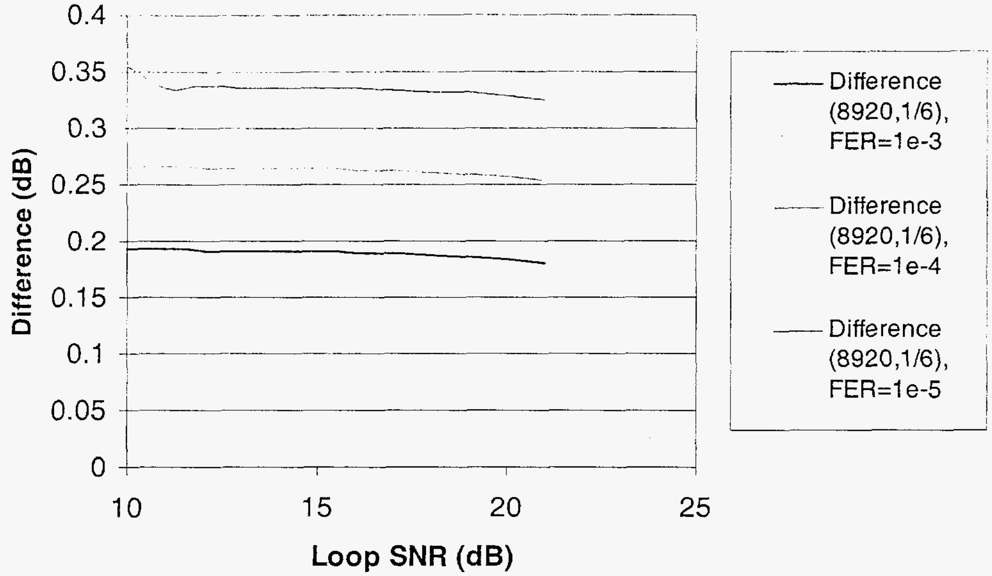

Figure 5. Difference Between "Brick Wall" Approximation and the Theoretical Radio Loss for (8920,1/6) Code vs. Loop Signal To Noise Ratio

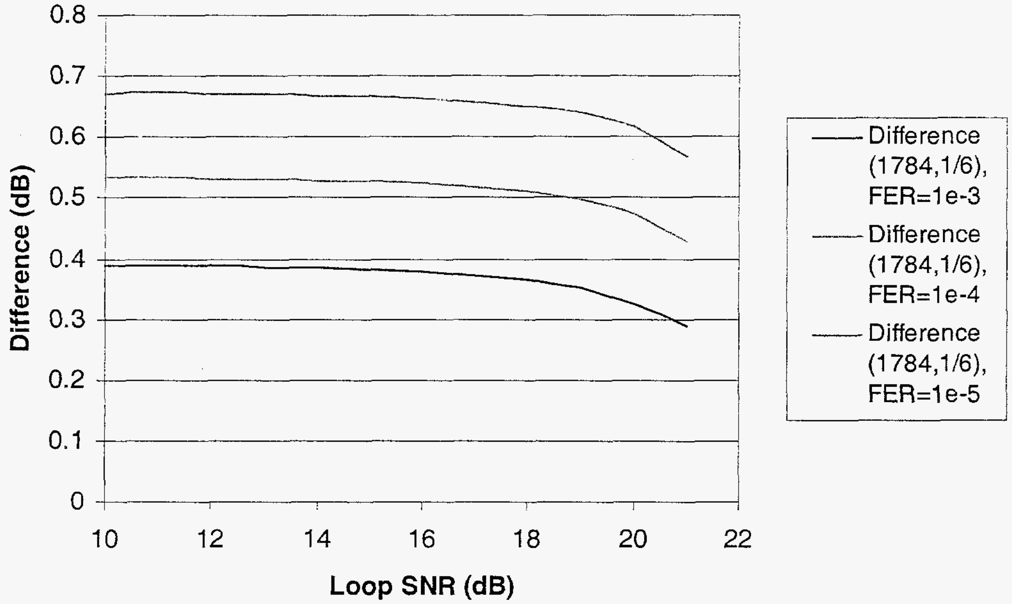

Figure 6. Difference Between "Brick Wall" Approximation and the Theoretical Radio Loss for (1784,1/6) Code vs. Loop Signal To Noise Ratio

IV. Advantages of the "Brick Wall" Approximation

The "brick wall" approximation indicates that if a channel code has a good performance over an Additive White Gaussian Noise (AWGN) channel then, it will have, in all likelihood, similarly good performance over a AWGN channel with imperfect carrier phase tracking. This means that a link designer should always use stronger channel code at higher data rates and that the only consideration should be the difficulty of implementation and acquisition of such a code over the link rather than its steady state performance.

V. Conclusions

In this paper we have introduced the concept of "brick wall" approximation of radio losses for strong channel codes. This concept involves approximating the frame error rate curve of a channel with a step function and calculating the radio loss based on that step function. We have shown that this approximation is reasonably accurate (less 0.7dB for the two turbo codes considered) over a wide range of PLL loop signal to noise ratios (10dB to 21dB). This accuracy indicates that other factors besides the steady state performance of a code (e.g., acquisition of the code and its implementation) should be considered when deciding what strong channel code to use.

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