Precision Formation Flight of Spacecraft in Non-Uniform Gravity Fields

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Try Lam
Jet Propulsion Laboratory
California Institute of Technology

Firdaus E. Udwadia
University of Southern California

Aaron Schutte
Aerospace Corporation

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Agenda

- Introduction
- Theory
- Formation Flying Example
  - Constraints
  - Numerical Results
- Conclusion

Introduction

- The problem of controlling and maintaining precise formation of spacecraft is a difficult one.
  - For collaborative observations it requires accurate actuation and sensing (or knowledge and control) by the system.
  - Although, with the combination of accurate sensors and knowledge it is not required to control the separation of the spacecraft, it is valuable to **understand what are the controls**.

- Our research focus is on **determining the dynamical force** necessary to control a formation precisely to some prescribed set of constraints around any non-spherical rotating body.

- We accomplish this by employing a general **constraint based control methodology**

- We assume perfect actuation (i.e., no misalignment or thrusting error) and knowledge (i.e., we know exactly where we are).

Introduction

- Benefits: (1) allows for the understanding of the system from a macro view, (2) method is simple to implement and exact (assuming perfect knowledge and control)

- Examples will be given for 2 different formation types, a **2-space-craft** formation and a **4-spacecraft** formation

- Objective is to keep the spacecraft in a very precise formation and analyze its dynamics

- Note, we do not try to optimize the configuration, or try optimize the number of spacecraft to best provide ground / space converge, but focus on the control required for a given formation.

- Not necessary trying to replace other control methods, but to complement and improve existing control methods.

Theory: EOM for Constrained System

- In general, the equations of motion of a system that is perturbed from its natural state has the form of

$$\boxed{M\ddot{x} = F + F_C}$$

where F_C is the perturbing or constraint force (Lagrange Multiplier approach).

- **The task is to find F_C** and there are a number of ways to solve for it depending on problem. Analytically, the problem is solved by the various form of the *Fundamental Equation.* The most famous is probably the concept of *Virtual Work* or the use of Lagrange multipliers.

- We apply here another form of the Fundamental Equation of Lagrangian mechanics, which we will call "**the Fundamental Equation**", based on **Gauss's principle of Least Constraint** (Udwadia and Kalaba [1991])

Theory: EOM for Constrained System II

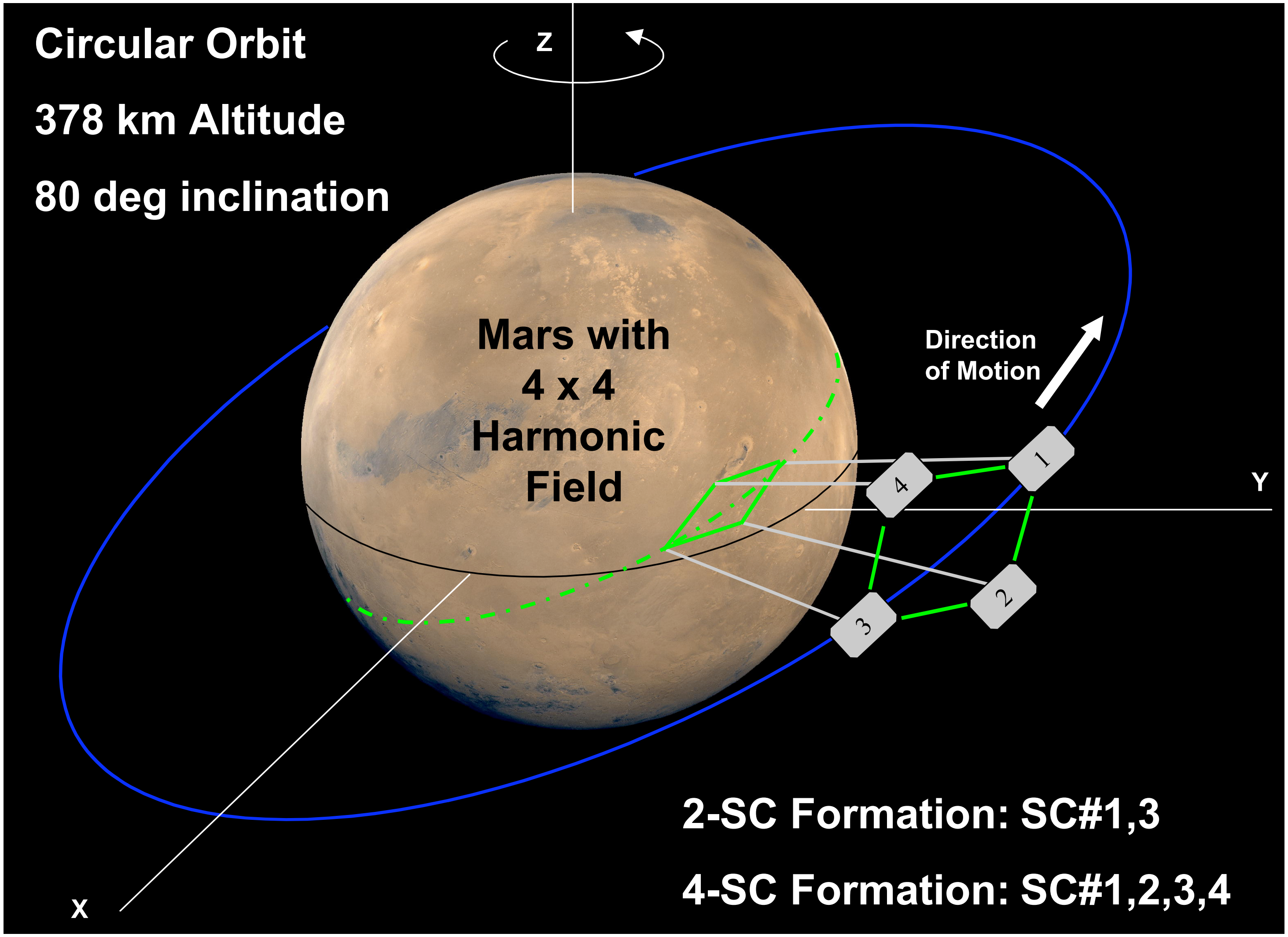
- Udwadia's and Kalaba's form of the Fundamental Equation:

$$\boxed{M\ddot{x} = F + \underbrace{M^{1/2}\left(AM^{-1/2}\right)^{+}}_{\text{Weighted Feedback Gain}}\underbrace{(b - Aa)}_{\text{Acceleration Error Signal}}}$$

F, a	= free-response force and acceleration
M	= diagonal mass matrix
A	= from the constraint equation **(will be discussed later)**
b	= from the constraint equation **(will be discussed later)**
$(\ )^{+}$	= pseudo-inverse

- All **constraints are solved in a least square sense analytically** at every time step

Formation Flying Example



Unconstrained Motion

- Spacecraft in a non-uniform gravity field
  - **Gravitational Potential**

$$U = -\frac{\mu}{r} + \frac{\mu}{r}\sum_{n=2}^{\infty} J_n \left(\frac{R}{r}\right)^n P_n(\sin\phi) - \frac{\mu}{r}\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^n P_n^m(\sin\phi)\left(C_{n,m}\cos m\lambda + S_{n,m}\sin m\lambda\right)$$

R	= body-centered reference radius
r	= geocentric distance
ϕ	= latitude of the spacecraft
λ	= longitude
$P_n(\sin\phi)$	= Legendre Polynomial of degree n in $\sin\phi$
$P_n^m(\sin\phi)$	= Legendre function of the first kind

  - **Unconstrained Acceleration**

$$a = -\nabla U$$

Unconstrained Motion II

- Spacecraft in a non-uniform gravity field
  - **Body Parameters**

Mars Physical Parameter	Value
Reference (Equator) Radius	3,397 km
GM	42,828.380415705753 km^3/s^2
Rotation Rate	7.08823595918567E-05 rad/s
Gravity Field	20 x 20
J_2 (normalized)	8.74554802878902E-04
J_3 (normalized)	1.18743368538763E-05

  - **Orbit Parameters**

Orbit Parameter	Value
Semi-major Axis, a	3,775 km
Eccentricity, e	0
Inclination, i	80°

Constraints

- Constraints applied between the spacecraft
  - **Relative Distance Constraints**

$$\boxed{\phi = L^2 - (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 = 0}$$

  - **Relative Radial Distance Constraints**

$$\boxed{r_i^{\,2} - r_j^{\,2} = c}$$

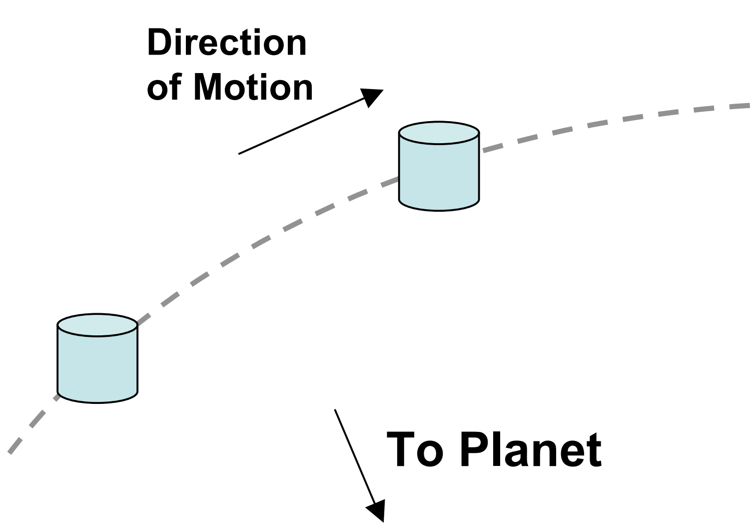
- For precision we use Baumgarte’s stabilization technique

$$\ddot{\phi} + \alpha\dot{\phi} + \beta\phi = 0$$

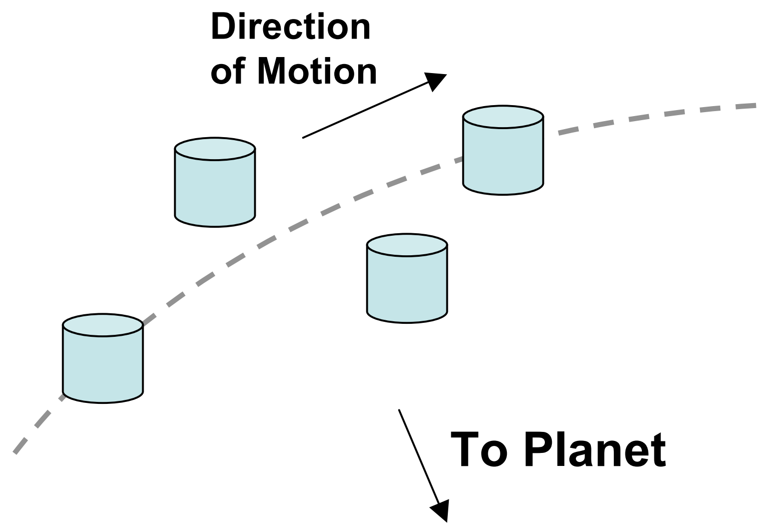
- Formation behaves as a **“virtual” rigid body**

Relative Distance Constraints

- **Circular obits** around a body



Two-Spacecraft Formation



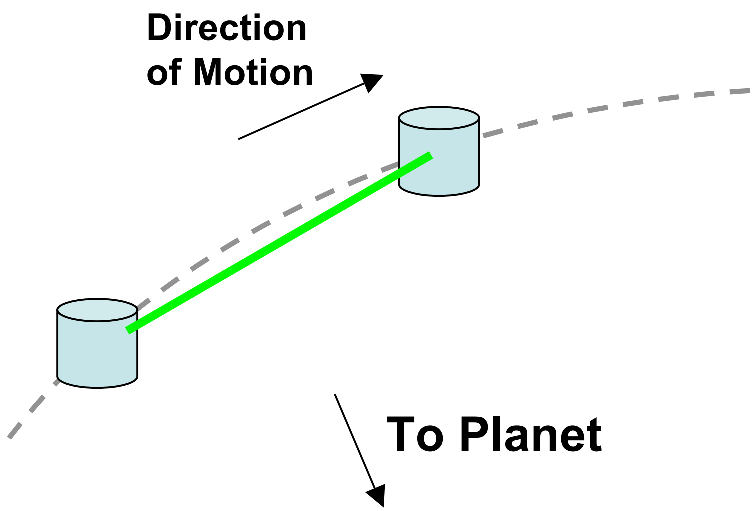
Four-Spacecraft Formation

$$\phi = L^2 - \left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2 + \left(z_i - z_j\right)^2 = 0$$

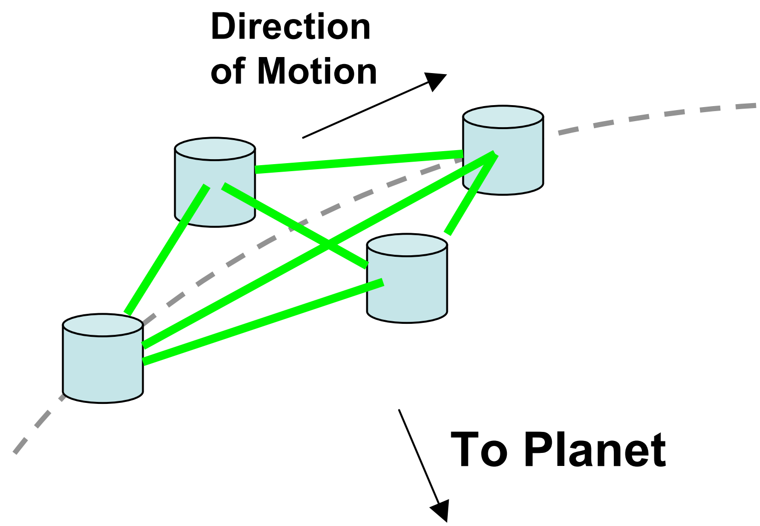
where *L* is a constant

Relative Distance Constraints

- **Circular obits** around a body



Two-Spacecraft Formation

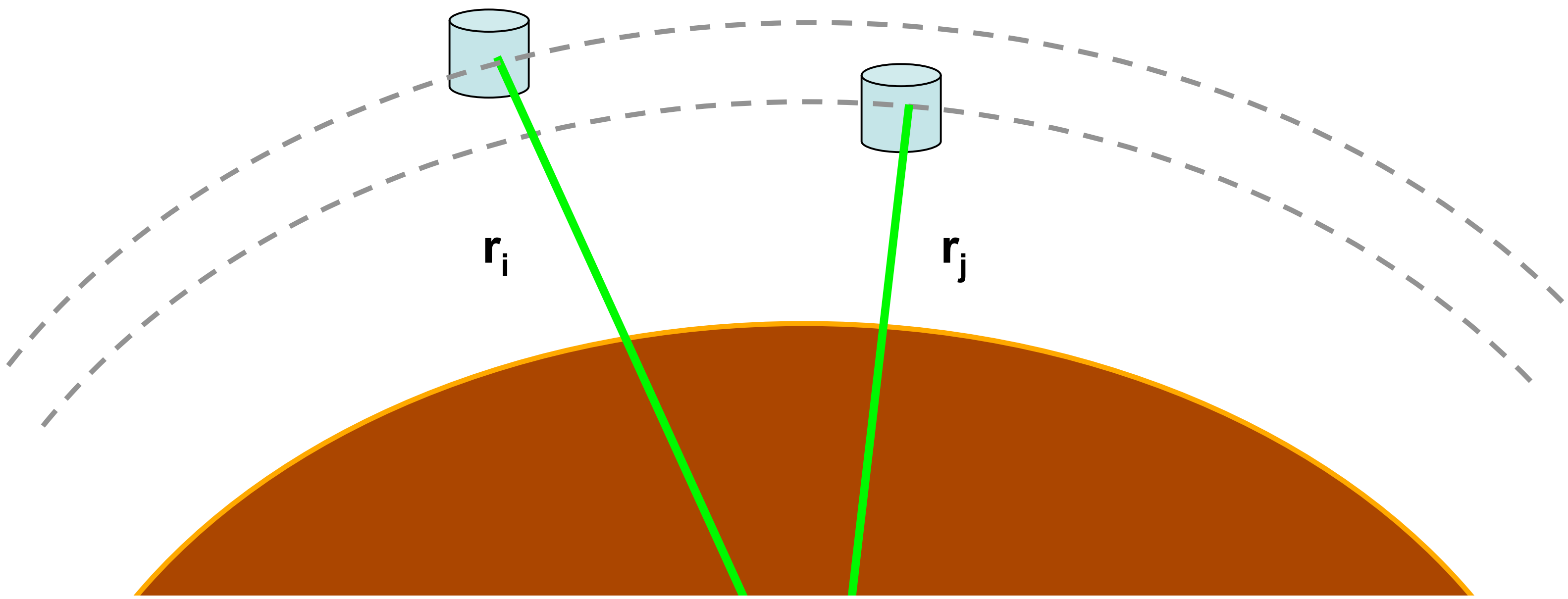


Four-Spacecraft Formation

$$\phi = L^2 - \left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2 + \left(z_i - z_j\right)^2 = 0$$

where *L* is a constant

Relative Radial Distance Constraints



$$r_i^{\,2} - r_j^{\,2} = c$$

where *c* is a constant

Finding A & b

m **constraints**

$$\varphi_m(x,\dot{x},t)=0 \quad \longrightarrow \quad A(x,\dot{x},t)\ddot{x}=b(x,\dot{x},t)$$

Differentiate twice for position constraints

Differentiate once for velocity constraints

- **Example:** $$r_1^{\ 2}-r_2^{\ 2}=const.$$

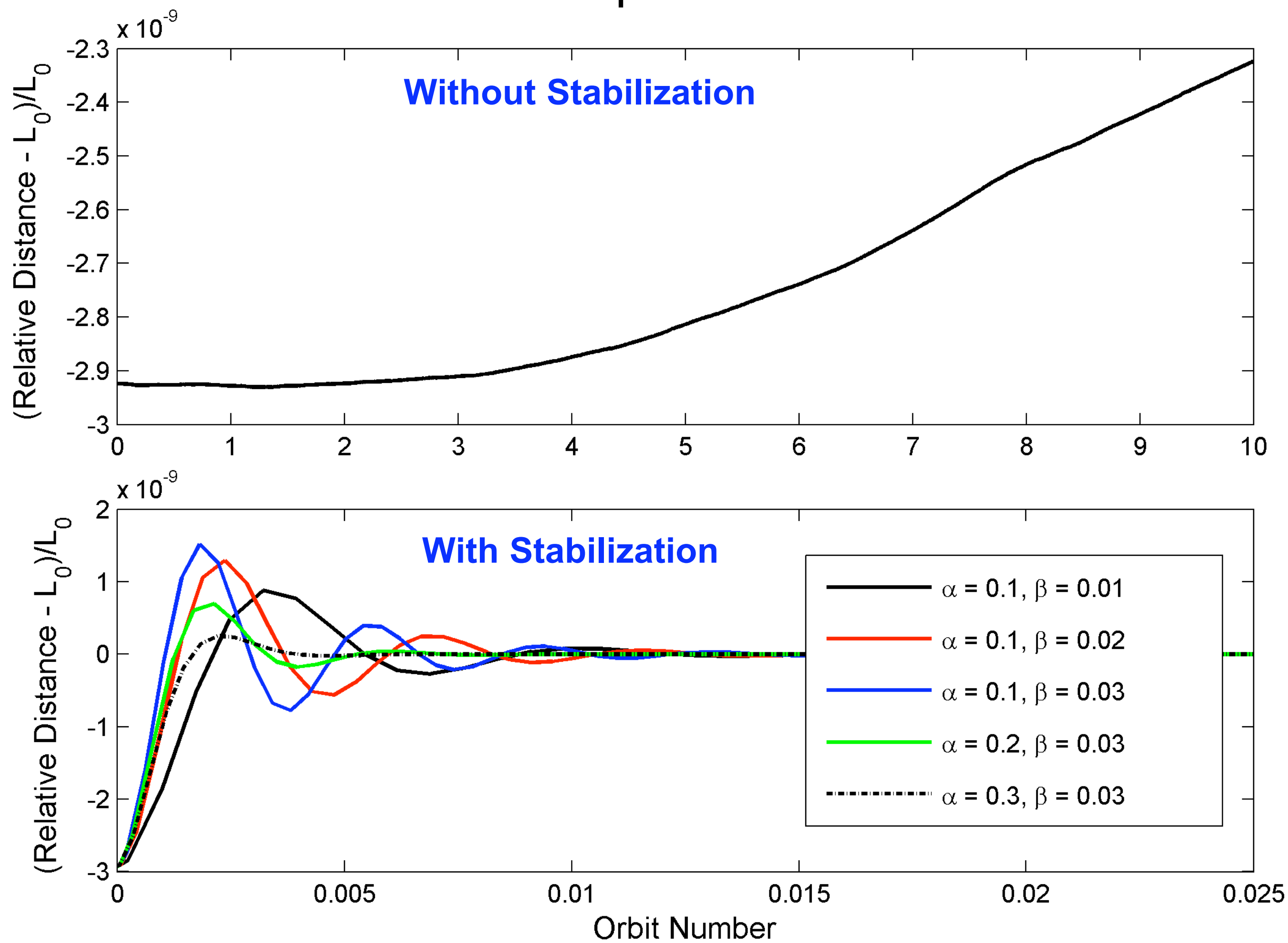
Differentiate twice

$$\begin{bmatrix}-x_1 & -y_1 & -z_1 & x_2 & y_2 & z_2\end{bmatrix}\ddot{\bar{x}}=\begin{bmatrix}x_1^2+y_1^2+z_1^2-x_2^2-y_2^2-z_2^2\end{bmatrix}$$

A **b**

Two-Spacecraft Example: Stabilization

- Relative Distance Between the 2 Spacecraft

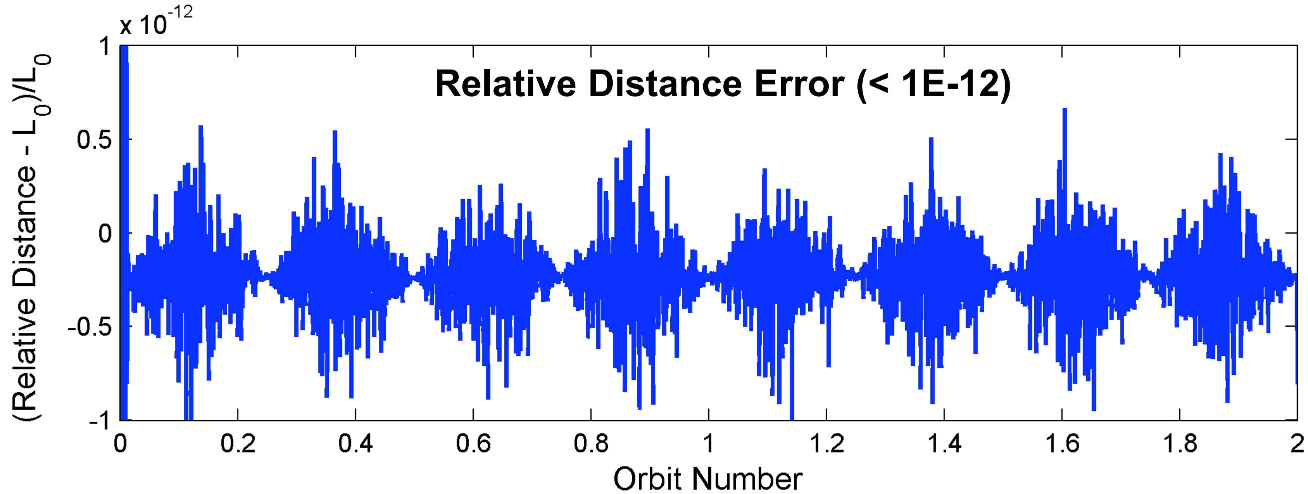


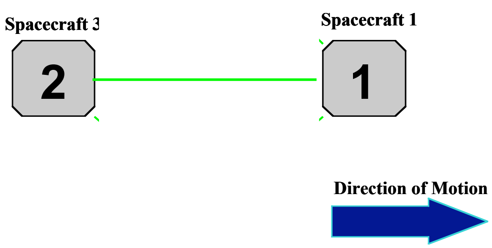
Two-Spacecraft Example: Simulation

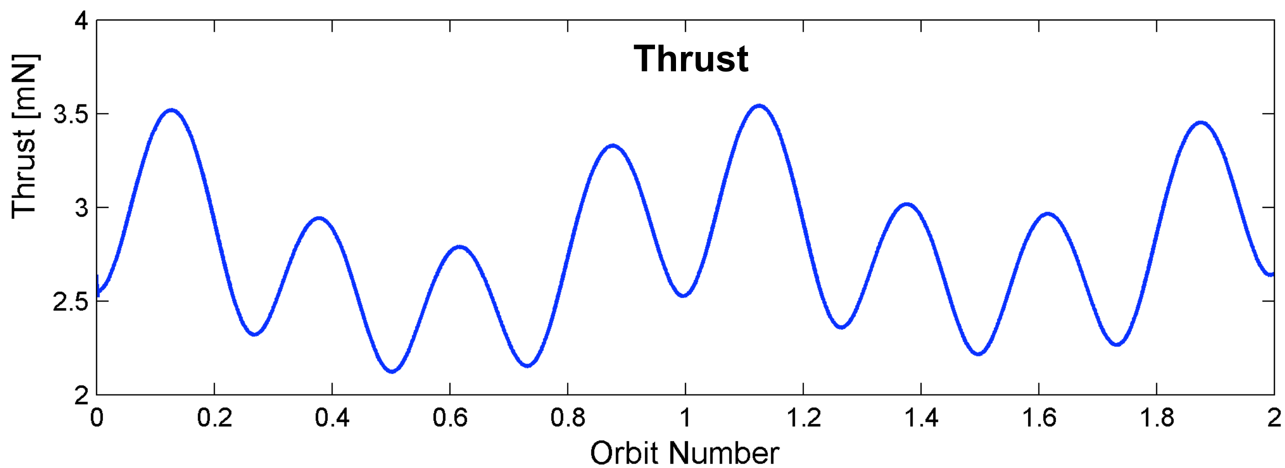
Circular Orbit

378 km Altitude

80 deg inclination





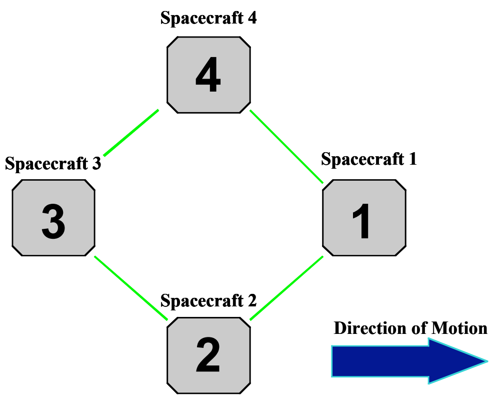


Four-Spacecraft Example: Simulation

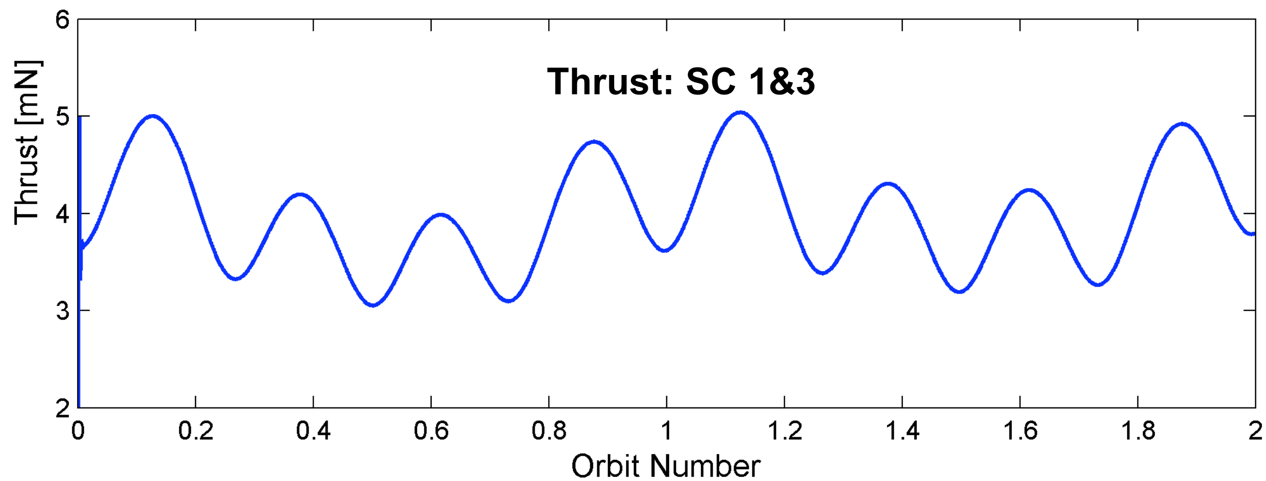
Circular Orbit

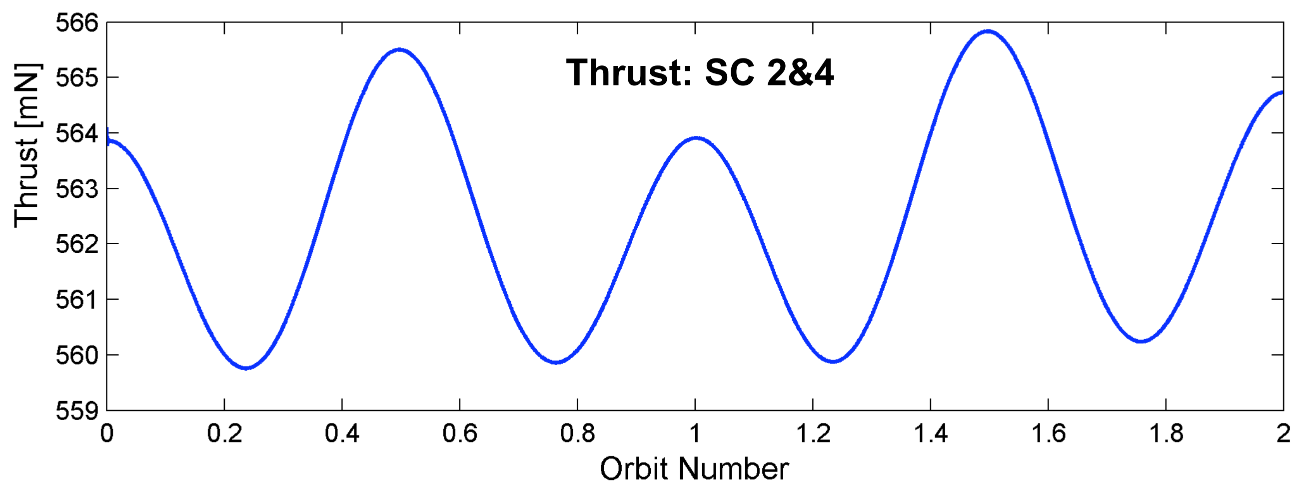
378 km Altitude

80 deg inclination



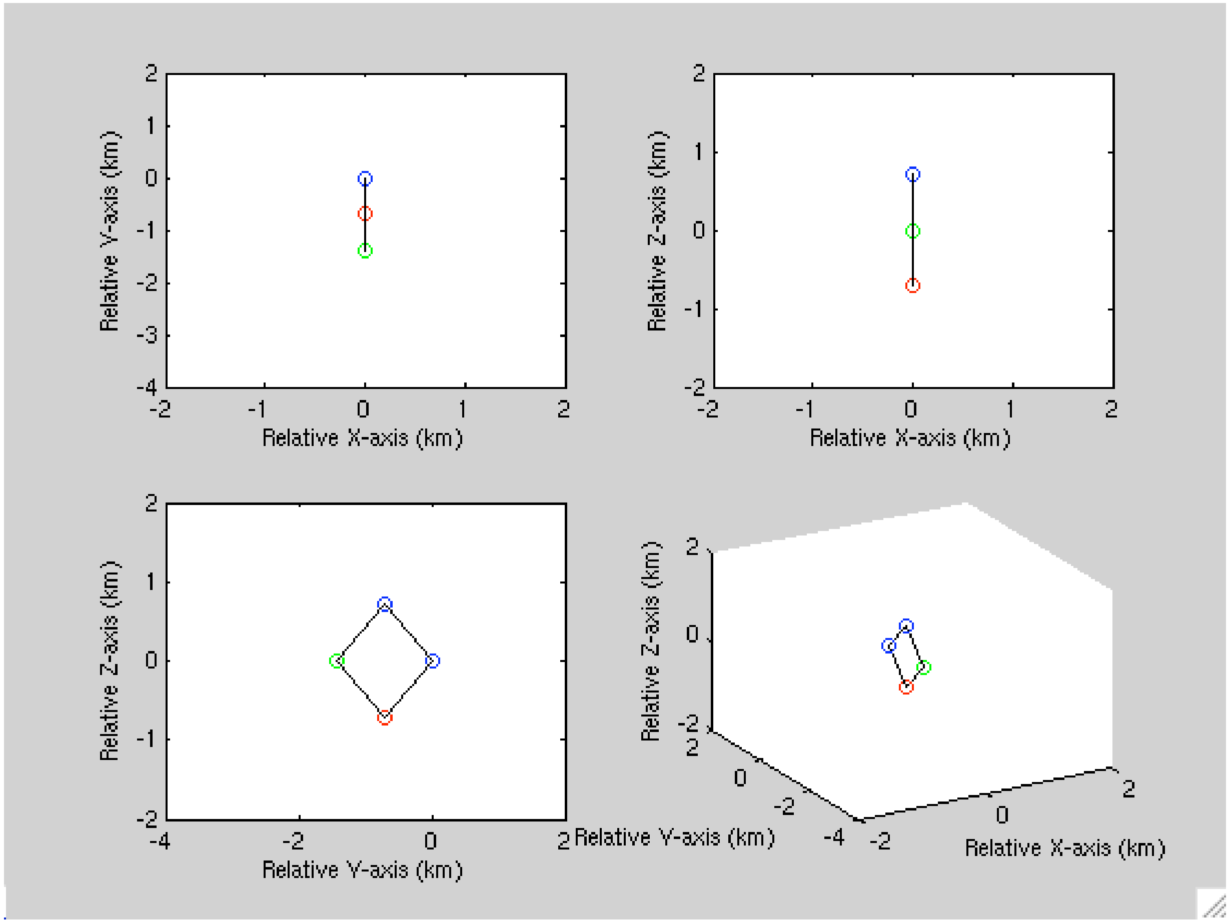
Relative Distance Error (< 1E-12)





Four-Spacecraft Example: Pulsating

- Time-varying constraint of the form $L(t) = L_0\left[abs\left(\sin(kt)\right)+1\right]$

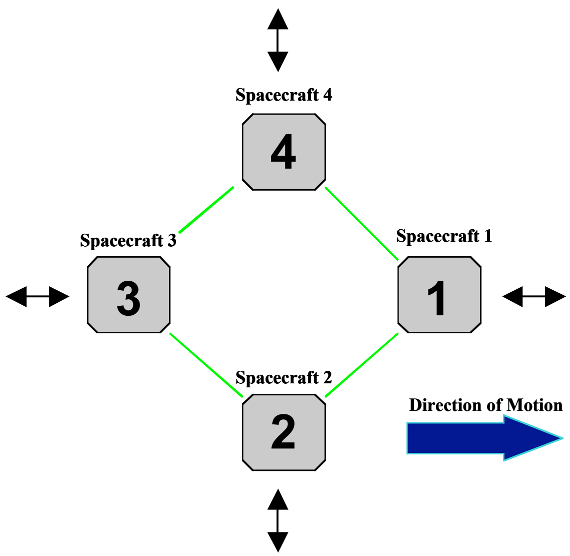


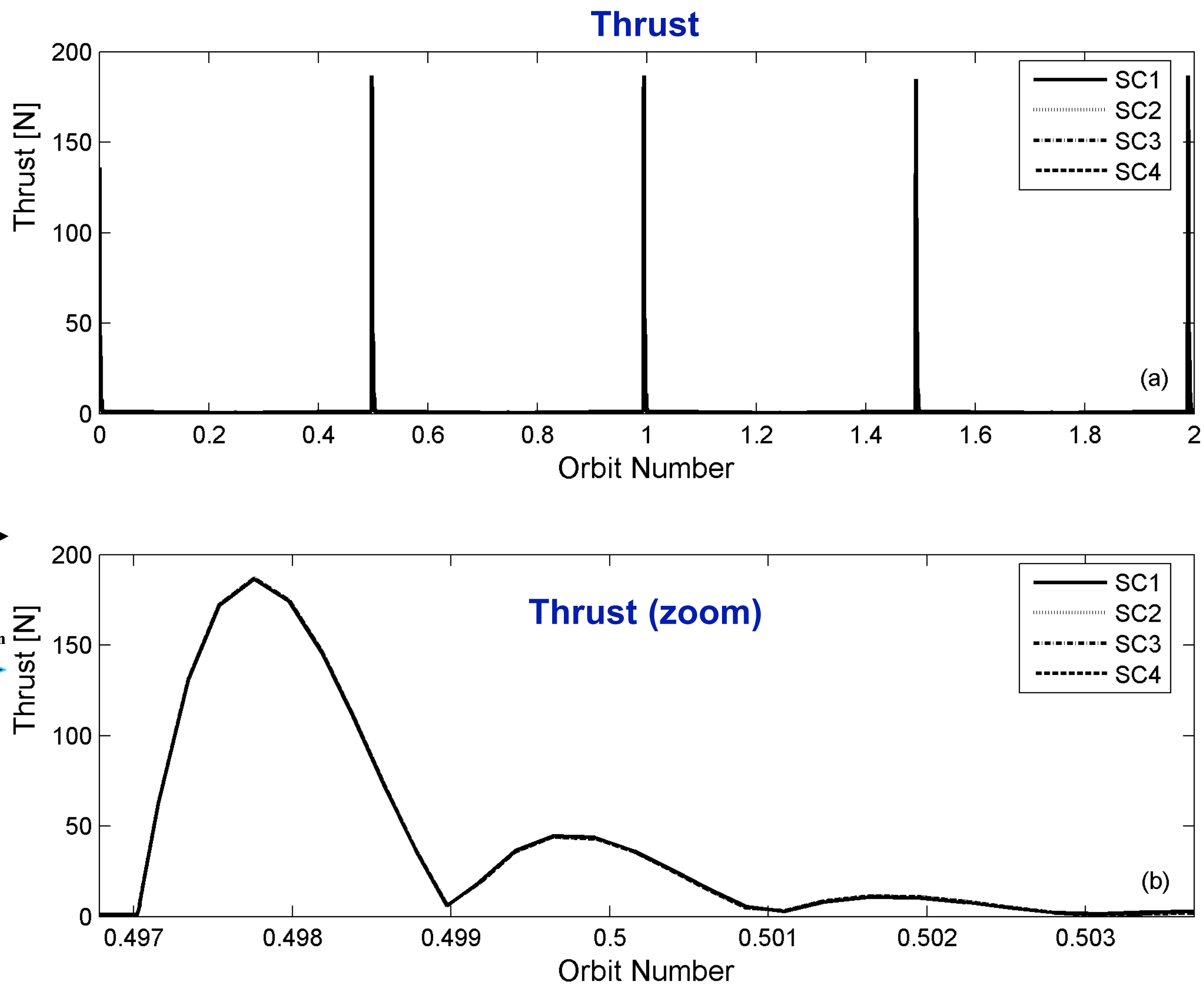
Four-Spacecraft Example: Pulsating II

Circular Orbit

378 km Altitude

80 deg inclination





Conclusion and Future Work

- Conclusion:
  - Introduced and applied different approach to the control of multiple spacecraft in precision formation flight
  - Method is based on a new form of the fundamental equations of motion for constrained system (Udwadia & Kalaba)
  - Solutions are explicit and equations are simple to derive
  - Good for analyzing and controlling multi-body systems
  - See paper, **AAS 06-122** for more details

- Future Work:
  - Improve numerical implementation
  - Add switching function to the thrusting
  - Model errors and noise
  - Compare method to existing method

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BACK UP

CHARTS

Theory: EOM for Constrained System III

- **Gauss's principle of Least Constraint** states that of all possible acceleration (including those that are non-physical) for the system, the "actual" acceleration is one which minimizes the Gaussian

$$G = (\ddot{x} - a)^T M (\ddot{x} - a)$$

- **Handling constraints**
  - Given a set of differentiable kinematic constraints

$$\phi(x, \dot{x}, t) = 0$$

holonomic or non-holonomic constraints

  - We can differentiate Eq. (2) to be of the form

$$A(x, \dot{x}, t)\ddot{x} = b(x, \dot{x}, t)$$

Theory: EOM for Constrained System IV

- Consider a system of *n* particles, then the free-response motion is

$$a = M^{-1}F\left(x, \dot{x}, t\right) \qquad (1)$$

- Given a set of differentiable constraints

$$\phi_i\left(x, \dot{x}, t\right) = 0 \qquad (2)$$

- We can differentiate (2) to be of the form

$$A\left(x, \dot{x}, t\right)\ddot{x} = b\left(x, \dot{x}, t\right) \qquad (3)$$

- The constraint force is found to be (next slide)

$$F_C = M^{1/2}\left(AM^{-1/2}\right)^{+}\left(b - Aa\right) \qquad (4)$$

- Thus,

$$M\ddot{x} = F + F_C \qquad (5)$$