Near-Earth Solar Sail Navigation: Preliminary Results

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Preliminary analyses have been conducted to assess the demands solar sails will make on the design of future navigational systems. A simulation of a 40mx40m solar sail used for raising the altitude of a 157 kg sun-synchronous Earth orbiter in a ~1000 km altitude, nearly circular, spiral orbit was performed. Though the forces and orbit perturbations introduced by the sail are quite small, we find that we can calibrate the accelerations acting on the spacecraft to within ~6x10^4 mm/s^2 in ~3 days, with one pass of Deep Space Network (DSN) tracking per day.

OVERVIEW

Preliminary analyses [Kakuda et al, 1994, Sims et al., 1999] and DS1 operational experience with Solar Electric Propulsion (SEP) [Wolff et al, 1998] show that low-thrust navigation is particularly sensitive to small errors in dynamical models and tracking data, particularly for unstable orbits [Koon et al, 2000]. Improper treatment of mis-modeled dynamical errors can, over time, result in large dispersions in the spacecraft trajectory. The DS1 experience has shown that concurrent design and implementation of the spacecraft and navigation systems is extremely valuable in terms of optimizing design and reducing operational risks. This is particularly true for emerging solar sail technologies that will present navigation with even more demanding challenges than SEP.

The overall objective of the analysis reviewed here is to assess the accuracy with which the solar pressure induced acceleration acting on a solar sail spacecraft in near-Earth orbit can be retrieved from ground-based tracking data. We considered a sun-synchronous orbit at an altitude of approximately 1000 km, where the semi-major axis steadily increases due to the thrust imparted by solar pressure. The sail itself is assumed to be a flat plate with a very large area-to-mass ratio. Simulations were conducted for best-case and worst-case tracking conditions in the presence of measurement noise, measurement biases, and stochastic non-gravitational forces.

MODELING ASSUMPTIONS

Orbit

The a priori Keplerian orbital elements and uncertainties, expressed in terms of Earth-true-equator-of-date coordinates, where chosen to be
\( t_0 = \text{October 1, 2005, 12:00:00:00 GMT} \) initial time (epoch)
\( a = 7378.14 \pm 100\text{km} \) semi-major axis
\( e = 0 \pm 0.01 \) eccentricity
\( \Omega = 270 \pm 2^\circ \) longitude of ascending node
\( \omega = 0 \pm 2^\circ \) argument of perigee
\( i = 99.479 \pm 2^\circ \) inclination
\( t_p = 0 \pm 100\text{s} \) time of periapsis

where the inclination is the critical inclination for sun-synchronous Earth orbiter, given by

\[
i = \cos^{-1} \left( -\frac{2}{3 r_{eq}^2} \frac{a^2}{\mu} \sqrt{\frac{J_2}{a^3}} \left( \frac{\delta \Omega}{\delta t} \right) \right)
\]

where \( r_{eq} \) is the mean equatorial radius, \( \mu \) the gravitational constant, and \( J_2 \) the gravitational oblateness for the Earth. The nodal rate is commensurate with the solar-year, i.e. \( \delta \Omega/\delta t = 2\pi (365.2423/86400) \), which initially keeps the orbit-normal directed toward the sun. Note that the inclination of the orbit will remain virtually constant over time so, as the semi-major axis \( (a) \) is slowly increased, the nodal rate will vary proportionately. As a result, the orbit will not remain sun-synchronous indefinitely.

**Solar Sail**

The solar sail is assumed to be a flat plate with the following properties

\[
\begin{align*}
m &= 157\text{kg} & \text{mass} \\
A &= 40\text{m} \times 40\text{m} \pm 5\% & \text{area} \\
\mu &= 0.415 \pm 10\% & \text{specular reflectivity coefficient} \\
\nu &= 0.040 \pm 10\% & \text{diffuse reflectivity coefficient} \\
\alpha &= 35.26^\circ \pm 0.1^\circ & \text{cone angle (surface normal relative to the sun line)} \\
\delta &= 0^\circ \pm 0.1^\circ & \text{clock angle (surface normal is in the orbit plane)}
\end{align*}
\]

Additional forces acting on the spacecraft include those due to Earth gravitational perturbations, drag, and unknown non-gravitational (stochastic) accelerations. An eighth degree and order spherical harmonic expansion is used to model gravity, an exponential atmosphere model is used to compute drag, and the stochastic accelerations are characterized by colored noise with a correlation time of 2 hr, which is approximately one orbital period. The a priori stochastic accelerations are assumed to be zero with a best-case noise figure of \( \pm 2 \times 10^{-12} \text{ km/s}^2 \) and a worst-case noise figure of \( \pm 20 \times 10^{-12} \text{ km/s}^2 \).
Tracking

Simulated tracking scenarios were based on S-band tracking from the 26m Deep Space Network (DSN) antennas at Goldstone (DSS16), Canberra (DSS46), and Madrid (DSS66). The 1-σ tracking errors are assumed to be

- **Measurements**
  - $\sigma_{\text{Range}} = \pm 5$ m
  - $\sigma_{\text{Doppler}} = \pm 1$ mm/s @60s
  - $\sigma_{\text{Angles}} = \pm 0.1$ deg
- **Calibration Errors**
  - $\sigma_{\text{Station locations}} = \pm 50$ cm/axis
- **Media**
  - $\sigma_{\text{Troposphere}} = \pm 1$ cm dry, $\pm 4$ cm wet
  - $\sigma_{\text{Ionosphere}} = \pm 5$ cm day, $\pm 1$ cm night

Case studies were performed for 3 tracking scenarios; 1 pass-per-day, 3 passes-per-day, and ‘continuous’ tracking from each station consecutively when the spacecraft is in view.

**COVARIANCE ANALYSIS**

Covariance analyses were performed to compute the expected error in the orbit and key solar sail parameters due to uncertainties in estimated parameters, *consider* parameters, and tracking data. *Consider* parameters are estimable parameters that are not included among the estimated parameters. For the studies discussed here, we chose the *estimated* parameters to be

- Keplerian orbital elements (*s/c* state)
- stochastic, non-gravitation forces
- flat Plate:
  - solar sail area
  - specular reflectivity
  - diffuse reflectivity
    - solar sail cone and clock angle

and the tracking and *consider* parameters to be

- measurement noise
- station locations
- media calibrations:
  - tropospheric refraction
  - ionospheric refraction

Orbit determination simulations were conducted for a number of tracking scenarios using
the estimation strategy, a priori uncertainties, and measurement noise described above. The standard deviations derived from the resulting consider-covariance matrix are taken to be the errors in the estimated parameters reported below.

RESULTS

Orbit Variations

The secular increase in semi-major axis (see Figure 1) was found to satisfy the relationship

\[ \Delta a = \frac{8\pi \kappa \mu}{3\sqrt{3}} = 0.339 \text{ km/orbit} \]

where \( \kappa \) is the along-track acceleration induced by the solar sail. The secular increase in eccentricity results from unbalanced, once-per-revolution variation in along-track accelerations also induced by the solar sail. Since the inclination is chosen to ensure that the orbit is sun-synchronous, the orbit-normal is initially \( \sim 100^\circ \) off the sun-line. This results in a once-per-revolution perturbation that causes a secular increase in eccentricity.

At constant inclination, the magnitude of the nodal rate decreases as semi-major axis increases (see figure below). Therefore, the orbit-normal slowly drifts away from the sun-line. This results in long period variations in the Earth-Probe-Sun (EPS) angle, semi-major axis, and eccentricity.

Figure 1. Perturbations in solar sail orbital parameters
The inclination remains virtually unperturbed, whereas the argument of periapsis decreases secularly due to unbalanced, once-per-revolution accelerations. (Note that the initial rapid variation in $\omega$ is due to the fact that it is undefined when $e = 0$, i.e. its initial rate of change is infinite.) To first order, the nodal variation is designed to keep the sail in a sun-synchronous orbit. However, at constant inclination, the magnitude of the nodal rate decreases as the semi-major axis ($a$) increases.

The Earth-Probe-Sun (EPS) angle varies initially by $\sim \pm 10^\circ$ each revolution due to the fact that the orbit plane is not exactly normal to the sun-line. The departure of the EPS from $90^\circ$ will steadily increase as the orbit-normal drifts away from the sun-line. (Note that the $\sim 11$ day cycle shown in the adjacent figure is actually and alias frequency resulting from sampling a once-per-revolution ($\sim 2 hr$) signal every 7 days.)

**Solar Pressure Accelerations**

The total acceleration resulting from solar pressure forces acting on the sail consists of two components in the $Z$-axis and $Y$-axis directions. The $Z$-axis is directed from the Probe to the Sun, therefore the acceleration is negative. The $Y$-axis is in a plane normal to the $Z$-axis and is directed along the velocity vector, and the $X$-axis points toward the Earth’s center.

![Figure 2. Accelerations acting on the spacecraft due to solar pressure](image-url)
To maximize the force in the velocity direction at all times, the clock and cone angle of the outward normal to the sail relative to the Z-axis are optimized. The clock angle is rotated once-per-revolution so that outward normal is always directed along the Y-axis and the cone angle is pitched at a constant 35.26° relative to the Z-axis [McGinnes, xxxx]. The accelerations vary slowly with time due to the fact that the orbit-normal is drifting away from the sun-line.

Uncertainties in Orbital Elements

Figure 3 depicts the uncertainty in solar sail state estimates as a function of tracking time. All three cases shown are for worst-case stochastic force errors of 20x10^{-12} km/s^2. Each curve represents 1 pass-per-day (blue/upper), 3 pass-per-day (magenta/middle), and continuous (green/lower) tracking. Observe that the best estimate can be obtained within a few days for continuous tracking, a week for 3 pass-per-day tracking, and two weeks for 1 pass-per-day tracking.

![Uncertainties on Eccentricity](image)

**Figure 3.** Typical uncertainties in estimate solar sail state parameters as a function of tracking time

Figure 4 depicts the uncertainty in solar sail acceleration estimates as a function of tracking time. All three cases shown are for worst-case stochastic force errors of 20x10^{-12} km/s^2. Each curve represents 1 pass-per-day (blue/upper), 3 pass-per-day (green/middle), and continuous (magenta/lower) tracking. Observe that the best estimate can be obtained within 3 days, regardless of the tracking scenario.
Uncertainty in Solar Radiation Pressure Acceleration

Figure 4. Uncertainties in estimate solar sail state accelerations as a function of tracking time

SUMMARY

Orbit Variations

Numerical integration verifies that, during the first few months of flight, the semi-major axis of the orbit increases in accordance with solar theory. At fixed inclination, the precession of the orbit slowly departs from the sun synchronous rate as semi-major axis increases. Eventually, after a number of months, the solar sail orientation will no longer be optimal in terms of orbit-raising capability. Perhaps this can be mitigated by periodically re-optimizing the sail's pointing program. Alternatively, the inclination could be slowly changed so that the orbit would remain sun synchronous as it spirals out from the Earth. The Earth-Sun-Probe (ESP) angle varies by $\pm 100^\circ$ each revolution because, for the orbit to be sun synchronous, it is inclined by $\sim 100^\circ$ relative to the sun-line. This results in an unbalanced, once-per-revolution along-track force that introduces a secular growth in eccentricity ($e$) and argument of periapsis ($\omega$). This will become even more pronounced as the orbit-normal precesses away from the sun-line. Though it was not considered here, it may be possible to modify the solar sail attitude program to balance these once-per-revolution along-track forces.

Orbit Estimate Uncertainty

The covariance analyses were performed based on the orbit and solar sail models described above and tracking from NASA's Deep Space Network (DSN). The DSN tracking types assumed were X-band, two-way Doppler and range data. The DSN has antenna complexes at three sites worldwide, one in Goldstone, California, USA, one in Madrid, Spain, and one in Canberra, Australia. For an earth orbiting sail, this means that there will be some gaps in coverage when the spacecraft is below the horizon. Note that the term 'continuous' tracking refers to tracking whenever the spacecraft is in view, not
that the spacecraft is being tracked at all times. For the three tracking scenarios considered, it was observed that the best estimate can be obtained within a few days for continuous tracking, a week for 3 pass-per-day tracking, and two weeks for 1 pass-per-day tracking. Overall best performance is obtained from 'continuous' tracking.

Our results show that the sail orbit is determined to well within 1 km after less than a week of continuous tracking and less than three weeks of once-per-day tracking. In addition, estimates for the solar sail acceleration can be obtained to within +1% after ~3 days of tracking, regardless of the tracking scenario. It should be noted that these results are preliminary and may be somewhat optimistic in that the stochastic non-gravitational forces were over-constrained and uncertainties in drag may be overly simplified.

FUTURE WORK

The current analysis needs to be extended to include solar sail pointing scenarios that reduce unbalanced, once-per-revolution forces. Also, methods that compensate for asynchronous precession relative to the sun-line are critical to accessing the feasibility of using solar sails for extended orbit-raising maneuvers. It is anticipated that more conservative estimates for the expected uncertainties in the orbit and key parameters will result from further study of errors introduced by non-gravitational stochastic forces and drag forces. In addition, more practical tracking requirements may result from analyzing the sensitivity to limited tracking data sets, e.g. Doppler-only and one-pass-per-week DSN tracking.

Solar sails are being considered for a number of new and demanding applications, e.g.

- Geo-synchronous orbit disposal systems
- Polar-Explorer remote sensing missions
- Earth orbit missions
- Solar-Polar probes
- Unstable libration-point probes
- Solar System Exploration (SSE) missions to comets and outer planets

each with their own distinct navigation requirements. The analysis tools and techniques used here need to be extended to meet the demands missions such as these will make on solar sail navigation.

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