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On April 16, 2005, the Cassini spacecraft performed its lowest altitude flyby of Titan to date, the Titan-5 flyby, flying 1027 km above the surface of Titan. This document discusses the development of a Titan atmospheric drag model for the purpose of the orbit determination of Cassini. Results will be presented for the Titan A flyby, the Titan-5 flyby as well as the most recent low altitude Titan flyby, Titan-7. Different solutions will be compared against OD performance in terms of the flyby B-plane parameters, spacecraft thrusting activity and drag estimates. These low altitude Titan flybys were an excellent opportunity to observe the effect of Titan's atmospheric drag on the orbit determination solution and results show that the drag was successfully modeled to provide accurate flyby solutions.

INTRODUCTION

Titan is the only known moon in the Solar System with a thick atmosphere, being 10 times denser than Earth's. And unlike Earth, a thick haze at 200 km above the surface obscures the entire satellite from view. Through ongoing observations from Earth as well as data collected by the Pioneer, Voyager and Cassini spacecraft, scientists now know that 95% of Titan's atmosphere is composed of nitrogen, while only 5% is composed of methane, cyanide and other hydrocarbons. One of the issues that scientists and engineers are facing when dealing with Titan's atmosphere is the variations in the densities observed from one encounter to another. In fact, for every close flyby so far, different density patterns were observed, Some examples are shown in Figure 1. Consequently, it has been decided to reassess the minimum safety altitude for the subsequent low-altitude Titan flybys, which was originally set to 950 km. It was decided recently to adopt a latitude-dependent density profile, although even longitudinal or time-variant models are being considered.

Due to the low altitude of these flybys, drag becomes an important perturbation which affects Cassini's trajectory. This document discusses the development of a Titan atmospheric drag model for orbit determination purposes and compares the OD solutions for different approaches taken during the actual navigation phase of the mission. The analysis is performed for 3 different flybys, Titan-A (10/26/2004), Titan-5 (04/16/2005) and Titan-7 (09/07/2005). For a more detailed analysis of the orbit determination results that covers

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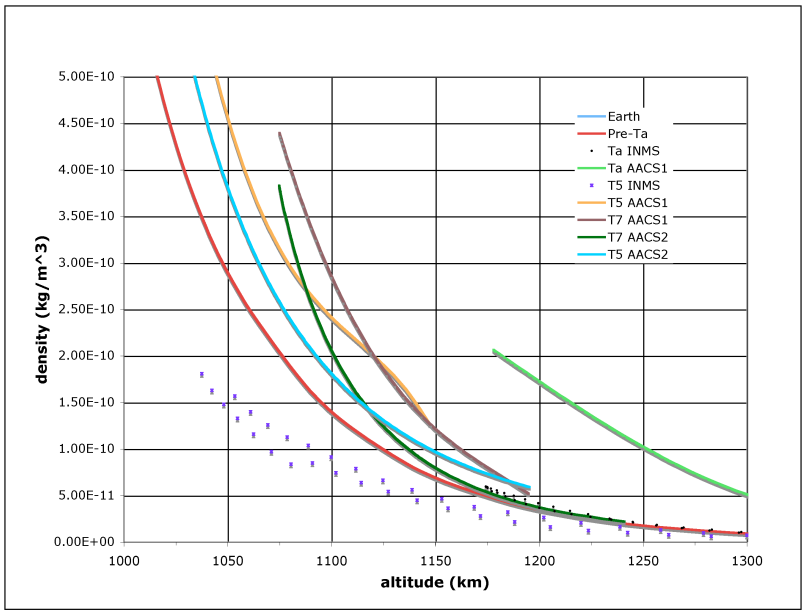


Figure 1 Various Density Profiles for Titan

these Titan flybys and provides the final reconstructed trajectory, the reader should refer to Antreasian *et al.* [1] , Roundhill *et al.* [2] , Stauch *et. al.* [3] , Stauch [4] and Ionasescu[5] .

DRAG INFLUENCE ON ORBIT DETERMINATION

On April 16, 2005, Cassini performed its lowest altitude flyby of Titan to date, the Titan-5 flyby (T5), flying at an altitude of 1027 km above the surface of Titan. Not only was it an excellent opportunity to observe the effect of Titan's atmospheric drag on the OD solution, it was found early on that the drag needed to be modeled to get an accurate OD solution. After processing the first few passes of tracking data after T5, the OD team discovered that small perturbations had to be taken into account in order to correctly model the spaecraft dynamics during the encounter. This was essential to get an accurate prediction of the trajectory ahead to design the maneuvers that targeted the next encounter with Enceladus in July. The orbit of Cassini from T5 to Enceladus is shown in Figure **??** together with a close up of the activity at T5. Picture to come

Figure 2 shows a representation of the accelerations (magnitudes) of the various perturbations that Cassini was subject to during the Titan 5 flyby. During this flyby, Titan's gravitational force grew to an order of magnitude larger than Saturn's, making it the predominant force acting on the spacecraft. It needs to be pointed out that no tracking data was acquired during any of the flybys discuss in this study. Cassini was pointed away from Earth to collect science data. This implies that small perturbations, such as drag and J2, cannot be directly evaluated and attributed to signatures in the Doppler data. The drag, however, is the largest of those perturbation and therefore the only one considered here. The J2 acceleration was not included in the model and is displayed for reference only. It

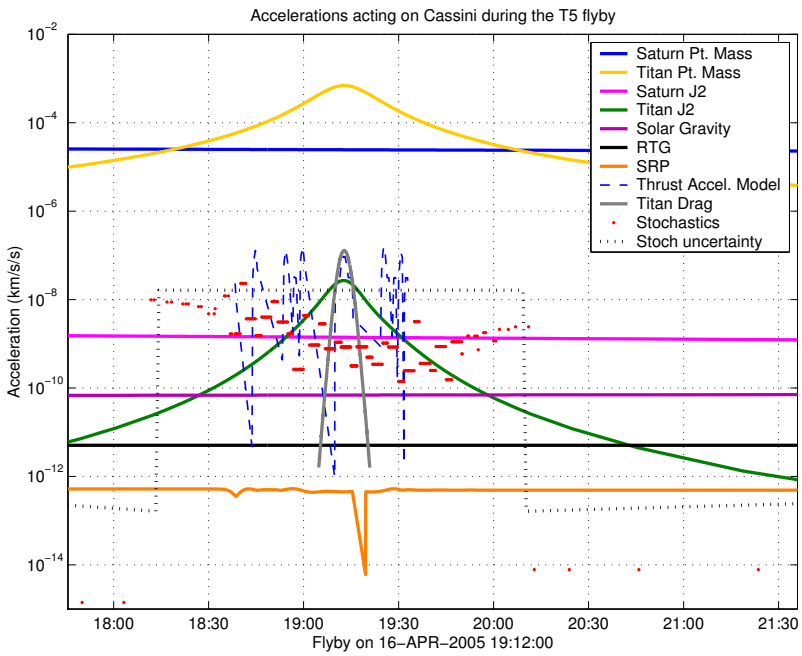


Figure 2 Accelerations acting on Cassini during the Titan 5 Flyby (from various in situ data)

represents a reasonable approximation based on geophysical shape assumptions of Titan[6] . Attempts to estimate J2 were not conclusive due to the fact that the spacecraft was not tracked during the encounter. The dotted blue line represents a thrusting acceleration sequence of considerable magnitude that was applied to Cassini during the flyby. This trusting activity is mainly attributed to a radar mapping experiment and was controlled by the Reaction Control System (RCS) thrusters. Note that the model shown is based on telemetry data and does not include the transitions with the reaction wheel control that occurred prior and after*. During the days that followed the T5 encounter, the OD team tried many models to best match the flyby dynamics. For instance, it was found that the pre-flyby prediction for the thrusting activities differed by approximately 30% with the telemetry model. This difference was mainly attributed to the fact that a higher density was used in the prediction than what was observed. For that reason, it was chosen to base our RCS model on the telemetry data rather than the predictions. Figure 3 shows the raw telemetry data for the RCS thrusting activity in the J2000 inertial coordinate system. The data is expressed in terms of an accumulative ΔV over time and is plotted against the predicted model (thin solid lines). From the telemetry points in Figure 3, we can see that Cassini experienced a series of periodic thrusting events that are quite distinct from each other. It was thus possible to approximate the whole sequence with a series of small impulsive forces occurring where the ΔV curve jumps. A time-line of the T5 flyby, including details on the small forces is shown in Table 1.

In the days that followed T5, two major observations were made. First, the orbit solution mapped to Enceladus was indeed very sensitive to any small change at T5 and second,

*This is why the stochastic uncertainties were increased from approximately 18:10 to 20:10 as shown in Figure 2

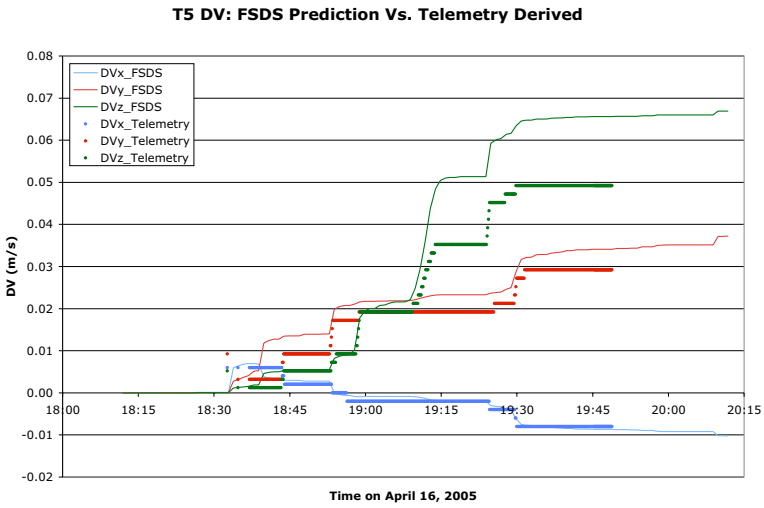


Figure 3 RCS Delta-V accumulation during the Titan 5 Flyby

Table 1 TITAN-5 ENCOUNTER TIMELINE (APRIL 16, 2005)

Event	Time (ET)	ΔV (mm/s)
Last Tracking Point	06:14	
Transition to RCS	18:11:59	4.472
Small Force	18:34:45	7.211
Small Force	18:43:41	8.246
Small Force	18:55:59	17.66
Small Force	19:11:49	16.00
Titan Periapsis	19:12:50	
Small Force	19:26:29	18.22
Transition to Wheels	19:49:20	4.898
Tracking Resumes	15:18	

when trying to estimate the small forces in Table 1, some of the ΔVs came out negative (non-physical estimates). Further consideration led us to believe that the drag, so far not modeled, had to be taken into account[†]. To resolve the problem rapidly, as operation dictates, it was chosen to include a small force at Titan periapsis in the opposite direction of the velocity vector and let the filter estimate it. The initial conditions for this force were chosen to be 19.14 ± 10 mm/s, based on a preliminary drag model. This approach has proven successful as all the other forces turned out positive and the solution became somewhat stable at Enceladus between our different models (The drag force has also been included in the RCS acceleration model runs).

[†]Because we will never be able to precisely model all the perturbations acting on a spacecraft, we will always have error in our solution. For this reason, our OD filter constantly estimates time-variant stochastic accelerations with given uncertainties. Those uncertainties are increased during a flyby since the modeling errors are more significant. In the previous flybys, the high level of uncertainty in Titan's ephemeris caused the drag and other mismodeling to be absorbed by the stochastic accelerations and this seemed to work quite well at that time. Prior to T5, the lowest flyby of Titan was Ta, where Cassini flew at 1200 km above the surface.

For this study, however, the impulsive forces were held fixed since there is no tracking data to help the estimate and because the telemetry is what represents the RCS activity best. The transition ΔV (events from reaction wheel to RCS attitude control an back) were estimated though because telemetry was not available for those times. An acceleration model that represents the thrusting activity instead of the discrete ΔV RCS model was also derived to produce a different solution for comparison. To account for mismodeling in both models, a white noise stochastic acceleration model was introduced with 5-minute batches using an uncertainty of 1.625×10^{-08} km/s^2 in the thrust-pointing direction[‡].

The Titan-5 encounter was the first time that the effect of drag had to be taken into consideration for orbit determination. Considering the number of low-Titan flybys to come and that it is likely that the drag force will continue to affect the OD performance, effort has been put into developing an atmospheric drag model to be included in the OD software. In addition to T5, the drag model was used to study two other past Titan flybys, Titan-A (1175 km) and Titan-7 (1075 km), which will be discussed later.

DRAG MODEL

The method chosen to model Titan's drag uses a single drag coefficient and computes the effective area of each spacecraft component in the drag direction. The resulting acceleration is given by

$$a_D = -\frac{\rho V^2}{2m} C_D \Sigma A_i \hat{V} \tag{1}$$

where

ρ	is the atmospheric density.
V	is the magnitude of the Titan body-fixed spacecraft velocity.
m	is the spacecraft mass.
C_D	is the coefficient of drag.
A_i	is the effective cross sectional Area of the spacecraft component i in the direction of the body-fixed spacecraft velocity vector.
$\hat{V}$	is the direction of the Titan body-fixed spacecraft velocity.

Because the density at Titan is highly uncertain and is likely to stay this way, the logical approach is to approximate it with an exponential model as a function of altitude and allow one parameter in the drag equation to be estimated. The density table used for this study was built from various data sources recorded before the Titan A flyby and correspond to the "Pre-Ta" curve in Figure 1.

The effective total cross sectional area ΣA_i is computed in the drag direction for each spacecraft component, where the computation of each A_i depends on the component's shape and orientation. Cassini was divided in eight components for the purpose of this study. The

[‡]The estimated stochastic accelerations and *a priori* uncertainties are also shown in Figure 2.

Table 2 CASSINI COMPONENTS

	Component	Model	Dimensions
1	High gain antenna	Antenna	Depth 0.760 m; Radius 2.007 m
2	Top of s/c bus	Cylinder	Length 3.023 m; Radius 1.397 m
3	Bottom of s/c bus	Cylinder	Length 0.635 m; Radius 1.397 m
4	Four thruster clusters beyond HGA	Flat Plate	Area 0.18 m^2
5	RPWS antenna 1	Cylinder	Effec. length 8.6 m; Radius 0.0143 m
6	RPWS antenna 2	Cylinder	Effec. length 8.5 m; Radius 0.0143 m
7	RPWS antenna 3	Cylinder	Effec. length 8.5 m; Radius 0.0143 m
8	Magnetometer boom	Cylinder	Length 9.627 m; Radius 0.140 m

coefficient of drag C_D was the only drag parameter estimated in this study. Its *a priori* value was set at 2.1. It is important to note that it is impossible to get an estimate of any given parameter in the drag equation unless we believe the others are right. This is not possible here because the coefficient of drag, the cross sectional area and the atmospheric density are all unknowns. The cross sectional areas are an approximation of a complicated shape and are dependent on an accurate reconstruction of the spacecraft attitude during the flyby since only the area with respect to the velocity vector is of interest. As for the C_D, 2.1 is an approximation and there simply isn't any data available that can predict what the exact value could be, given the complexity of the Cassini spacecraft and the air flow that it is subject to. Further studies might help determine better what the exact cross sectional area was during any given flyby, but until we get a better understanding of the density profile at Titan, there is little we can do to distinguish between C_D, A_i and ρ with OD. What is important, however, is that the overall drag acceleration defined in Equation 1 and shown in Figure 2 can be reasonably well determined and the estimated parameter, in this case C_D, becomes a scale factor. The drag acceleration estimates are reported using a drag scale factor determined from the estimated C_D normalized by its nominal value (2.1).

FLYBY RESULTS

This section will present OD results and drag estimates for the T5 and T7 flybys. The TA flyby is also reviewed when the drag is taken into account using a new satellite model. Solutions compared in the encounter target B-plane[§] and their corresponding drag ΔV estimates will be presented. For each orbit, the deep space network in Goldstone, CA, Madrid, Spain and Canberra, Australia, tracked Cassini and provided coherent X-band radio metric data for about 6 hours per day for orbit determination. The tracking data used for OD consist of two-way range and Doppler data and optical navigation (opnav) data. the Doppler data were generally compressed to 5-minute points, except during Orbit Trim Maneuvers (OTM), where the Doppler data were compressed to 60 seconds. During operations, it was observed that the drag estimate was very much dependent on the amount

[§]A description of the B-plane is provided in Appendix.

of data processed after the flyby. Figure 4 shows an example of T7 where the ΔV solution improves as more data is added, which resolves to a smaller drag ΔV than initially estimated.

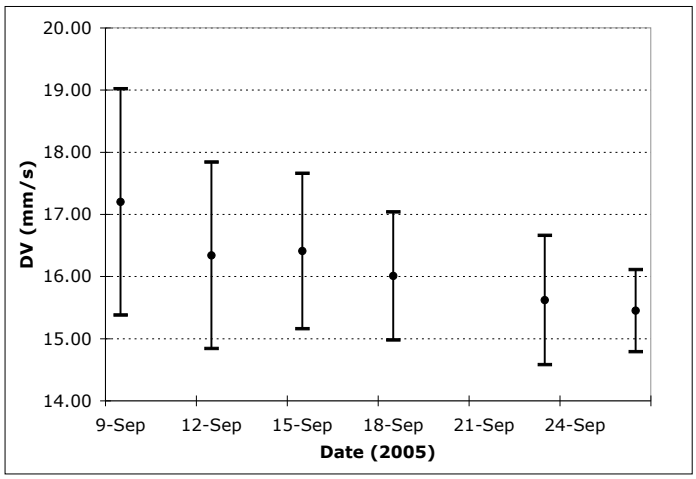


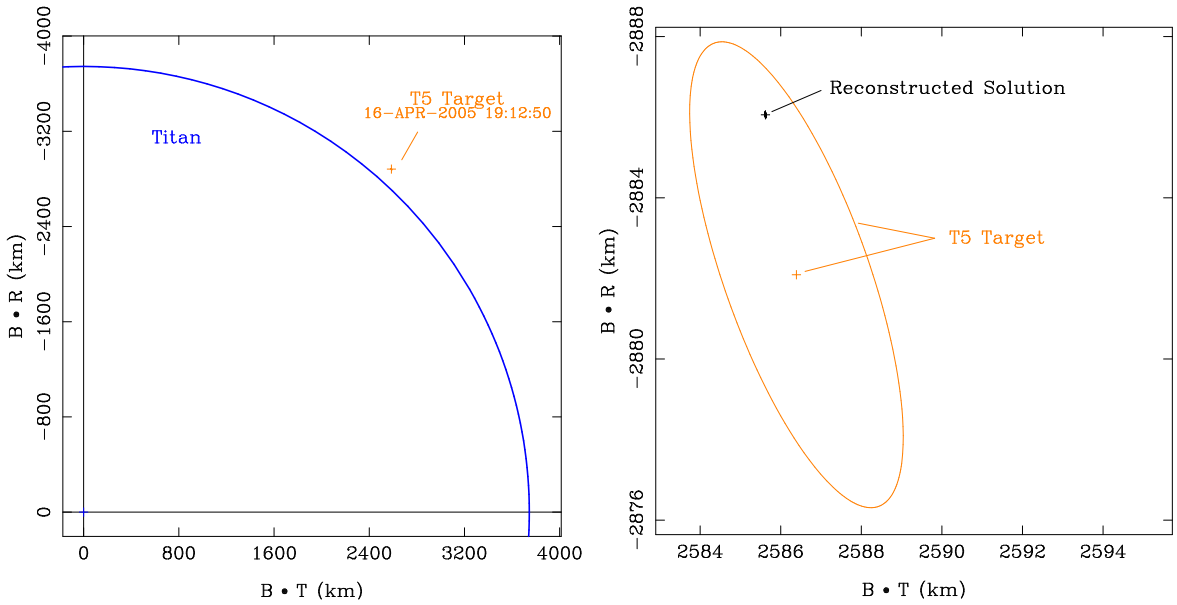
Figure 4 T7 Drag ΔV Estimate History as a Function of the Data Cut-off

Titan-5

The Titan-5 flyby occurred on April 16, 2005 at approximately 19:12 UTC. The data arc evaluated for this study contains two OTMs. OTM-21 was executed on April 10 near the orbit apoapsis and was about 5.86 m/s while OTM-22, executed on April 14, was about 64.8 mm/s. Both OTMs were used in order to correctly target Cassini's trajectory to the T5 encounter at 1025 km altitude above Titan's surface. The data arc starts on April 07, 2005 at 07:00 and stops on April 28 at 05:11, right before the next scheduled maneuver, OTM-24[¶]. To properly assess the impact of the drag modeling, Cassini's trajectory will be compared at the Titan-5 B-plane periapsis and also at the next targeted encounter, Enceladus-2 (E2), since most of the errors at T5 will be amplified at E2. E2, was Cassini's second targeted visit to Enceladus on July 14, 2005.

The next Figures 5 to 8 show various solutions mapped to the T5 or E2 periapsis. Each solution is shown on the B-planes with the corresponding two-dimensional one-sigma uncertainty ellipses. Figure 5 shows the Titan-5 B-plane. The solution labeled "T5 Target" corresponds to the targeted aim point on the B-plane at 2586.4 km in $B \cdot T$ and -2882.1 km in $B \cdot R$ (Titan is at 0,0) for a flyby on April 16, 2005 at 19:12:50 Spacecraft Event Time. On the right is shown the maneuver dispersion ellipse for the OTM-22 design on April 12. The final reconstructed solution is marked at about 4 km above the targeted point. Note

[¶]OTM-23 scheduled for April 20 was canceled due to a reevaluation of the maneuver requirements through an optimization technique.



(a) Target wrt Titan

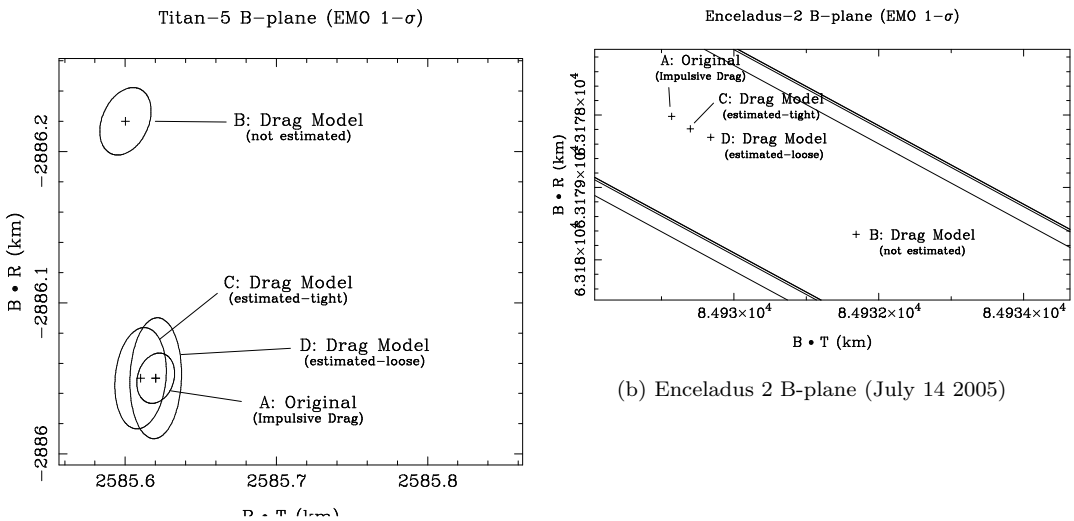
(b) Flyby Solution wrt Target

Figure 5 Titan 5 Target & Flyby B-plane Solutions

that the reconstructed uncertainty compared to the pre-flyby solution is too small to show on this scale. Figures 6(a) and 6(b) show the effect of changing the drag modeling on both the T5 and E2 B-planes from the unique small force to the more complete drag acceleration model as described on page 5. The solution marked "A: Original" used the original impulsive force for the drag. Solution "B" was obtained when the drag acceleration model replaced the estimated small drag force, although nothing was estimated. The filter was then allowed to estimate the C_D (solution "C"), which brought back the solution close to the expected value. It was observed that the C_D's *a priori* sigma was too tight in the "C" solution (set to 0.5) and when loosened up to 1.5, the new "D" solution coincides almost perfectly with the original case at T5, although with a larger uncertainty. It is important to note that the initial condition used for the drag small force case is dependent on a preliminary assessment of the drag acceleration profile at T5 and thus it was expected that both the A and D solutions agree with each other. Choosing the acceleration modeling approach is nonetheless much more robust since the small force method (A) is dependent on the drag acceleration model in the first place.

As discussed on page 4, the RCS events at T5 were modeled using the telemetry data with two different models. A comparison is presented in Figures 7 & 8 and Table 3. They show the differences between the following four possible cases:

- Case 1 is the original baseline solution, using impulsive small forces to model the RCS events at T5. The Drag force is estimated as an impulsive ΔV.
- Case 2 is similar to case 1 except that it uses the drag acceleration model. The drag is estimated through the C_D.



(a) T5 B-plane

(b) Enceladus 2 B-plane (July 14 2005)

Figure 6 Effect of Drag modeling on the B-planes for T5 and E2

- Case 3 uses an acceleration model for the RCS events at T5 and an impulsive ΔV to model the drag. The drag ΔV is estimated.
- Case 4 uses both the RCS acceleration model and the drag acceleration model. The drag is estimated through the C_D.

Note that the RCS thrust models were not estimated. This makes sense because no tracking data was available during the flyby to estimate the forces individually and the resolution of the telemetry data was 2 mm/s. Any mismodeling is taken into account via the stochastic accelerations.

The effect of considering the drag in the model can be seen in the B-planes solutions in Figures 7 and 8. The differences between solutions with and without drag are approximately 200 m at Titan and 150 km at Enceladus. From Figure 7, one can see that all the drag solutions agree within less than 40 m of each other at T5 and the four nominal cases are all inside a one-sigma error, which is encouraging. The same is true for the predictions at Enceladus-2. In Figure 8, the cases 1 to 4 solutions are within 500 m, which is very small considering that Cassini had more than 4 orbits to go at Saturn before the encounter in mid July. Also shown on the plots is an updated solution to case 4, which uses an updated Titan density profile based on the T5 science results (see the T5 curves in Figure 1). This case represents the best reconstruction of the flyby, but since the densities are only available well after the fact, we cannot expect to achieve that accuracy during operations.

The most interesting results are seen in Table 3, where an estimate of the drag force is shown in terms of ΔV together with the estimated drag scale factor, when applicable. The ΔV estimates are also compared graphically in Figure 9, where the estimated drag accelerations (cases 1 & 2) were integrated. All the cases agree with each other in the one-sigma

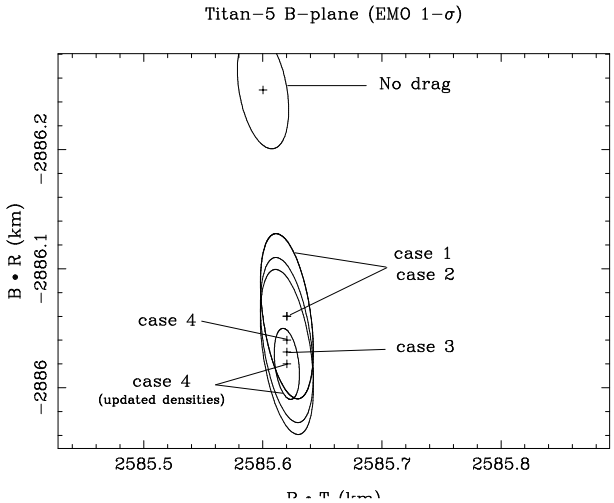


Figure 7 Titan 5 B-plane Final Solutions

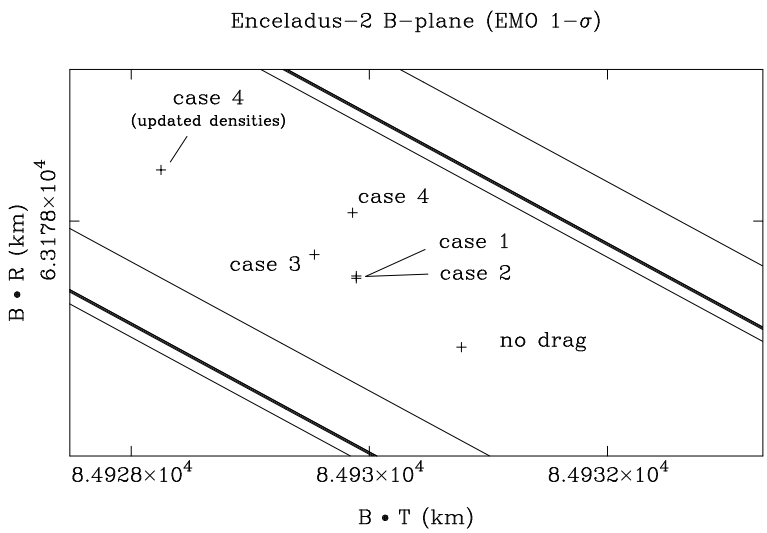


Figure 8 Enceladus 2 B-plane Final Solutions

sense. In fact, even though cases 3 & 4 are a little apart on the B-plane, all the solutions show ΔVs close to each other. The scale factor is constant around 1.4 ± 0.7, indicating a slight mismodeling in the drag parameters. One significant result is the case 4 update, which shows that the uncertainty in the ΔV dropped from about 6.6 mm/s to 2.9 mm/s. The drag scale factor also came out close to 1.0, indicating that the C_D of 2.1 is close to the truth with an uncertainty of only 0.2.

Another interesting point to analyze is the effect of the drag model on the amplitude of the stochastic accelerations estimated during the encounter period. If everything was to be modeled perfectly, these would be 0. Stochastic accelerations during the entire orbit were set up in 8-hour batches with uncertainties at 0.5×10^{-12} km/s^2 in all three directions. During the encounter period under RCS control, the stochastic accelerations were estimated every 5 minutes while the uncertainties were raised to 1.625×10^{-08} km/s^2 in the thrust pointing

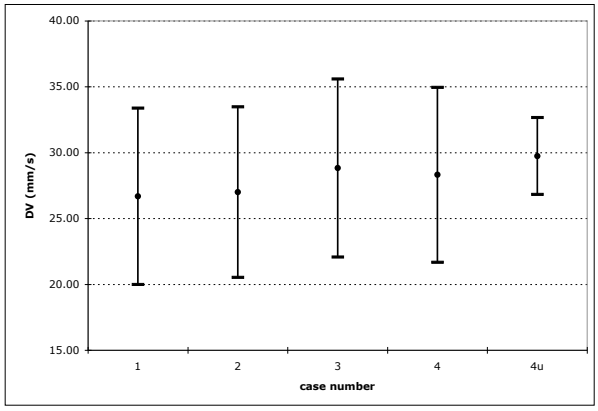


Figure 9 Drag ΔV Solutions for T5

Table 3 DRAG ΔV AND SCALE FACTOR ESTIMATES FOR THE T5 FLYBY

Case	RCS Thrust Model		Drag Model		Estimated Drag ΔV (mm/s)	Estimated Drag Scale Factor
	SMF	ACC	IMP	EXP		
1	X		X		26.69 ± 6.7	N/A
2	X			X	27.01 ± 6.5	1.42 ± 0.7
3		X	X		28.83 ± 6.8	N/A
4		X		X	28.33 ± 6.6	1.45 ± 0.2
4u	4 w/ updated densities				29.75 ± 2.9	1.03 ± 0.2

SMF: Small forces model (impulsive ΔVs)
ACC: Acceleration model
IMP: Impulsive ΔV model
EXP: Exponential acceleration model

direction. Figure 10 shows the estimated stochastic accelerations during the encounter period for the different case studies. A net improvement is noticeable between solutions with and without drag. The cases where the RCS activity was modeled as impulsive forces (1 & 2) seem to indicate a better fit, although the differences are too small to be conclusive. More importantly, it can be concluded that the drag model can be successfully used for OD during low Titan flybys. The best result was undoubtedly achieved when the density model was updated with the values from the T5 flyby. Effectively, the stochastic accelerations are near zero for this case. This result, together with the estimated scale factor for the drag being close to 1.0, show that the overall dynamics at the encounter were well modeled and validate the reconstructed densities at T5.

Titan-7

The Titan-7 encounter occurred on on September 7, 2005, where Cassini flew at 1075 km above the surface. Although 50 km higher than T5, the flyby was low enough to experience

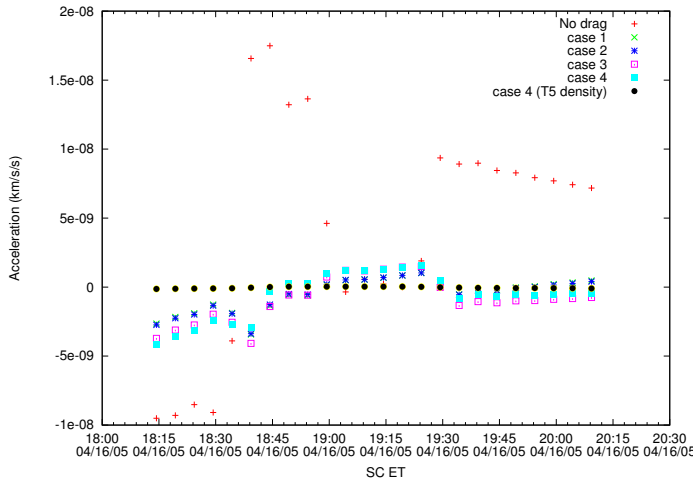


Figure 10 Stochastic Accelerations during the Titan 5 Encounter (Thrust direction)

drag and modeling it indeed made a difference on Titan's ephemeris and hence a different B-plane solution. The accelerations that Cassini experienced during the flyby are shown in Figure 11. The Titan point mass gravity is again the predominant force at periapsis. The thrusting activity is shown with a mean acceleration around 10^{-8} km/s^2 and the estimated drag acceleration is the short solid gray line in the middle, which peaks at about 10^{-7} km/s^2. An interesting aspect is the low values of the estimated stochastic accelerations. This seems to indicate that the flyby was well modeled, even though the results are independent of the model chosen, as we will see later. The data arc that covered the T7 encounter started on September 2 near apoapsis and contained two maneuvers on September 3 and September 19. Similar to the T5 cases, the drag and the thrusting activity were modeled with the four cases described earlier using either the impulsive forces or acceleration models. A summary of the events near T7 is shown in Table 4, together with the breakdown of the RCS impulsive force model.

Figure 12 shows the four Titan-7 B-plane solutions. Case 1 & 2 are off by more than 1 sigma with respect to case 3 & 4 due to the fact that the different models used for the RCS activity where not exactly equivalent. If more flexibility is given in the solutions (i.e. more uncertainty in the model), one can demonstrate that the case 1 & 2 solutions move closer to the 3 & 4. Even though the differences are small, we believe the acceleration model better since it is a more complete representation of the truth.

The results for drag ΔV and scale factors are shown in Table 5 and Figure 13. One can notice much smaller uncertainties overall for both parameters compared to the T5 cases. Again there is good agreement between all cases, although the ΔV came out higher when using the updated densities. Note also that the scale factor goes to 1.0 for this case, indicating once again that the coefficient of drag and the cross-sectional area are close to the values put in the model.

The stochastic acceleration estimates (shown in Figure 14) are quite small (10^{-14} km/s^2)

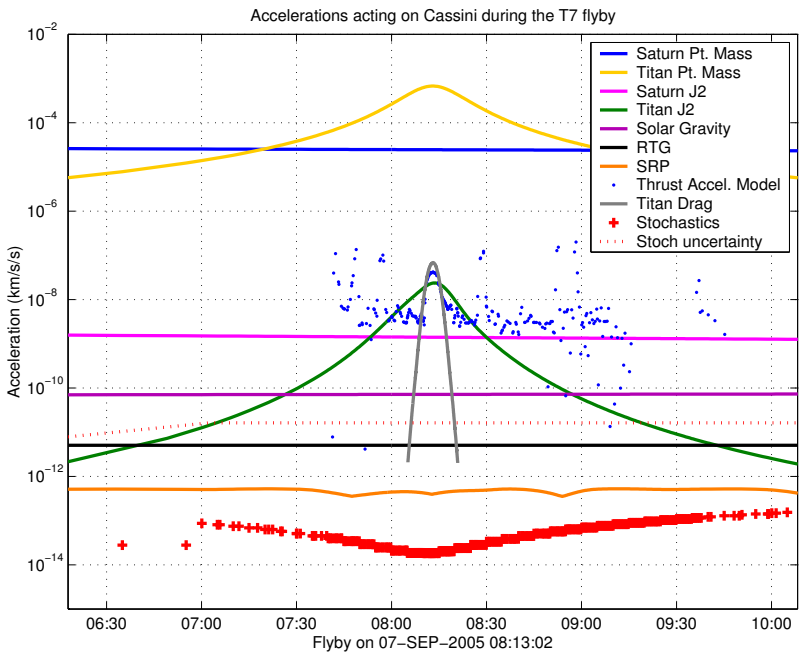


Figure 11 Accelerations acting on Cassini during the Titan 7 Flyby

and the differences from one case to the other is irrelevant. This may be explained by the fact that the resolution in the telemetry recorded during the T7 flyby was improved from 2 mm/s to 0.02 mm/s. It is thus safe to say that the flyby was well modeled and that, unlike T5, the density profile used had little impact on the solution. In any case, errors in the model were adjusted by either the drag scale factor or the stochastic acceleration to yield good B-plane solutions.

Titan-A

Before T5 and T7, the only Titan flyby where Cassini could experience drag significantly was Titan-A (TA)[‖] The TA encounter occurred on October 26, 2004 at approximately 1175 km above Titan's surface. It is important to note that since TA was the first Titan flyby, the satellites ephemerides, more specifically Titan's, were not as precisely known at that time as they were for T5 and T7. Therefore, too much uncertainty existed in the satellites to allow for the estimation of the drag acceleration, which explains why any attempt was unsuccessful at the time. Details on the TA flyby OD results can be found in Stauch *et al.*[3] . Considering the successful results obtained for the T5 and T7 studies, it is of interest to revisit the TA flyby using the exponential drag model and determine whether the drag can be distinguished from the thrusting events or not. For both the T5 and T7 arcs, the satellite *a priori* covariance used were created using a collection of data from Pioneer, Voyager as well as the data from Cassini available prior to the flybys. The TA flyby, on the other hand, was reconstructed using an older *a priori* satellite covariance, which was created in November 2004 and contained fewer Cassini observations. More details on the Saturnian

[‖]Please refer to Antreasian *et al.*[1] for more complete details on the Cassini Tour.

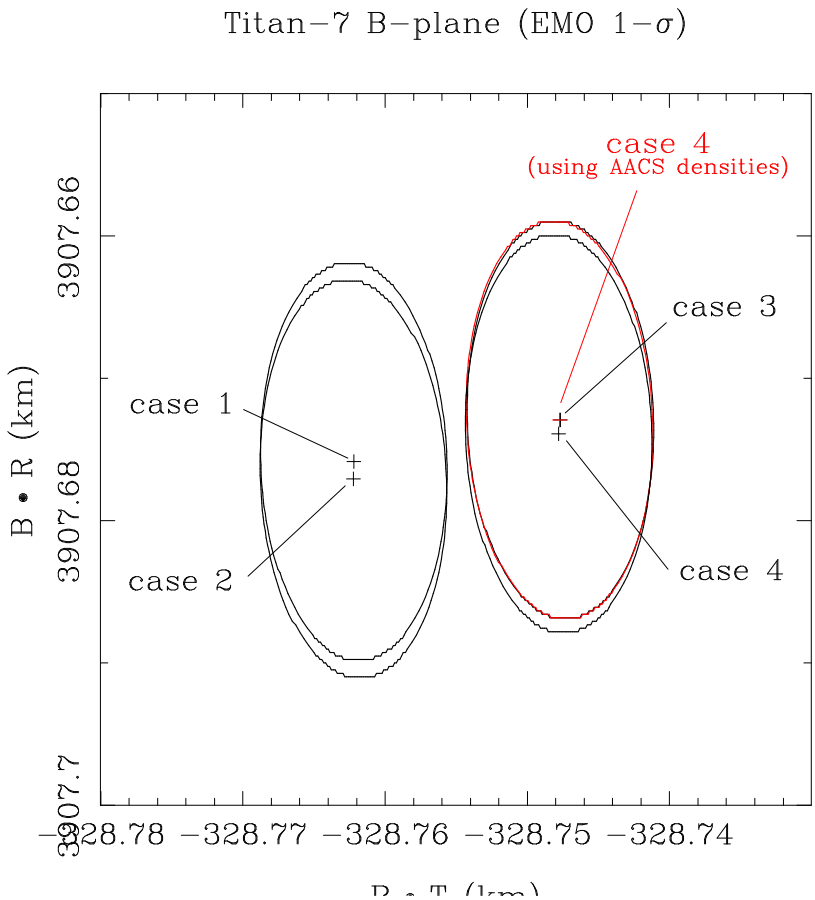


Figure 12 Titan 7 B-plane Final Solutions

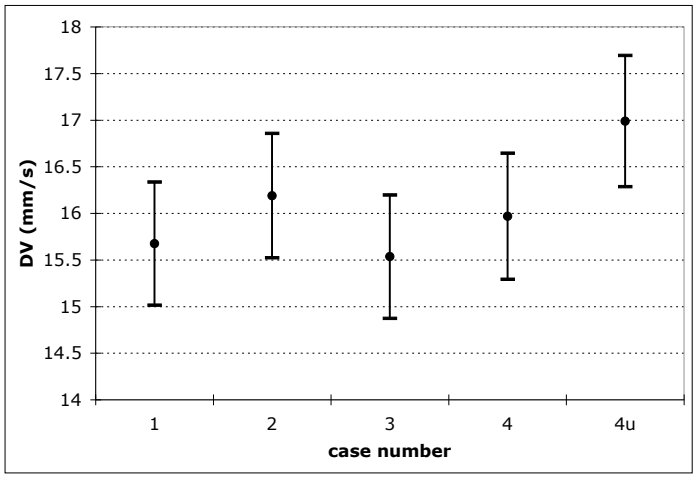


Figure 13 Drag ΔV Solutions for T7

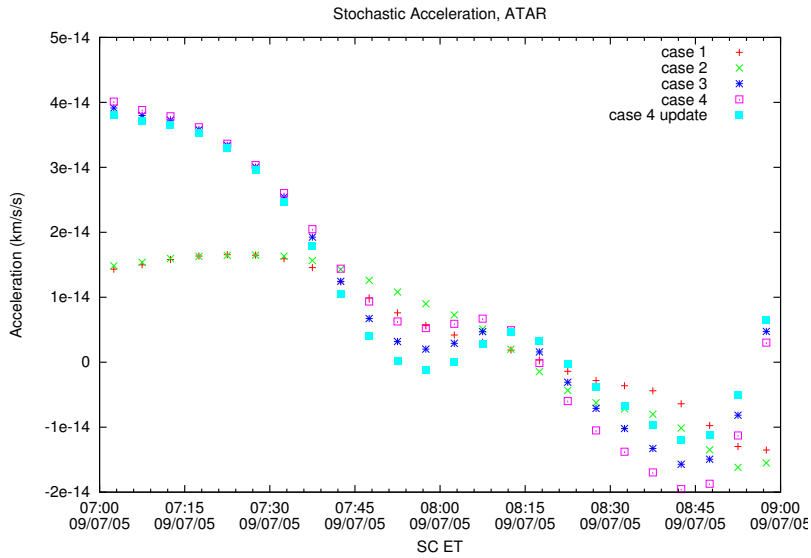


Figure 14 Stochastic Accelerations during the Titan 7 Encounter (Thrust direction)

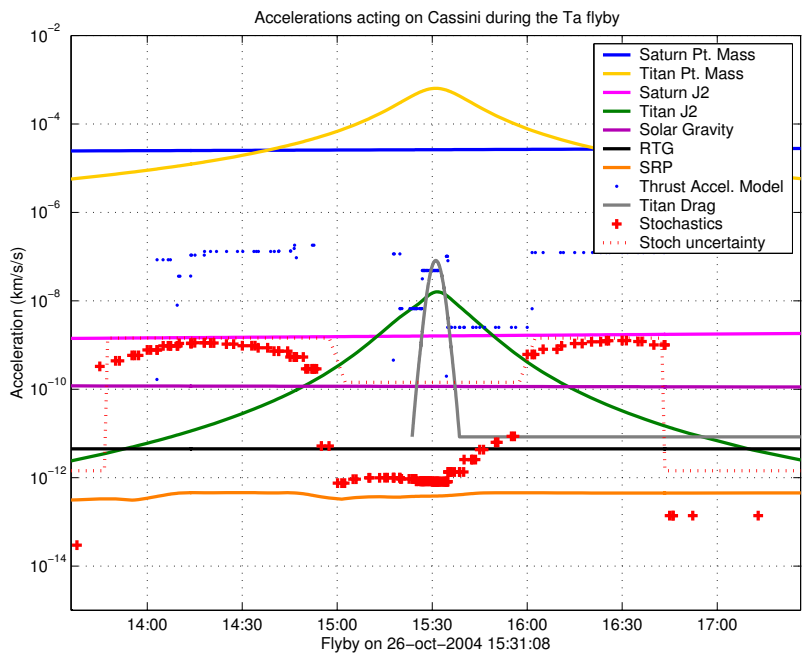


Figure 15 Accelerations acting on Cassini during the Titan A Flyby

Table 4 TITAN-7 ENCOUNTER TIMELINE (SEPTEMBER 7, 2005)

Event	Time (ET)	ΔV (mm/s)
Last Tracking Point	20:23 (Sep 6)	
Transition to RCS	07:25	55.0
Small Force	07:41	75.0
Small Force	07:47	10.8
Small Force	07:55	10.8
Titan Periapsis	08:13	
Small Force	08:15	13.1
Small Force	08:27	14.1
Small Force	08:50	4.4
Small Force	08:55	8.4
Small Force	09:15	11.8
Transition to Wheels	09:40	6.9
Tracking Resumes	06:53 (Sep 8)	

Table 5 DRAG ΔV AND SCALE FACTOR ESTIMATES FOR THE T7 FLYBYB

Case	RCS Thrust Model		Drag Model		Estimated Drag ΔV (mm/s)	Estimated Drag Scale Factor
	SMF	ACC	IMP	EXP		
1	X		X		15.67 ± 0.7	N/A
2	X			X	16.19 ± 0.7	1.55 ± 0.13
3		X	X		15.54 ± 0.7	N/A
4		X		X	15.97 ± 0.7	1.53 ± 0.14
4u	4 w/ updated densities				16.99 ± 0.7	1.01 ± 0.09

SMF: Small forces model (impulsive ΔVs)
ACC: Acceleration model
IMP: Impulsive ΔV model
EXP: Exponential acceleration model

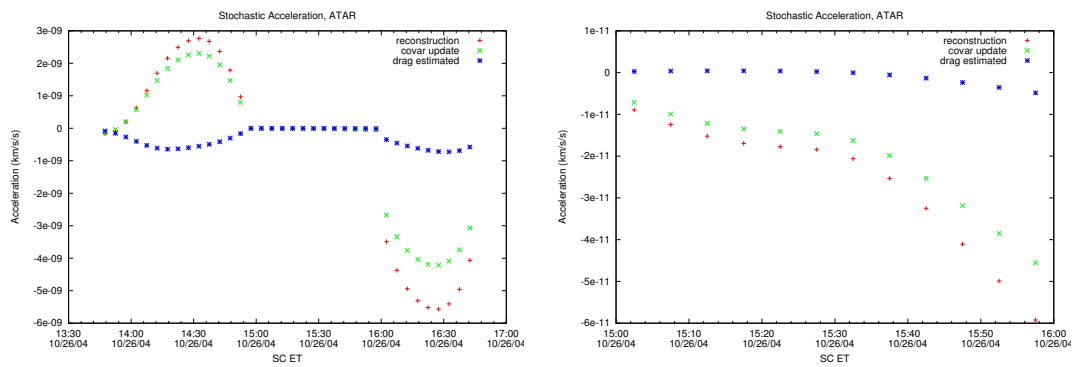
satellites models can be found in Jacobson[7] .

The TA arc analyzed here starts on August 27, 2004 and has a data cut-off after the TA flyby on Nov 20, 2004. Initial tests using the original satellite covariance quickly determined that Titan's ephemeris was not known precisely enough to include the drag in the model. (Stauch *et al.*[3]). However, better results were obtained when the satellite ephemeris was updated with a more recent set. Note that since the new covariance contains data beyond the TA arc epoch, the satellite uncertainties were left to the old numbers for more flexibility. This way, even though we know that the satellites are better determined, we avoid over-constraining the solution and processing the data twice, since that data later served to build up the new covariance. It was then observed that the drag model was too weak and the filter wanted to increase the drag acceleration by changing the C_D estimate. However this time, it seems that the force needed to be corrected by a factor of almost 8, as indicated in Table 6. While this higher drag force is consistent with science predictions (the densities observed during the TA encounter were about 4 times larger than the predictions), the fact that the model converged is encouraging.

Table 6 DRAG ΔV AND SCALE FACTOR ESTIMATES FOR THE TA FLYBY

ID	Drag ΔV mm/s	C_D
1	55.5 ± 8.4	7.90 ± 2.5

Figure 15 shows the magnitude of the accelerations that were observed. Again, the predominant force during the flyby is Titan's gravity, and the spacecraft was subject to a major thrusting activity, which was modeled this time as an acceleration model and compensated by looser stochastic accelerations. The drag curve shown is the resulting estimated acceleration, which peaks at approximately 1×10^{-7} km/s^2, about one order of magnitude higher than the original prediction (Stauch *et al.*[3]). Comparing the stochastic accelerations during TA between the old reconstruction and the new updates is also interesting. Figure 16 shows that the update in the satellite ephemeris as well as the drag model both helped the model, as expected.



(a) Encounter Period

(b) Close-up

Figure 16 Stochastic Accelerations during the Titan A Encounter (Thrust direction)

CONCLUSION

The development of an acceleration drag model for low-altitude Titan flybys was successfully validated during the Titan-A, the Titan-5 and the Titan-7 flybys. Consequently, the drag model is now set up for the future low altitude encounters, such as Titan-16 scheduled for July 22, 2006. For the three flybys studied, better solutions were obtained on the encounter target B-plane when the drag was estimated. For instance, differences on the order of 200 m at closest approach were seen for T5 when the drag was taken into account. The results from the drag estimate show that the densities at Titan were denser than originally predicted, which concurs with the density model acquired in-situ during the encounter. The best results were undoubtedly achieved when the new density model were used, as the stochastic accelerations were near zero for these cases. In addition, the drag

scale factor estimates for these cases were close to 1.0, indicating that the overall dynamics at the encounters were well modeled and that the reconstructed densities are validated.

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APPENDIX: B-PLANE DESCRIPTION

Planet or satellite approach trajectories are typically described in aiming plane coordinates referred to as "B-plane" coordinates[8] (see Figure 17). The B-plane is a plane passing through the target body center and perpendicular to the asymptote of the incoming trajectory (assuming 2 body conic motion). The "B-vector" is a vector in that plane, from the target body center to the piercing-point of the trajectory asymptote. The B-vector specifies where the point of closest approach would be if the target body had no mass and did not deflect the flight path. Coordinates are defined by three orthogonal unit vectors, **S**, **T** and **R**, with the system origin at the center of the target body. The **S** vector is parallel to the spacecraft V_∞ vector (approximately the velocity vector at the time of entry into the gravitational sphere of influence). **T** is arbitrary, but it is typically specified to lie in the ecliptic plane (the mean plane of the Earth's orbit), or in a body equatorial plane. Finally, **R** completes an orthogonal triad with **S** and **T** (i.e., $\mathbf{R} = \mathbf{S} \times \mathbf{T}$).

Trajectory errors in the B-plane are often characterized by a 1-σ dispersion ellipse, shown in Figure 17. SMAA and SMIA denote the semi-major and semi-minor axes of the ellipse; θ is the angle measured clockwise from the **T** axis. The dispersion normal to the B-plane is typically given as a 1-σ time-of-flight error, where time-of-flight specifies what the time to encounter would be from some given epoch if the magnitude of the B-vector were zero. Alternatively, this dispersion is sometimes given as a 1-σ distance error along the **S** direction, numerically equal to the time-of-flight error multiplied by the magnitude of the V_∞ vector.

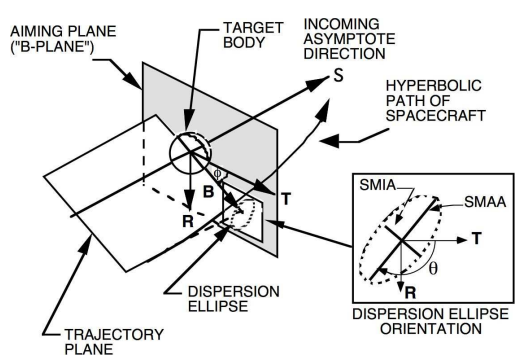

Figure 17 B-Plane Coordinate System

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