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Hires: Super-resolution for the Spitzer Space Telescope

C. Backus, T. Velusamy, T. Thompson, and J. Arballo

Jet Propulsion Laboratory, Pasadena, CA 91109-8099, Email: backus@jpl.nasa.gov

Abstract. We present a description of hires, a super-resolution program based upon the Richardson-Lucy algorithm, generalized to the case of redundant coverage, with higher order optical image distortion, implemented for the Sptizer Space Telescope.

1. Introduction

Spitzer is approximately critically sampled and has good SNR, but the telescope is relatively small (85cm). To compensate for this there are various observing strategies employing redundant coverage. Hires was implemented to reassemble the information from overlapping observations into a single image with finer sampling, super-resolution, justified by the increased u-v plane coverage provided by the redundancy, in conjunction with reliable knowledge of the point spread functions.

2. The Hires Algorithm

The algorithm is based upon the Maximum Correlation Method (MCM) developed at the Caltech Infrared Analysis Center (IPAC) (Aumman et al 1990) for use with IRAS data. MCM is a generalization of the Richardson-Lucy (R-L) algorithm (Richardson 1974; Lucy 1972) to redundant coverage. We reformulated the MCM calculations so that convolutions could be evaluated using the Convolution Theorem, which enables the use of FFT's.

Let

D:	Acquired image
P:	Point spread function (psf)
$\tilde{P}$:	Reflected psf, $\tilde{P}(v) = P(-v)$
f^n:	Current trial image estimate

then the single coverage R-L algorithm can be described as a series of iterations shown in vectorized form as

$$f^{n+1} = f^n \frac{D}{f^n * P} * \tilde{P}, \tag{1}$$

a procedure which consists of three main steps: Simulate acquisition of the data by convolving the current image with the psf and create an image of correction

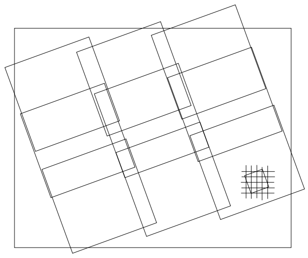

Figure 1. The trial image with data coverage, or correction factors.

factors by dividing the simulated data into the acquired data pixel by pixel. Second, convolve the correction factor image with the reflected psf. Finally, the smoothed correction factors are applied to the current image to produce the next estimate.

The initial image estimate is customarily uniformly set to 1's, in accordance with a Bayesian assumption commonly referred to as "uniform ignorance." *A priori* knowledge can be introduced by placing delta functions at the locations of known point sources[1], in hopes of solving the point source removal problem connected with stellar debris disks and galactic nuclei. But, this is difficult because the $\tilde{P}$ convolutions cause a spike amplitude to adjust slowly.

Extension to the multiple coverage case involves what is essentially weighted averaging.

$$f^{n+1} = f^n \frac{\sum_j \left[\frac{D_j}{f^n * P_j}\right] * \tilde{P}_j}{\sum_j U_j * \tilde{P}_j} = f^n \frac{\sum_{j,k} c_{jk}(u_{jk} * \tilde{P}_j)}{\sum_{j,k} u_{jk} * \tilde{P}_j} \tag{2}$$

where c_{jk} is the kth pixel of the jth data correction factor image, and U_j is a unit 2D pulse function coaligned with D_j with (unit data) pixels u_{jk}. The right hand side of this equation is quite similar to the formulation given in the appendix of (Aumann et al 1990), because the $u_{jk} * \tilde{P}_j$ are response functions.

The extension retains the key attributes of the R-L method: no negative flux, conservation of flux[2], and increasing likelihood[3]. It should be remarked with clarity that the above expressions of the R-L algorithm in hires depend upon two important (yet caveated) assumptions:

1. Psf Isotropy. The psf does not vary over the detector array. The D_j could be decomposed into subarrays small enough to assume needed stationarity with a different psf on each one, but hires presently does not handle separate psf's.

[1] John Fowler, IPAC.

[2] Provable in closed form by Yu Cao, in informal communication with IPAC, but with our edge effect treatment, conservation is no longer exact near the edges.

[3] The proof is difficult. (Lucy 1974) proof is close. In informal communication Lucy referred us to (Dempster et al 1977), very deep.

2. Noise Isotropy. Hires assumes that the rms measurement error of each pixel in each observation is constant. Noise weighting each pixel is feasible, as in (Aumann et al 1990), by dividing each c_{jk} and each u_{jk} by a σ_{jk}^2.

If either of these assumptions is vitiated significantly then the linearities inherent to the above formulations can not presently be employed to use FFT's to evaluate convolutions.

2.1. Implementation of the Formalism

Figure 1 shows the overall setting for the above calculations, using the middle member of eq (2). The trial image is sampled at a level appropriate to the amount of redundant coverage. The data images are in general not aligned with the trial image, and they are not necessarily coaligned with each other, although for a given Spitzer observation set taken on the same day they would be co-oriented to within one degree of rotation. In this case it is important to note that equal (or nearly equal) $\tilde{P}_j$ can be factored out of partial sums, greatly reducing the number of FFT's that need to be performed. The same is true of the simulation convolutions: resampling to the detector image resolution and division into each member of a coaligned subset of the D_j can be performed using the trial image convolved once with one of the P_j oriented with the subset.

2.2. Services Performed by Hires

The iterative procedure itself is a relatively simple implementation. Hires performs various calculations to prepare for it:

1. Resampling. Striping is mitigated by correcting accumulations with ratios computed from pixel areas and inclusion counts.
2. Edge effects. Abrupt flux transitions at the image boundary can cause high frequency ringing artifacts. We mitigate this by adopting the included psf solid angle weighting technique discussed in (Aumann et al, 1990).
3. Projection calculation. Some observation scenarios can cover as much as 4 degrees of sky, mandating an exact calculation to perform tangent plane transfers. This is done with 3D homogeneous linear coordinate transformations followed by renormalization to the destination tangent plane.
4. Distortion. Spitzer optical distortion is recorded in the form of FITS keywords giving coefficients for quadratic or cubic bivariate Taylor series. These polynomials need to be evaluated for each trial image pixel for each observation image to determine pixel inclusions for resampling. We use the recursive Horner algorithm[4], which we generalized to the multivariate case with a second recursion on the coefficients, which in this conception are themselves polynomials, with the number of variables reduced by 1. The Jacobian of the distortion transformation is needed because pixel areas in Spitzer images can vary by as much as 10%.

For FFT's a suitable convolution pad is added to the trial image. Then dimensions are further increased slightly, up to the nearest higher numbers which

[4]Horner published the technique in 1819. It involves successive factorizations of the independent variable to reach a form with nested multiplications, familiar to most students of computer science.

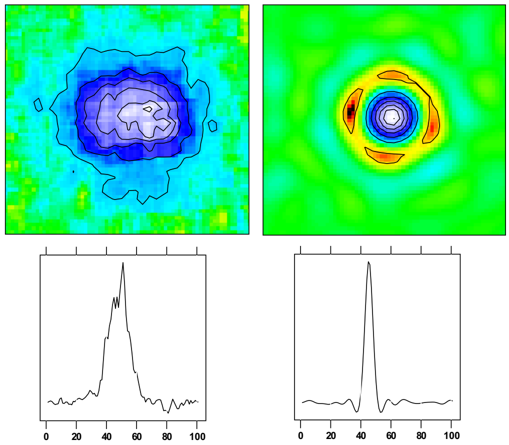

Figure 2. Response to Isolated 24μm Point Source. Left is coadd, right is 100 hires iterations with 10x redundancy.

are products of powers of the first few primes. A very substantial percentage of the execution time is spent evaluating the Fourier transforms, which are RAM and CPU intensive. We use as much memory as we can and use an FFT package that takes advantage of threading.

3. Invocation of hires

All of the image files used are in FITS format. The program is invoked with a script run in background mode, and information about progress is piped to a log file. Various switches set output image registration, resolution, dimensions, and orientation, the latter being important in cases where coverage is long and narrow, and diagonal to the local celestial frame. The trial images and correction factor images from successive iterations can be examined with a FITS viewer during execution. Correction factor images are useful for diagnosis of bad input pixels and poorly matched psf's. At convergence they should uniformly be within a percent or so of all 1's. Our largest runs, 4096x4096 pixel trial image and 300 256 x 256 input images, take 5 minutes per iteration on a modern 2 CPU PC with 2 GB of RAM running linux.

The number of iterations to run is an imposition of *a priori* knowledge on the part of the user. There is a switch to determine the iteration count, but the user can examine intermediate output and decide which image to use[5].

4. Examples

Figure 2 shows the 3:1 reduction in the half-power width of the point source response which is typical for many Spitzer observations. The ringing is due to

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[5]The literature is extensive on stop rules for R-L. Cf (Prasad 2002).

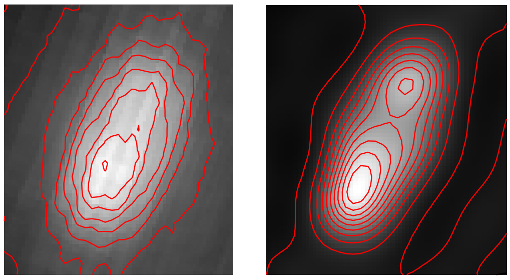

Figure 3. Fomalhaut Debris Disk. Coadd on left, 50 hires iterations on right. The central source, not visible at 70μm, is between the peaks.

the high spatial frequency limitation inherent to the process. It can be mitigated by running fewer iterations , or, for cosmetic purposes, close background removal.

Figure 3 shows the effect of 50 hires iterations on a Spitzer 70μm observation of the debris disk around Fomalhaut, where a single bulge of ice and dust emission is resolved into a nearly edge on view of a toroidal shape, with an asymmetry possibly due to an unseen planet (Stapelfeldt et al 2004).

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