# Mode-Expansion Method for Predicting Radar Signature above Rough Ocean Surfaces at Low-Grazing Angle

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The Mode-Expansion Method (MEM) is introduced to calculate the electromagnetic (EM) waves scattered by 2-D rough water surfaces at low-grazing angles. The Electric Field Integral Equation (EFIE) is used in defining the problem and is simplified by using the Impedance Boundary Condition (IBC). The surface currents are expressed as the sum of modes expanded as the Fourier series with incident wave as the dominant mode. It is shown that, by the MEM and for the geometry with transmitting and receiving waves at low-grazing angles, very few modes are needed in solving the forward scattering field with reasonable accuracy.

## Introduction

The attempts to understand radar signatures of ocean surfaces have attracted much attention in the development of fast computer code for calculating EM waves scattered by highly conducting dielectric surfaces. In theory, the standard Method of Moments (MoM) technique can be used to solve for the unknown surface currents on the rough surfaces; however, the required dense discretization of the rough surfaces makes the conventional numerical method impractical because of extremely large memory requirement and long computational time.

Several methods have been developed to render the problem tractable. For example, iterative Method of Moments such as the Generalized Forward-Backward (GFB) method and the banded matrix iterative algorithm has been developed based on matrix splitting techniques. Computational efficiency has been improved for the GFB method by introducing the spectral acceleration scheme for evaluating the impedance elements. Fast methods include the Fast Multipole Method and the Fast Far Field Algorithm, all of which rely on grouping schemes and approximations for group interactions. However, to solve for surface currents, point matching sub-domain basis functions are usually used that must sample the surface with ten or twenty points per wavelength. Hence, for large 2-D problems or any 3-D problems, the computational complexity can still be significant.

Recently the MEM was introduced to calculate the EM waves scattered by perfectly electrically conducting (PEC) object on rough ocean surfaces [1] where the rough surface is also modeled as PEC for EM waves in the GHz bound. In this paper, the MEM is extended to the highly conducting dielectric surfaces for both transverse electric (TE) and transverse magnetic (TM) polarized waves scattered by rough water surfaces at low grazing angles.

## Mode-Expansion Method

The geometry of the problem is illustrated in Fig. 1 where an EM wave incident upon 2-D dielectric rough water surfaces. The EFIE with IBC for TE and TM waves is rewritten in a compact form as

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$$\alpha \frac{1}{2} J_S(\bar{r}) - \int_{S'} dS' J_S(\bar{r}')\left[\alpha \hat{n}' \cdot \nabla' g(\bar{r},\bar{r}') + \beta i \omega \mu_o g(\bar{r},\bar{r}')\right] = \psi_{inc}(\bar{r}) \tag{1}$$

where $\alpha = \eta_1$ and $\beta = 1$ for TE wave, $\alpha = 1$ and $\beta = \eta_1/\eta_o^2$ for TM wave with the intrinsic impedances expressed as $\eta_o = \sqrt{\mu_o/\varepsilon_o}$ and $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$ for free space and water, respectively. $\psi_{inc}(\bar{r})$ is the unit surface-normal vector, $\psi_{inc}(\bar{r})$ is the incident wave, and $J_S$ denotes the surface current on the rough surface.

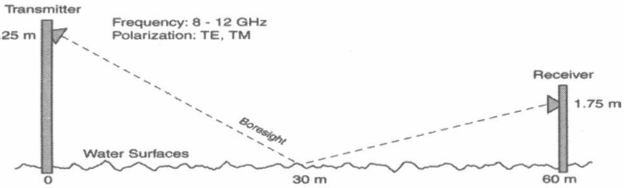


**Figure 1**. Geometry of the problem: EM wave, transmitted and received at low-grazing angles, scattered by highly conducting dielectric rough surfaces.

The Mode-Expansion Method aims to solve for the unknown function $J_S$ on the rough surface by performing the following expansion

$$J_S(\bar{r}') = \sum_p \sum_m a_{pm} f_p(\bar{r}') e^{i\left(k_{ms} + \bar{k}_i \cdot \hat{s}'\right)s'} \tag{2}$$

where $k_{ms} = 2\pi m/L_p$ with $m = 0, \pm1, \pm2, \ldots$ specifying mode numbers, $a_{pm}$ is the unknown amplitude, $f_p(\bar{r}') = 1$ (for $\bar{r}' \in$ patch $p$) or $0$ otherwise, and $L_p$ is the patch length. Upon plugging Eq. (2) into (1), and multiplying the test function $T_q(\bar{r}) = e^{-ik_{ns}s} f_q(\bar{r})$ on both sides of Eq. (1), the matrix equation is obtained by performing integration over the entire surface as following

$$\sum_p \sum_m \left( \delta_{qp} \delta_{nm} \alpha \frac{1}{2} L_q + Z_{qn,pm} \right) a_{pm} = V_{qn} \tag{3}$$

where $\delta_{qp} = 1$ (if $q = p$), otherwise $\delta_{qp} = 0$; and $\delta_{nm} = 1$ (if $n = m$), otherwise $\delta_{nm} = 0$. The impedance $Z_{qn,pm}$ is expressed as the following integration

$$Z_{qn,pm} = \frac{\omega \mu_o}{4} \int_0^{L_q} \int_0^{L_p} ds ds' e^{ik_{ms}s'} e^{ik_{ns}s} \left[ \beta H_0^{(1)}\left(k\left|\bar{r} - \bar{r}'\right|\right) - i \frac{\alpha}{\omega \mu_o} \hat{n}' \cdot \nabla' H_0^{(1)}\left(k\left|\bar{r} - \bar{r}'\right|\right) \right] \tag{4}$$

and the right-hand side of Eq. (3) is given by

$$V_{qn} = \int_0^{L_q} ds \, \psi_{inc}(\bar{r}) e^{-ik_{ns}s} \tag{5}$$

The integration of Eq. (4) can be evaluated efficiently by expressing the Hankel function as the integral in spectral space and integrating it along the Steepest Descent Path (SDP). Eq. (5) can be evaluated directly by numerical integration. Upon solving the amplitude $a_{pm}$ in Eq. (3), the scattering field is calculated numerically as

$$\begin{aligned} \psi_s(\bar{r}) = \frac{i}{4} \sum_{p,m} a_{pm} \int_0^{L_p} ds' e^{ik_{ms}s'} &\left\{ \beta i \omega \mu_o H_0^{(1)}\left(k\left|\bar{r} - \bar{r}'\right|\right) \right. \\ &\left. + \alpha k \left|\bar{r} - \bar{r}'\right|^{-1} \left[ -S_z'(x - x') + S_x'(z - z') \right] H_1^{(1)}\left(k\left|\bar{r} - \bar{r}'\right|\right) \right\} \end{aligned} \tag{6}$$

where $H_1^{(1)}$ is the first-order Hankel function of the first kind, $S_x'$ and $S_z'$ are cosine and sine functions of the surface tilt angle, respectively.

## Numerical Results

Fig. 2 shows the simulation result for the multi-path field with TE polarization over rough water surfaces of the 1/10th-scaled ocean state 3. Plot (a) shows the multi-path field as functions of frequency and receiver's (Rx) height. Plot (b) is the Fourier transform of the data shown in plot (a), representing the multi-path field in time-domain. Plot (c) shows the power spectrum for the rough water surfaces, and plot (d) is the probability density function (PDF) for the data shown in plot (b) with the bin of 2 dB. Agreement is achieved in comparison with the measurement results [2].

Fig. 3 shows the simulation result for the multi-path field with TM polarization in the same format as shown in Fig. 2. From the simulation result, it is found that the contrast of the multi-path field of the TM wave (as shown in plot (a) in Fig. 3) is much smaller than the TE wave. This is due to the fact that, at the low grazing angle of the incident wave depicted in Fig. 1, the forward scattering angle is near the Brewster angle (about 5 degrees of grazing), where the TM scattering wave has very low intensity.

The computer codes for the simulation are written in MATLAB, and the simulations are performed in Linux system on the Dell Precision 650 Workstation with dual Xeon 2.0 GHz processor and 2.0 GB RAM. The CPU time used for performing the simulation per frequency is approximately $50N_s + 0.3N_r$ (sec), where $N_s$ is the number of rough surface profiles, and $N_r$ is the number of receiving points.

## Conclusion

The Mode Expansion Method has been developed for TE and TM waves scattered by highly conducting dielectric rough water surfaces. It shows that only the dominant modes (incident waves) are needed in solving for the surface current with reasonable accuracy for the geometry stated in this paper. Future work includes the optimization of the code for speeding up the simulation, and extended the method to 3-D rough ocean surfaces.

## Acknowledgement

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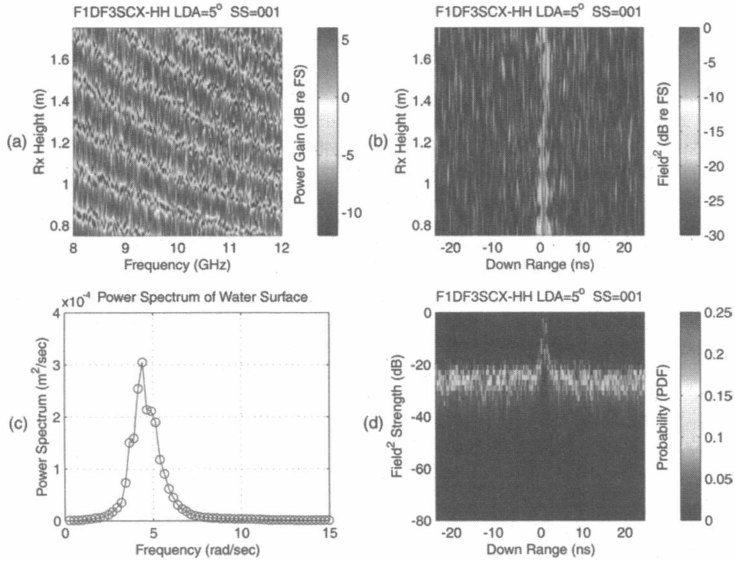


**Figure 2**. TE Multi-path field over rough water surfaces of the 1/10$^{th}$ scaled ocean state 3.

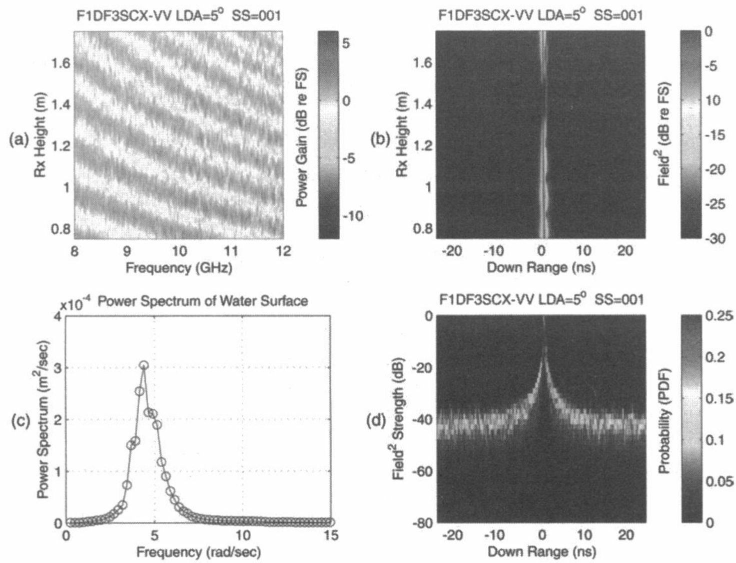


**Figure 3**. TM Multi-path field over rough water surfaces of the 1/10$^{th}$ scaled ocean state 3.