P23D-01: Saturn's Gravity Field and Interior Structure: First Results from the Cassini Mission

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References

1. J.K. Campbell and J.D. Anderson, Gravity field of the Saturnian system from Pioneer and Voyager tracking data, *Astron. Jour.* **97**, 1485-1495, 1989.

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Saturn's External Gravitational Field

We use a truncated form of the standard Legendre expansion of the potential function V in spherical harmonics.

$$V(r,\phi) \;=\; \frac{GM}{r}\left[1-\sum_{n=1}^{5}\left(\frac{a}{r}\right)^{2n} J_{2n}P_{2n}\left(\sin\phi\right)\right]$$

The spherical coordinates (r, ϕ) are referred to the center of mass, with r the radial distance and ϕ the latitude. P_n is the Legendre polynomial of degree n, and J_2, J_4, J_6 are the corresponding harmonic coefficients inferred from the data. The reference equatorial radius a is 60,330 km. The coefficient J_8 is fixed at an adopted value of -10^{-5} for the data analysis, as in previous work[1], and the coefficient J_{10} is extrapolated to 2×10^{-6} for fifth order calculations.

Table 1. Results for Saturn Gravity Field

	1989[1]	2004[2]	2005[3]
J_2	16298(10)	16292(7)	16290.6(0.4)
J_4	-915(40)	-931(31)	-936(3)
J_6	103(50)	91(31)	86(10)
GM	-64(200)	-76(59)	0.0(1.2)

a. Saturn GM in units km^3/s^2 is referenced to 37931207.8

b. Units are 10^{-6} for all gravity coefficients

c. 1989[1] is from Pioneer and Voyager data

d. 2004[2] is from all data pre Cassini tour

e. 2005[3] is from all data including Cassini tour

f. J_8 is fixed at -10, all other harmonics fixed at zero

Hydrostatic Equations for Saturn Interior

$$\frac{1}{\rho}\frac{dp}{ds} = -\frac{GM}{s^2} + \frac{2}{3}\omega^2 s$$

$$\frac{dM}{ds} = 4\pi\rho s^2$$

Define dimensionless variables in terms of the mean radius $R = 58301.9$ km at the surface, the mean density $\rho_0 = 684.635$ kg/m^3, the total mass $M_T = 5.68324 \times 10^{26}$ kg, a characteristic internal pressure $p_0 = GM_T\ \rho_0/R = 4.45423$ Mbar, and the ZT smallness parameter $m = \omega^2\ R^3/GM_T = 0.136825$.

Hydrostatic Equations for Saturn Interior (Continued)

The dimensionless variables are defined by $\alpha = \mathrm{M}/\mathrm{M_T}$, $\beta = \mathrm{s}/\mathrm{R}$, $\xi = \mathrm{p}/\mathrm{p_0}$, and $\eta = \rho/\rho_0$. In terms of these variables, the hydrostatic equations are,

$$\frac{1}{\eta}\frac{\mathrm{d}\xi}{\mathrm{d}\beta} = -\frac{\alpha}{\beta^2} + \frac{2}{3}\mathrm{m}\beta$$

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\beta} = 3\eta\beta^2$$

$$\xi = \left(\frac{\rho_0^2}{\mathrm{p_0}}\mathrm{K}\right)\eta^\gamma = \mathrm{k}\eta^\gamma$$

Numerical Integration of Hydrostatic Equations

Substitute polytropic EOS into pressure ODE to obtain a density ODE. The two ODE for numerical integration are,

$$\gamma \mathrm{k} \eta^{\gamma-2} \frac{\mathrm{d}\eta}{\mathrm{d}\beta} = -\frac{\alpha}{\beta^2} + \frac{2}{3}\mathrm{m}\beta$$

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\beta} = 3\eta\beta^2$$

The boundary conditions are $\alpha = 1$, $\xi = 0$, $\eta = 0$ at $\beta = 1$.

Table 2. Best Fit Polytrope to Measured Harmonics

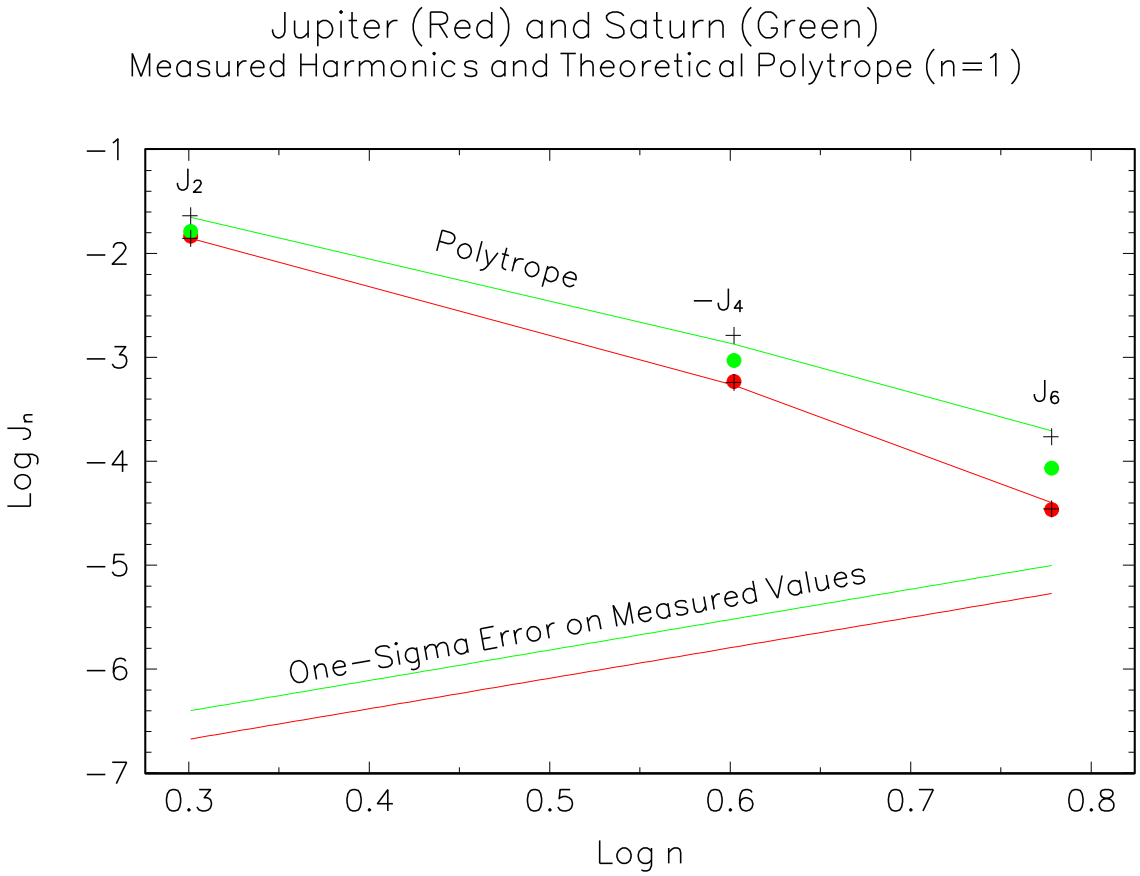
Harmonic	Measured	No Core	With Core
J_2	16290.6(3)	16294.60	16287.92
J_4	-936(3)	-885.47	-936.42
J_6	86(10)	74.97	83.77

a. Without core, $\gamma = 1.78586045$, k = 0.15032921

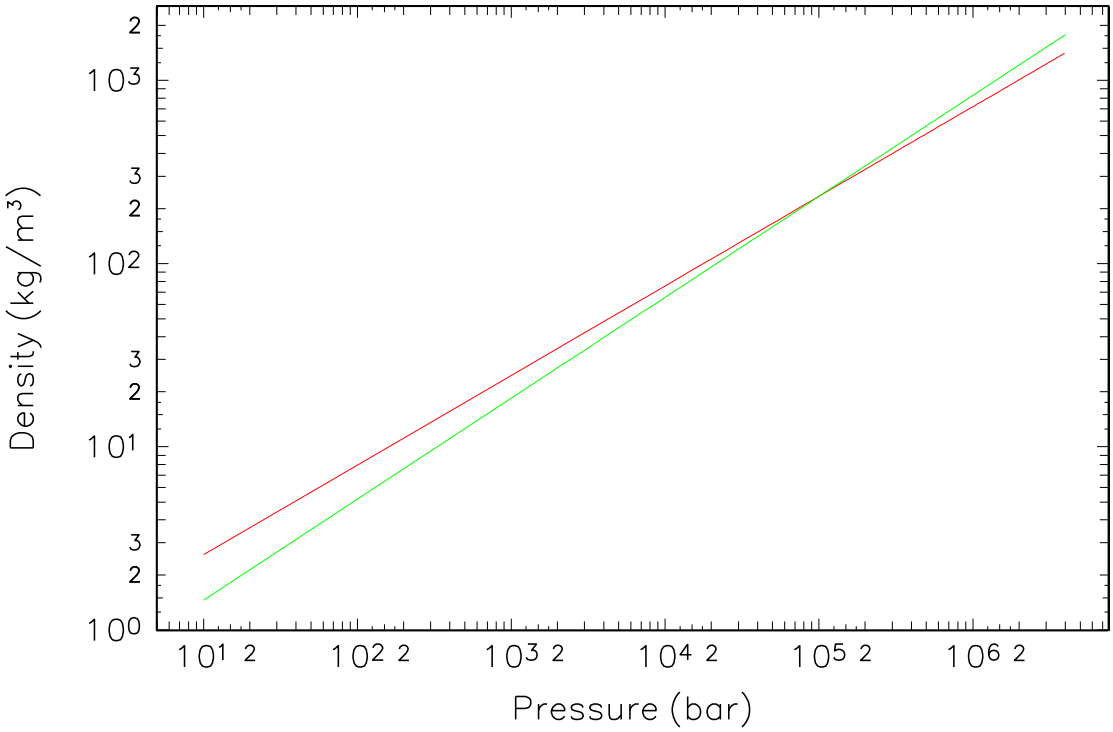
b. With core, $\gamma = 1.81496792 \pm 0.00172$, k = .158966966 $\pm$ 0.000511

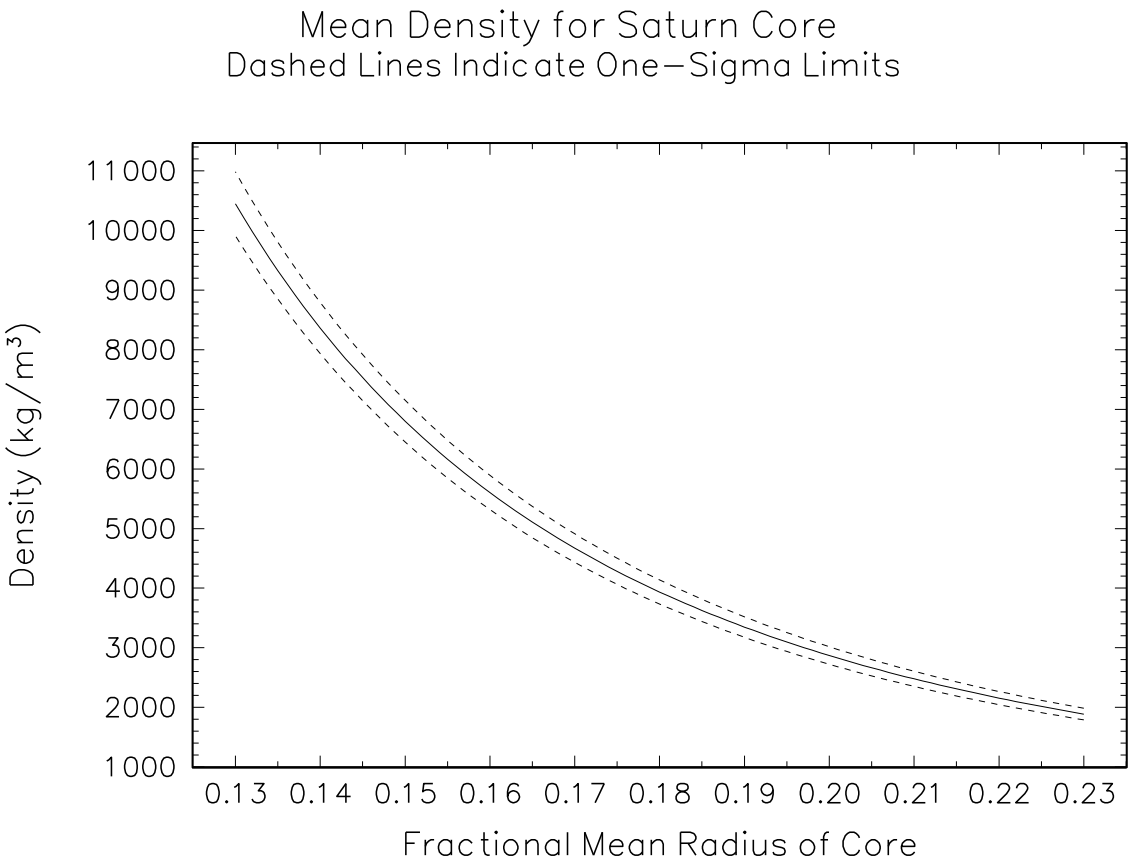
c. With core, central point mass is M_{core}/M_{total} = 0.03352 $\pm$ 0.00172

d. Hydrostatic integration to O(m), harmonics computed to $O(m^3)$


Jupiter (Red) and Saturn (Green)
Measured Harmonics and Theoretical Polytrope (n=1)
Log J_n
Log n
J_2
$-J_4$
J_6
Polytrope
One−Sigma Error on Measured Values
−1
−2
−3
−4
−5
−6
−7
0.3
0.4
0.5
0.6
0.7
0.8

Empirical EOS for Jupter (Red) and Saturn (Green)




Mean Density for Saturn Core
Dashed Lines Indicate One−Sigma Limits
Density (kg/m³)
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.20
0.21
0.22
0.23
Fractional Mean Radius of Core

Summary of Preliminary Results

- Updated data for gravity harmonics and total mass from Cassini gravity data, and rotation period from Cassini magnetic field data.

- Surface shape, mean radius, mean density, smallness parameter m computed to fifth order.

- Software developed for numerical hydrostatic equations to first order, computation of level-surfaces and J_2,J_4,J_6 to third order, and nonlinear least squares fitting procedure.

- Best-fit Saturn models with and without a core (best-fit core mass equal to about 3.2 Earth masses, core radius on the order of 0.18 R for core density of 4000 kg/m^3).