National Aeronautics and
Space Administration
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

TPF Terrestrial Planet Finder Interferometer

Removing instability noise from nulling interferometers

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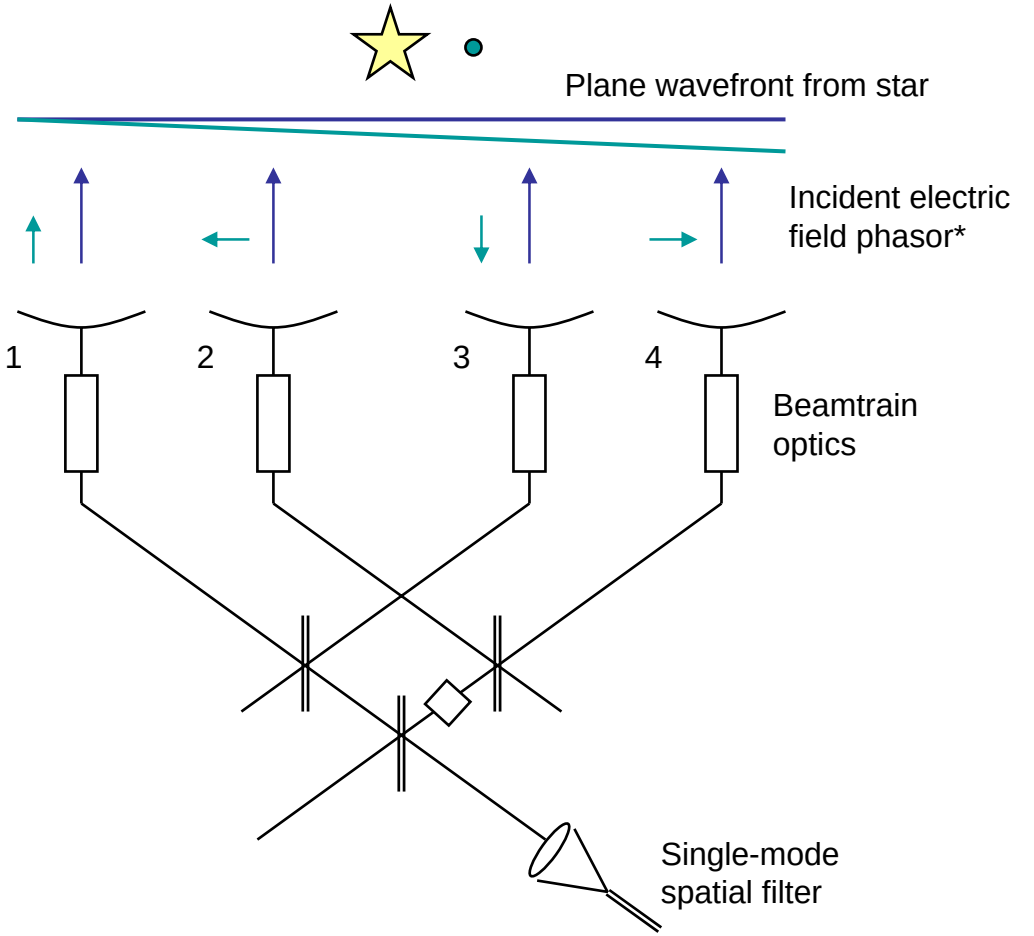
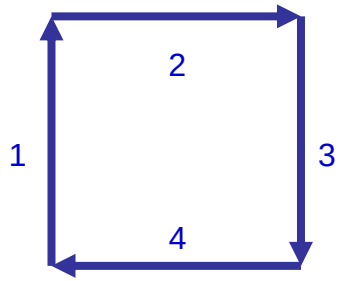
Outline

- Systematic error: background
- Current approach
- New approach
- Benefits
- Negative impacts
- Potential future work
- Summary

Electric field phasors



Ideal stellar electric fields



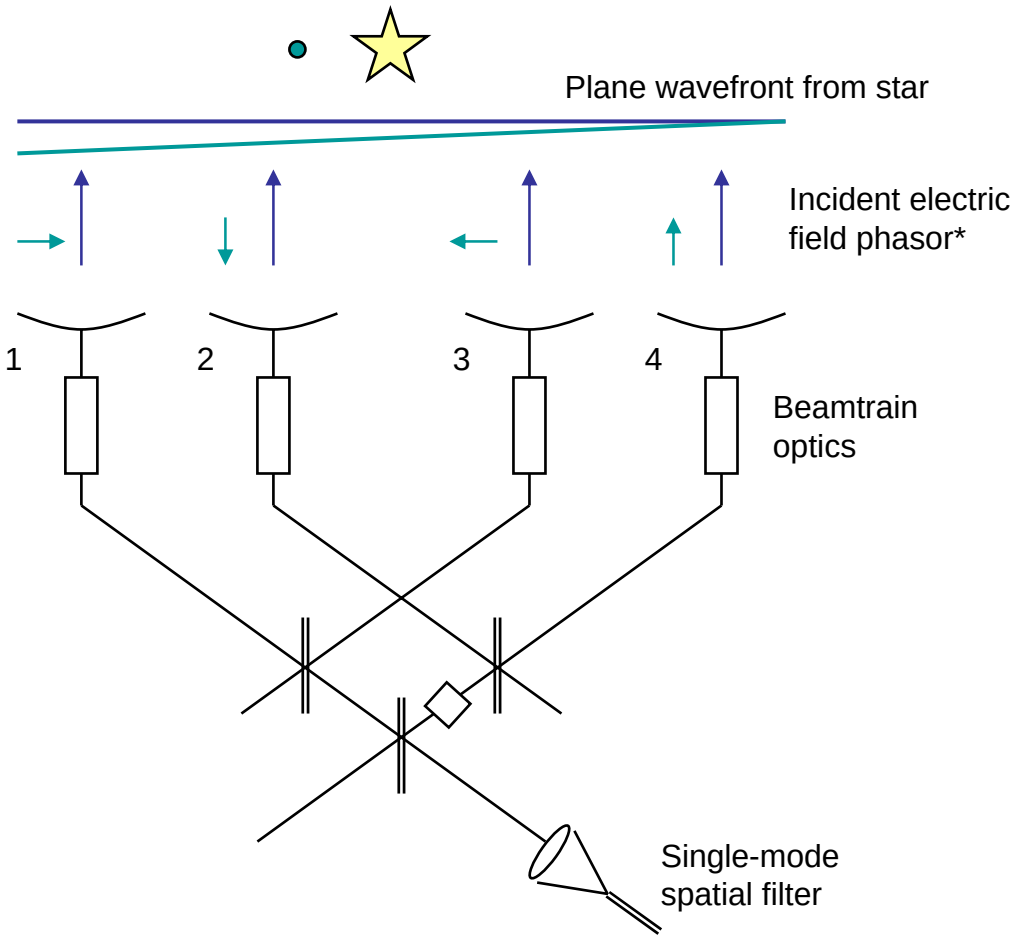
Planet electric field
~ 1/3000 of star

Electric fields* in filter:

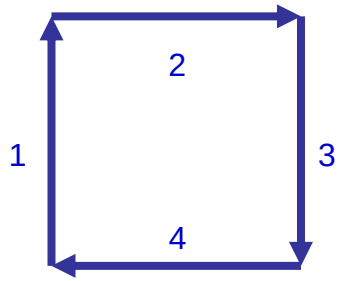
- Star nulled
- Planet transmitted

* Phasor angle represents electric field phase, not polarization

Array rotation



Ideal stellar electric fields



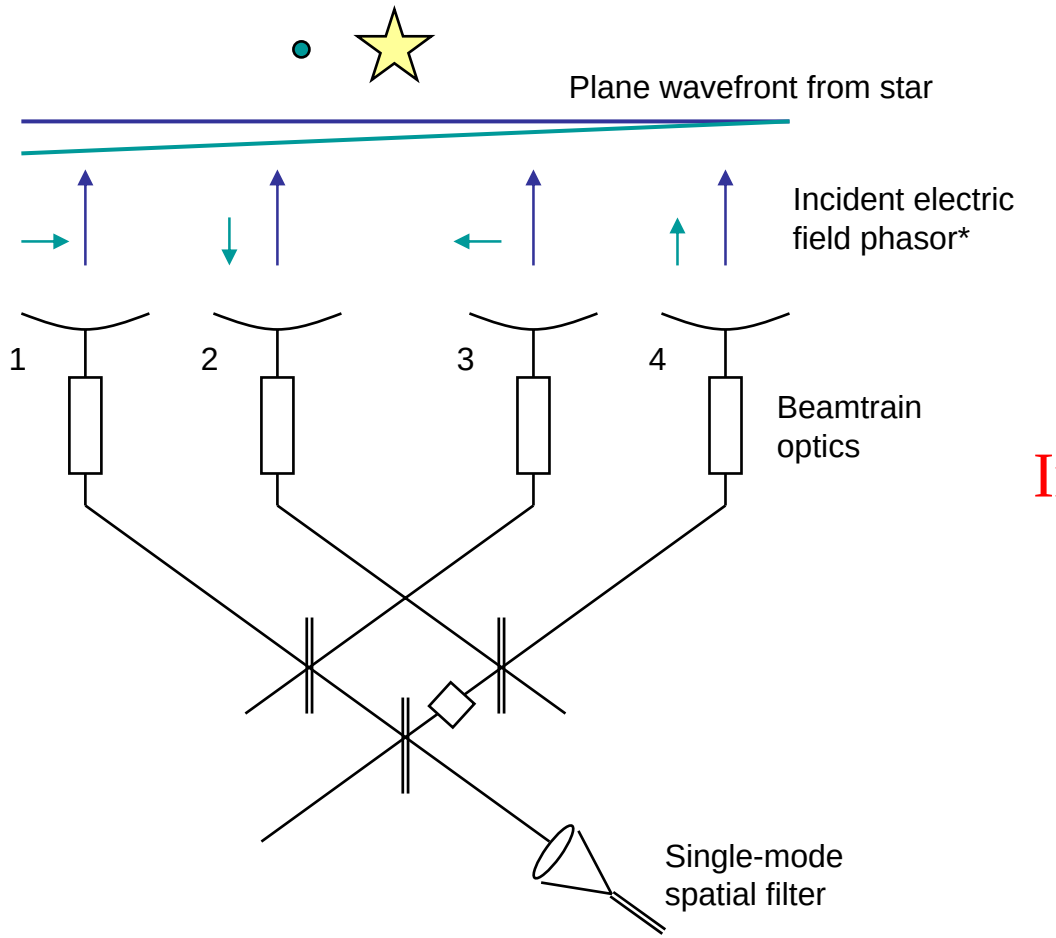
Planet electric fields after half turn rotation

Electric fields* in filter:

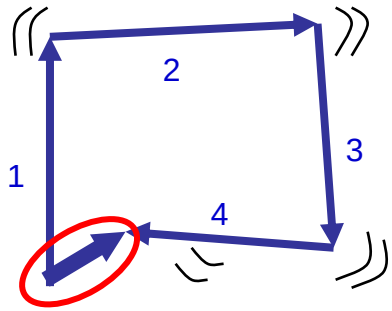
- Star nulled
- Planet nulled

* Phasor angle represents electric field phase, not polarization

Real arrays are not stable

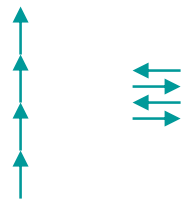


Real stellar electric fields



Instability noise

Planet electric fields



Electric fields* in filter:

- Time-variable leakage
- Time-variable planet

* Phasor angle represents electric field phase, not polarization

The nature of instability noise

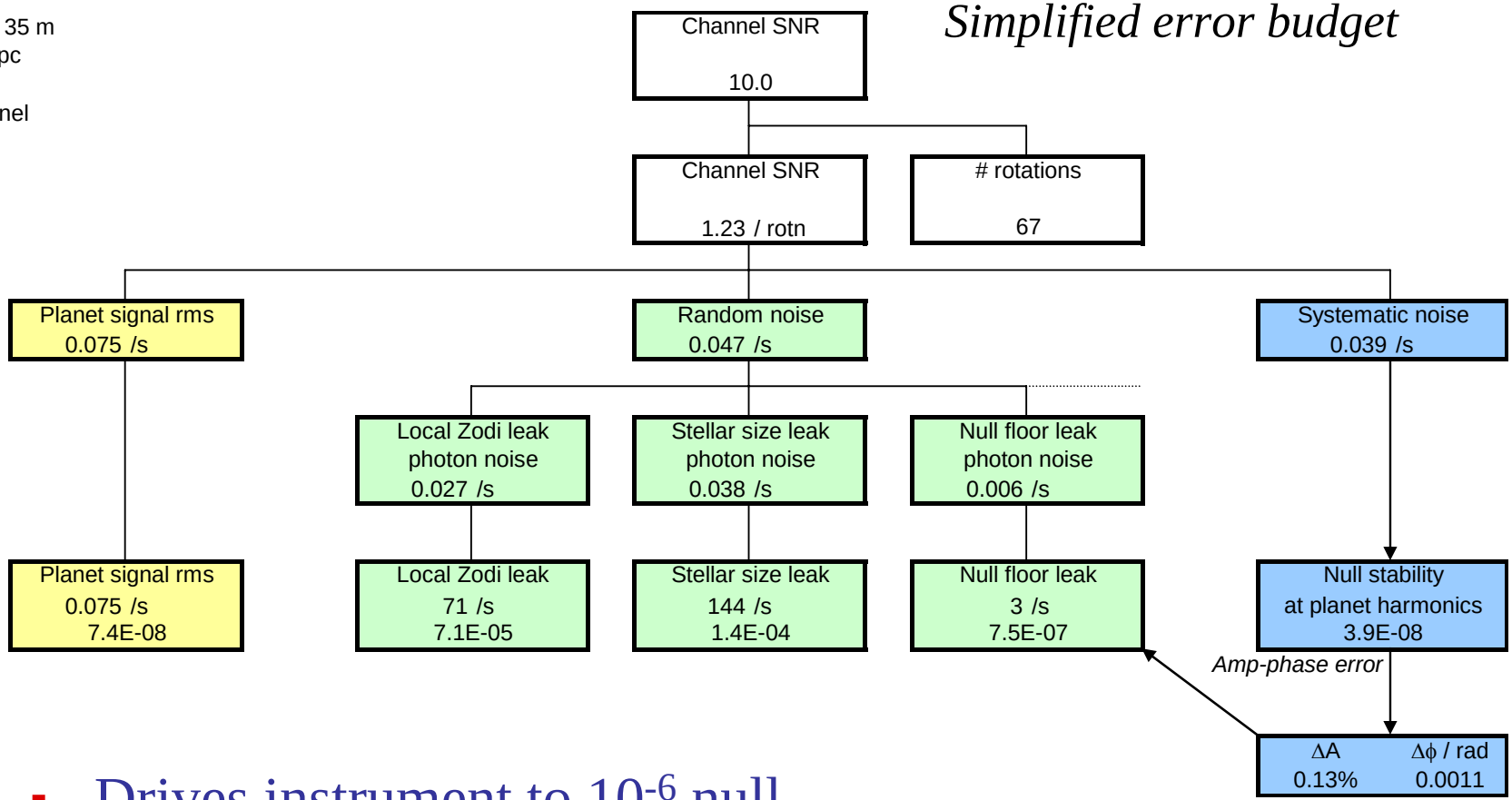
Phase			Amplitude		
Mechanism	**Spectrum**	**Static / dynamic**	**Mechanism**	**Spectrum**	**Static / dynamic**
OPD Vibration	λ^{-1}	dynamic	Tip / tilt	λ^{-2}	dynamic
Fringe tracker offset	λ^{-1}	dynamic	Focus	λ^{-2}	dynamic
Control noise	λ^{-1}	dynamic	Higher order	λ^{-2}	dynamic
Dispersion mismatch	$f_1(\lambda)$	static	Beam shear	λ^{0}	dynamic
Birefringence mismatch	$f_2(\lambda)$	static	Reflectivity / transmittivity	$f_3(\lambda)$	static

- Combinations not removed by phase chopping:
  - 'Amplitude-Phase cross-terms', $\delta A_i \delta\phi_j \sim (a_1\lambda^{-1} + a_3\lambda^{-3})F_*$
  - 'Co-phasing terms', $\delta\phi_i \sim b_1\lambda^{-1} F_*$
  - 'Chop imbalances', $\sim (c_0\lambda^{0} + c_1\lambda^{-1} + c_2\lambda^{-2})F_*$
- F_* is the stellar spectrum
- Adaptive Nuller fixes static terms
- a_1, a_3, b_1, c_1 etc. are random variables, varying from second to second

Impacts of instability noise

X-Array 2:1, 70 x 35 m
Earth-Sun @ 15 pc
50 mas offset
9.5 - 10 um channel

Simplified error budget

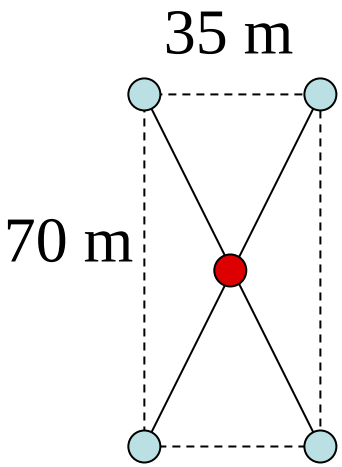


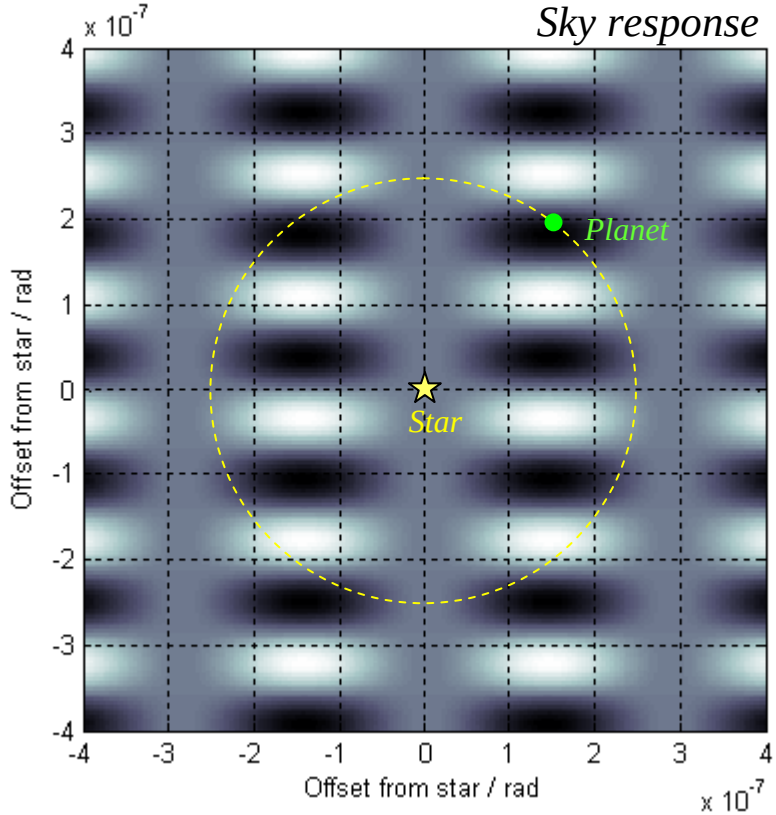
- Drives instrument to 10^{-6} null
- Roughly doubles time for spectroscopy and detection
- Requires significant fraction of time (~ 15%) for calibration

Current approach

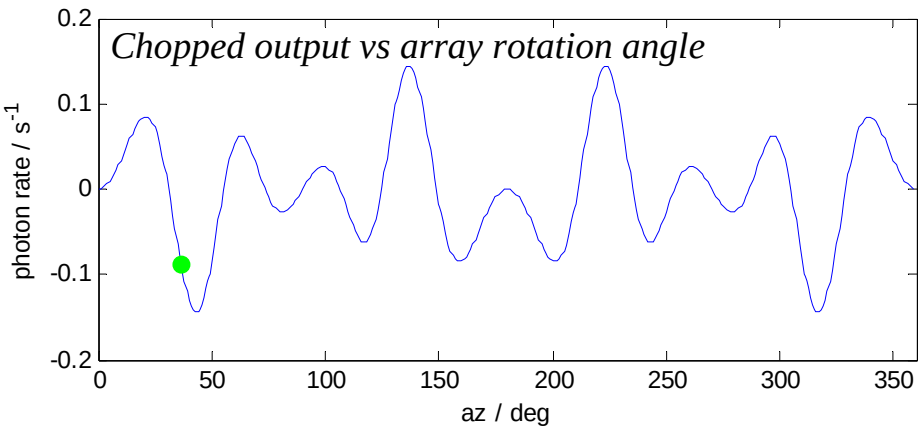
Planet signal

X-Array 2:1

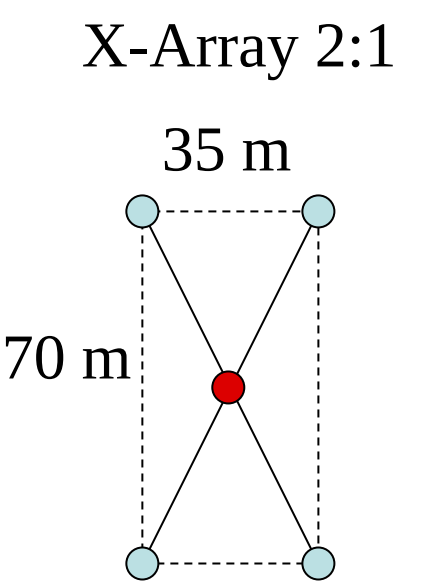


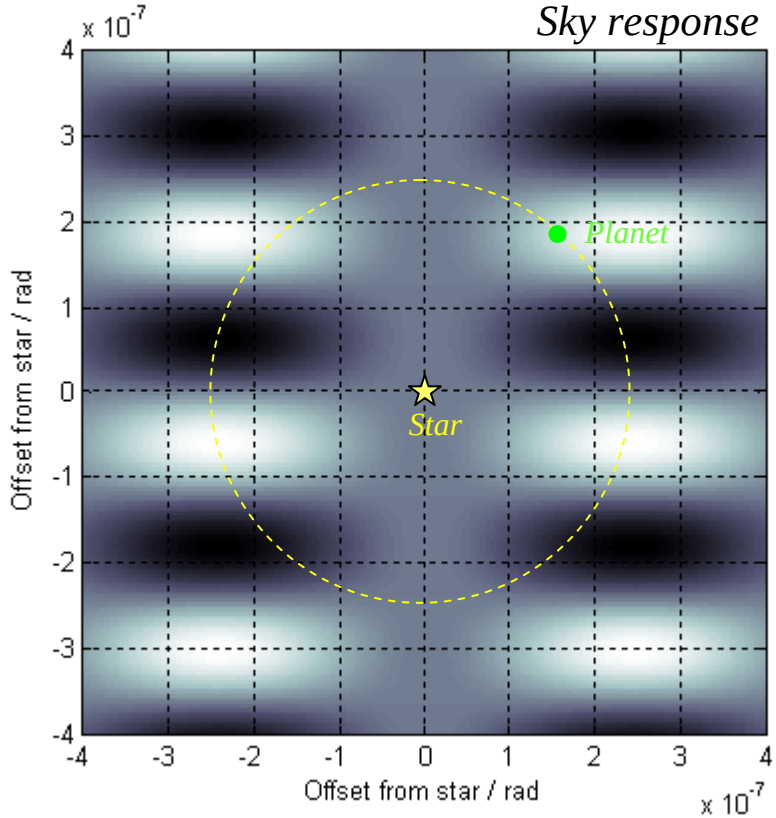


- λ = 10 μm
- Earth @ 50 mas = 2.5 x 10^{-7} rad



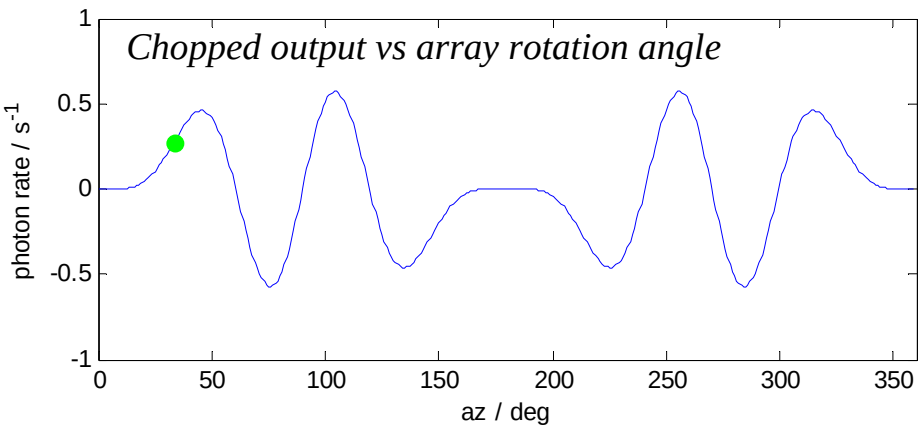
Planet signal, longer wavelength





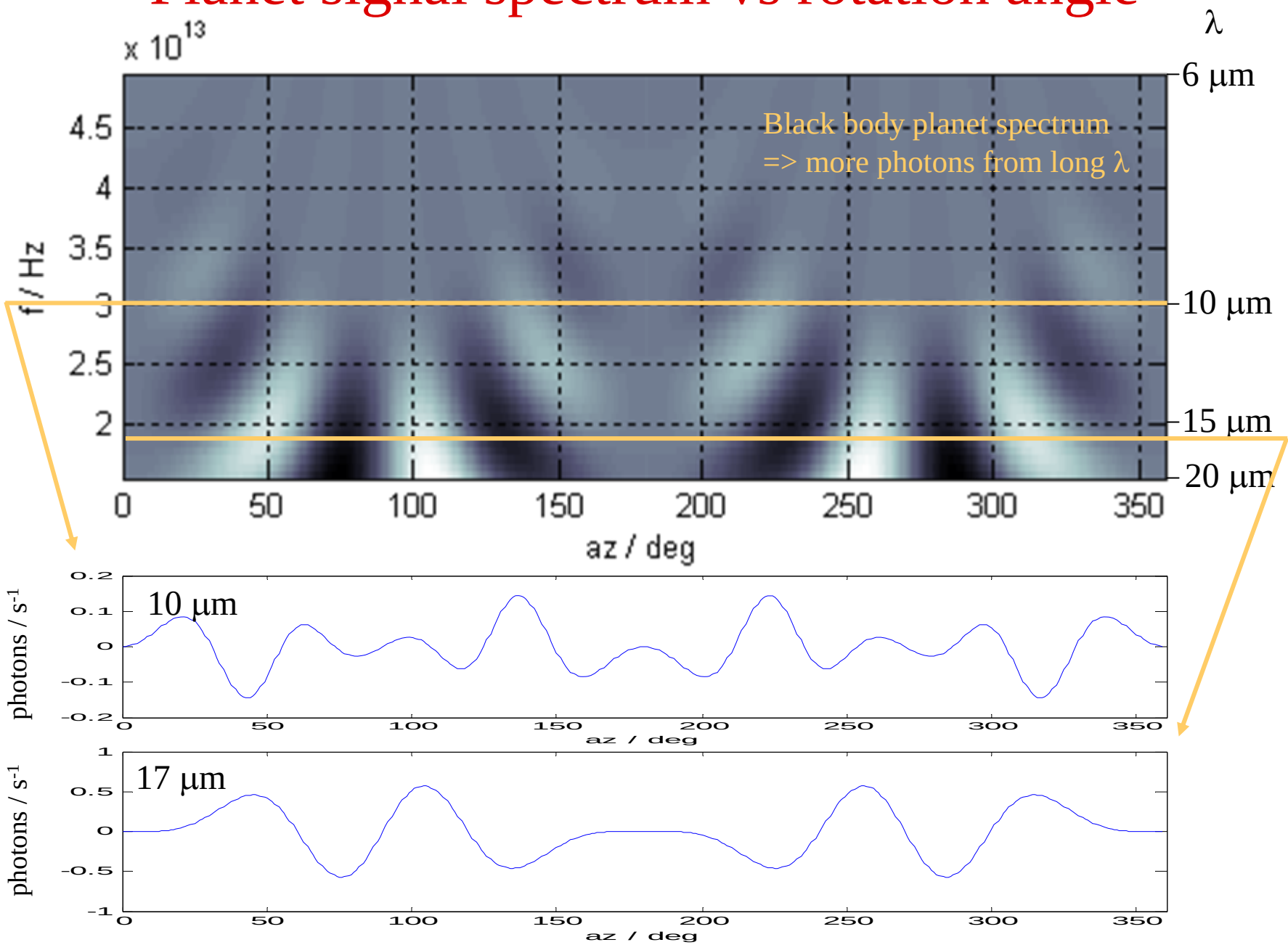
- λ = 17 μm
- Earth @ 50 mas = 2.5 x 10^{-7} rad

- Sky response is stretched out version of shorter wavelength

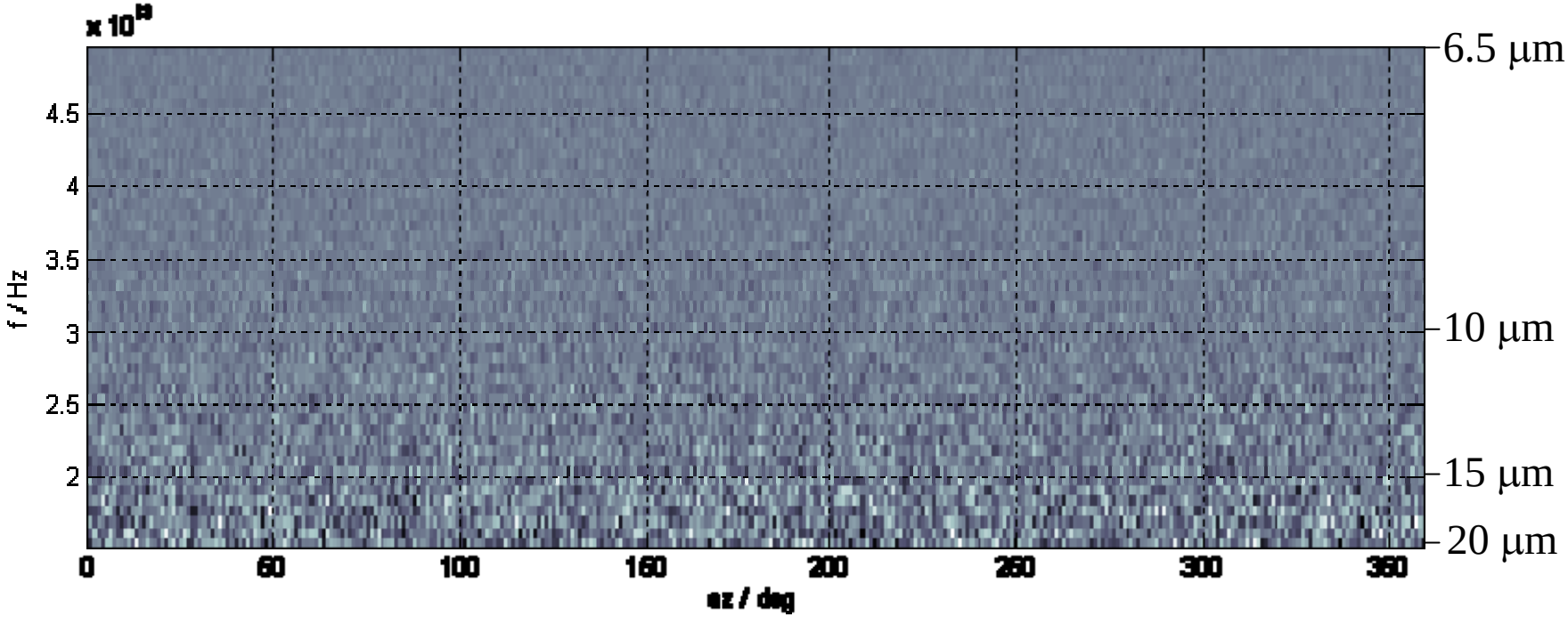


- Reduced modulation frequency vs array rotation angle

Planet signal spectrum vs rotation angle

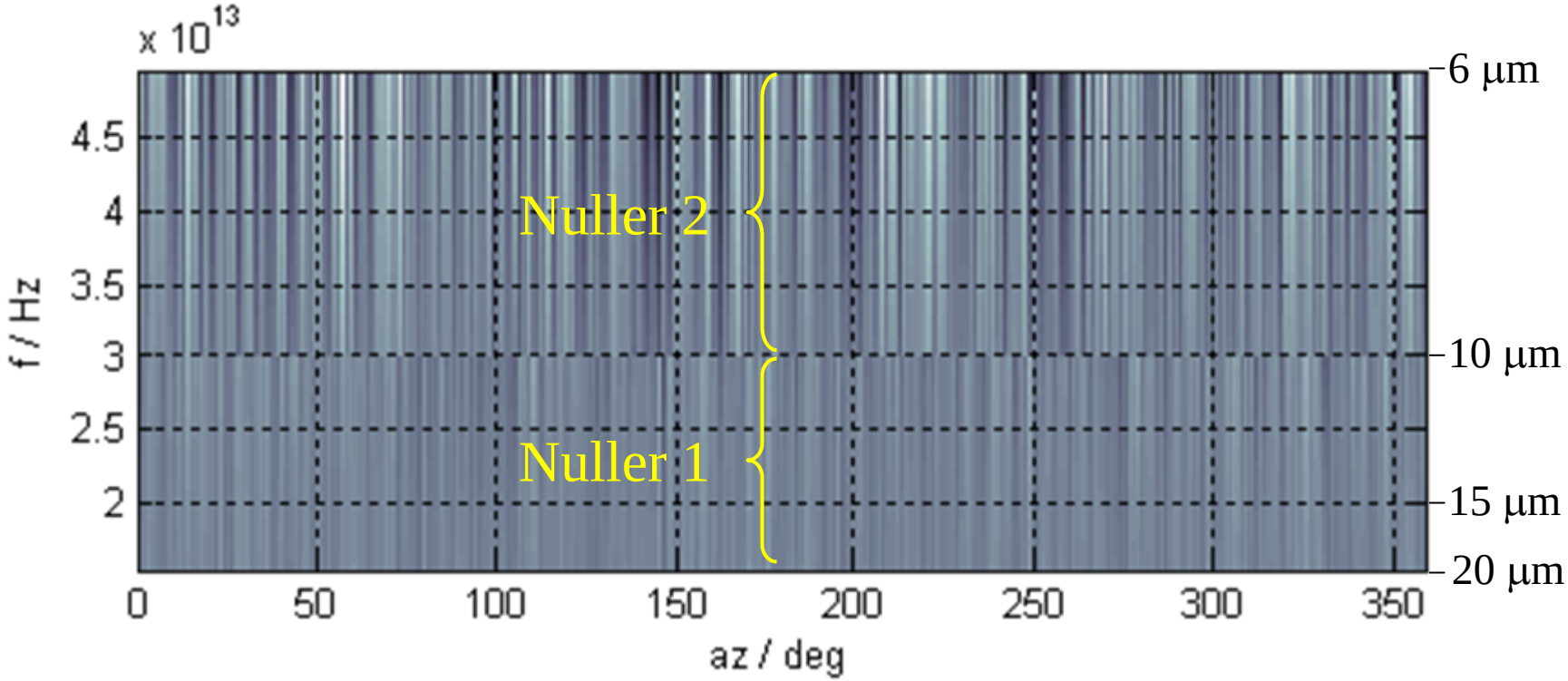


Random photon shot noise



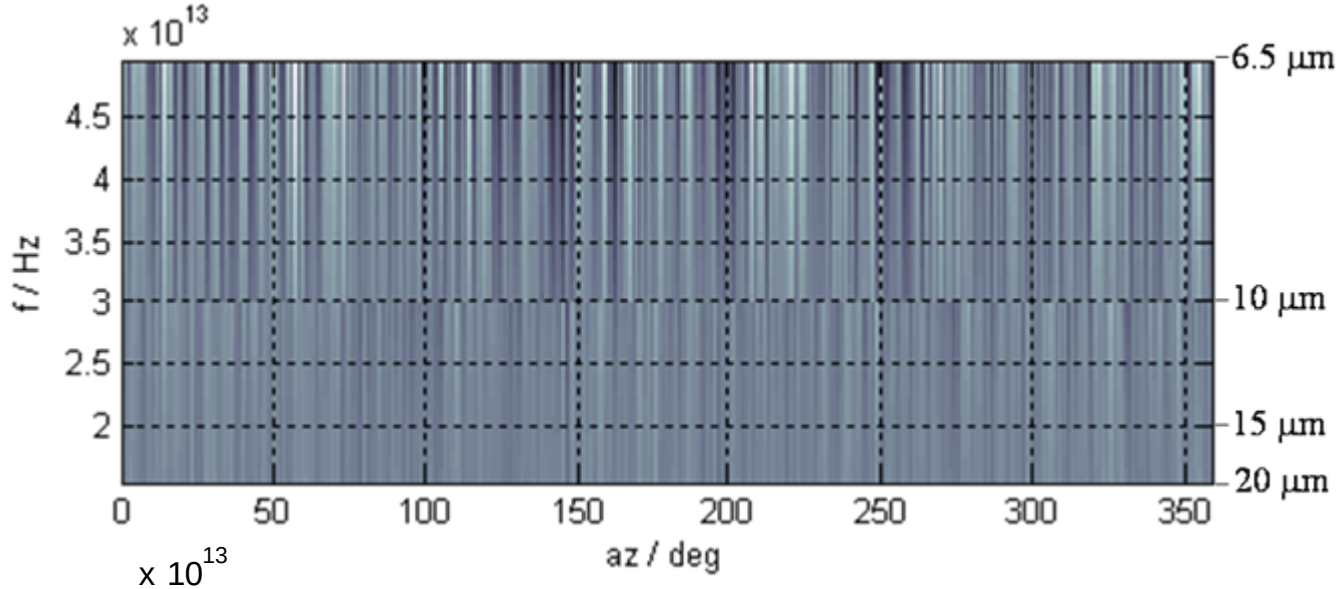
- Shot noise proportional to (photon rate)$^{0.5}$
- Local & Exo-Zodi at long wavelengths
- Stellar leakage at short wavelengths (not included in this example)
- White noise spectrum in azimuth (time) domain at a given wavelength

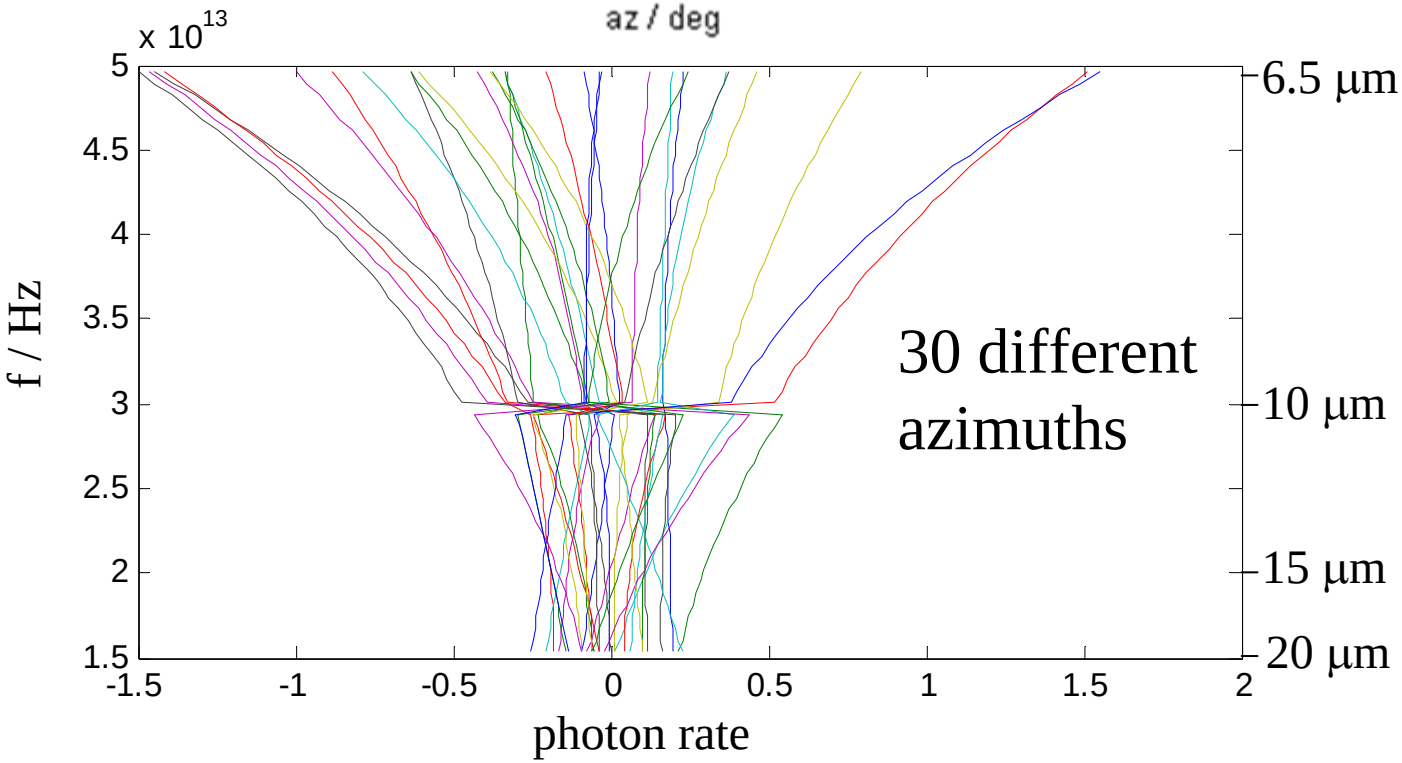
Instability noise



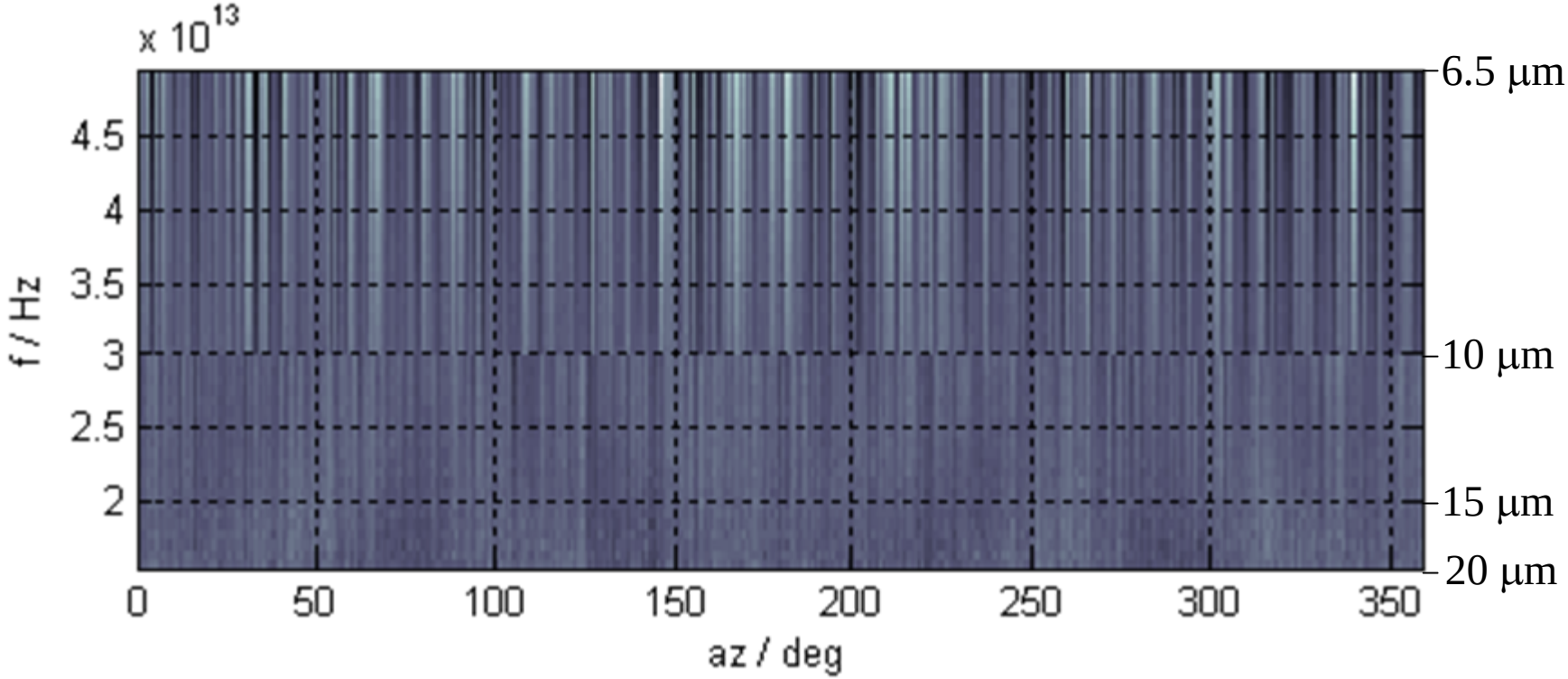
- Superposition of power laws: $a_1\lambda^{-1} + a_2\lambda^{-2} + a_3\lambda^{-3}$
- a_1, a_2, a_3, are Gaussian random variables, varying from one azimuth to the next
- Independent errors for each nulling beam combiner
  - 6 – 10 μm, 10 – 20 μm
- In practice there will be some correlation, both in time and between spectral bands, but we do not depend on it here

Instability noise profiles





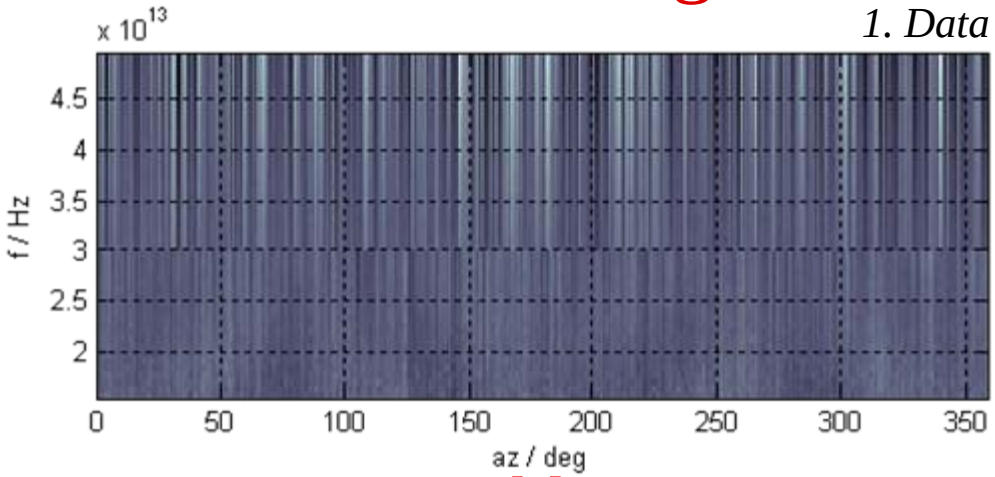
Combined signal and noise



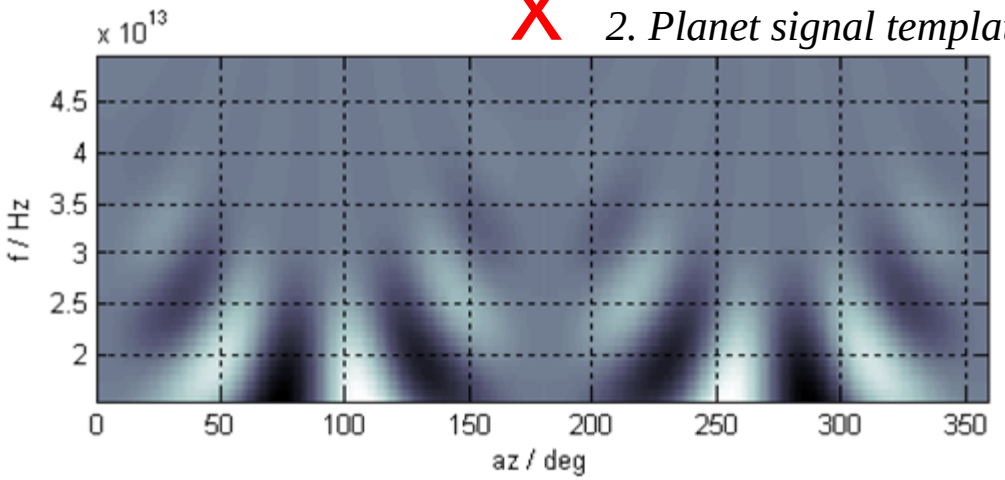
- Superposition of planet signal, photon shot noise and instability noise
- Relative levels not to scale (signal is over-represented)

Signal extraction

1. Data



X *2. Planet signal template*



Sum over spectral chans and azimuth

3. Output signal

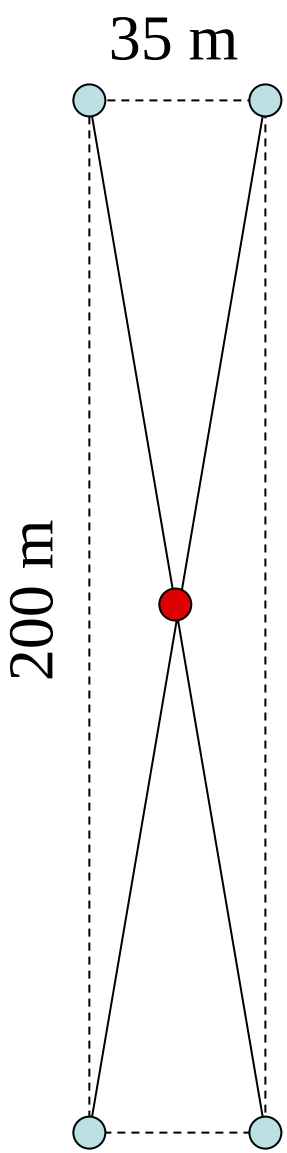
repeat for all possible planet locations

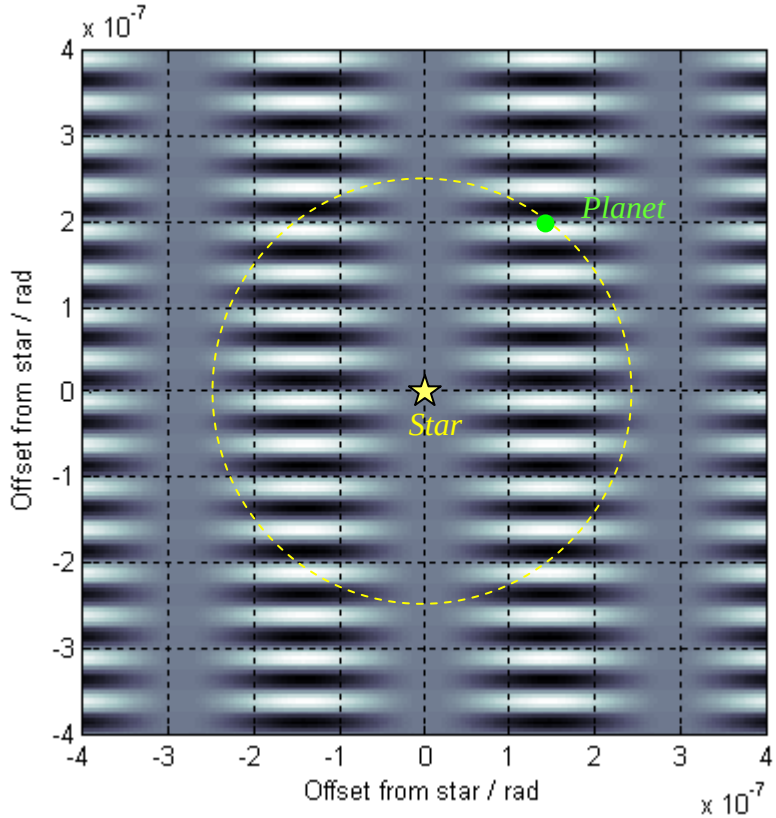
- Template weighted in wavelength axis to maximize SNR
- Essentially an optimally-weighted matched filter for the planet signal
- Approx equal contributions from photon shot noise and instability noise
- There is a component of each noise source that looks exactly like the planet signal

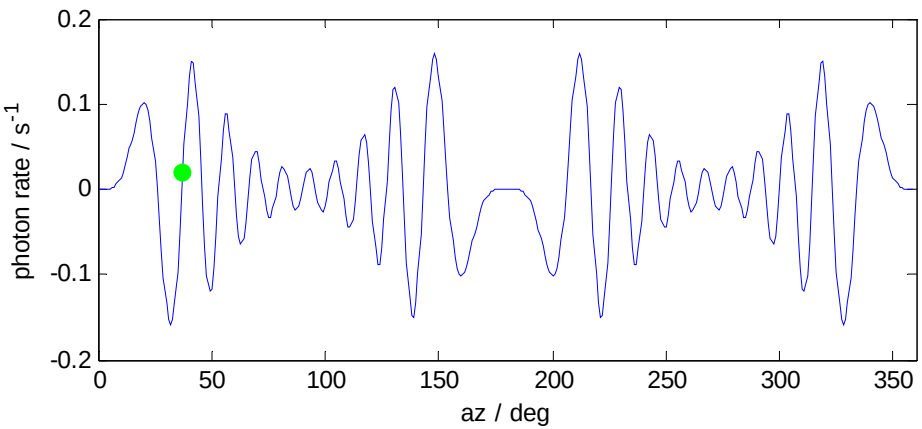
New approach:
‘stretch & fit’

Planet signal – array stretched x3

X-Array 6:1





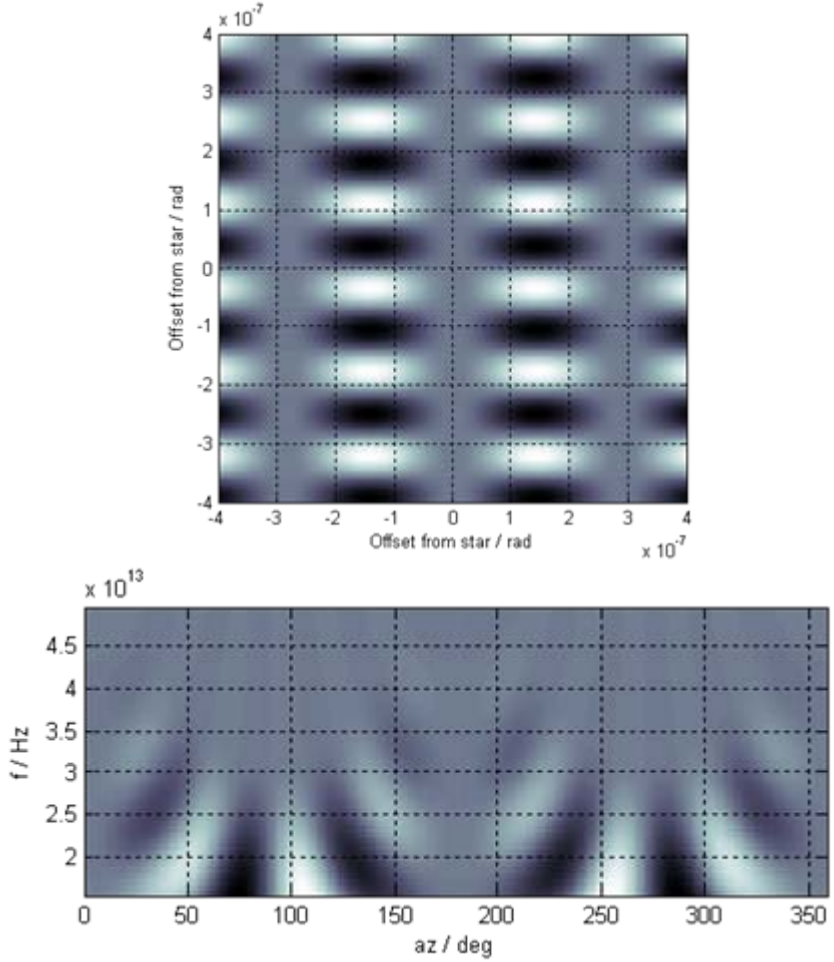


- λ = 10 μm
- Earth @ 50 mas = 2.5 x 10^{-7} rad

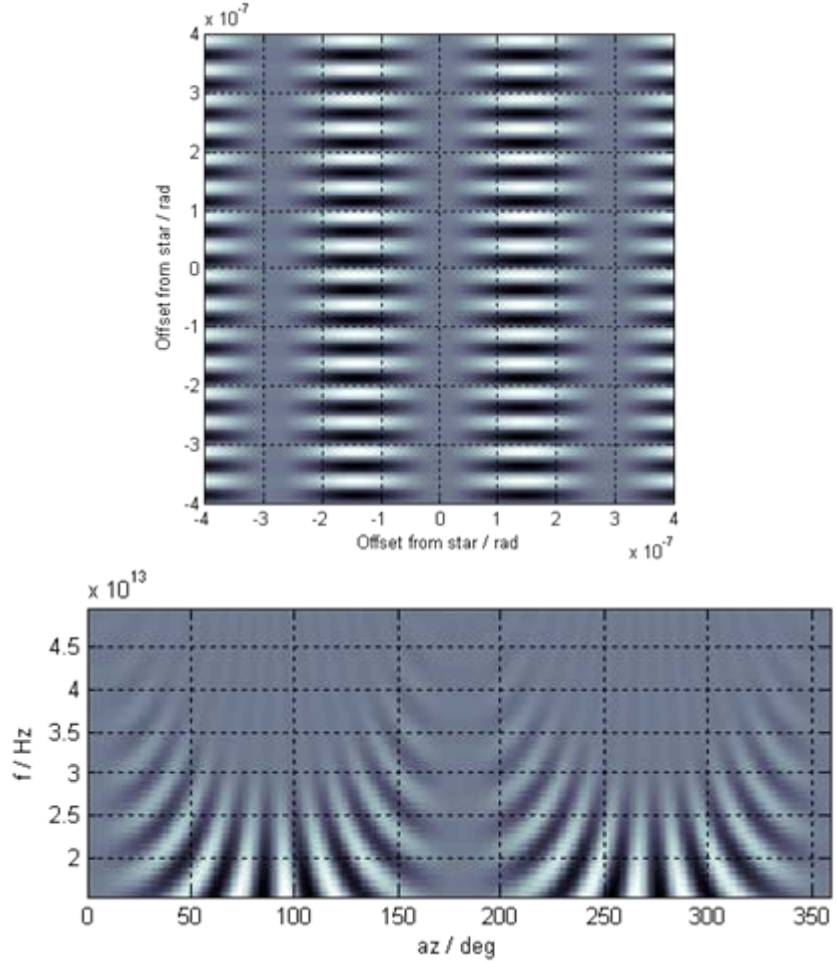
- 3-fold increase in planet modulation frequency

Comparison with unstretched case

70 x 35 m

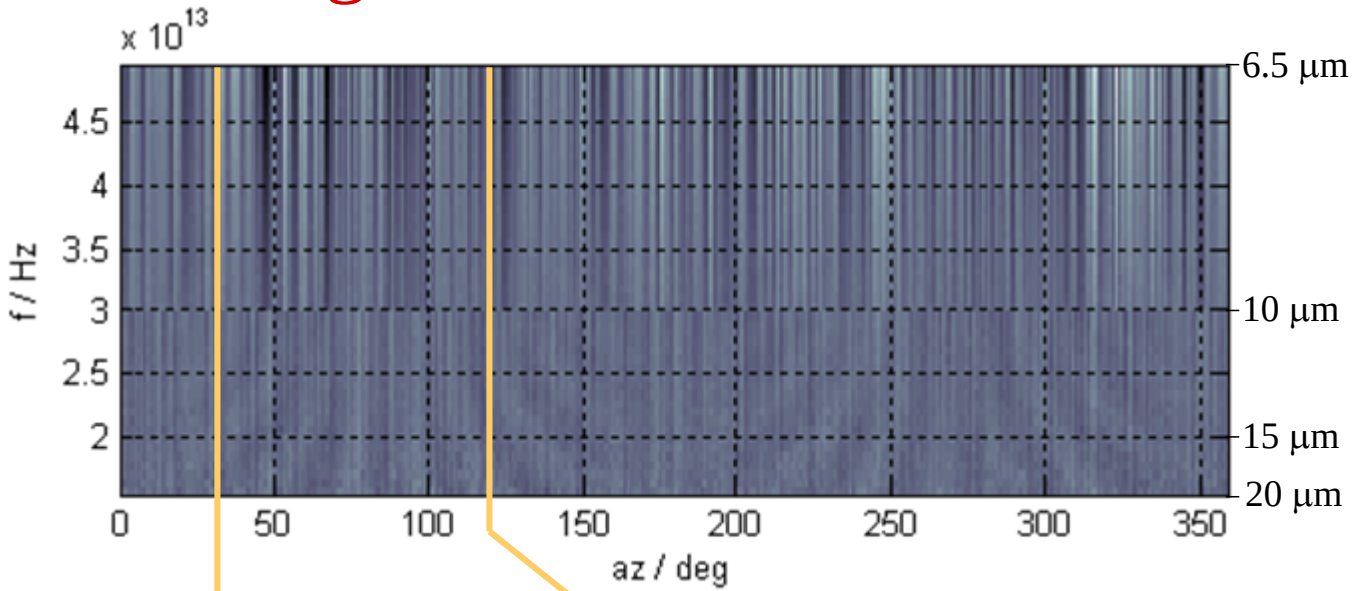


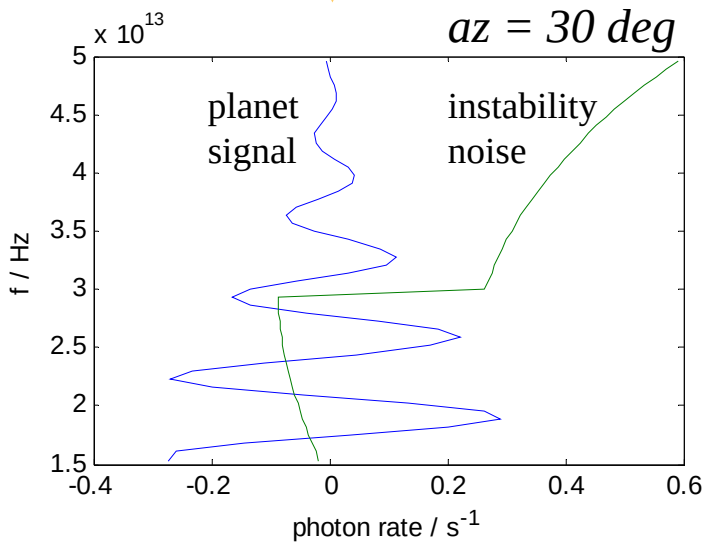
200 x 35 m

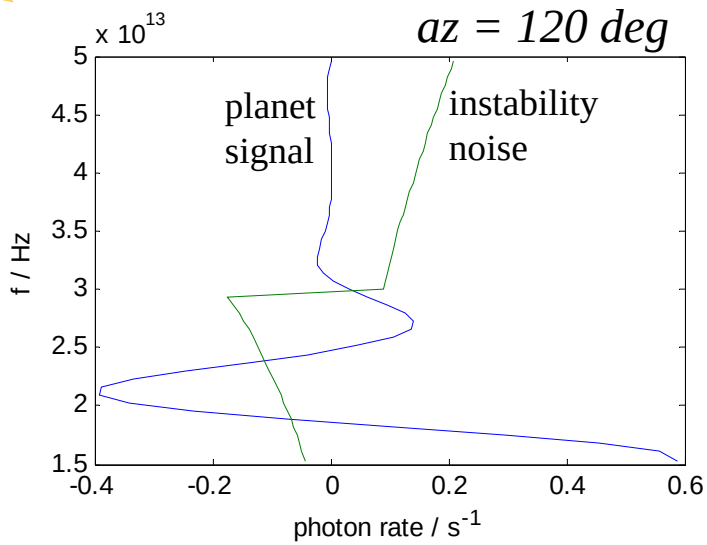


- 3 times as many ‘fringes’ in both types of plot

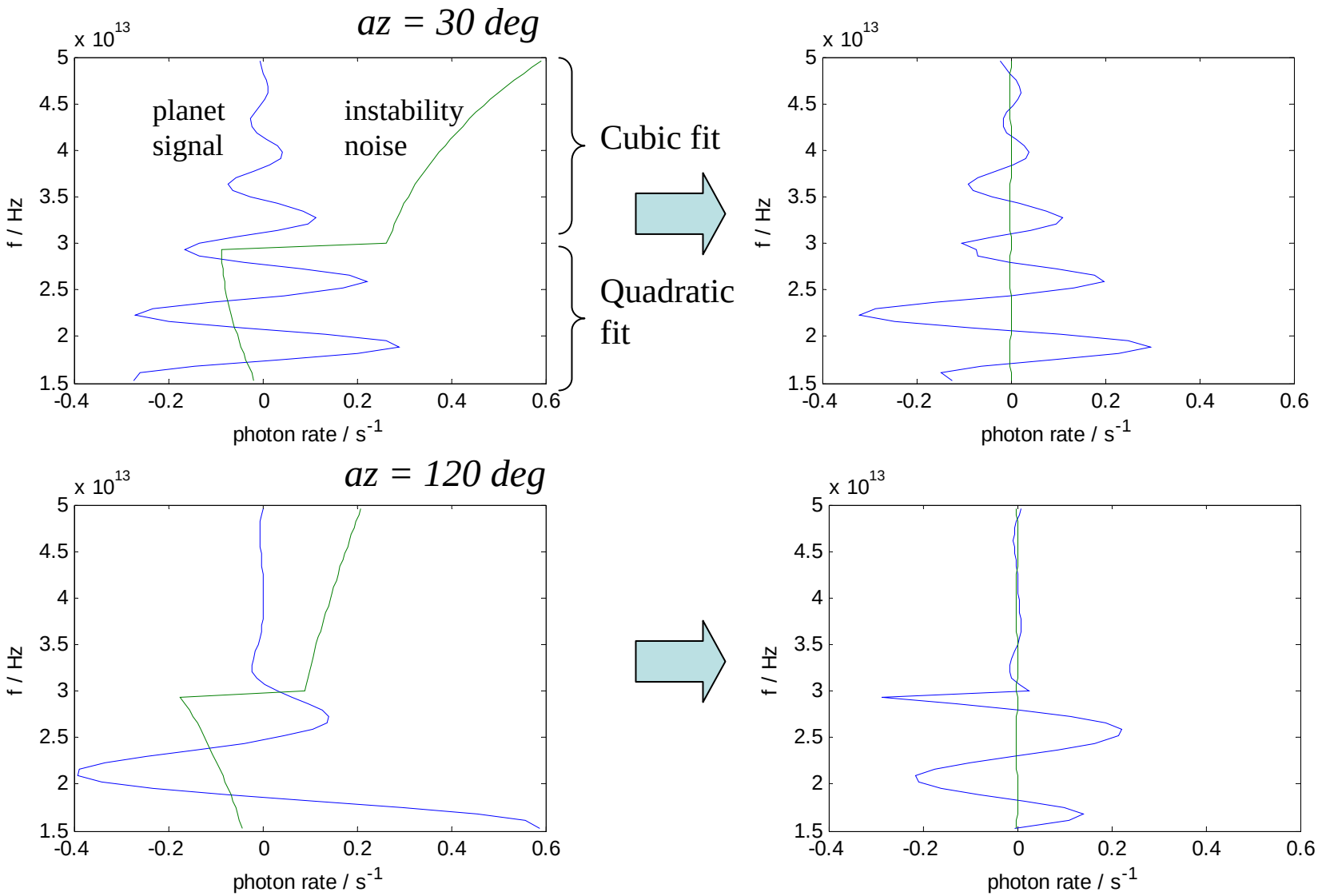
Combined signal and noise – stretched array





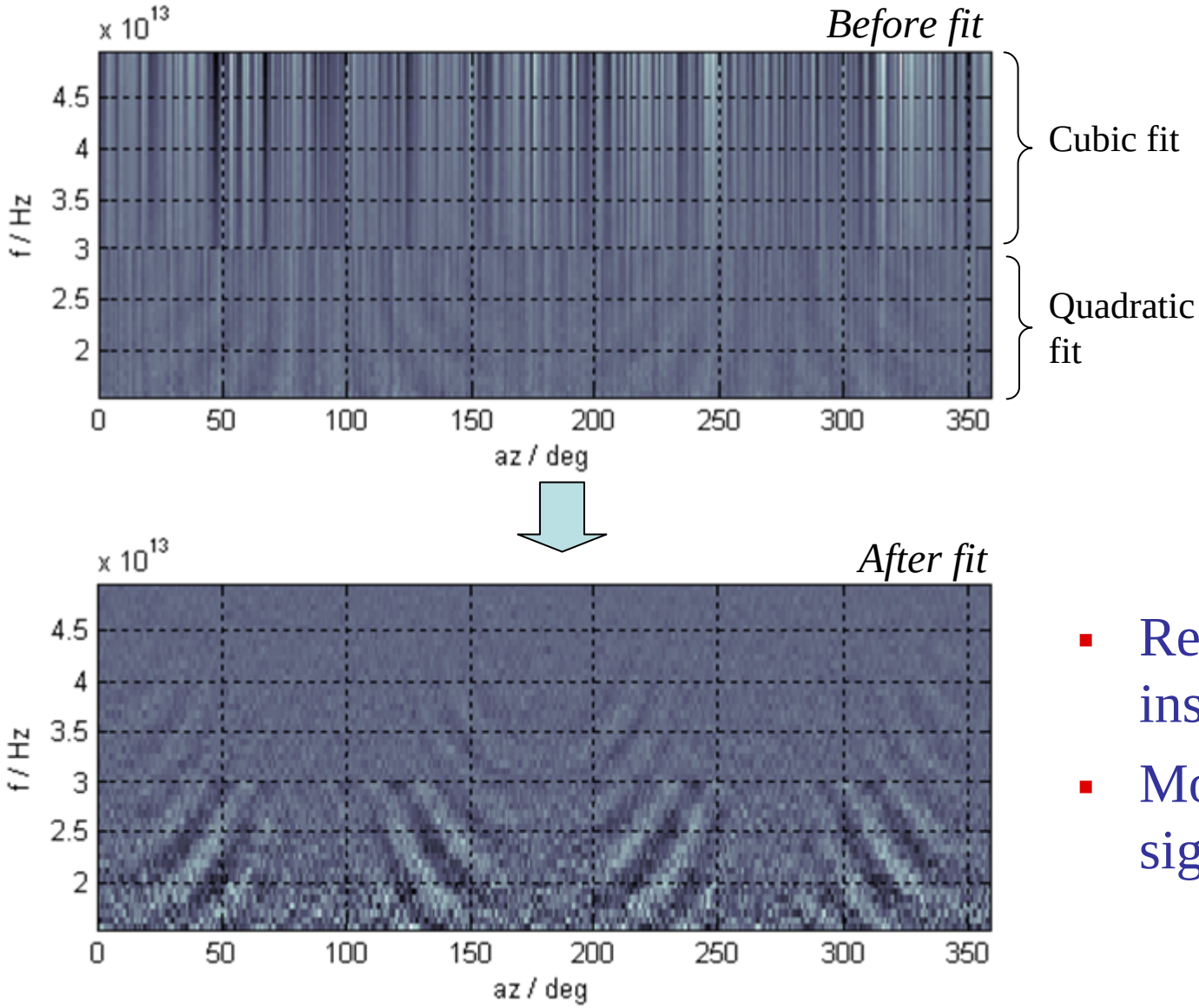


Apply polynomial fit to remove instability noise



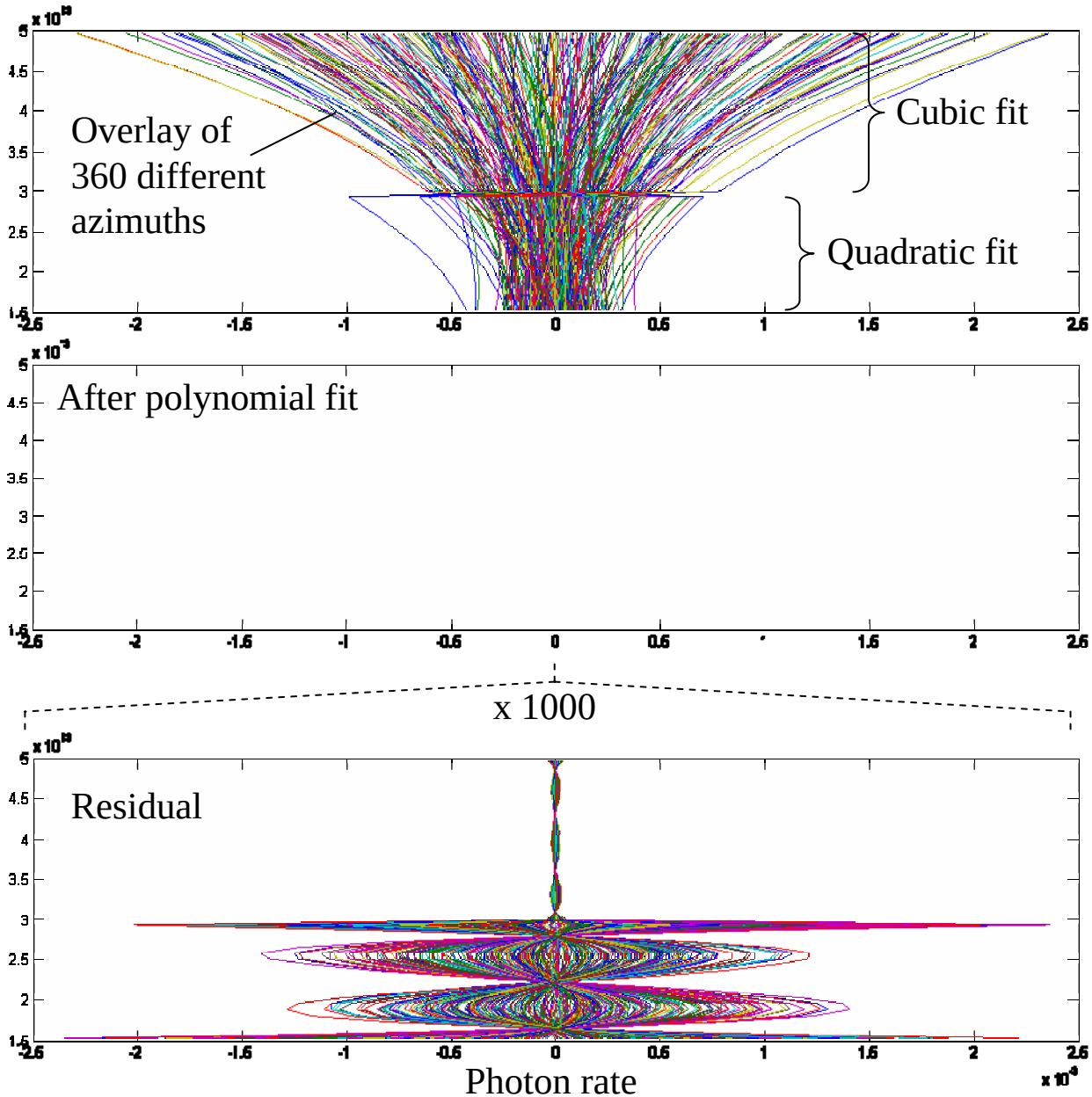
- Removes instability noise; modifies planet signal

Polynomial fitting



- Removes instability noise
- Modifies planet signal

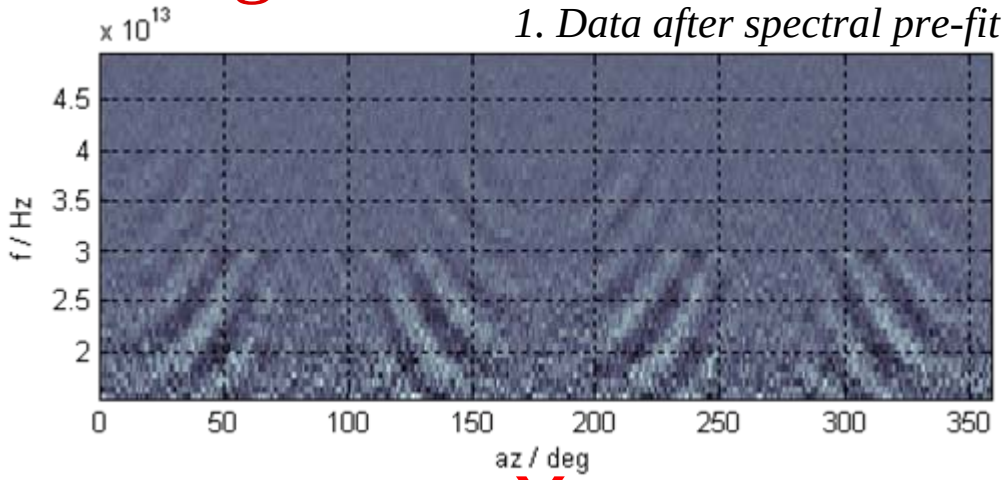
2D plot of systematic error before and after



- Spectral Pre-Fit reduces systematic error by factor ~ 1000
- Exact result depends on distribution of power law index
  - this case a mix of $\lambda^{+0.5}$, $\lambda^{-1.5}$, $\lambda^{-2.5}$, λ^{-3}

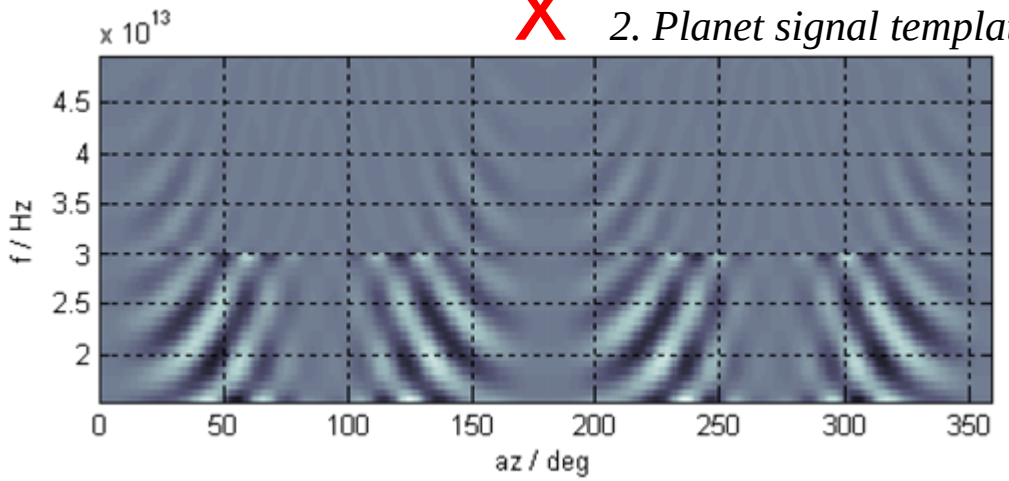
Signal extraction – stretched & pre-filtered

1. Data after spectral pre-fit



X

2. Planet signal template



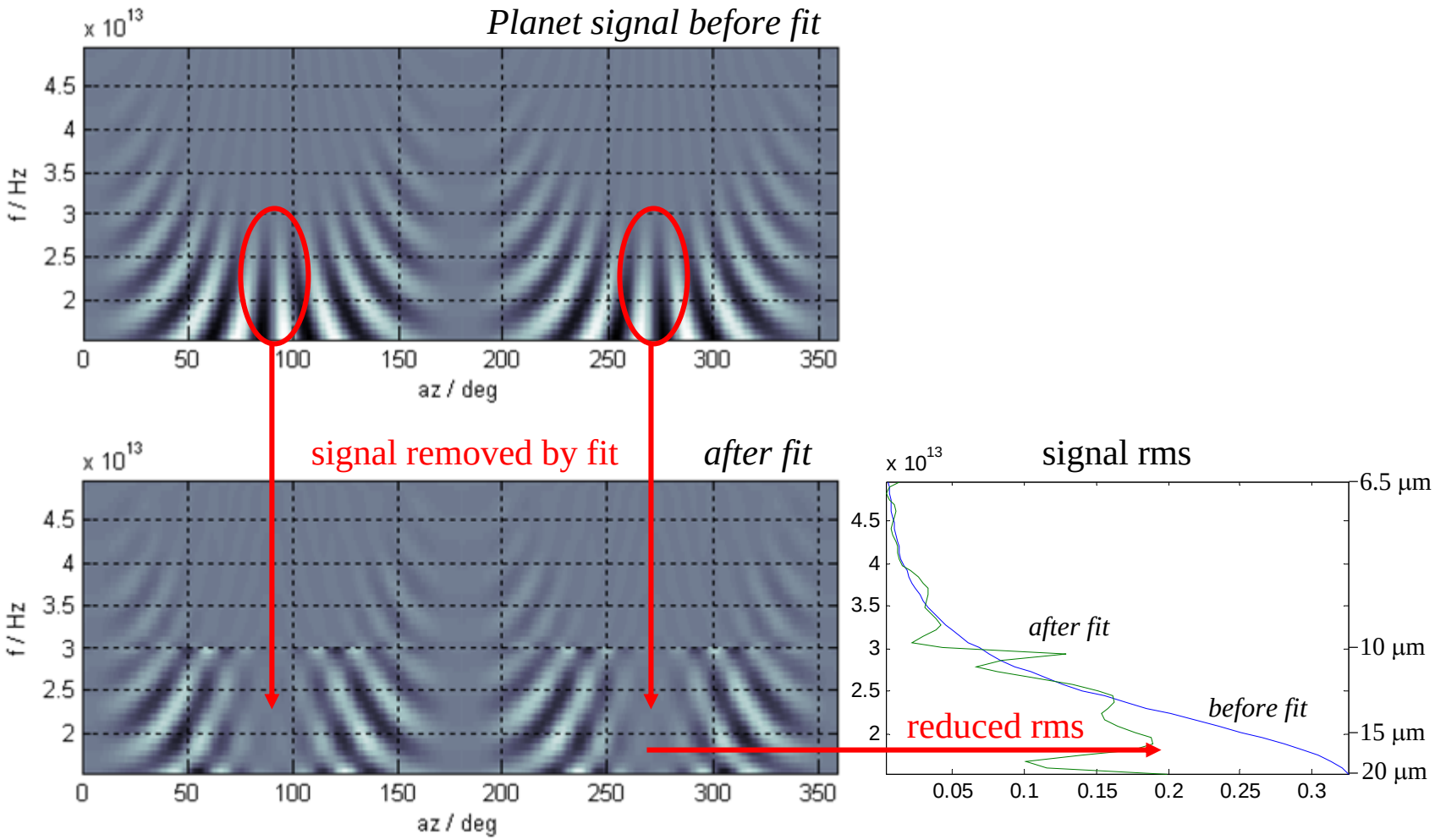
Sum over spectral chans and azimuth

3. Output signal

repeat for all possible planet locations

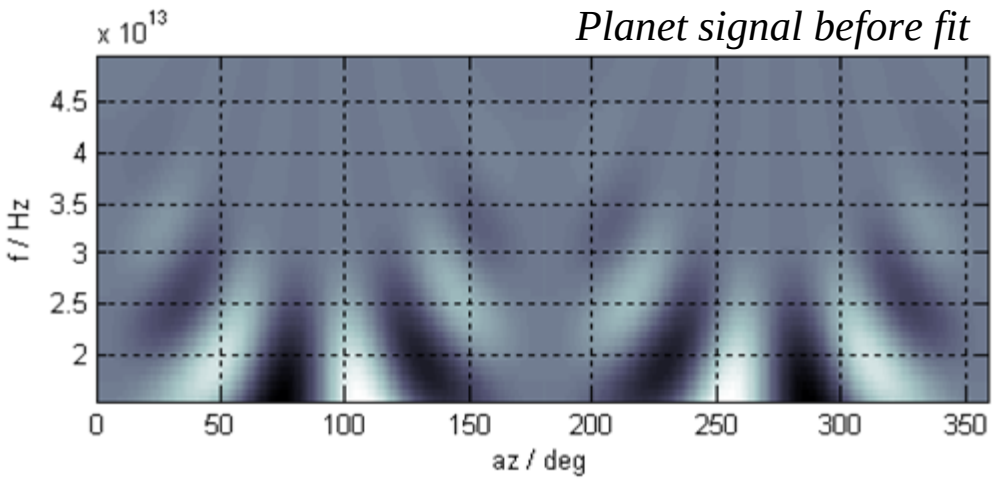
- Template is modified to account for spectral fitting
- Impact of instability noise removed almost completely
- Some signal removed
- Random noise contribution reduced

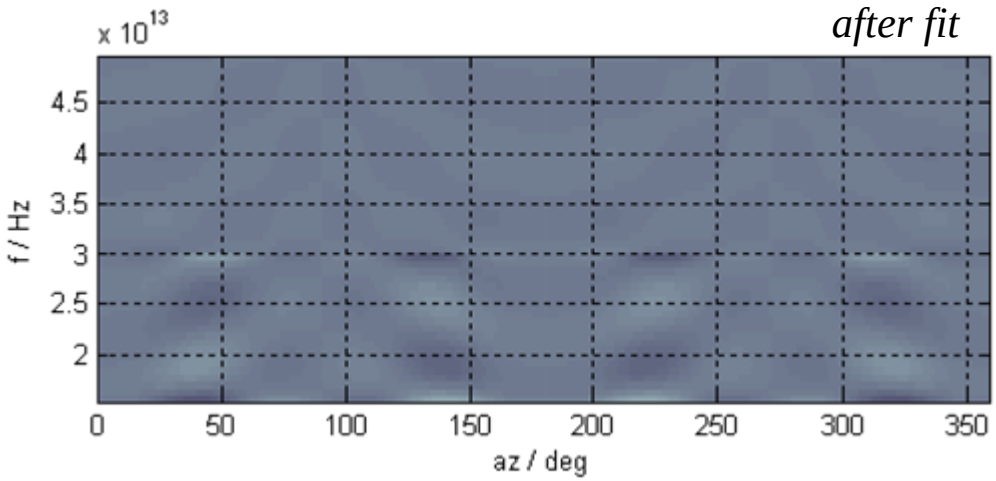
Impact of pre-fitting on signal – 200 x 35 m

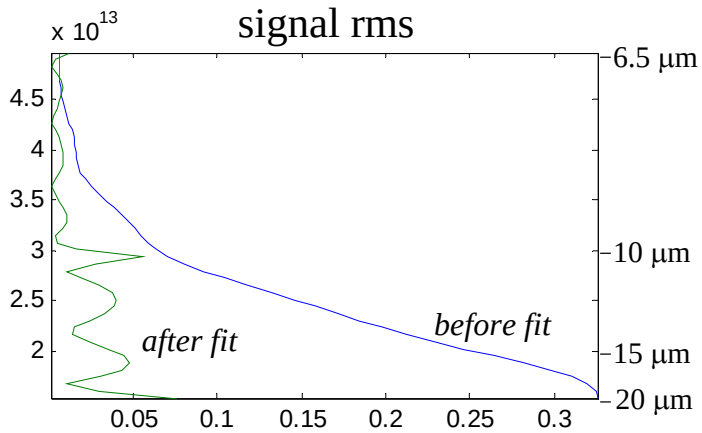


- Signal with low modulation in spectral domain is removed by polynomial fit => reduced sensitivity

Impact of pre-fitting on signal – 70 x 35 m



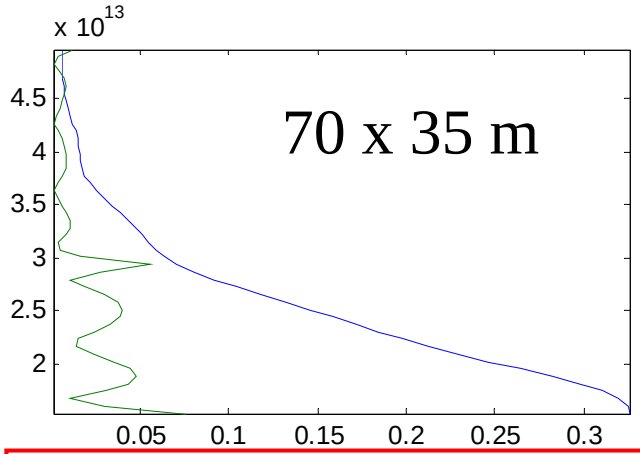


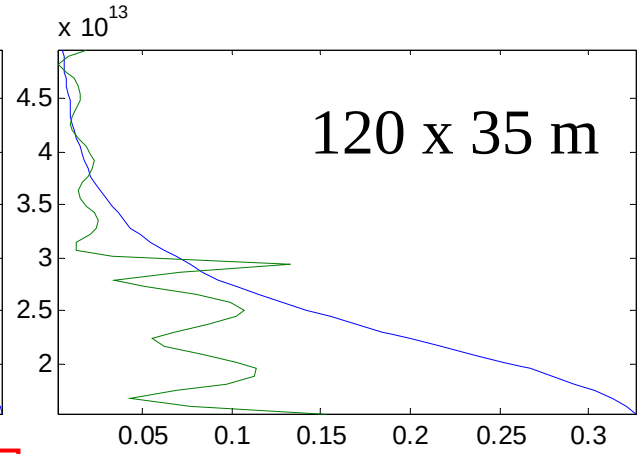


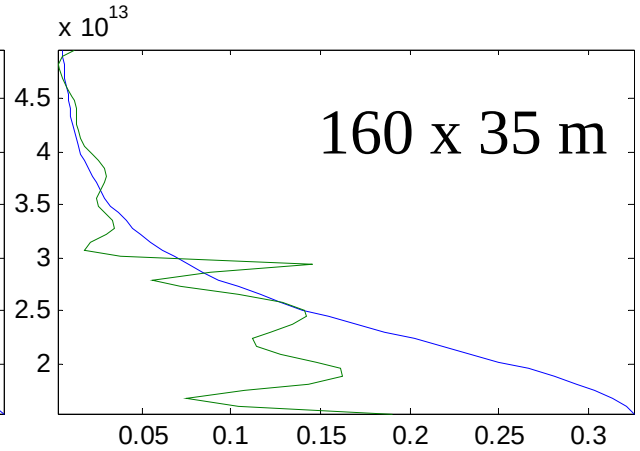
- Signal almost entirely removed by fitting for 70 x 35 m array
- Large loss of sensitivity

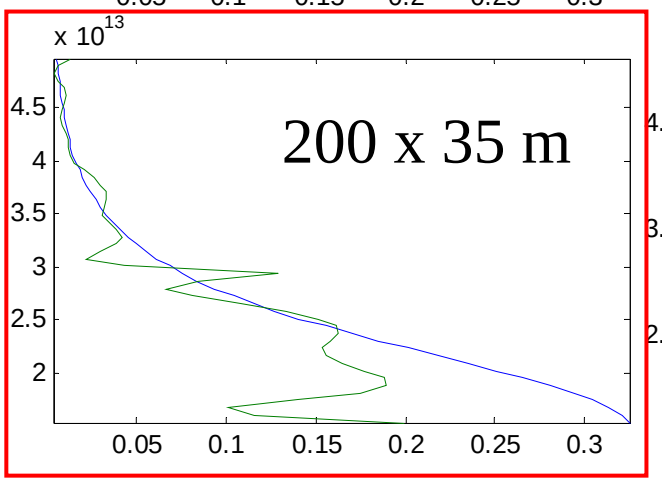
Impact of pre-fitting on signal vs array size

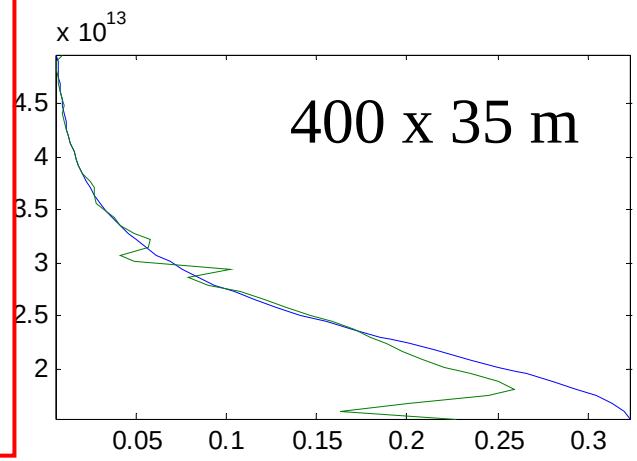
- Increasing the stretch increases the number of modulation fringes and reduces the signal lost to the polynomial fit
- Here we adopt the 200 x 35 m case



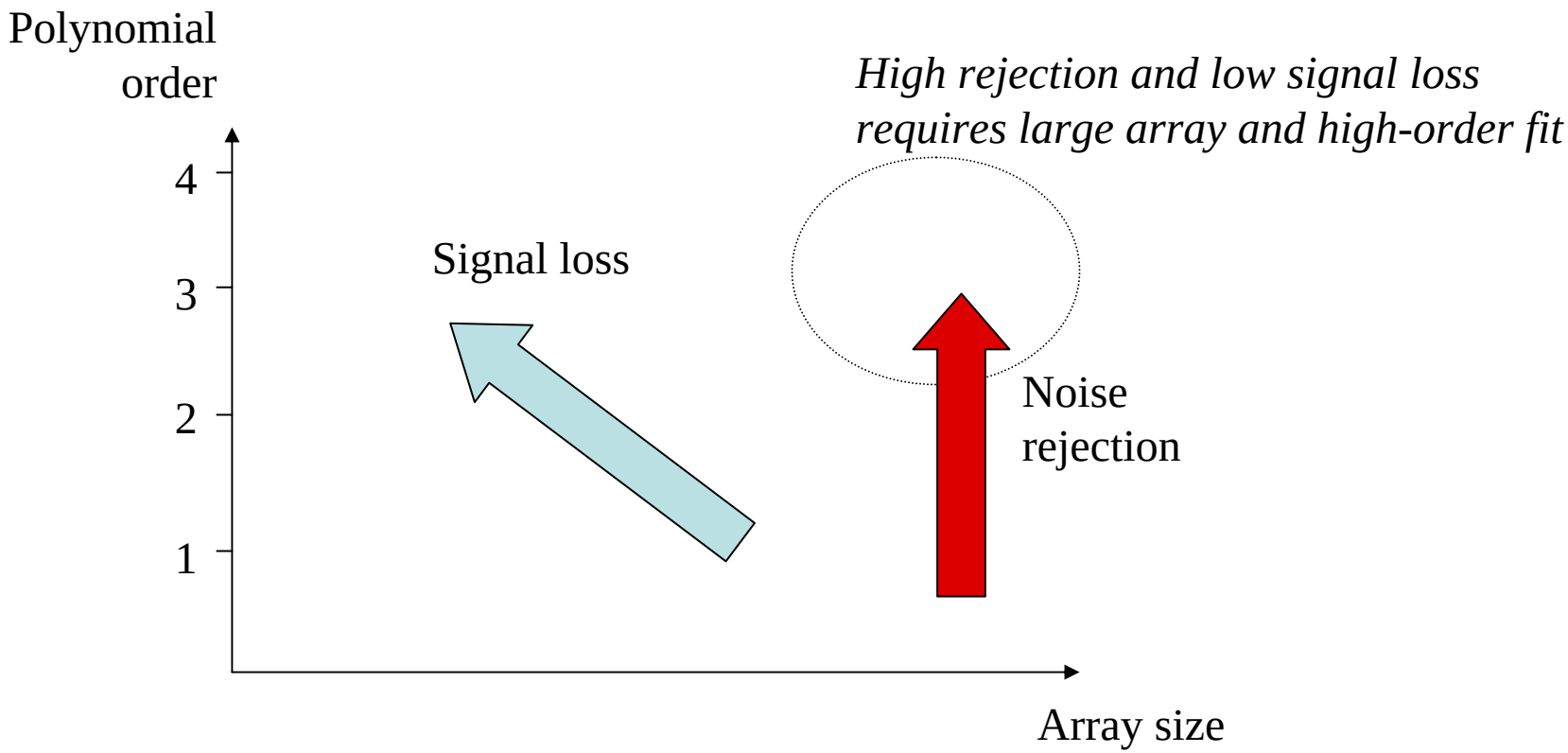






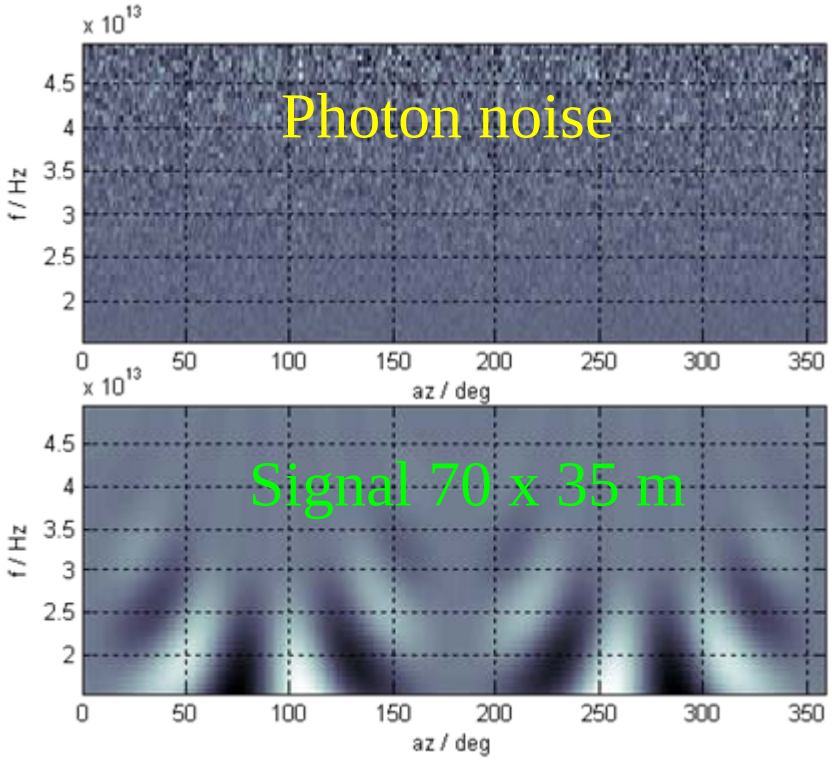


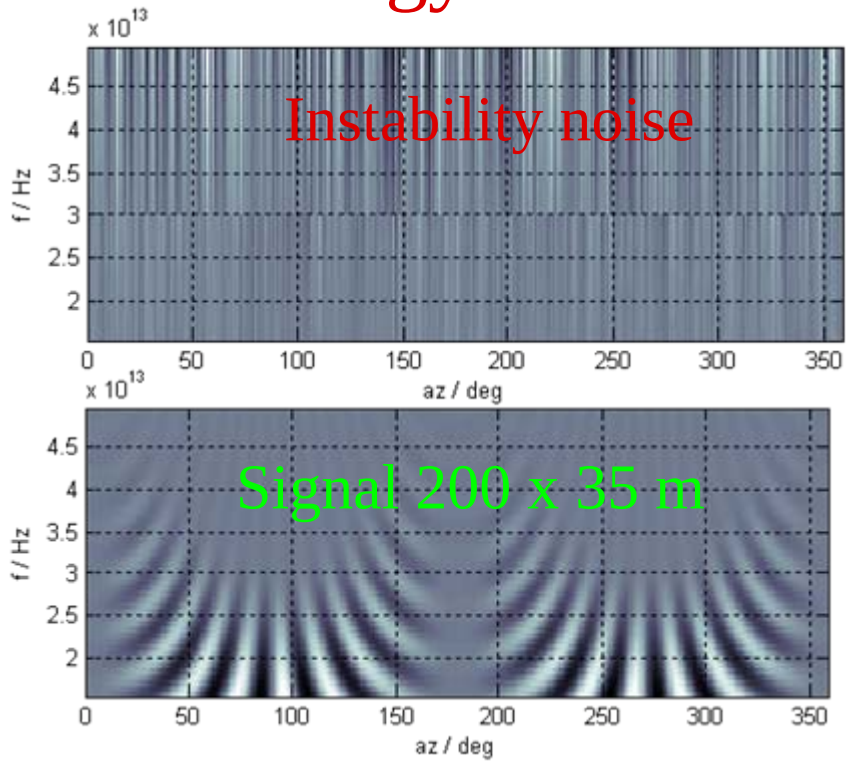
Trade space

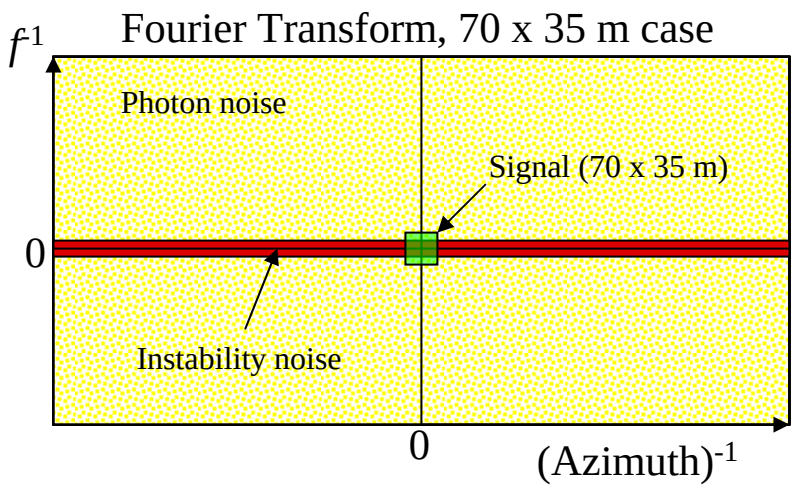


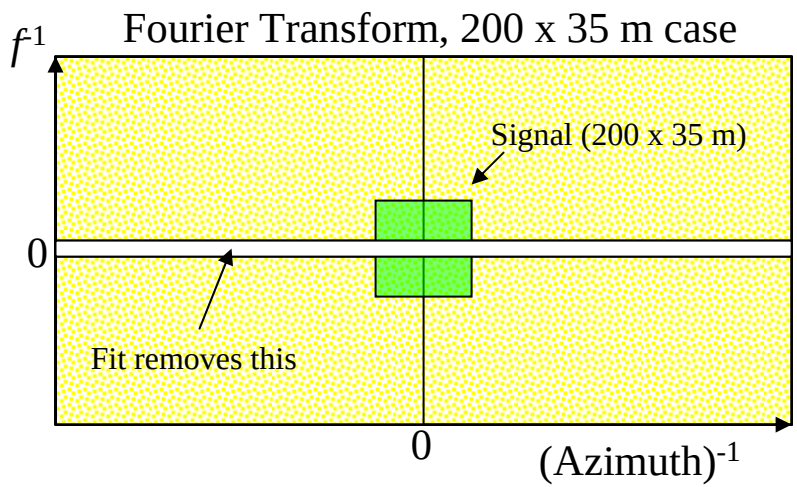
- Width of spectrum with common instability errors also important
- If instability errors can be fit in one go over full 6 – 20 μm, instead of two independent subbands, then array size needed is approximately halved

Fourier Transform analogy





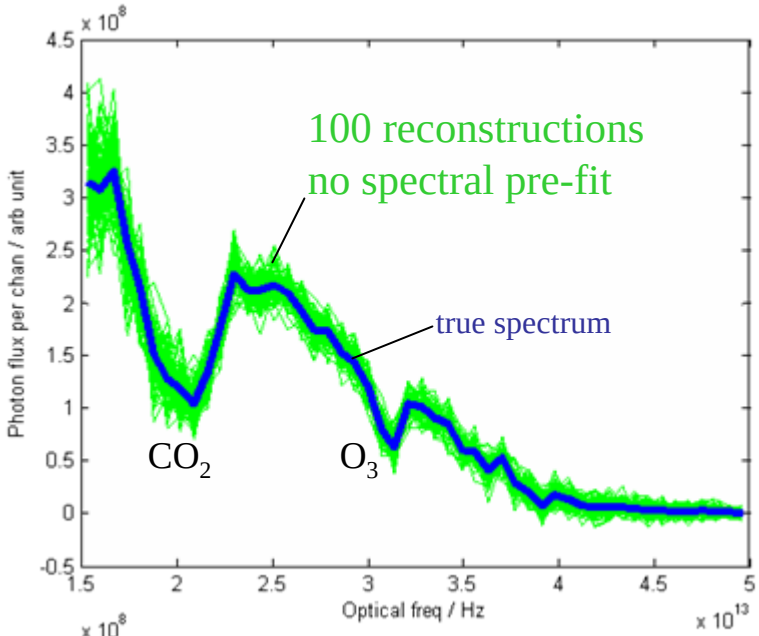


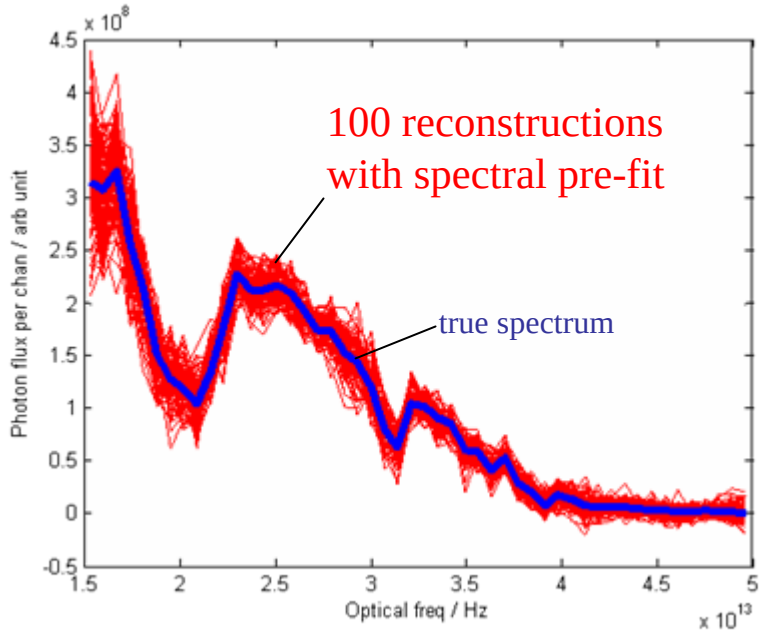


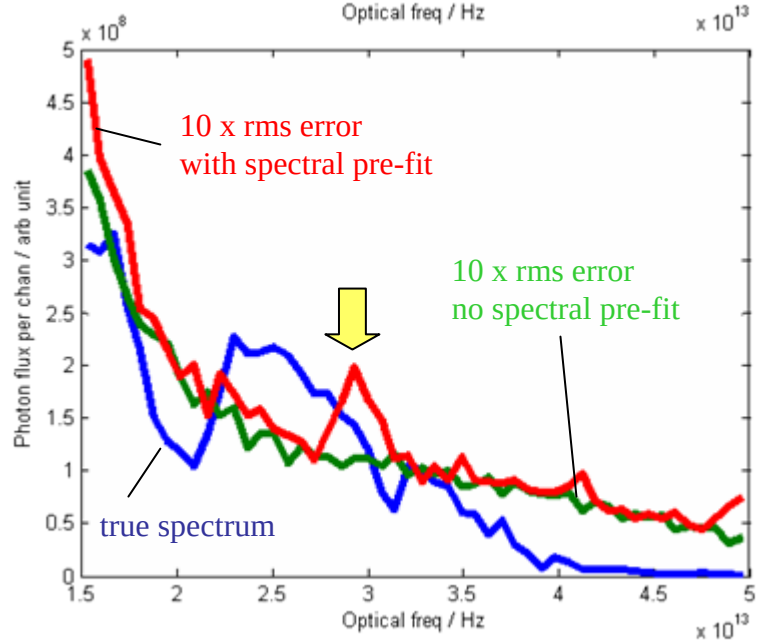
Spectroscopy

- Previous case was for broadband detection (black body spectrum)
- Spectrum can be recovered after spectral pre-fitting using Singular Value Decomposition
- Makes no assumptions about spectrum
- No iteration required

Spectroscopy







- Reconstruction in presence of photon noise
- Loss of sensitivity at boundary between spectral bands
- In case with no pre-fitting, instability noise must be added
  - increases net noise for ozone by ~ 40%

Summary of positive and negative impacts

+ Relaxation in null depth requirement
+ Improved sensitivity
+ Greatly increased angular resolution

- Stray light requirement
- Fuel consumption
- Increased # spectral channels
- Does not work with all array configurations
- Increased signal processing complexity

Impact of null depth on shot noise

Null depth @ 10 μm	Broadband SNR* (relative)	Ozone SNR* (relative)
10^{-6}	1.00	1.00
10^{-5}	0.97	0.92
10^{-4}	0.80	0.60

*X-Array, shot noise only, 200 x 35 m, Earth-Sun @15 pc, 50 mas

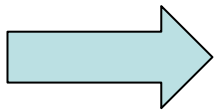
- Null depth is one of several contributions to photon noise; also LZ, EZ, thermal, stellar size leak
- Mission is still viable with even a 10^{-4} null depth
- Much easier to monitor and control a shallower null

Improved Sensitivity

- Initial calculations show sensitivity is improved despite a factor of 10 relaxation of null depth:

~ 20 – 30% higher sensitivity

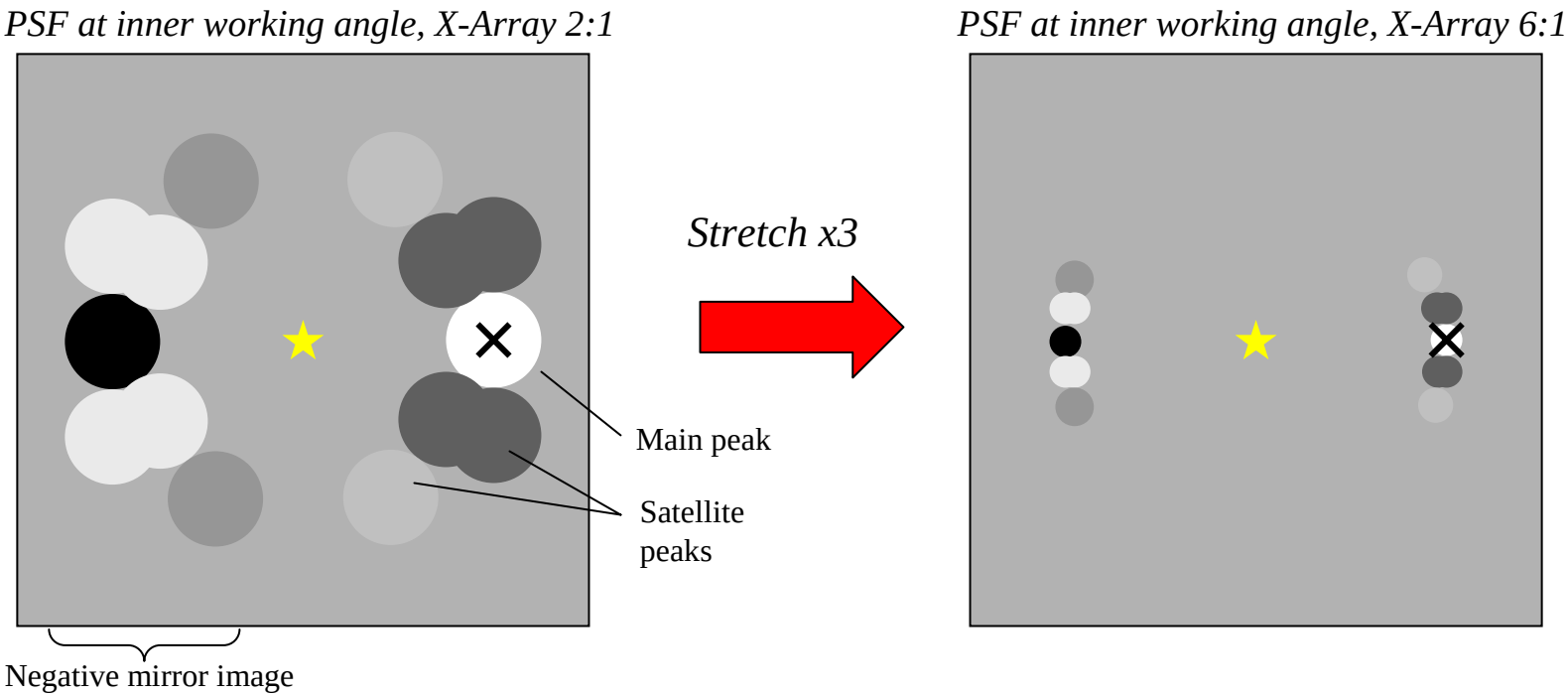
X-Array 2:1
10^{-6} null
no Spectral Pre-Fit



X-Array 6:1
10^{-5} null
with Spectral Pre-Fit

- Increases number of targets observable during mission
- Includes:
  - Removal of instability noise (+++)
  - Signal reduction from fitting (--)
  - Random noise reduction from fitting (+)
  - Increased detection threshold needed higher angular resolution (-)
  - Reduced calibration time (+)
- More work needed

Greatly increased angular resolution



- Eases multiple planet extraction
  - 9 times lower chance of overlap of sidelobes from different planets
- More robust to Exo-Zodi structure
- 3 times faster orbit & proper motion determination
  - More rapid screening of background sources

Summary of positive and negative impacts

+ Relaxation in null depth requirement
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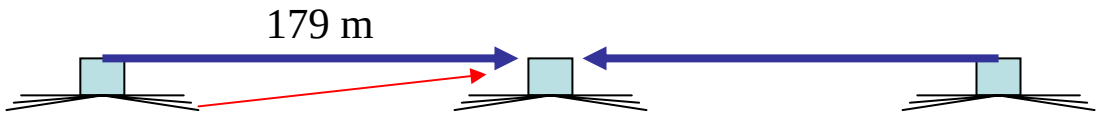
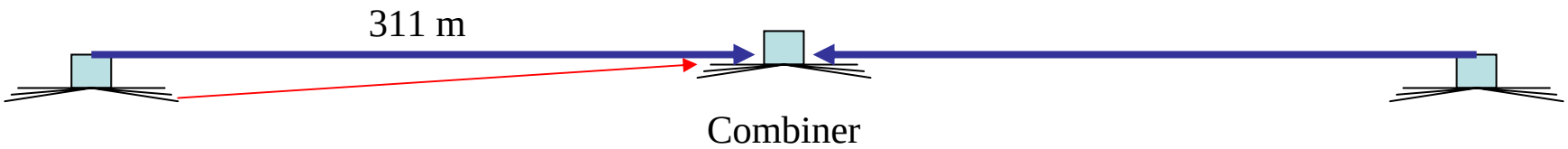
Stray light

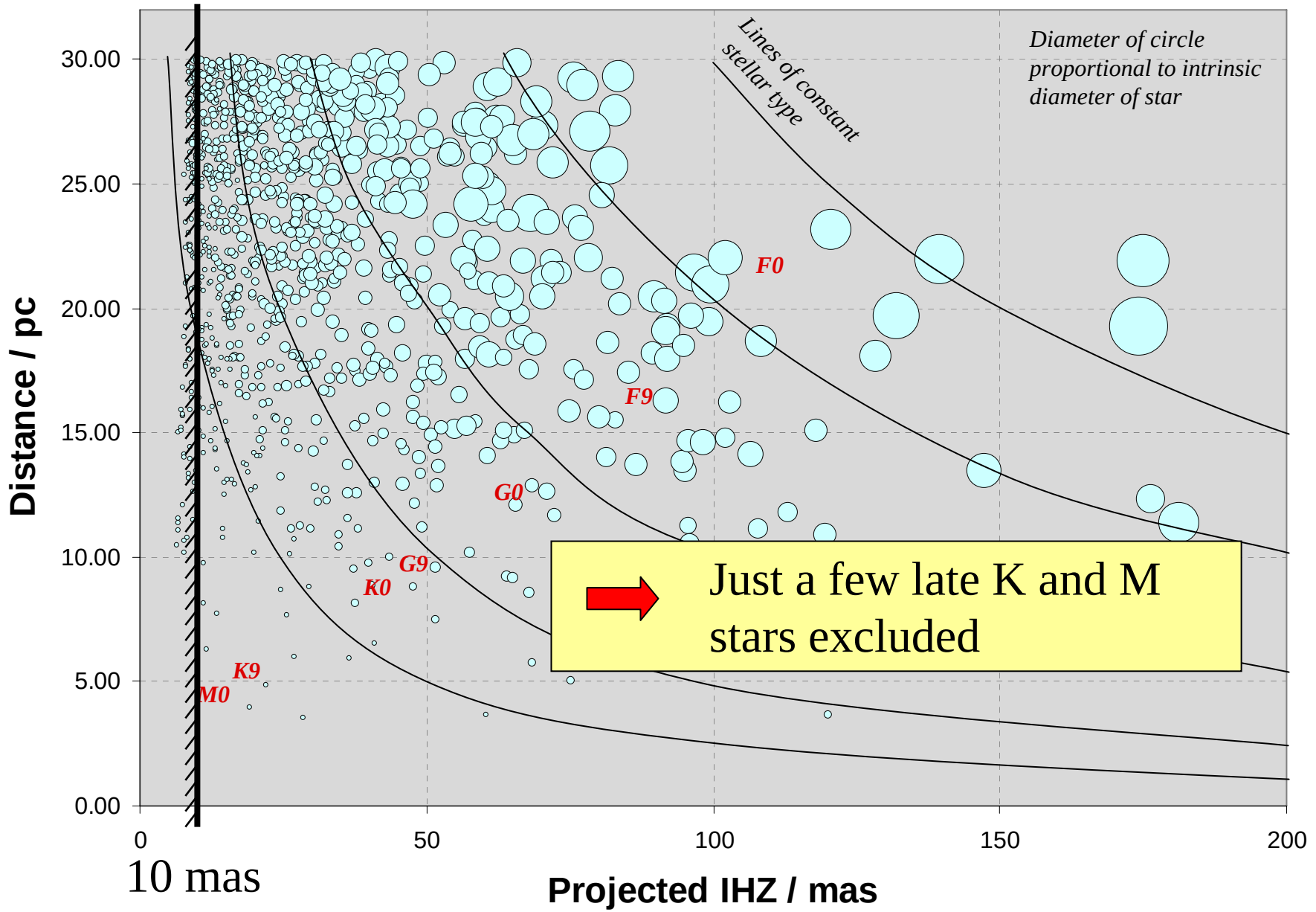


- Current design imposes a limit of ~ 179 m between spacecraft
  - collector – collector (see Noecker & Leitch, SPIE 5905)
- Limits Inner Working Angle of stretched array to ~ 17 mas



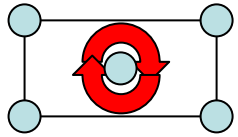
- Noecker points out that combiner-collector separation can be ~ 311 m
- Thermal penalty for out-of-plane combiner minor, since large separation between combiner and collectors
- Goes some way to mitigating constraint; IWA ~ 10 mas

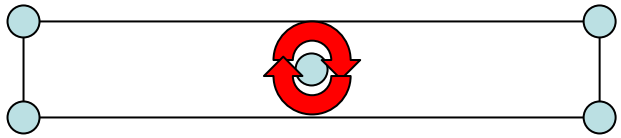
Star count plot



Fuel consumption

- Fuel consumption scales as $\frac{mrT_{obs}}{T_{rot}^{2}}$
  - m = spacecraft mass
  - r = radius from center of rot
  - T_{obs} = total observation time
  - T_{rot} = time for 360 deg rotation

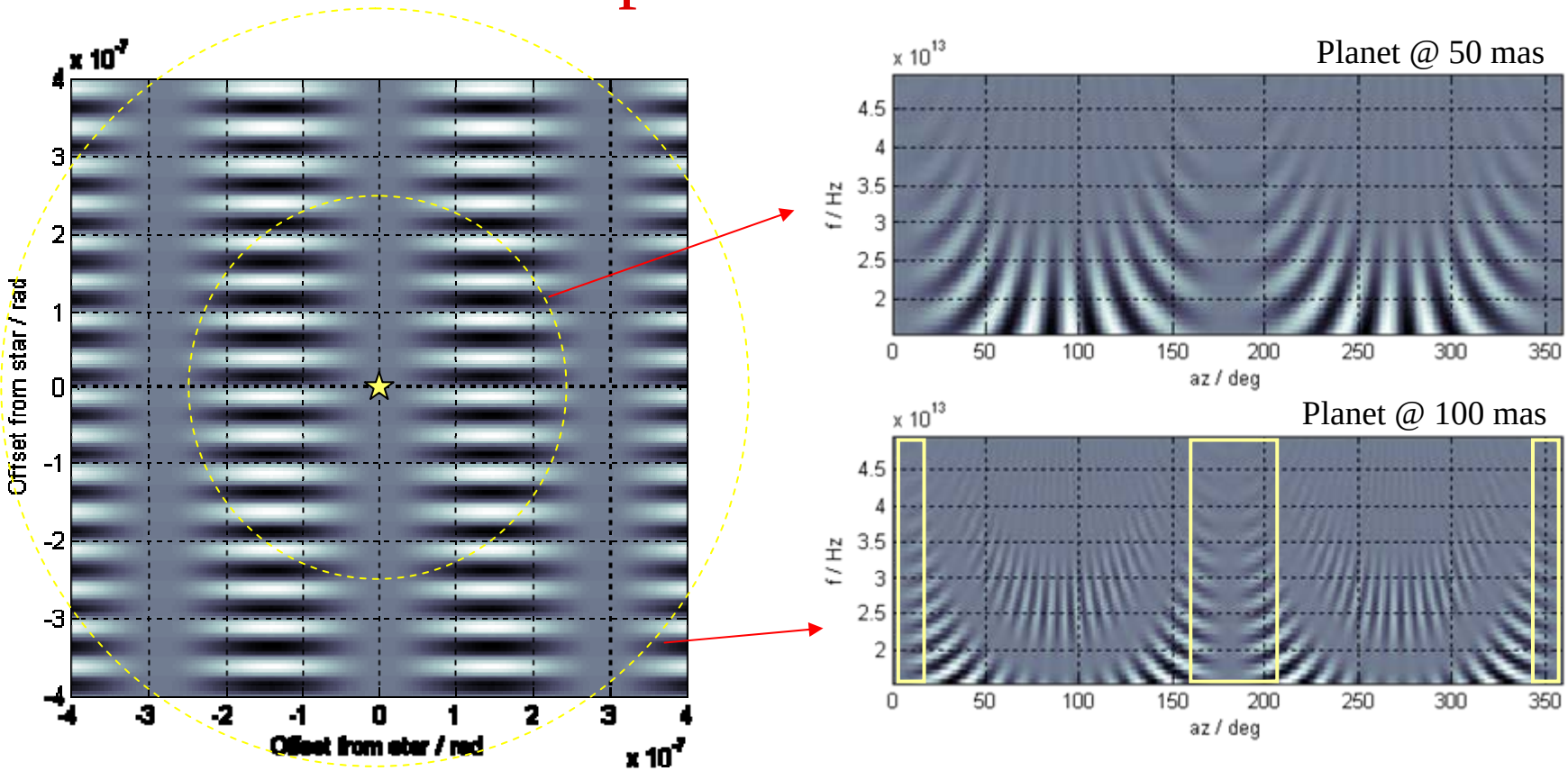




- Stretching array increases r by factor 3
- What drives T_{rot}?
  - Need at least half a rotation (nearby targets)
  - Minimize instability noise (distant targets & spectroscopy) - slower rotation is more susceptible to noise
- High specific impulse microthrusters mean that fuel usage is not currently a major issue

Removing the instability noise allows longer rotation times, mitigating the impact of the larger array

spectral channels

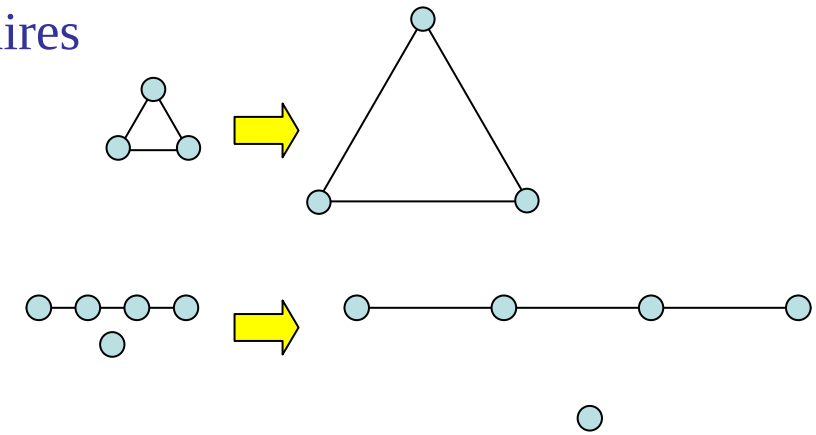


- As array size and planet radial offset increased, period of fringes vs wavelength decreases
- Spectrometer should ideally have enough channels to avoid smearing at edge of field of interest
- For $\theta_{max} = 10\theta_{IWA}$, desire ~ 120 spectral channels (vs 40 for X-Array 2:1)
- Increase in read noise offset by relaxed requirement on readout rate (which was driven by desire to remove systematic error with chop)

$$\frac{\lambda}{\Delta\lambda} \geq 2\frac{\theta_{max}}{\lambda/B}$$

Other array configurations

- In principle Spectral Pre-Fitting can be used with any array configuration
- In practice Spectral Pre-Fitting requires
  - ⇨ 3 x array size
  - ⇨ 3 x nulling baseline length
  - ⇨ 9 x stellar leakage
  - ⇨ greatly reduced sensitivity



- Only the X-Array has independent scaling for imaging and nulling baselines
  - Can increase array size without increasing stellar leakage

Impact summary

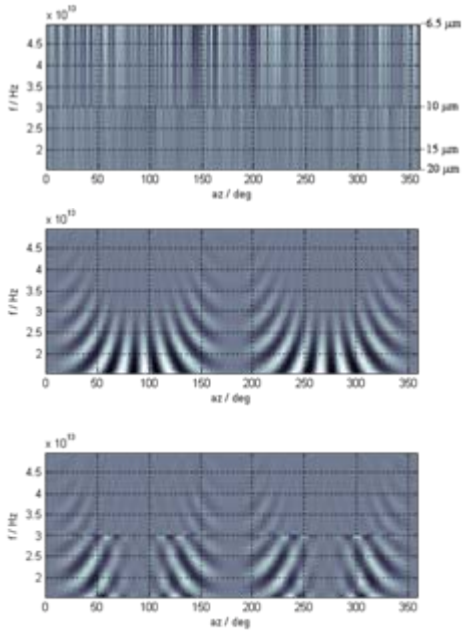
+ Relaxation in null depth requirement	*10^{-5}; mission viable @ 10^{-4}*
+ Improved sensitivity	*~ 20 – 30 %*
+ Greatly increased angular resolution	*factor 3*

- Stray light requirement	*Miss a few late K and M stars*
- Fuel consumption	*Mitigated by reduced rate*
- Increased # spectral channels	*40 => 120*
- Does not work with all array configurations	*X-Array only*
- Increased signal processing complexity	*Need to investigate further*

Potential future work

- Numerical simulation with higher fidelity noise and spectral properties
- Are there dynamic instrument instabilities with spectra that are not well fit by a polynomial?
- Simulations of signal extraction / imaging
- Study variants of the fitting approach
- Integrate into a Bayesian-style planet extractor?

Summary



- Instability noise drives the instrument to a 10^{-6} null
- Instability noise has a power-law spectrum
- Stretching the array gives the signal a distinct modulated spectrum at each rotation angle
- 'Spectral pre-fitting' removes the instability noise without a major loss of signal
- Relaxes null requirement from 10^{-6} to 10^{-5} (10^{-4} null may even be viable)
- And increases sensitivity
- And increases angular resolution
- But needs more spectral channels, and complicates the signal processing
- Only possible with stretched X-Array

Back-up

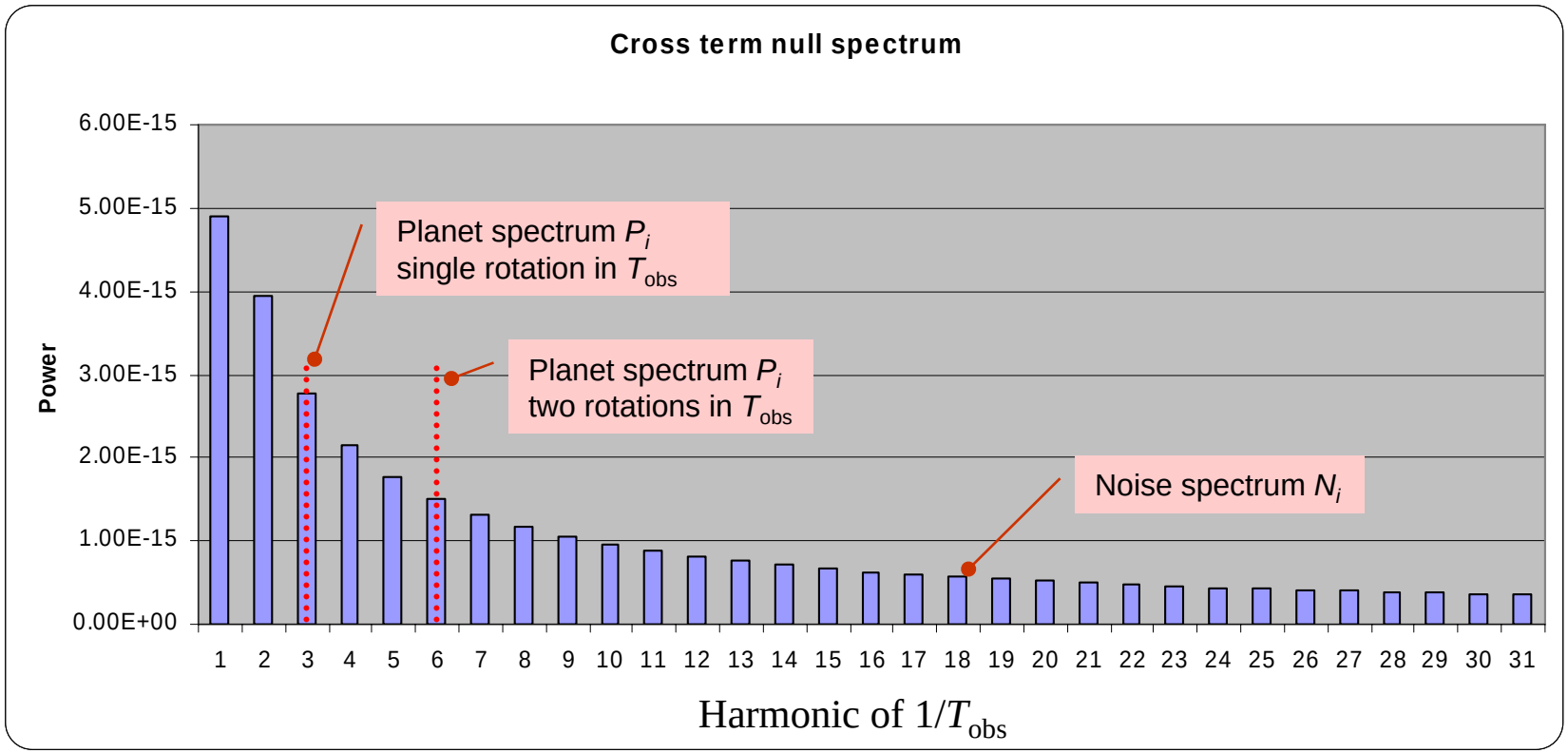
SNR summary for stretch and fit

10^{-6} null	X-Array 2:1	Stretched x3	Stretch + Fit
Signal	1	1	0.8
Shot noise	0.1	0.1	0.09
Instability noise	0.1	0.03	0.00015
SNR	7.1	9.6	8.9

10^{-5} null	X-Array 2:1	Stretched x3	Stretch + Fit
Signal	1	1	0.8
Shot noise	0.103	0.103	0.092
Instability noise	1	0.3	0.0015
SNR	1.0	3.2	8.7

50,000 s rotation; Values approximate only, pending more detailed models

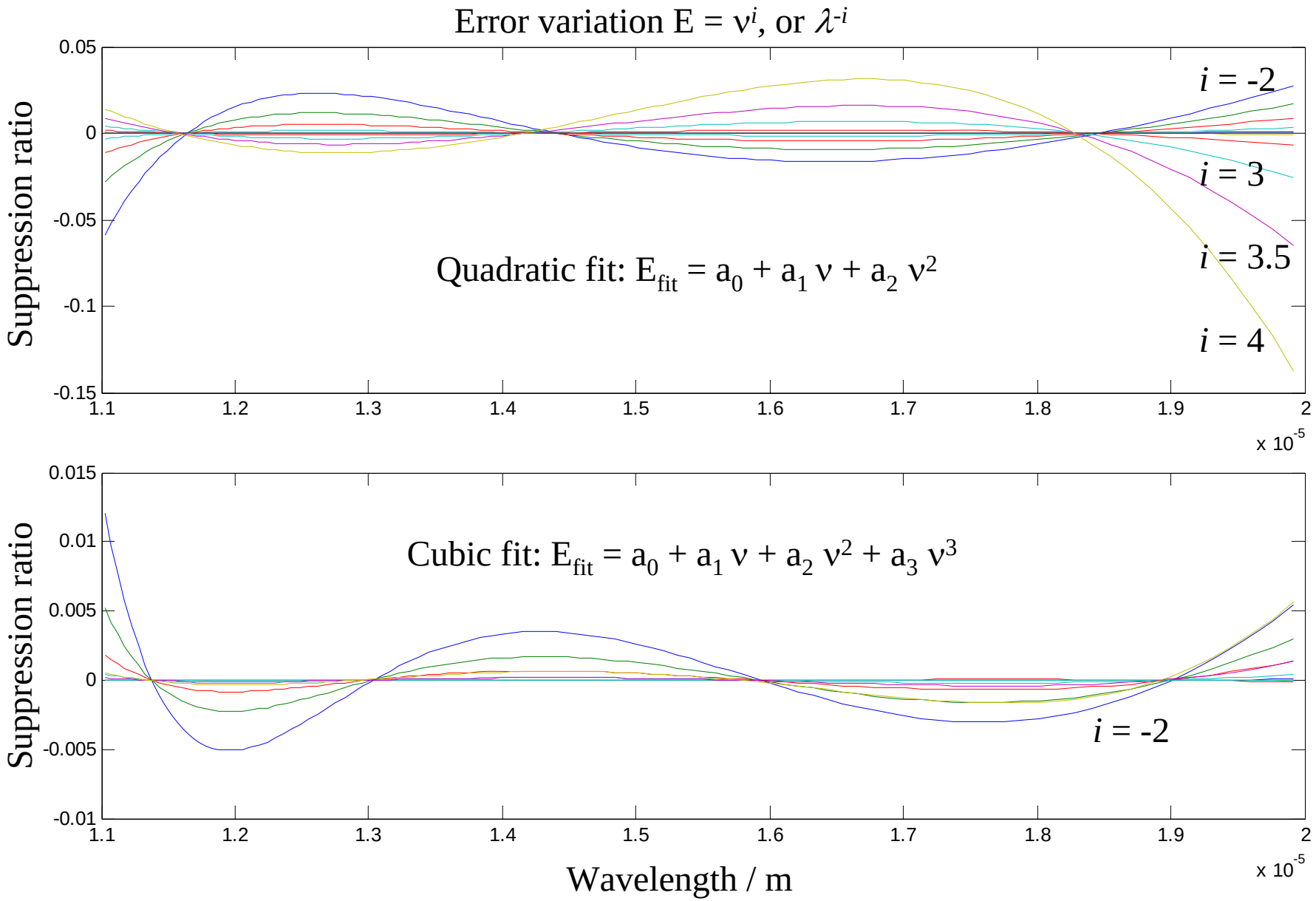
Instability noise and array rotation period



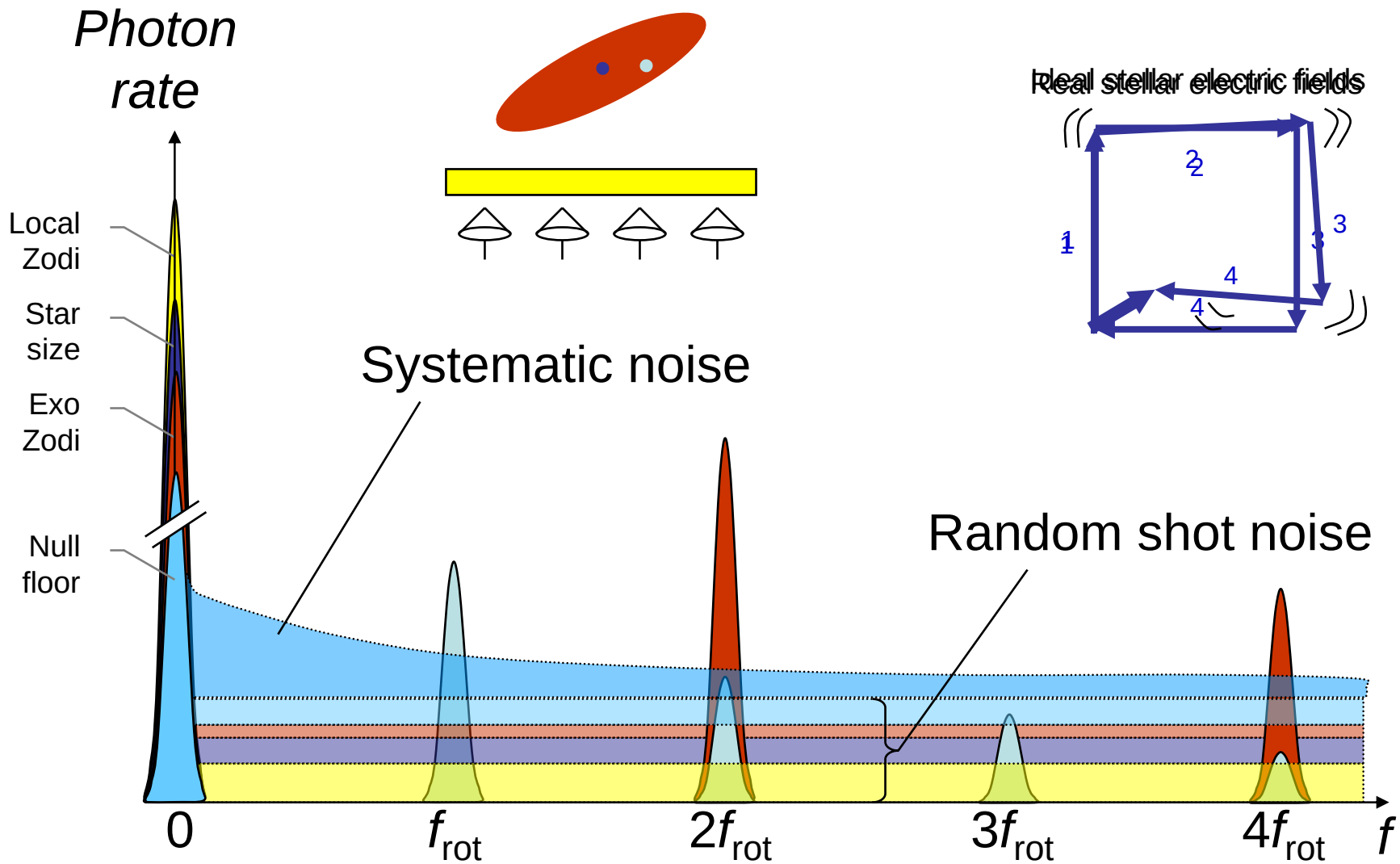
$$Noise \propto \left\{ \frac{\sum_i P_i N_i}{\sum_i P_i} \right\}^{\frac{1}{2}}$$

- Null spectrum is not white
- Faster rotation means planet signal competes with lower instability noise

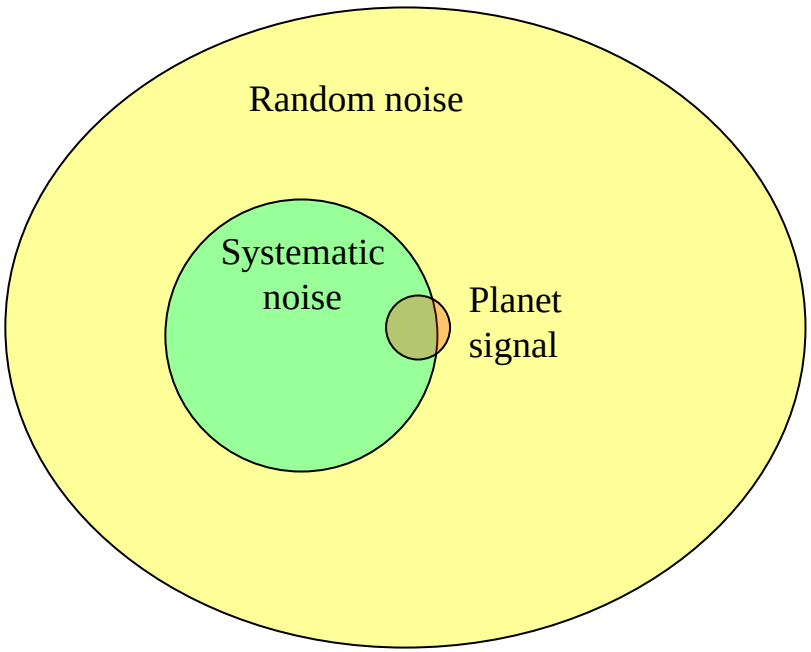
Fit effectiveness vs spectral index of error

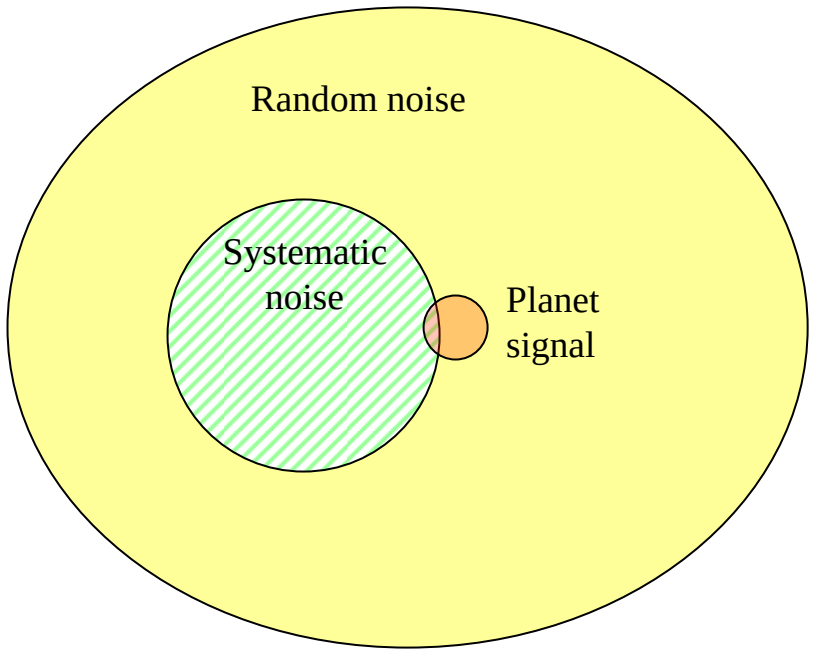


Power spectrum



Signal & Noise Venn diagram





Nomenclature

- ✘ Systematic error
  - a misleading name, suggests a non-stochastic quantity, something that does not average down
- ✘ Variability noise
  - ESA's term, but suggests astronomical origin
- ✓ Instability noise
  - captures both instrumental origin and stochastic nature

Change history

- Dec 15, 2005: First version presented
- Dec 29, 2005: Changed title; added slides on spectroscopy
- Mar 14, 2006: Changed fuel usage slide; misc minor changes