

# Spacecraft Inertia Estimation Via Constrained Least Squares

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**Abstract**— This paper presents a new formulation for spacecraft inertia estimation from test data. Specifically, the inertia estimation problem is formulated as a constrained least squares minimization problem with explicit bounds on the inertia matrix incorporated as LMIs (linear matrix inequalities). The resulting minimization problem is a semidefinite optimization that can be solved efficiently with guaranteed convergence to the global optimum by readily available algorithms. This method is applied to data collected from a robotic testbed consisting of a freely rotating body. The results show that the constrained least squares approach produces more accurate estimates of the inertia matrix than standard unconstrained least squares estimation methods.

## I. INTRODUCTION

This paper presents a new formulation for spacecraft inertia estimation from data. The data are the spacecraft quaternion and the reaction wheel speeds. Initially, a filtered version of the spacecraft attitude dynamics equation of motion is used to set up a least squares parameter estimation problem. The filtered version of the differential equations eliminates numerical differentiation of noisy data to obtain the angular acceleration of the spacecraft and reaction wheels. Then, explicit bounds on the inertia matrix are presented and incorporated into the least squares optimization problem as LMIs (linear matrix inequalities). The resulting minimization problem is a semidefinite optimization problem that is solved efficiently with a guaranteed convergence to the global optimum by readily available algorithms [1].

This research is motivated by NASA-JPL's Formation Control Testbed (FCT). FCT is a robotic testbed being developed to validate technology for future Autonomous Formation Flight missions, such as Terrestrial Planet Finder Interferometer [2] and Stellar Imager [3]. Three robots will comprise the FCT when completed in 2006. Each robot is made up of two bodies: the Translation Platform (TP) and the Attitude Platform (AP). See Figure 1. The motion of the robot must accurately emulate a spacecraft in deep space. The TP emulates spacecraft motion via four air bearings, three linear and one spherical. The linear air bearings allow the robot to freely translate across the floor and the spherical air bearings enables the AP to freely tip, tilt and spin (see Figure 2). The spin is unconstrained, physical stops limit tip and tilt to a range of sixty degrees.

Gravity influences rotation of the AP when the platform center-of-gravity (CG) does not coincide with the spherical air bearing center-of-rotation (CR). The CG-CR offset

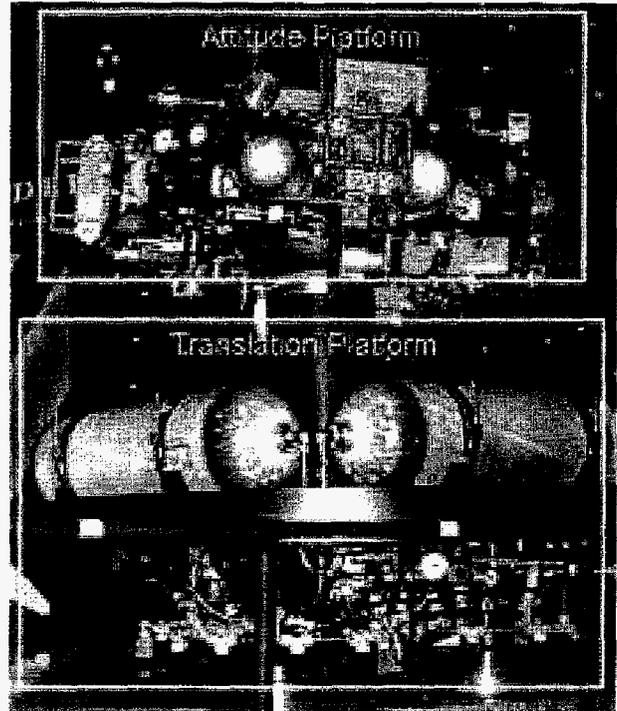


Fig. 1. Image of an FCT Robot

induces a gravitational torque and deteriorates closed loop control performance. For this reason, the CG-CR offset must be minimized. Additionally, in order to achieve precision control of the AP attitude, the AP inertia must be accurately known. The estimation and elimination of the CG-CR offset and the identification of the AP inertia matrix motivates this work.

Recent works on mass property estimation utilize standard least squares and total least squares [4], [5], [6]. These methods require large data sets in order to produce accurate estimates. The physical constraint on tip and tilt of the AP and saturation of the reaction wheels due to CG-CR offset result in small data sets. In order to make accurate estimates based on these data sets, a new *constrained least squares* approach is developed.

In the following sections, we first derive the equation of motion that is the basis for inertia and CG-CR imbalance estimation. The equation of motion is then reformulated in a standard least squares form. To improve estimation accuracy, constraints are introduced in the form of LMIs. Next, the experimental setup is presented, followed by a discussion on data processing. Finally, we estimate the inertia matrix and

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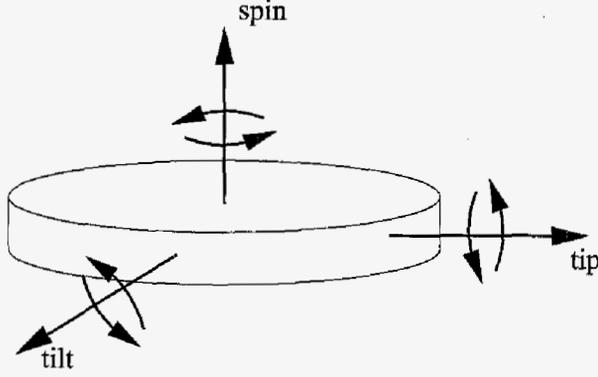


Fig. 2. Illustration of the AP body rotations: Tip, tilt and Spin

the CG-CR offset using constrained least squares, standard least squares [7], and the total least squares methods [8]. The comparison of the methods indicates that the constrained least squares approach produces significantly more accurate inertia estimates with respect to an independent estimate of the inertia matrix from a CAD model.

## II. SYSTEM MODEL

In this section, we present the equations of motion describing the attitude dynamics of the AP with reaction wheels. The total angular momentum of the AP with the reaction wheels  $\vec{H}$  relative to CR is given by,

$$\vec{H} = \mathbf{J}_a \vec{\omega} + \vec{r}_a \times (\vec{\omega} \times m_a \vec{r}_a) + \sum_{j=1}^M (\mathbf{J}_j (\vec{\omega} + \vec{\omega}_j) + [\vec{r}_j \times (\vec{\omega} \times m_j \vec{r}_j)]), \quad (1)$$

$\mathbf{J}_a$  is the AP inertia tensor without wheels relative to its center of mass,  $\mathbf{J}_j$  is the inertia tensor for the  $j$ th reaction wheel about its center of mass (assumed to be coincident with its center of rotation),  $\vec{\omega}$  is the angular velocity vector of the AP relative to an inertial frame,  $\vec{\omega}_j$  is the angular velocity vector of the  $j$ th reaction wheel relative to the AP,  $\vec{r}_a$  is the position vector of the AP relative to the CR,  $\vec{r}_j$  is position vector of the  $j$ th wheel relative to the CR,  $m_a$  is the mass of the AP and  $m_j$  is the mass of the  $j$ th wheel. See Figure 3. The notation  ${}^I d\vec{H}/dt$  denotes an inertial time derivative of the angular momentum. It can be expressed in

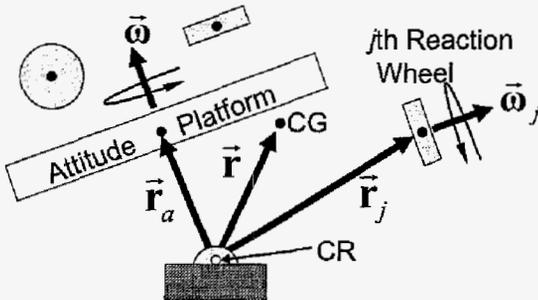


Fig. 3. Diagram of the AP and Reaction Wheels. CG is the center of gravity of the AP and the wheels. CR is the center of rotation.

terms of the time derivative in a body frame  ${}^B d\vec{H}/dt$  as follows

$${}^I d\vec{H} = \frac{{}^B d\vec{H}}{dt} + \vec{\omega} \times \vec{H}.$$

Then, the equation of motion for the attitude dynamics of AP is

$$\frac{{}^B d\vec{H}}{dt} + \vec{\omega} \times \vec{H} = \vec{\tau}, \quad (2)$$

where  $\vec{\tau}$  is the net external torque applied to AP. Once a coordinate frame attached to the body of AP is chosen, referred to as the AP body frame, we represent all vectors in this frame and (2) becomes

$$\dot{H} + \omega \times H = \tau, \quad (3)$$

where  $\dot{v} \equiv {}^B d\vec{v}/dt$  for any vector  $\vec{v}$  with coordinates given by the column vector  $v \in \mathbb{R}^3$ . Letting matrix  $J_a \in \mathbb{R}^{3 \times 3}$  be the representation of  $\mathbf{J}_a$  and  $J_j$  be the representation of  $\mathbf{J}_j$  in the body frame, and defining

$$S(x) = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} \quad (4)$$

for any vector  $x \in \mathbb{R}^3$ , (3) can be written as

$$J\dot{\omega} + \omega \times \left( J\omega + \sum_{j=1}^M J_j \omega_j \right) = \tau - \sum_{j=1}^M J_j \dot{\omega}_j,$$

where  $J$  is the inertia of the system relative to CR give by

$$J = J_a - m_a S^2(r_a) + \sum_{j=1}^M (J_j - m_j S^2(r_j)). \quad (5)$$

In the equation above, it is assumed that the reaction wheels rotate around a principal axis of inertia,  $\hat{n}_j$ , that is assumed to be an axis of symmetry for the reaction wheel and fixed in the AP body frame, that is,

$$J_j \omega_j = I_j \omega_j \hat{n}_j, \quad (6)$$

where  $I_j \in \mathbb{R}_+$  is the corresponding principal moment of inertia,  $\omega_j = \|\omega_j\|$  for the  $j$ th reaction wheel, and by symmetry of the wheel

$$\dot{J}_j = 0. \quad (7)$$

By using (6) and (7), (3) can now be expressed as

$$J\dot{\omega} + \omega \times \left( J\omega + \sum_{j=1}^M I_j \omega_j \hat{n}_j \right) = \tau - \sum_{j=1}^M I_j \dot{\omega}_j \hat{n}_j. \quad (8)$$

Because thrusters are not used during inertia identification maneuvers, the only external torque is due to the CG-CR offset. See Figure 3. The gravitational torque  $\tau_g$  is given by

$$\tau_g = m\vec{r} \times C(q)g, \quad (9)$$

where the total mass of the system is

$$m = m_a + \sum_{i=1}^M m_i,$$

$g$  is the gravity vector expressed in inertial frame, i.e.,  $g = [0, 0, -9.807]^T$  ( $m/s^2$ ), and  $C(q)$  is the coordinate transformation from the inertial to the AP body frame given as a function of the quaternion  $q \in \mathbb{R}^4$  describing the orientation of the AP frame relative to the inertial frame. Summarizing the discussion above and considering the kinematics of the AP rotation via a quaternion description, we have the following differential equations that are the basis for inertia and CG-CR imbalance estimation,

$$J\dot{\omega} + \omega \times J\omega + z \times C(q)g = \sum_{j=1}^M [w_j I_j \hat{n}_j \times \omega - I_j \dot{w}_j \hat{n}_j] \quad (10)$$

$$\dot{q} = \frac{1}{2} \Omega(\omega)q \quad (11)$$

where

$$z = mr, \quad (12)$$

$$\Omega(\omega) = \begin{bmatrix} 0 & -\omega_3 & \omega_2 & \omega_1 \\ \omega_3 & 0 & -\omega_1 & \omega_2 \\ -\omega_2 & \omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix},$$

$$C(q) = \begin{bmatrix} 1 - 2(q_2^2 + q_3^2) & 2(q_1 q_2 + q_3 q_4) & 2(q_3 q_1 - q_2 q_4) \\ 2(q_1 q_2 - q_3 q_4) & 1 - 2(q_1^2 + q_3^2) & 2(q_2 q_3 + q_1 q_4) \\ 2(q_3 q_1 + q_2 q_4) & 2(q_2 q_3 - q_1 q_4) & 1 - 2(q_1^2 + q_2^2) \end{bmatrix},$$

and  $q = [q_1, q_2, q_3, q_4]^T$ .

*Remark 1:* Note that  $J$  and  $z$  are the unknown constant parameters that will be identified as a part of inertia identification. Here  $\omega$  is obtained from the quaternion measured by the celestial sensor [9], and  $w_j$  are measured by tachometers. Numerical differentiation of  $w_j$  obtains  $\dot{w}_j$  for each reaction wheel.  $\diamond$

### III. ESTIMATION OF INERTIA AND MASS IMBALANCE

In this section, we derive the equation that leads to the least squares parameter estimation problem. Letting

$$J_v = [J_{11} \ J_{22} \ J_{33} \ J_{12} \ J_{13} \ J_{23}]^T \quad (13)$$

the equation describing the dynamics of AP (10) can be rewritten as

$$[Q(\dot{\omega}) + S(\omega)Q(\omega) \quad S(C(q)g)] \phi = \sum_{j=1}^M [w_j I_j \hat{n}_j \times \omega - I_j \dot{w}_j \hat{n}_j], \quad (14)$$

where

$$\phi = \begin{bmatrix} J_v \\ z \end{bmatrix}, \quad (15)$$

$S$  is given by (4), and

$$Q(x) = \begin{bmatrix} x_1 & 0 & 0 & x_2 & x_3 & 0 \\ 0 & x_2 & 0 & x_1 & 0 & x_3 \\ 0 & 0 & x_3 & 0 & x_1 & x_2 \end{bmatrix}. \quad (16)$$

Equation (14) is written more compactly as following,

$$H(t)\phi = \eta(t), \quad (17)$$

where

$$H(t) = [Q(\dot{\omega}(t)) + S(\omega(t))Q(\omega(t)) \quad S(C(q(t))g)]$$

and

$$\eta(t) = \sum_{j=1}^M w_j(t) I_j \hat{n}_j \times \omega - I_j \dot{w}_j(t) \hat{n}_j.$$

When a sampled data set is provided for  $\omega$ ,  $\dot{\omega}$ ,  $g$ , and  $w_j$ , for  $j = 1, \dots, M$ , equation (17) directly implies a least squares problem to estimate  $\phi$ . However, AP sensors directly measure  $\int \omega$ ,  $q$  and  $w_j$  from which  $\dot{\omega}$  and  $I_j \dot{w}_j \hat{n}_j$  must be derived. Since  $\dot{w}_j$  and  $I_j$  is known very accurately, the above discussion implies that  $\dot{\omega}$  and  $\dot{w}_j$  must be obtained numerically when (17) is used to formulation the least squares estimation problem.<sup>1</sup>

To do so, we use a filtered version of (14) to avoid numerical differentiation to obtain  $\dot{\omega}$  and  $\dot{w}_j$ . The second-order causal filter,

$$F(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}, \quad 0 < w_n, \quad 0 < \xi \leq 1, \quad (18)$$

applied to (14) gives rise to

$$\begin{bmatrix} Q(\tilde{\omega}_f(t) - \delta(t)\omega(0)) + \psi_f(t) & S(\eta_f(t)) \end{bmatrix} \phi = \sum_{j=1}^M I_j [\hat{n}_j \times \rho_{f,j}(t) - w_{f,j}(t) + \delta(t)w_j(0)], \quad (19)$$

where

$$\begin{aligned} \tilde{\omega}_f(t) &= \tilde{\mathcal{F}}\{\omega\}(t), \\ w_{f,j}(t) &= \mathcal{F}\{w_j\}(t), \quad j = 1, \dots, M, \\ \rho_{f,j}(t) &= \mathcal{F}\{w_j \dot{\omega}\}(t), \quad j = 1, \dots, M, \\ \psi_f(t) &= \mathcal{F}\{S(\omega)Q(\omega)\}(t) \\ \eta_f(t) &= \mathcal{F}\{C(q(t))g\}(t), \end{aligned}$$

and  $\mathcal{F}\{\cdot\}$  is the filtering operator with transfer function  $F(s)$ ,  $\tilde{\mathcal{F}}\{\cdot\}$  is the filtering operator with transfer function  $\tilde{F}(s) = sF(s)$  and  $\delta(t)$  is the impulse response of  $F(s)$ . Note that we use the fact that  $S$  and  $Q$  are linear operators in the derivation of 19. The filtered version of the dynamics (14) is then written in the following compact form,

$$G(t)\phi = g(t), \quad (20)$$

where

$$G(t) = [Q(\tilde{\omega}_f(t) - \delta(t)\omega(0)) + \psi_f(t) \quad S(\eta_f(t))] \quad (19)$$

and

$$g(t) = \sum_{j=1}^M I_j [\hat{n}_j \times \rho_{f,j}(t) - w_{f,j}(t) + \delta(t)w_j(0)].$$

The experimental data is obtained at discrete time instances,  $t_0, t_1, \dots, t_N$ , that is, the data is sampled. Therefore (20) describes an equality at each sampled time instance  $t_k$ . Consequently, we have the following set of equalities

$$G(t_k)\phi = g(t_k), \quad k = 0, 1, \dots, N. \quad (21)$$

<sup>1</sup>Note that, if the external torque due to gravity does not exist, one can use conservation of momentum equation rather than (2) to obtain an equation that does not contain the derivatives of the angular rates [5]. This is not applicable in our case.

Additionally, the filtering required to obtain  $G(t_k)$  and  $g(t_k)$  is performed digitally by using a discrete approximation of the filter (18). That is, for any signal  $v(t)$

$$\mathcal{F}\{v\}(t_k) \approx \mathcal{F}_d\{v_0, \dots, v_N\}_k, \quad k = 0, \dots, N,$$

where  $\mathcal{F}_d$  is the filtering operator for the discrete approximation of  $\mathcal{F}$ , and  $\mathcal{F}_d\{v_0, \dots, v_N\}_k$  is the value of the filtered signal at  $k$ th sample obtained by filtering the data  $v_0, \dots, v_N$ . Then, the least squares problem parameter estimation is given by

$$A\phi = b, \quad (22)$$

where  $A \in \mathbb{R}^{3(N+1) \times 9}$  and  $b \in \mathbb{R}^{3(N+1)}$  are given by

$$A = \begin{bmatrix} G(t_0) \\ G(t_1) \\ \vdots \\ G(t_N) \end{bmatrix}, \quad b = \begin{bmatrix} g(t_0) \\ g(t_1) \\ \vdots \\ g(t_N) \end{bmatrix}.$$

At this point one can use standard least squares techniques [7] to obtain a solution for  $\phi$ . However, such an approach fails to capture known constraints on the estimated parameters. One obvious constraint is  $J = J^T > 0$ , that is, the inertia matrix is symmetric positive definite. Indeed more specific bounds are generally known about the inertia matrix, such as

$$\alpha I \leq J \leq \beta I, \quad (23)$$

where  $\alpha$  and  $\beta$  are positive scalars bounding the maximum and minimum eigenvalues of  $J$ . Then a general constrained least squares problem can be written as

$$\begin{aligned} & \min_{\phi} (A\phi - b)^T W (A\phi - b) \quad \text{subject to} \\ & \alpha I \leq \sum_{i=1}^6 \phi_i P_i \leq \beta I \\ & c_i \leq \phi_i \leq d_i, \quad i = 1, \dots, 9 \end{aligned} \quad (24)$$

where  $c_i$  and  $d_i$ ,  $i = 1, \dots, 9$ , bound the individual components of  $\phi = [\phi_1, \dots, \phi_9]^T$ ,  $W = W^T > 0$  is a weight matrix, and  $P_1, \dots, P_6$  are symmetric matrices that are used to impose minimum and maximum eigenvalue constraints on the inertia matrix. They are given by

$$\begin{aligned} P_1 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ P_4 &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad P_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \end{aligned}$$

Equation (24) introduces LMI constraints and is a semidefinite programming problem [10], [11] that can be solved very efficiently with guaranteed convergence to the unique optimum solution by existing algorithms [1]. Additional constraints can be accommodated in this framework to further improve the inertia estimate with small data sets. For

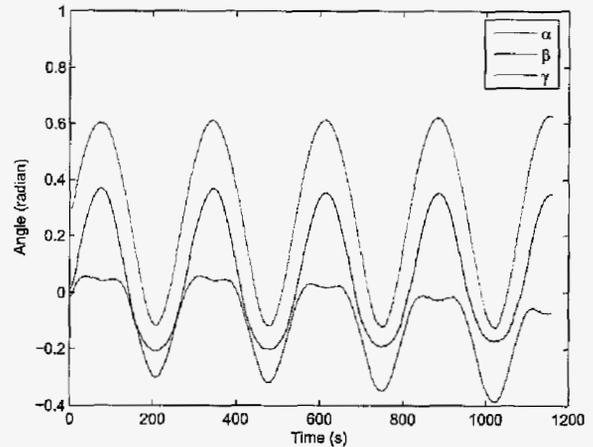


Fig. 4. Time History of the AP Attitude

example, bound quantifying the diagonal dominance of the inertia matrix and be added:

$$\begin{aligned} -\gamma_{41}\phi_1 &\leq \phi_4 \leq \gamma_{41}\phi_1, \\ -\gamma_{42}\phi_2 &\leq \phi_4 \leq \gamma_{42}\phi_2, \\ -\gamma_{51}\phi_1 &\leq \phi_5 \leq \gamma_{51}\phi_1, \\ -\gamma_{53}\phi_3 &\leq \phi_5 \leq \gamma_{53}\phi_3, \\ -\gamma_{62}\phi_2 &\leq \phi_6 \leq \gamma_{62}\phi_2, \\ -\gamma_{63}\phi_3 &\leq \phi_6 \leq \gamma_{63}\phi_3, \end{aligned}$$

where  $\gamma_{41}$ ,  $\gamma_{42}$ ,  $\gamma_{51}$ ,  $\gamma_{52}$ ,  $\gamma_{62}$ ,  $\gamma_{63}$  are positive scalars.

#### IV. DATA COLLECTION AND POSTPROCESSING

To compare standard least squares, total least squares, and constrained least squares, we acquired test data from the FCT. The FCT avionics closely emulate those of a spacecraft. Torques produced by three orthogonally mounted reaction wheels and sixteen cold gas thrusters rotate the AP. Optical gyroscopes and an in-house developed celestial sensor (CS) [9] measure the rotation of the AP. Using closed loop control with the gyroscopes and reaction wheels, the AP was rotated without violating the physical constraints imposed on tip and tilt. The controller oscillated the platform  $\pm 15$  degrees in tip and tilt and  $\pm 20$  degrees in spin. As part of a frequency sweep, the period of the oscillations continuously decreased at a rate 11.25 seconds per minute. Figure 4 illustrates the rotation as measured by the CS. Here,  $\alpha$ ,  $\beta$  and  $\gamma$  are tip, tilt and spin, respectively.

The filtered estimation approach of (20) requires measurement of the AP angular velocity and the reaction wheels speeds relative to the AP body. Wheel speeds are directly measured by tachometers. Rotation of the platform is measured by two devices: the CS and gyroscopes. The gyroscopes of the FCT run in accumulated angle mode producing measurements as integrated angles. The celestial sensor measures the AP attitude in the form of a quaternion.

Processing either the integrated angles or the quaternions is necessary to resolve the body angular velocity. Both process entail comparable amounts of work and complexity [12]. Here, we derive the AP angular velocity from the measured quaternions, because the CS measurement frame defines the AP body frame and CS noise is stationary. As part of preprocessing the data, the quaternion time history is smoothed to reduce noise via methods presented in [10]. From the smoothed quaternion data set, we determine the angular velocity of the AP by methods found in [12].

## V. RESULTS FROM EXPERIMENTAL DATA

Standard least squares, total least squares and constrained least squares are applied to the derived body angular velocity and the measured reaction wheel spin rates to estimate the AP inertia and mass imbalance. The estimates of the AP inertia and CG-CR offset are:

$$\begin{aligned}
 J_{ls} &= \begin{bmatrix} 5.7928 & -0.7568 & -0.9744 \\ -0.7568 & 10.8692 & 0.7670 \\ -0.9744 & 0.7670 & 15.3069 \end{bmatrix} \text{ kg} \cdot \text{m}^2 \\
 J_{tls} &= \begin{bmatrix} 5.8399 & -0.7533 & -0.9346 \\ -0.7533 & 10.8726 & 0.7733 \\ -0.9346 & 0.7733 & 15.3611 \end{bmatrix} \text{ kg} \cdot \text{m}^2 \\
 J_{cls} &= \begin{bmatrix} 7.5691 & -0.5298 & -0.0300 \\ -0.5298 & 11.6034 & 0.8122 \\ -0.0300 & 0.8122 & 16.0833 \end{bmatrix} \text{ kg} \cdot \text{m}^2 \\
 z_{ls} &= \begin{bmatrix} -0.5515 & -0.6313 & 0.4282 \end{bmatrix}^T \times 10^{-4} \text{ kg} \cdot \text{m} \\
 z_{tls} &= \begin{bmatrix} -0.5515 & -0.6313 & 0.4283 \end{bmatrix}^T \times 10^{-4} \text{ kg} \cdot \text{m} \\
 z_{cls} &= \begin{bmatrix} -0.6071 & -0.6786 & 0.0413 \end{bmatrix}^T \times 10^{-4} \text{ kg} \cdot \text{m}.
 \end{aligned}$$

An independent estimate of the AP inertia relative to the CR exists as the result of a comprehensive SolidWorks model. This estimate is

$$J_{sw} = \begin{bmatrix} 8.0105 & -0.2330 & -0.5063 \\ -0.2330 & 11.4476 & 0.0163 \\ -0.5063 & 0.0163 & 14.9564 \end{bmatrix} \text{ kg} \cdot \text{m}^2.$$

From this model, the lower bound on  $J$  were chosen to be

$$J > \begin{bmatrix} 7.5 & 0.0 & 0.0 \\ 0.0 & 7.5 & 0.0 \\ 0.0 & 0.0 & 7.5 \end{bmatrix} \text{ kg} \cdot \text{m}^2$$

Using the SolidWorks estimate as the basis for comparison, the relative errors of the least squares estimates are defined as:

$$\begin{aligned}
 e_{ls} &= \|J_{ls} - J_{sw}\| / \|J_{sw}\| = 0.2036, \\
 e_{tls} &= \|J_{tls} - J_{sw}\| / \|J_{sw}\| = 0.1988, \\
 e_{cls} &= \|J_{cls} - J_{sw}\| / \|J_{sw}\| = 0.1259.
 \end{aligned}$$

## VI. CONCLUSIONS

A constrained least squares approach based on LMIs has been developed. This approach can incorporate the positive definiteness of the inertia matrix directly. Furthermore, additional bounds on the inertia matrix can also be considered. These bounds are particularly useful in obtaining

more accurate estimates of the inertia matrix when there is a limited amount of angular motion allowed (in terms of angular velocities and displacements) as is the case for the FCT robots. This technique, along with standard least squares and total least squares, was then applied to estimate inertia matrix of the FCT robot AP using experimental data. Of the three estimates, the estimate using the technique developed here most closely agrees with the pre-existing, independently-derived estimate of the AP inertia.

Although the characteristics of the FCT robot necessitated this work, the algorithm developed here can produce more accurate estimates of the inertia matrix than standard least squares with smaller data sets. Consequently, it will be useful in other engineering applications beyond FCT. In particular, this method can provide accurate inertia estimates for flight missions with a minimal consumption of fuel and time. Additionally, the constrained least squares approach is a natural extension of the standard least squares methods for inertia estimation via the inclusion of the matrix constraints as LMIs.

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