Atom-interferometer Gravity Gradiometer for Space and UGS Detection Applications

Nan Yu

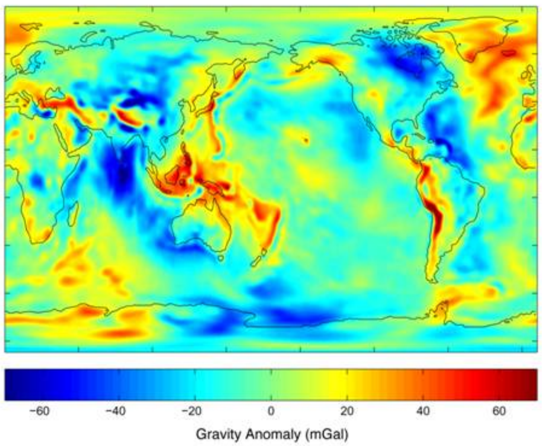
Jet Propulsion Laboratory

Outline

1) Gravity measurement and Applications
2) Atom interferometry (AI) introduction
3) Study for space and for underground structure (UGS) detection
4) JPL technology development for gravity measurement based on AI
5) Summary

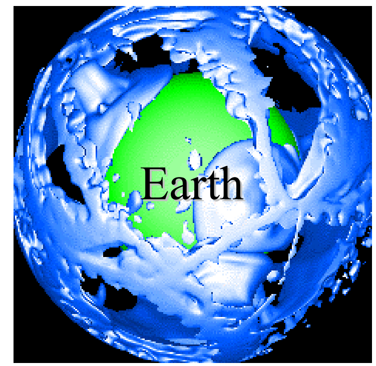
Atom Interferometer Inertial Sensors in Space

Earth Observatory for Climate Effects



Gravity anomalies from 111 days of GRACE data

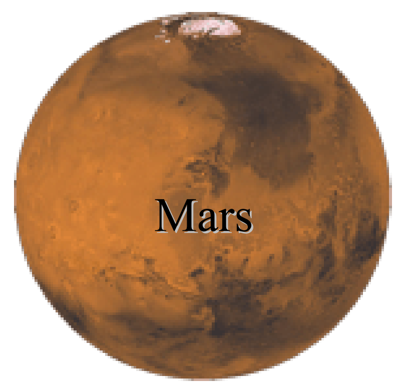
Solis Earth and planetary interior modeling



3-D simulation of compressible mantle convection

Science

Cs Rb $\Delta g = g_{Cs} - g_{Rb} = ?$



Mars gravity field mapping, Supporting Mars exploration.

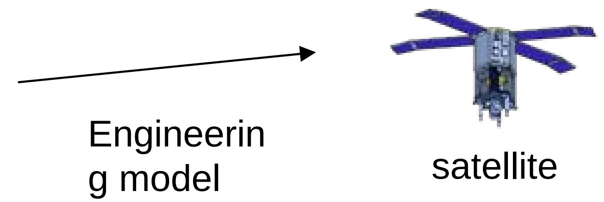
NASA ESD Technology Program Synergy

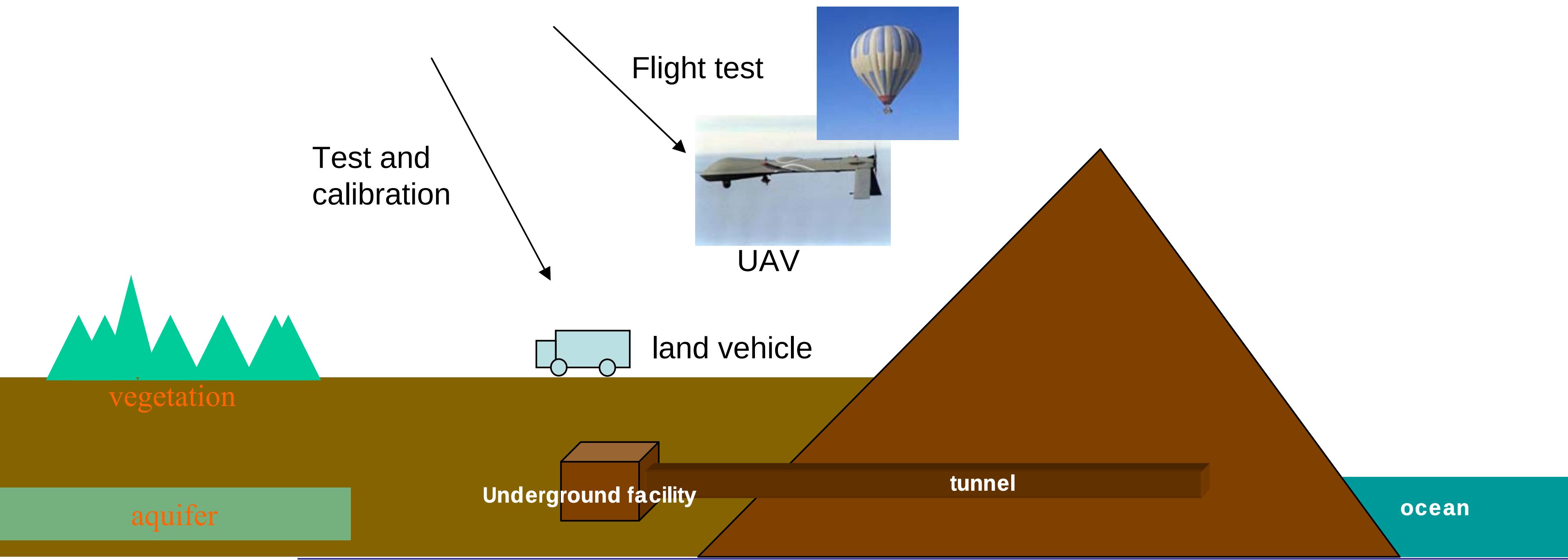
Completed: Advanced Technology Component - laboratory experiment and component technology.

Currrent: Instrument Incubator Program.

Objective: developing a portable gradiometer instrument prototype.

3 Year program.





Forward Estimation

Gravity potential

$$\phi(\vec{r}) = \sum_i G \frac{m_i}{\left|\vec{r} - \vec{r}_i\right|}$$

Gravity gradient **G**

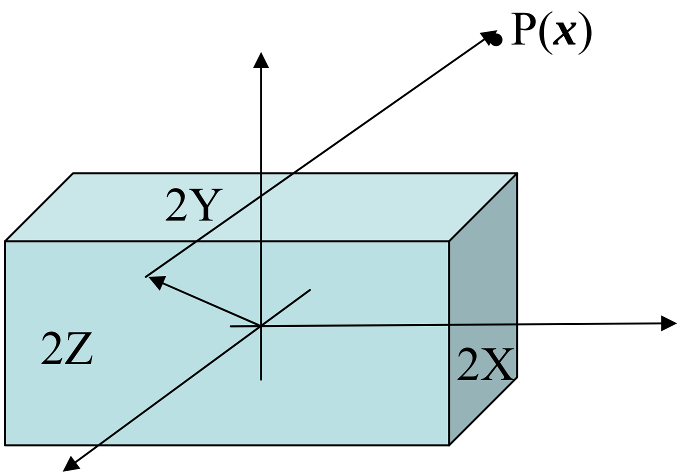
$$G_{ij} = \partial_j g_i = -\partial_{ij}^2 \phi(r)$$

Measurable directly by gradiometer

Forward problem: the gravity field $\Psi(r)$ produced by a mass distribution $m_i(r_i)$.

Inversion problem: obtain information about $m_i(r_i)$ from $\Psi(r)$.

Gravity Field of a Prism Mass



$$G_{ii}(\vec{x}) = -G\rho \int_{-Z}^{Z}\int_{-Y}^{Y}\int_{-X}^{X} \frac{[3(x_i - x_i')^2 - r^2]d^3\vec{x}'}{r^5}$$

$$G_{ij}(\vec{x}) = -G\rho \int_{-Z}^{Z}\int_{-Y}^{Y}\int_{-X}^{X} \frac{3(x_i - x_i')(x_j - x_j')d^3\vec{x}'}{r^5} \qquad (i \neq j)$$

$$\begin{aligned}
G_{ij}(\vec{x}) &= -G\rho \int_{-Z}^{Z}\int_{-Y}^{Y}\int_{-X}^{X} \frac{3(x_i - x_i')(x_j - x_j')d^3\vec{x}'}{r^5} \\
&= G\rho\big[K_{ij}(x_1 - X, x_2 - Y, x_3 - Z) - K_{ij}(x_1 + X, x_2 - Y, x_3 - Z) \\
&\quad - K_{ij}(x_1 - X, x_2 + Y, x_3 - Z) - K_{ij}(x_1 - X, x_2 - Y, x_3 + Z) \\
&\quad + K_{ij}(x_1 + X, x_2 + Y, x_3 - Z) + K_{ij}(x_1 + X, x_2 - Y, x_3 + Z) \\
&\quad + K_{ij}(x_1 - X, x_2 + Y, x_3 + Z) - K_{ij}(x_1 + X, x_2 + Y, x_3 + Z)\big]
\end{aligned}$$

where ($i \neq j$) and $K_{ij}(\vec{x}) = \ln(x_k + R(\vec{x}))$

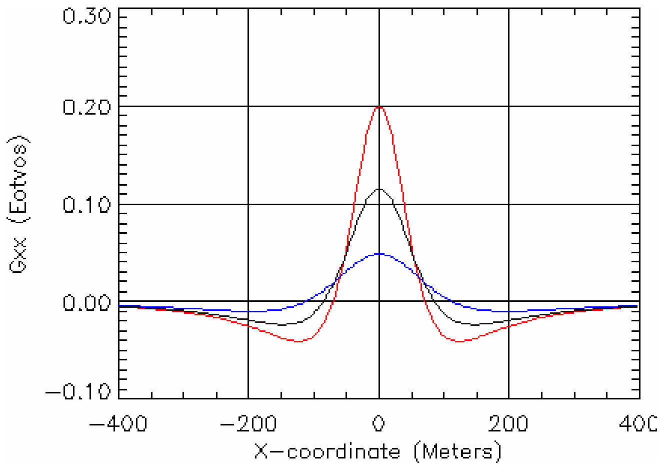
and $R(\vec{x}) = \sqrt{x_i^2 + x_j^2 + x_k^2}$

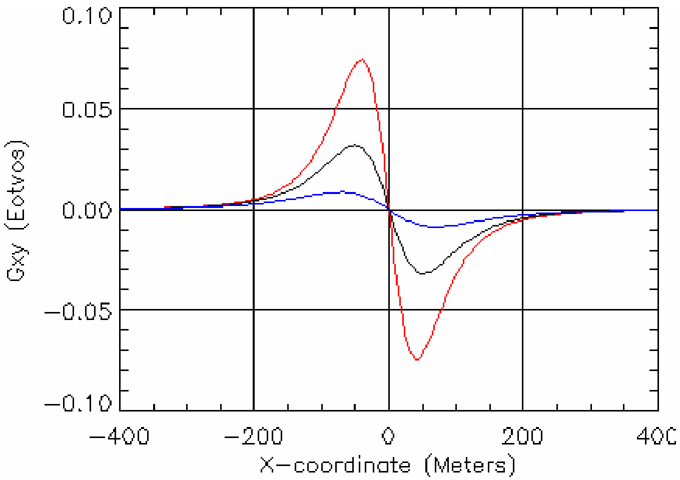
$$\begin{aligned}
G_{ii}(\vec{x}) &= -G\rho \int_{-Z}^{Z}\int_{-Y}^{Y}\int_{-X}^{X} \frac{[3(x_i - x_i')^2 - r^2]d^3\vec{x}'}{r^5} \\
&= G\rho\big[K_{ii}(x_1 - X, x_2 - Y, x_3 - Z) - K_{ii}(x_1 + X, x_2 - Y, x_3 - Z) \\
&\quad - K_{ii}(x_1 - X, x_2 + Y, x_3 - Z) - K_{ii}(x_1 - X, x_2 - Y, x_3 + Z) \\
&\quad + K_{ii}(x_1 + X, x_2 + Y, x_3 - Z) + K_{ii}(x_1 + X, x_2 - Y, x_3 + Z) \\
&\quad + K_{ii}(x_1 - X, x_2 + Y, x_3 + Z) - K_{ii}(x_1 + X, x_2 + Y, x_3 + Z)\big]
\end{aligned}$$

where $R(\vec{x}) = \sqrt{x_i^2 + x_j^2 + x_k^2}$ and

$$K_{ii}(\vec{x}) = -\arctan\left[\frac{x_j x_k}{x_i R}\right] + \frac{x_i x_j x_k R}{x_i^2 R^2 + x_j^2 x_k^2} + \frac{x_i^3 x_j x_k}{x_i^2 R^3 + x_j^2 x_k^2 R} + \frac{x_i x_j}{x_k R + R^2} + \frac{x_i x_k}{x_j R + R^2}$$

Gravity Gradient Plots





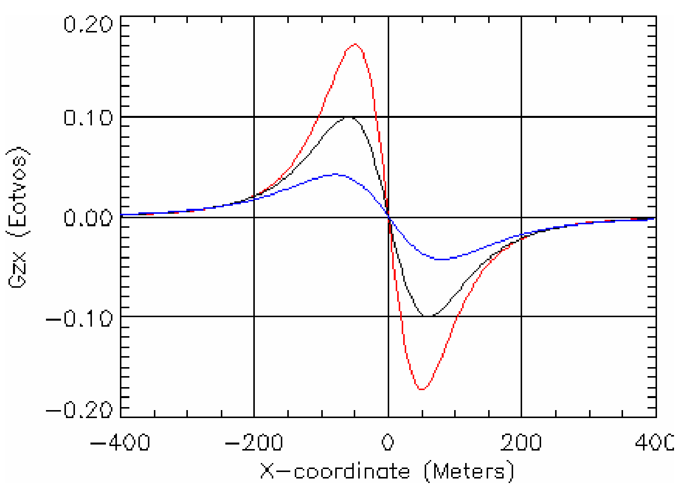
Volume:
(12x6x14) m^3

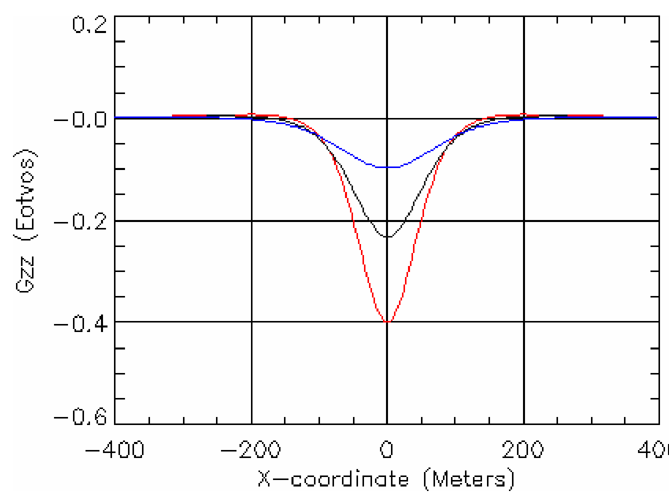
Heights:

-- 100 m

-- 120 m

-- 160 m.





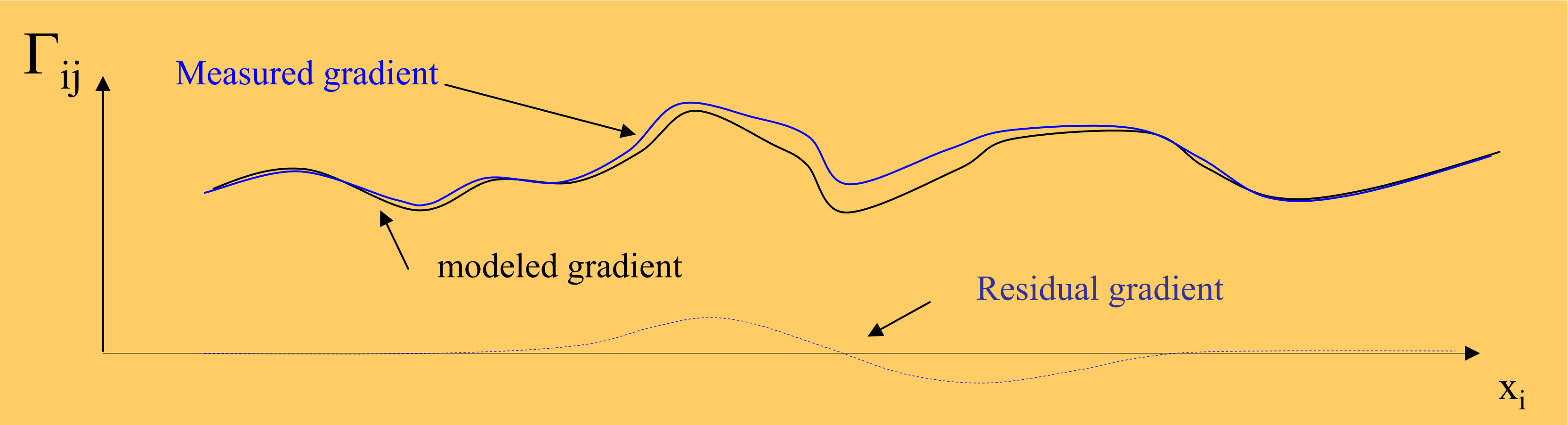
Gravity Gradient Field Property

- Since $G_{ij} \propto (r_0/z_0)^3$, the plot can be viewed as the normalized plot with height to linear target size ratio $R_{r0/z0}$ of 10. Of course, the overall signal strength falls off with $(1/z_0)^3$.
- As the detector is further away from the source, the signal is weaker and also more spread (over the distance ~ altitude z_0).
- The diagonal component G_{zz} provides twice the signal size as rest of the tensor components. But other components may be necessary to derive other information about the target or discriminate against unwanted noise.
- At $r_0/z_0 = 0.1$, small (<10%) difference between a prism and a sphere mass distributions.

Inversion Based on Local Gravity Anomaly

Assumed inversion strategy for the estimation

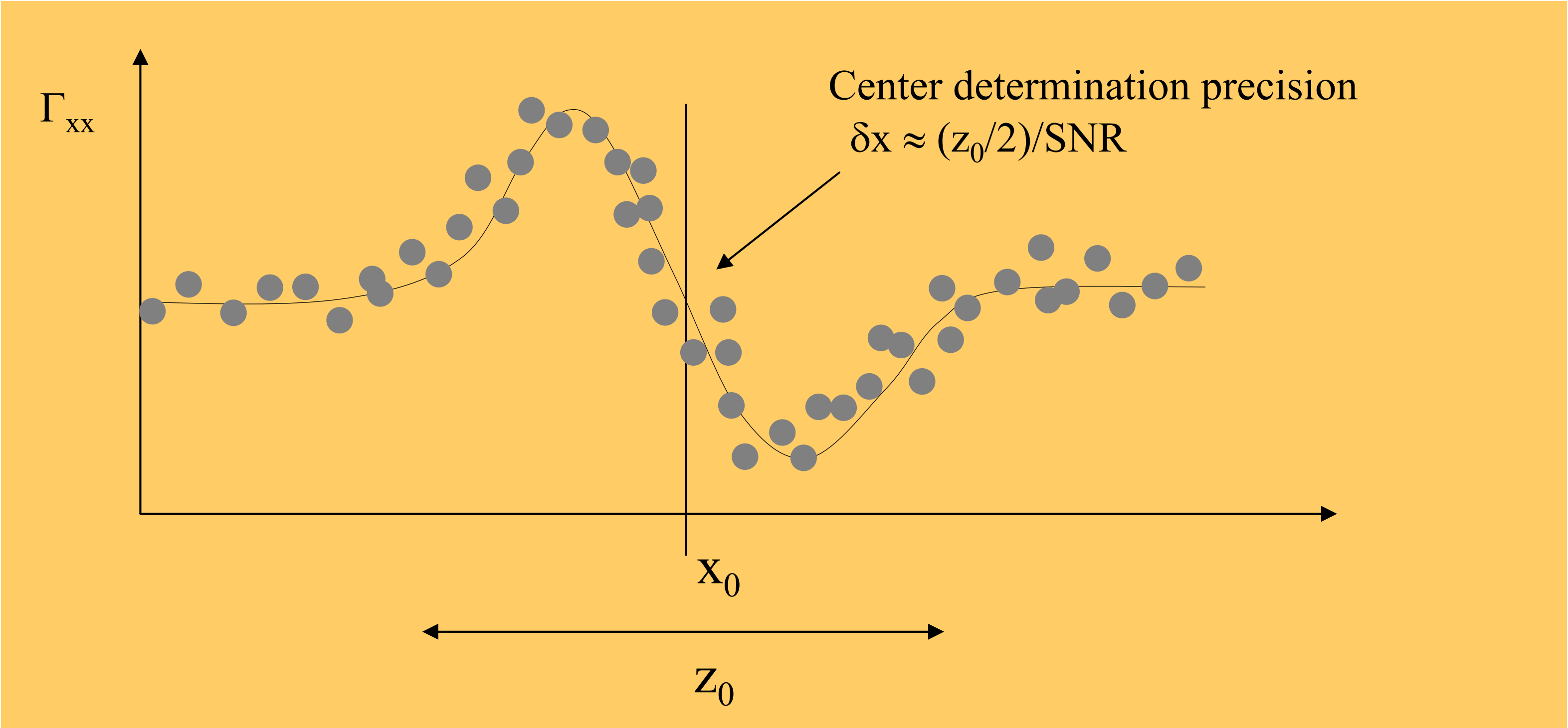
In order to estimate the required sensitivity, we assume that the inversion is done by taking the residual of what's measured from what's modeled from surface measurements and known local geology (i.e. mean density of the mass distribution underneath.



The residual is used to determine the unknowns underground. Much sophisticated inversion algorithms can be developed to extract the unknown from the residual. But to the 1st order, there must be enough detected residual signal that is comparable to the gradient amplitude due to the source of interest as a monopole.

Target Location Estimate

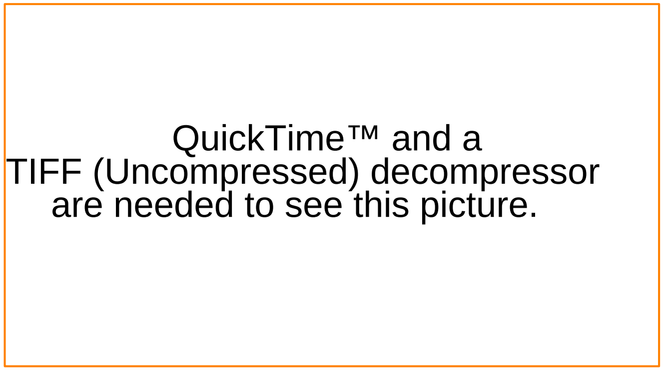
If one requires the knowledge of target location to $\delta x \approx (\delta\Gamma/\Gamma)z_0$, it must have the gradient resolution on the order of $\delta x/z_0$, where z_0 is the altitude or the gradient signal spread.



Note that the location estimation accuracy and the SNR of the measurement are also directly related to the resolving power of the different targets. When the data is sufficient in determining the single target to δx, it should be close in resolving the two targets separated by δx to the first order.

Location Positioning Resolution

To scale the detection with the criteria such that the location determination is the linear size of the target, then we have the condition: $\delta x = r_0 \approx (\delta\Gamma/\Gamma_e) z_0$, where Γ_e is the gradient near Earth surface. This results in:



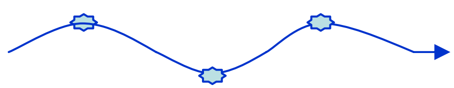
Because of the square root 4th power in the gradient sensitivity, the resolving power is a very slow function of the instrument sensitivity.

Use 0.1 EU as the instrument resolution at 100 m height,

$\delta\Gamma/\Gamma_e = 0.1/3300$, $r_0/z_0 = 0.074$, r_0 = 7 m.

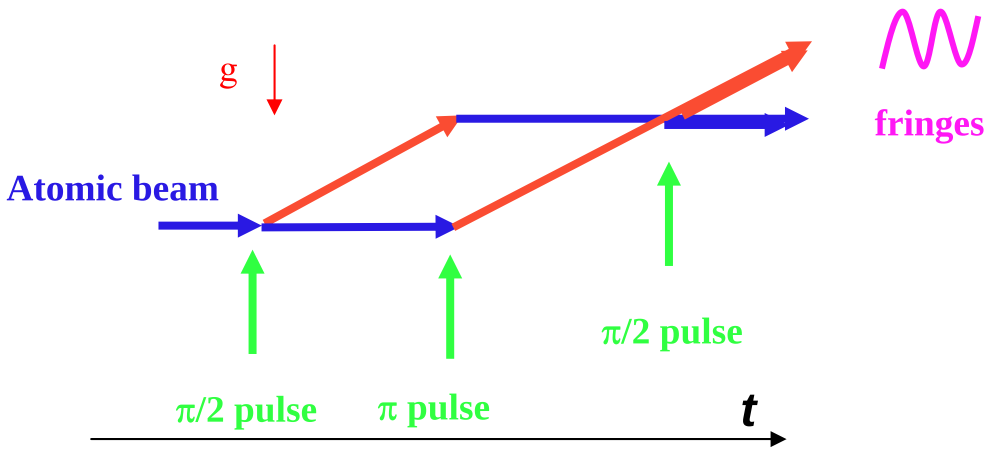
Atom Interferometer with Light Pulses

Quantum particle-wave duality



de Broglie wave: $\lambda_{dB}=h/mv$

Atom-wave Mach-Zehnder Interferometer



Splitter/mirror functions are accomplished by interaction with laser pulses.
(M. Kasevich and S. Chu *Phys. Rev. Lett*. vol. 67, p.181, 1991)

AI as an Accelerometer

No acceleration, total phase shift difference is $\Delta\Phi = 0$;

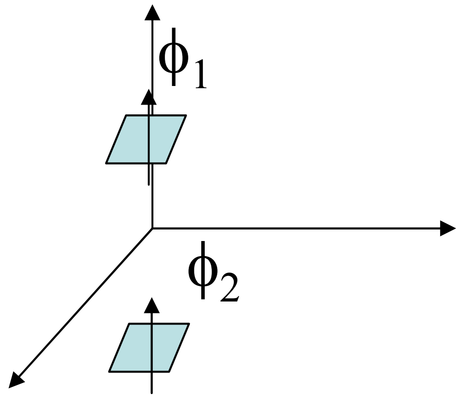
With an acceleration g, the phase difference is

$$\Delta\Phi = 2kgT^2$$

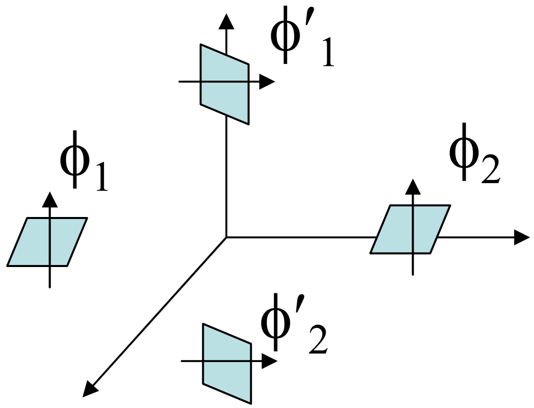
where k is the laser wavenumber and T the time interval between laser pulses.

1 rad. phase shift corresponds to $7\text{x}10^{-9} g/T^2$ acceleration

Gradient Extraction From Acceleration Measurements



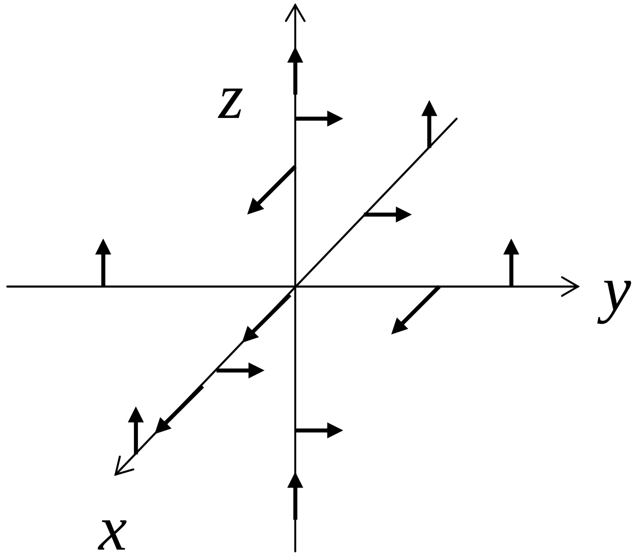
$$\Delta\varphi_1 - \Delta\varphi_2 = T^2 \vec{k}_{eff} \vec{\Gamma} \vec{\rho} - T^2 \vec{k}_{eff} (\vec{\Omega} \times \vec{\Omega} \times + \dot{\vec{\Omega}} \times) \vec{\rho}$$
$$+ (2m/\hbar) \vec{\Omega} (\vec{A}_1 - \vec{A}_2)$$
$$\Gamma'_{\alpha\beta} = (\Delta\varphi_1 - \Delta\varphi_2) / k_{eff\,\alpha} L_\beta$$



$$\Gamma_{xz} = \{\Gamma'_{xz}\} + \Omega_x \Omega_z$$
$$\Gamma_{xz} = \{\Gamma'_{xz}\} + \Omega_x \Omega_z$$
$$\Gamma_{xy} = \{\Gamma'_{xy}\} + \Omega_x \Omega_y$$
$$\Gamma_{xx} = \Gamma'_{xx} - (\Omega_y^2 + \Omega_z^2) = \Gamma'_{xx} - (\Omega^2 - \Omega_x^2)$$
$$\Gamma_{yy} = \Gamma'_{yy} - (\Omega_x^2 + \Omega_z^2) = \Gamma'_{yy} - (\Omega^2 - \Omega_y^2)$$
$$\Gamma_{zz} = \Gamma'_{zz} - (\Omega_x^2 + \Omega_y^2) = \Gamma'_{zz} - (\Omega^2 - \Omega_z^2)$$
$$\dot{\Omega}_x = [\Gamma'_{yz}], \quad \dot{\Omega}_y = -[\Gamma'_{xz}], \quad \dot{\Omega}_z = [\Gamma'_{xy}]$$

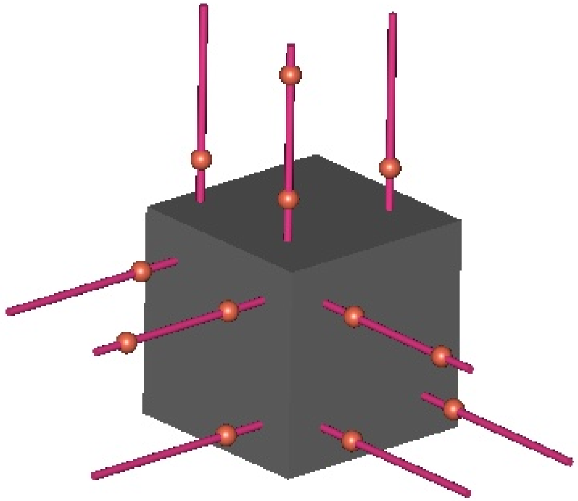
$$\{\Gamma'_{\alpha\beta}\} = (\Gamma'_{\alpha\beta} + \Gamma'_{\beta\alpha})/2 \qquad [\Gamma'_{\alpha\beta}] = (\Gamma'_{\alpha\beta} - \Gamma'_{\beta\alpha})/2$$

Full Tensor Configuration



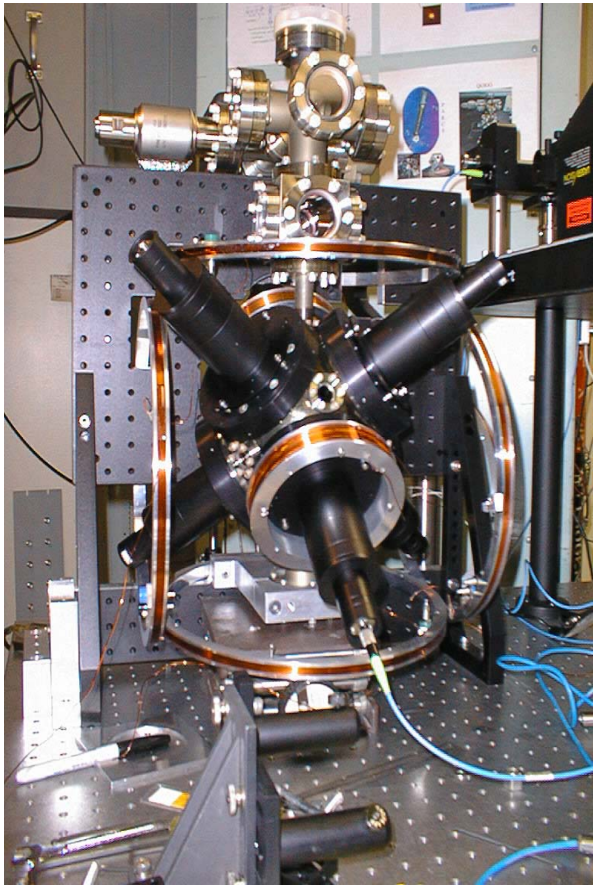
For a full tensor determination, as many as fourteen (14) interferometers may be necessary.

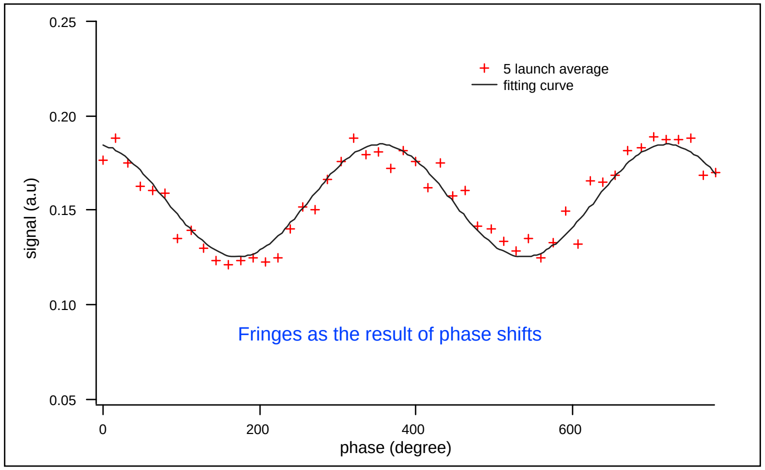
Possible implementation with 12 interferometers with simultaneous all-component acquisition.



Atom Interferometer Development at JPL

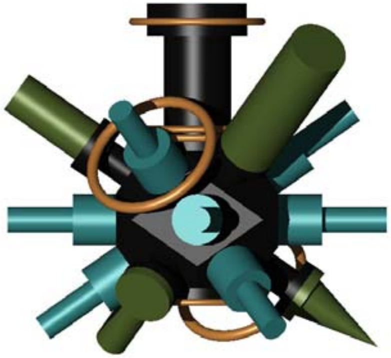
First JPL experimental setup



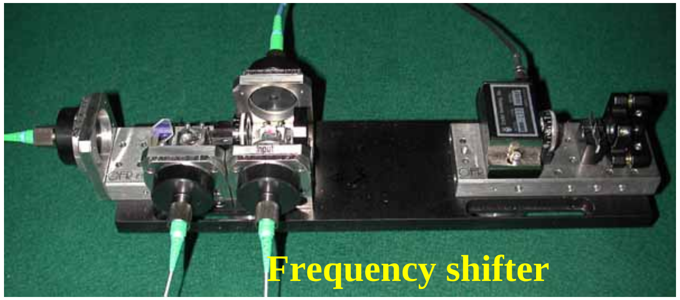


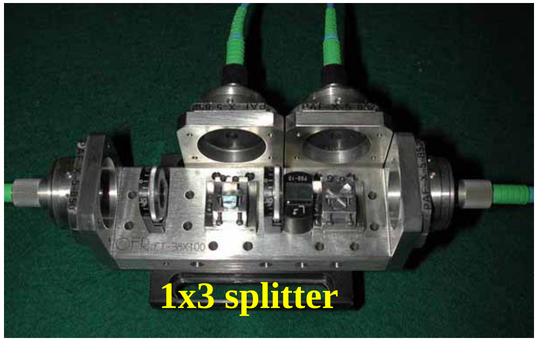
Observed atom interferometer fringe

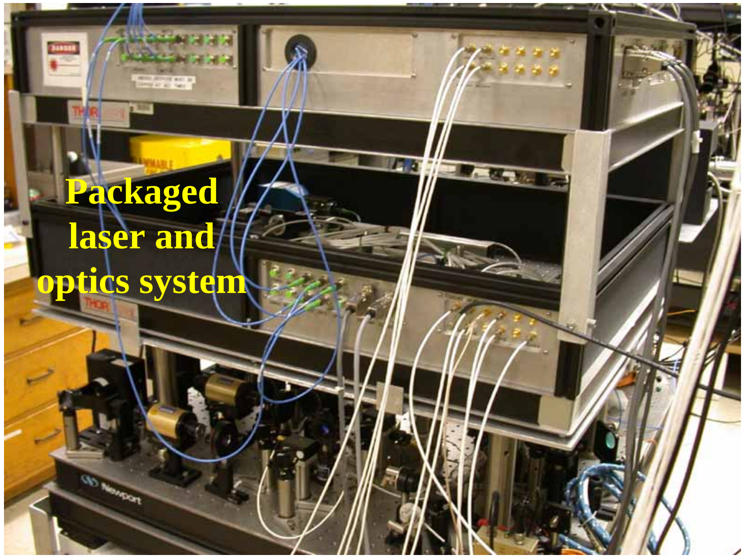
Model of laboratory atom interferometer physics package



Instrument Development: Laser System



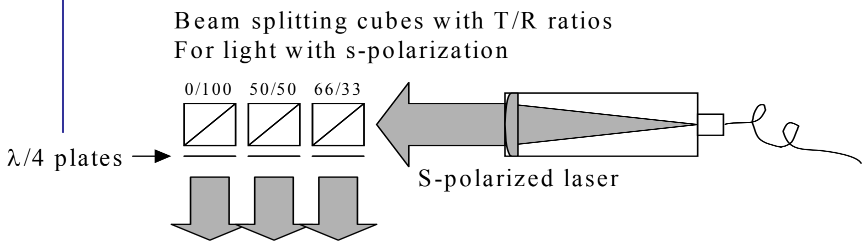


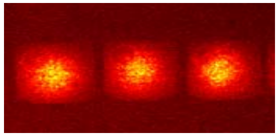


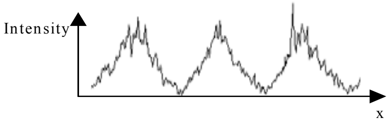
The laser system consists of functional modules interconnected with fiber optics

Instrument Development: Compact Cold atom source

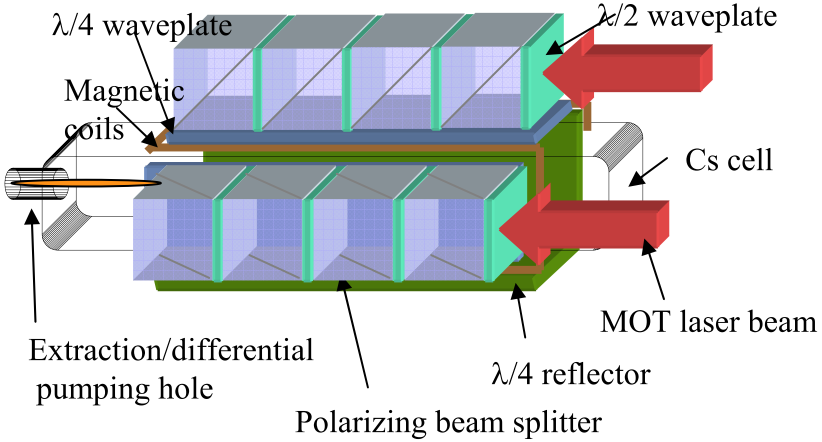
A multi stage 2D-MOT source

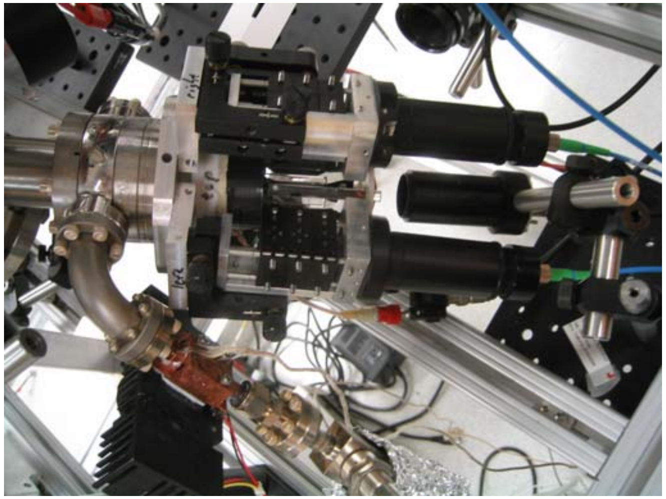




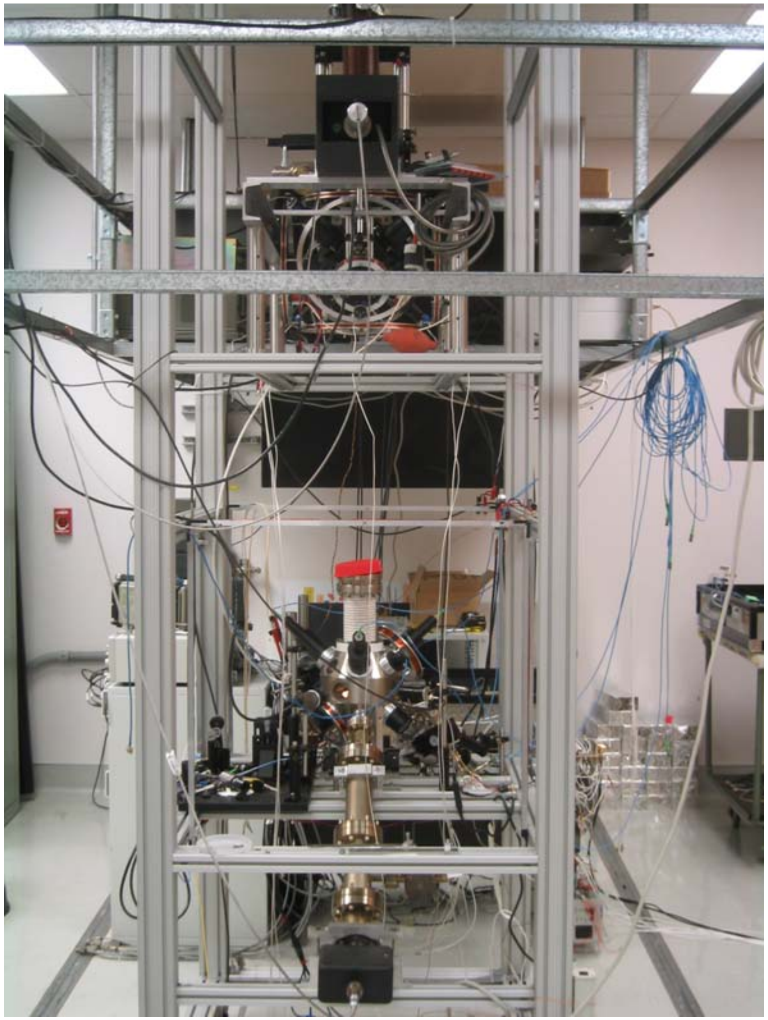


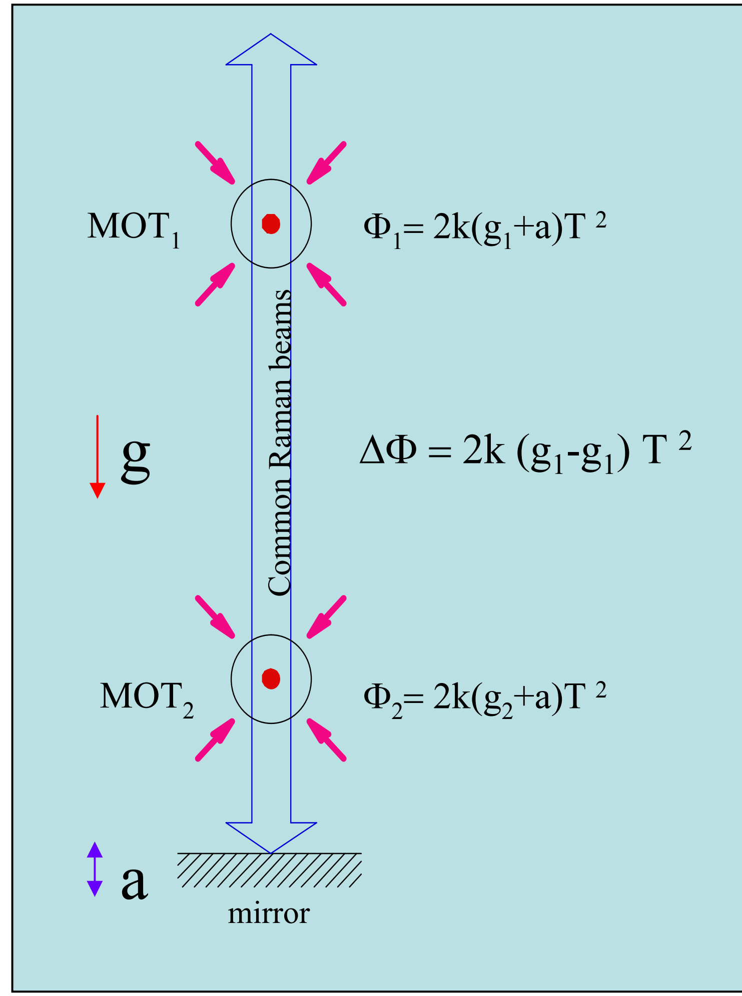
Compact size: 3 cm × 3 cm × 5 cm





Laboratory System





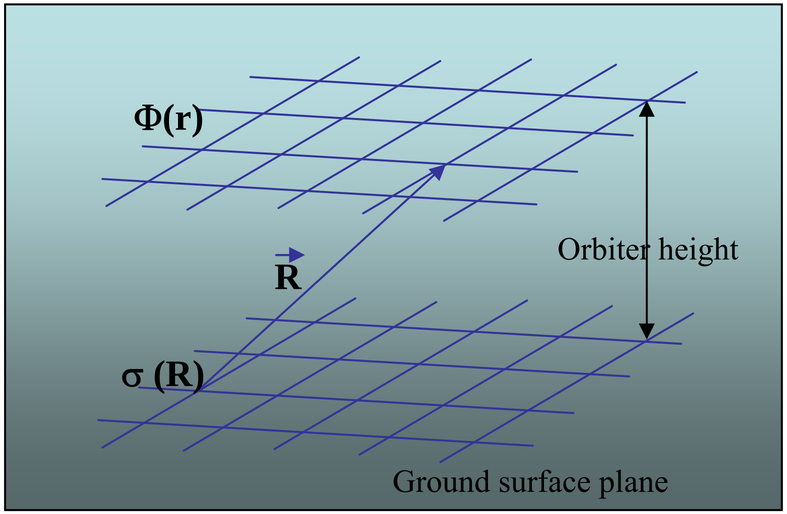
Planar Inversion Approach

The gravity gradient arising from a three-dimensional distribution of mass density takes the form

$$\Phi_{ij}(\vec{r}) = \int K_{ij}(\vec{r}-\vec{R})\,\rho(\vec{R})\,d^3\vec{R}$$

$$K_{ij}(\vec{r}) = G\frac{r_i r_j - r^2\delta_{ij}}{r^5}$$

One useful constraint on the inversion, which usually results in a well-posed problem, is to restrict the sources to a two-dimensional surface



$$\Phi_{ij}(\vec{r}) = \int K_{ij}(\vec{r}-\vec{R})\,\sigma(\vec{R})\,d^2\vec{R}$$

One formal approach to the inversion is to take the Fourier transform of each side.

$$\tilde{\Phi}_{ij}(\vec{k}) = \tilde{K}_{ij}(\overrightarrow{-k})\,\tilde{\sigma}(\vec{k})$$

Spectral deconvolution involves solving this equation for the Fourier transform of the source distribution:

$$\tilde{\sigma}(\vec{k}) = \frac{\tilde{\Phi}_{ij}(\vec{k})}{\tilde{K}_{ij}(\overrightarrow{-k})}$$

Source: Larry Roman, JPL

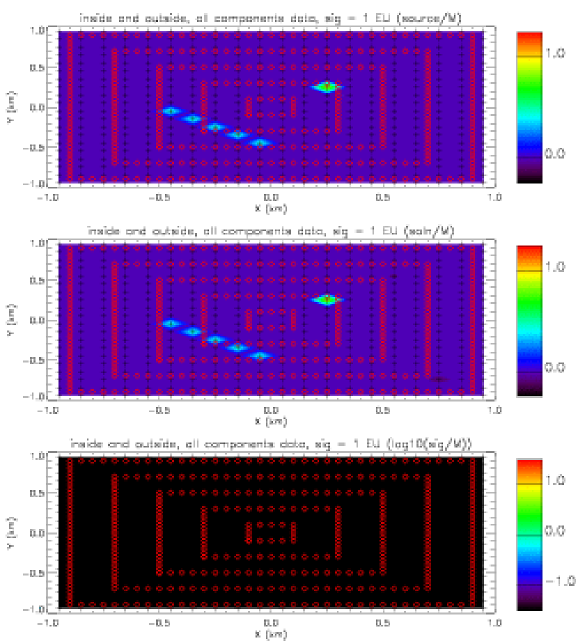
Ground measurements

Ground measurements inside and outside

Source scale = 10^4 m^3, 50m deep

All components, sigma = 1 Eotvos

(good recovery)

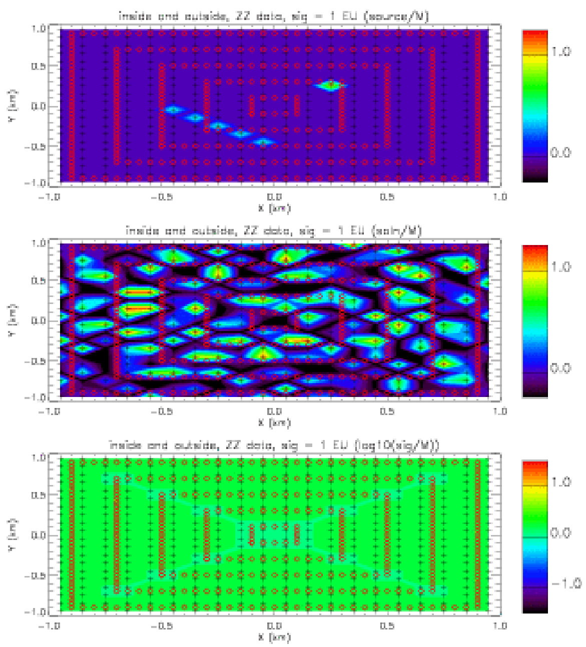


Ground measurements inside and outside

Source scale = 10^4 m^3, 50m deep

ZZ component, sigma = 1 Eotvos

(poor recovery)

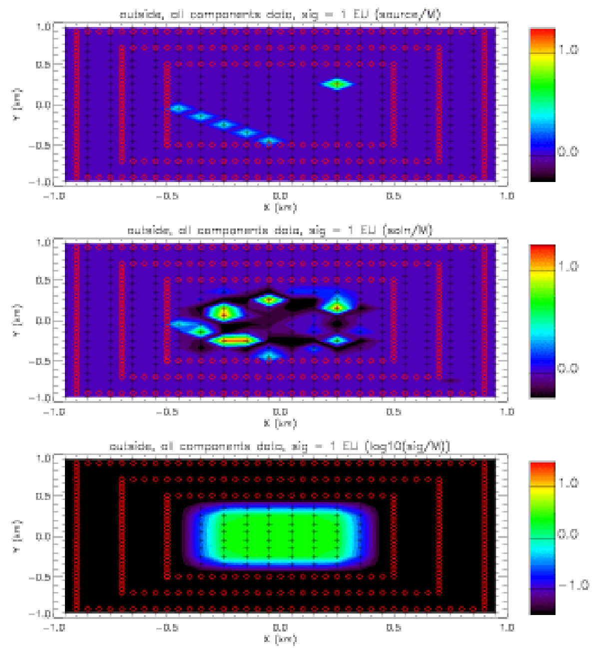


Ground measurements outside perimeter

Source scale = 10^4 m^3, 50m deep

All components, sigma = 1 Eotvos

(poor recovery inside)



Ground+Airborne measurements

Ground measurements outside + airborne at 0.25 km

Source scale = 10^4 m^3, 50m deep

All components, sigma = 1 Eotvos

(poor recovery inside)

Ground measurements outside + airborne at 0.25 km

Source scale = 10^4 m^3, 50m deep

ZZ component, sigma = 0.1 Eotvos

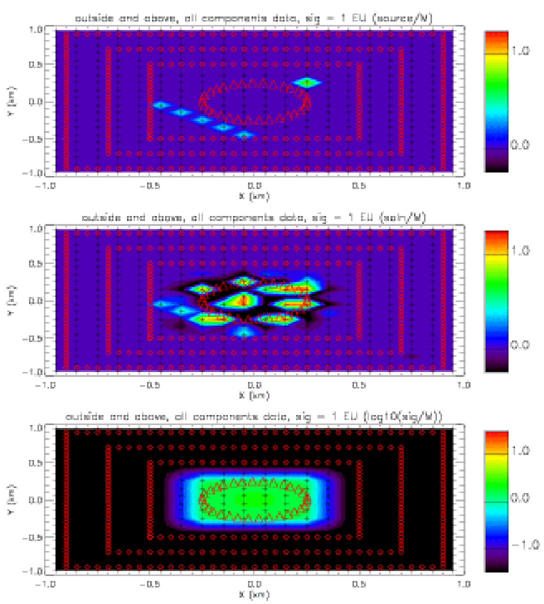
(somewhat better recovery)

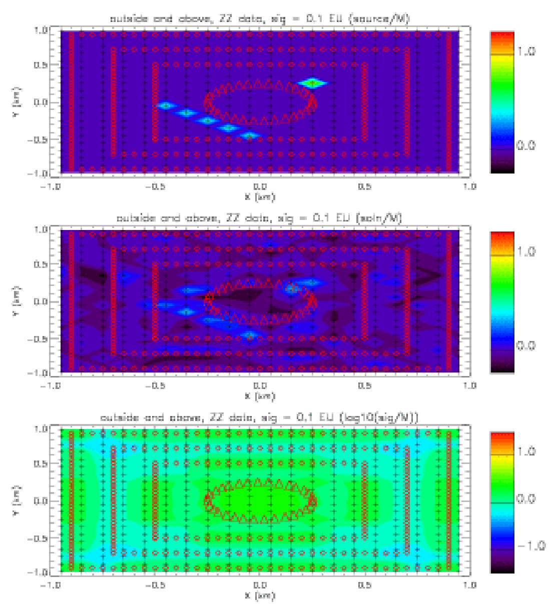
Ground measurements outside + airborne at 0.25 km

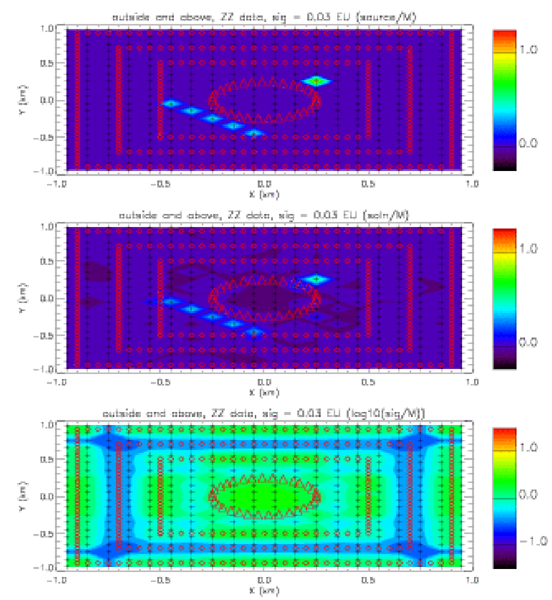
Source scale = 10^4 m^3, 50m deep

ZZ component, sigma = 0.03 Eotvos

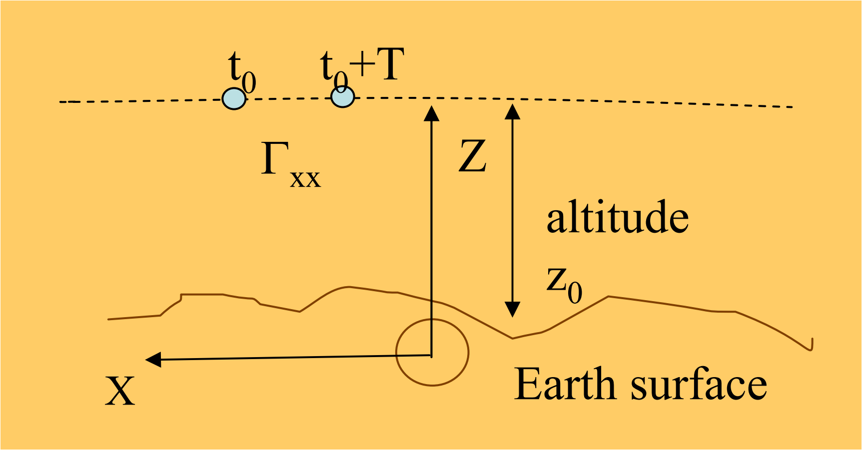
(good recovery)







Data Rate, Sensitivity and Spatial Resolution



The moving platform issue:

Platform speed at v.
Acceleration sensitivity $\Delta\phi_n/2kT^2$,
($\Delta\phi_n = \pi/SNR$).
Acceleration res. $\Delta g = \Delta\phi_n (T/\tau)^{1/2}/2kT^2$
(where τ is the total time at single measurement point).
Spatial resolution $\Delta x \propto vT$.

$$\Delta g \bullet \Delta x^{3/2} = \pi v^{3/2}/2k\,\tau^{1/2}(SNR)$$

Slow platform speed really helps!

Summary

• **Atom interferometer based inertial sensors hold great promises for applications in space and for UGS detection from land and airborne measurements.**

• **JPL is developing towards a cold-atom gravity gradiometer prototype. The instrument will be portable and designed for airborne test.**

• **Technology synergy and engineering heritage exist from other NASA cold atom space instrument development.**