Dilution of Precision-Based Lunar Navigation Assessment for Dynamic Position Fixing

Obed S. Sands, *NASA Glenn Research Center*
Joseph W. Connolly, *NASA Glenn Research Center*
Bryan W. Welch, *NASA Glenn Research Center*
James R. Carpenter, *NASA Goddard Space Flight Center*
Todd A. Ely, *NASA Jet Propulsion Laboratory*
Kevin Berry, *NASA Goddard Space Flight Center*

BIOGRAPHY

O. Scott Sands, Joseph Connolly and Bryan Welch perform research in systems analysis, communications architecture, and signal processing within the Communications Technology Division at NASA's Glenn Research Center. J. Russell Carpenter has a long history of accomplishments in navigation while working in NASA Goddard's Flight Dynamics Analysis with Kevin Berry. Todd Ely is with the NASA JPL's Guidance, Navigation, and Control section and has worked on Mars and lunar navigation architectures; he developed a class of inclined elliptical frozen orbits for the Moon.

ABSTRACT

The NASA vision of exploration is focused on the return of astronauts to the Moon [1]. While navigation systems have already been proven in the Apollo missions to the moon, the current exploration campaign will involve more extensive and extended missions requiring new concepts for lunar navigation. In contrast to Apollo missions, which were limited to the near-side equatorial region of the moon, missions under the Exploration Systems Initiative will require navigation on the moon's limb and far-side. As these regions have poor Earth visibility, a navigation system comprised solely of Earth-based tracking stations will not provide adequate navigation solutions in these areas. In this paper, a Dilution of Precision (DoP) based analysis of the performance of a network of Moon orbiting satellites is provided. The analysis extends previous analysis of a Lunar Network (LN) of navigation satellites by providing an assessment of the capability associated with a variety of assumptions regarding the navigation receiver and satellite visibility. The assessment is accomplished by making appropriately formed estimates of DoP, with different adaptations of DoP (i.e. GDoP, PDoP, etc.) being associated with a different set of assumptions regarding augmentations to the navigation receiver or transceiver.

A significant innovation described in this paper is the "Generalized" Dilution of Precision. In the same sense that the various versions of DoP can be represented as a functional of the observability grammian, Generalized DoP is defined as a functional of the *sum* of observability grammians associated with a batch of radiometric measurements. Generalized DoP extends the DoP concept to cases in which radiometric range and range-rate measurements are integrated over time to develop an estimate of user position (referred to here as a 'dynamic' solution.) Generalized DoP allows for the inclusion of cases in which the receiver location is underdetermined when assessed in the usual 'kinematic' sense. The Generalized DoP concept is thereby a method to assess the navigation capability associated with constellations with sparse coverage without the burden of performing a full "covariance analysis" for each point on the surface of the Moon.

INTRODUCTION

In support of NASA's vision for space exploration [1], extension of the position fixing capability provided by the GPS constellation [2] to the moon is being considered. This extension would be provided through the introduction of a Lunar Network (LN) of spacecraft orbiting the Moon [3]. This study provides a Dilution of Precision-based analysis of the navigation performance associated with a LN for a user located on the lunar surface. The current study is similar to a prior study on the subject [4] with the main difference being in the use of newly developed DoP technique referred to as "Generalized DoP" [5].

Generalized DoP provides the ability to assess the navigational performance associated with a receiver that is able to integrate radiometric measurements over time. Such an analysis method provides the ability to directly compare the navigational capability associated with sparse constellations to that provided by constellations supporting full coverage of an appropriate fold. Estimates of user state that are derived from multiple radiometric measurements collected over a period of time are referred to here as being `dynamic' whereas those provided by full constellations and that do not employ integration over time in the receiver are referred to as being `kinematic.' As opposed to standard measures of DoP that are restricted to kinematic position fixing capabilities, the use of Generalized DoP further allows assessment of the constellation to be performed in terms of the latency associated with obtaining a specified level of performance.

Several different options for the LN are considered in this study including standard Walker constellations, Polar/circular constellations, Lang-Myer constellations and special constellations that include navigation spacecraft in highly elliptical orbits [6-9]. Also included in the study are assessments of a number of augmentations to the system such as highly stable clocks within the receiver, good knowledge of the terrain, and the integration of radiometric measurements over periods of time. Comparisons of the system performance under the different systems assumptions indicate that system availability performance is significantly improved and latency is reduced by the prescribed augmentations. In particular, while using a highly stable clock for the user receiver brings an improvement in performance, the improvement in performance brought by the knowledge of user height alone is significantly greater than that brought by a stable user clock. Additionally it is shown that using a stable user clock together with knowledge of user height provides significant improvements over knowledge of user height alone. It is further shown that the use of time integration of radiometric measurements is an effective way to improve system availability to required levels.

The Generalized DoP approach can be applied along with a variety of assumptions regarding navigation receiver and satellite visibility, for versions of DoP (i.e. GDoP, PDoP, etc.) with varying requirements of the number of satellites in view to obtain a solution. For example, for a two-way mode of operation the basis for assessment, is the Positional Dilution of Precision (PDoP), which assumes that the navigation transceiver only needs to solve for the users position in three dimensions. Appropriate versions of DoP (or Generalized DoP) are applied according to the assumptions regarding the nature of the radiometric measurements that are available as well as assumptions regarding the availability of collateral information such as synchronized clock or height above the lunar geoid. User height is assumed to be obtained from accurate knowledge of terrain coupled with user latitude and longitude. User latitude and longitude would be obtained from radiometric measurements. Results are derived from temporally and spatially averaged system availability numbers. Results are also provided in terms of system latency associated with pre-specified levels of system availability.

The results of this analysis illustrate some interesting points on the performance of Polar, Walker, Lang-Meyer and Hybrid-Elliptical constellations. General performance trends of the LN in a Walker formation are best in Equatorial Regions, while in polar formation the LN performs well in Polar Regions. The number of orbiters in the LN are very dependent on the system availability for the kinematic solutions, however this becomes less true as the dynamic solution is integrated for longer durations. The reduction in the required navigation satellite coverage by assuming clock synchronization and good knowledge of the terrain greatly improves the system availability.

CONSTELLATIONS

Four main categories of LN constellations are considered including Polar [6], Walker [7], Lang-Meyer [8], and Hybrid Elliptical [9]. The variations of the LN investigated all meet the requirement of providing continuous coverage by at least one satellite anywhere on the lunar surface. The notation for the LN subsequently used, such as Lang-Meyer N/p/f + x is defined as N the number of satellites, p the number of orbital planes, f the phasing in the mean anomaly between satellites in adjacent planes, and + x denotes possible added lunar satellites for equatorial coverage. Table 1 lists the parameters of the constellations are considered here.

Table 1. Lunar network constellations

Constellation	# Satellites	# orbital planes	SMA (km)	Inclination
Polar 12/4/1	12	4	9250	90^o
Polar 8/2/1	8	2	9250	90^o
Polar 6/2/1	6	2	9250	90^o
Walker 6/2/0	6	2	8050	52.2^o
Walker 5/5/1	5	5	9150	43.7^o
Lang-Meyer 4/4/1 +2	4	4	8050	58.9^o
	2	1	8050	0^o
Hybrid Elliptical 4/2/1 +3	4	2	6541.4	62.9^o
	3	1	11575	27.1^o

The Hybrid Elliptical and Lang-Meyer constellations are illustrated in Figures 1 and 2 respectively. These Figures show the satellites in equatorial orbital plane for enhanced equatorial coverage.

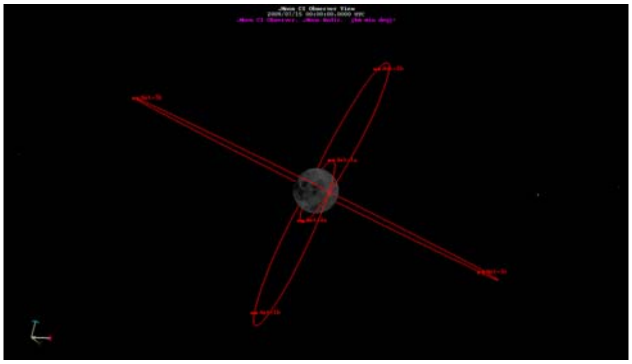

Figure 1. Hybrid Elliptical 4/2/1 +3

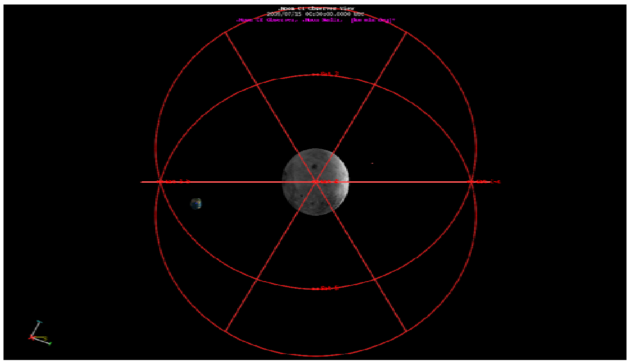

Figure 2. Lang-Meyer 4/4/1 +2

Each of the constellations has specific reasons for consideration in this study. The Polar constellations are considered for providing a focus of coverage over the polar region. The Polar 6/2/1 has the minimum number of satellites need for a circular polar orbit constellation to provide single fold global coverage. The Polar 8/2/1 provides improved navigation performance and adds significant robustness, because it can experience a loss of two satellites and maintain global coverage. The Polar 12/4/1 is chosen for its ability to provide nearly continuous 4-fold coverage over the lunar poles. Walker constellations provide a focus of coverage over the equatorial regions. The Walker 5/5/1 constellation provides the absolute minimum number of satellites in circular orbit planes to provide global coverage, while the Walker 6/2/0 maximizes the elevation angle at edge of coverage. To reduce the semi-major axes of the LN a Lang-Meyer is considered. The Hybrid Elliptical constellation provides a focus of polar coverage and minimal orbital maintenance by placing the elliptical satellites into "frozen orbits."

ANALYSIS

Generalized DoP

The analysis performed is a generalized version of the Dilution of Precision metric [12], of which several forms are subsequently used for analysis. The generalized DoP is derived from of the observability grammian, which is obtained by using the navigation user equations of motion and the associated sequence of measurements. The equations of motion and the measurement sequence are given by [10]

$$\dot{X}(t) = F(t, X) \tag{1}$$

$$Y_i = G(t_i, X_i) + v_i \tag{2}$$

By assuming some prior knowledge of the nominal trajectory $x_{nom}(t)$, and using the Taylor series one can obtain the partials where the higher order terms of the expansion are ignored.

$$A(t) = \frac{\partial F}{\partial X}\Big|_{x_{nom}(t)} \tag{3}$$

$$H_i = \frac{\partial G}{\partial X}\Big|_{X_{nom}(t)} \tag{4}$$

This can then be used to establish an approximation of the linear time varying system of equations, where $\dot{x}(t)$ and y_i are the deviations from $\dot{X}(t)$ and Y_i.

$$\dot{x}(t) = A(t)x(t) \tag{5}$$

$$y_i = H_i x_i + v_i \tag{6}$$

The linear time varying system results in a state transition described by

$$\dot{\Phi}(t, t_o) = A(t)\Phi(t, t_o) \tag{7}$$

$$\Phi(t_o, t_o) = \mathrm{I} \tag{8}$$

The homogeneous solution for $x(t)$, is then described by

$$x(t) = \Phi(t, t_o)x_o \tag{9}$$

This results in the system of equations of y_i

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_l \end{bmatrix} = \begin{bmatrix} H_1\Phi(t_1, t_o)x_o \\ H_2\Phi(t_2, t_o)x_o \\ \vdots \\ H_l\Phi(t_l, t_o)x_o \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_l \end{bmatrix} \tag{10}$$

$$y = \tilde{H}_o x_o + v \tag{11}$$

The estimate is obtained using a least squares solution, where you want to minimize the cost function

$$J = (y - \tilde{H}_o \hat{x}_o)^T W (y - \tilde{H}_o \hat{x}_o) \qquad (12)$$

Taking the partial derivative of the cost function with respect to $\hat{x}_o$ and setting it equal to zero will result in the expression of the estimate

$$\hat{x}_o = (\tilde{H}_o^T W \tilde{H}_o)^{-1} \tilde{H}_o^T W y \qquad (13)$$

The 'W' is a diagonal matrix with relative weights associated with the expected accuracies of the measurements. The observability grammian is defined by

$$\tilde{H}_o^T W \tilde{H}_o \qquad (14)$$

The inverse of this matrix is a covariance matrix and the usual definition of DoP is the trace of this covariance matrix. In this paper the concept of DoP is generalized to multiple measurements by summing the covariance matrices associated with the various measurements and using an appropriate matrix norm. The use of this generalized form of DoP provides an approximate measure of the navigational performance associated with a navigational receiver that integrates information from multiple radiometric measurements, as would be collected over a period of time. This feature, in turn, allows comparisons to be made among and between constellations that are fully populated and thereby enable kinematic position fixes with those that are sparse and require the use of multiple measurements, integrated against estimates of user motion on the surface. Generalized DoP allows for comparisons to be made without conducting computationally intensive Monte-Carlo of full covariance analysis simulations. In this paper, Generalized DoP therefore takes the form:

$$\sqrt{\max\left(eig\left(\left(\sum_{t_o}^{t_n} \tilde{H}_o^T W \tilde{H}_o \right)^{-1} \right) \right)} \qquad (15)$$

The concept of Generalized DoP can be applied to the various versions of DoP including Positional DoP, or PDoP, Horizontal DoP or HDoP, etc. as will be specified in the next section. The matrix norm usually associated with DoP is the trace and not the maximum eigenvalue [11, 12]. The maximum eigenvalue is used here because it is felt that the trace metric overestimates the DoP. Note that if the summation of the inverse of the observability grammian is over a single time instance then eq (15) reduces to the more familiar DoP.

Variations of the Generalized DoP

In order to relax the constraint of satellite coverage to invert the observability grammian, a number of augmentations to the lunar navigation system are considered in the analysis. These augmentations constrain the navigation solution and thereby reduce the number of required satellites in view. These augmentations include clock synchronization and good knowledge of the terrain. This results in several forms of DoP. The selected form of DoP used not only affects the required satellites in view, but also the state transition and H matrixes used in the calculation.

Geometric Dilution of Precision (GDoP) is used in the Global Positioning System where the solution solves for position of the user in three dimensions and the time bias, resulting in the requirement of four navigation signals. Positional Dilution of Precision (PDoP) provides an estimate of user positioning accuracy for the case in which there is no time bias between orbiter clocks and user clocks, such as the case in a two-way mode of operation. PDoP results in the requirement of three navigation signals.

Horizontal/Time Dilution of Precision (HTDoP) is applied when a user has knowledge of their height above the center of the moon but a time bias exists resulting in the requirement of three navigation signals. Horizontal Dilution of Precision (HDOP) provides an estimate of user positioning accuracy when both time and user altitude are known, only requiring two navigation signals, such as the case of a two-way mode of operation with good knowledge of terrain. A more detailed discussion is found in Understanding GPS, Kaplan [12].

System Availability

The underlying Figure of Merit (FOM) used for evaluating the performance associated with a navigation system is 'system availability'. System availability is defined here as the proportion of time that the navigation system is predicted to provide performance at or below a specified level of DOP. In other words the navigation system is defined as 'available' when the appropriately chosen version of DOP falls below a certain threshold. System availability is calculated here for a large number of points on the surface of the moon. Results provided below are in terms of system availability as well as system latency. System latency results are based on the given system availability FOM.

The DoP threshold for the chosen definition of system availability is set to 10. The value 10 was chosen because studies of the variation of spatially averaged system availability thresholds have shown that a 'knee' in the curves exists near this threshold of 10. Spatially averaged system availability is sensitive to DoP thresholds between 1 and 10 while the sensitivity drops above 10. Additionally, the relative rankings of the constellations are not strongly affected by the choice of DoP threshold. A value of 6 is typically used when defining system availability for the GPS system. Note that a value of 6 is close to 10 when considered in the context of how DoP values are typically distributed. The threshold operation is applied to the DoP values, followed by an averaging operation performed on the points in time. This results in an estimate of the percentage of time that the 'system available' condition has been satisfied.

Assumptions

Navigation signal

The navigation signal requirements are outlined in Table 2.

Table 2. Navigation Signal Assumptions

Frequency used for Doppler Measurements	GPS L1 (1.57545 GHz)
URE (user range error)	1 m
URRE (user range rate error)	0.1 mm/sec
Minimum Elevation Angle	5^{o}

User burden

Receivers that support a reduced number of satellites will have associated with them an increased level of processing or other sensing equipment. This leads to increased user burden in terms of the mass and power the host platform must provide to the navigation receiver. In order to provide knowledge sufficient to infer user height given a horizontal location a large digital elevation map would have to be available to the user. In order to provide error comparable to the 1-m URE assumed for the system, the user is required to store approximately 1 Terabyte of terrain data for global coverage. For the user to have knowledge of terrain within a 30-km radius of a starting point, approximately 100 megabytes is required for storage.

For a navigation system using one-way radiometric signals as a mode of operation the clock synchronization assumption implies the clocks would have to be stable enough to have the ability to 'free-wheel' for a number of hours after synchronization. User clocks would then require periodic synchronization with orbiting clocks. The threshold used to synchronize the clock is a GDoP $\leq$ 5 with no knowledge of the terrain, or HTDoP $\leq$ 5 with good knowledge of terrain, which in turn would impose a requirement of four and three satellites respectively. The reduced DoP value from 10 to < 5 is assuming that the transfer of time would require a more accurate solution than is nominally needed. The availability analyses are performed assuming a clock resynchronization period of 3 hours. The low mass, volume and power expected for highly stable oscillators will make this a viable option. The clock synchronization is not a requirement when using a two-way radiometric navigation signals for the system's mode of operation. Table 3 lists the forms of DoP used in the analysis here together with their corresponding assumed system requirements.

Table 3. DoP Assumptions Summary

Knowledge of Terrain	Synchronized Clock	DoP Requirements	# Satellites Required
No	No	GDoP 10	4
Yes	No	HTDoP 10	3*
No	Yes /(2way)	PDoP 10**	3
Yes	Yes/(2way)	HDoP 10***	2*

(*) Terrain knowledge of latitude and longitude
(**) If one way GDoP 5 required to synchronize clock
(***) If one way HTDoP 5 required to synchronize clock

RESULTS

Results are reported as system availability, which is defined here as the percentage of time over one sidereal lunar month that a DoP value is less than 10 for a given point on the lunar surface. System availability is evaluated in five-minute epochs. The latency associated with achieving spatially averaged system availability of 90% or better is given in tabular form for selected areas on the face of the moon.

These areas include:

1. Global: All latitudes and longitudes, entire lunar surface coverage
2. South Pole: Latitudes within 10° of the lunar south pole, all longitudes
3. Front Equatorial: Latitudes between 45°N and 45°S, and longitudes between 90°W and 90°E on the nearside

The South Pole analysis is performed to determine the system availability in the context of Lunar Outpost missions that are expected to focus on concentrated exploration of the South Pole. The Front Equatorial analysis is provided in the context of 'Apollo-like' missions.

The term 'no terrain' indicates that there is no detailed cartography of the terrain that would allow, for example, determining the altitude of the ground, where as the term 'good terrain' indicates there is such knowledge and an accurate estimate of user height above the lunar datum is available to the navigation receiver. The term 'no clock' indicates that the user clocks and orbiter clocks are not synchronized, and the term 'good clock' indicates that the clocks are synchronized and remain like that for a specific number of hours (indicated by τ) given a GDoP or HTDoP less than or equal to 5. If a two-way mode of

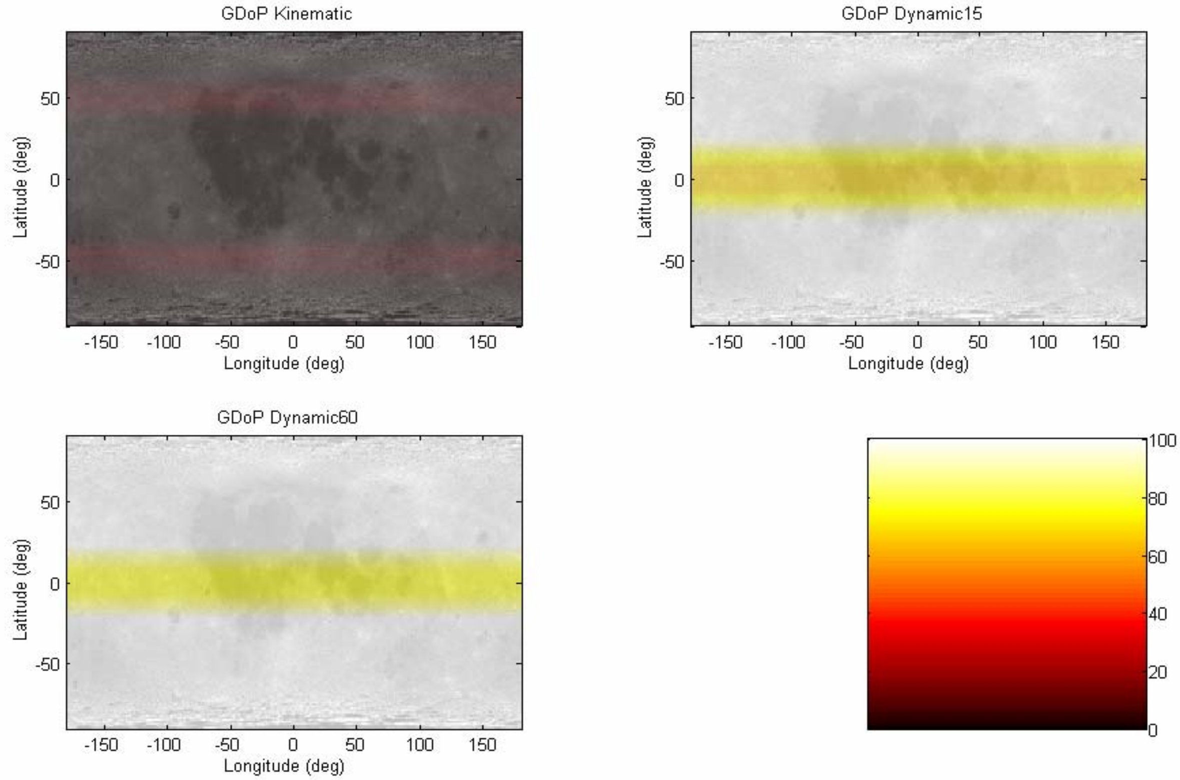

Figure 3: System availability for Polar 6/2/1 constellation,

operation then the concepts associated with GDoP or HTDoP do not apply.

Figure 3 illustrates the improved performance using a dynamic navigation solution over that provided by kinematic solutions for the Polar 6/2/1 constellation under the assumption that no knowledge of user height is available nor that a stable oscillator is available. In Figure 3 system availability performance for the constellation is shown with pseudo-color graphics for kinematic position fixing (upper left), dynamic position fixing, with 15 minute integration time (upper right) and dynamic position fixing with one hour integration time (lower left.) These graphics are superimposed on a gray scale image of the moon's surface for reference purposes. The color bar in the lower right portion of Figure 3 provides a scale for system availability with white indicating 100 % and black indicating 0 %. To get adequate system availability performance the kinematic solution requires more satellites in-view at a given time instant than a dynamic solution. Restriction to kinematic solutions would then lead to consideration of only larger constellations such as the Polar 8/2/1 and Polar 12/4/1. However, using a dynamic solution of only 15 min the system availability improves to 100% over most of the lunar surface. There is only a small band of reduced performance in the equatorial region. The system availability improves still greater for a dynamic solution of 1 hr.

Figure 4. shows the performance of each of the systems proposed in this paper in terms of the latency required to achieve 90% system availability over a specified region of the surface of the moon. In this table a green box indicates that the criterion is met in a kinematic sense with zero latency. If the criterion is not met with kinematic measurements, but is met with a dynamic fix of 15 minutes, the box is shaded yellow. If the criteria are not met by either of these metrics, but is met with a dynamic fix of 1 hour, it is shaded red. Finally, if the criterion is not met with either kinematic or dynamic fixing the box is shaded gray. Inspection of the latency result summary provided in Figure 4 reveals two overall general trends. These trends are apparent in each of the identified lunar regions (i.e. `global', `front equatorial', etc.). In general, system latency improves for a given constellation as the augmentations are added. In particular the improvement in performance brought by knowledge of user height alone is significantly greater than that brought by a highly stable user clock alone. Additionally, note that using a highly stable user clock together with knowledge of user height provides significant improvements over knowledge of user height alone. The other general trend observed for each identified region is that the system performance

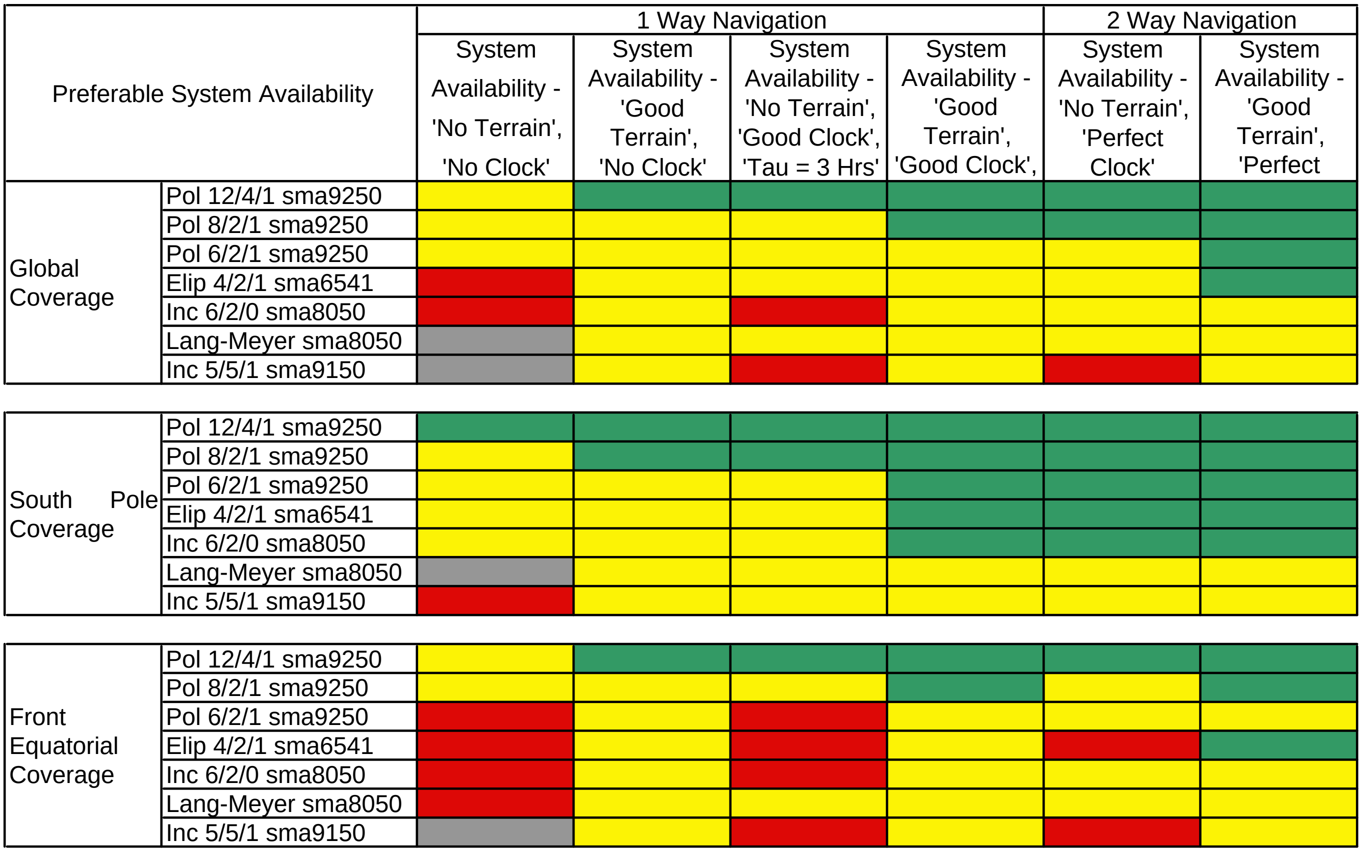

Figure 4. System latency based for selected lunar surface regions.

improves with the number of satellites in the constellation. Notable exceptions to this trend are present for the Hybrid Elliptical. For example, the polar and inclined 6 satellite constellations provides better latency than the elliptical case which contains 7 satellites using no knowledge user height and without using an onboard clock when global coverage is required.

The general trend for the one-way and two-way mode of operation is that the two-way mode of operation is better able to provide a navigation solution in all of the regions. This is apparent in the global region for the Polar 8/2/1 and the Walker 6/2/0 where even when clock synchronization with a τ of 3 hrs is used to simulate the performance of a two-way system the one-way measurement is not able to meet the two-way performance. The analysis shows when using a two-way system the Polar 8/2/1 constellation can give kinematic navigation solutions 90% of the time over the lunar globe. The Polar 6/2/1 can provide a 15 minute dynamic solution for global coverage, and a kinematic solution for the polar region given a two-way system or augmentations to a one-way system.

The results of this analysis illustrate some interesting points on the performance of Polar, Walker, Lang-Meyer and Hybrid-Elliptical constellations. The number of orbiters in the LN are very dependent on the system availability for the kinematic solutions, however this becomes less true as the dynamic solution is integrated for longer durations. The reduction in the required navigation satellite coverage by assuming clock synchronization and good knowledge of the terrain greatly improves the system availability, while a two-way mode of operation gives superior performance when compared to one-way.

Depending upon requirements, the Polar 6/2/1 can provide acceptable performance. This constellation can provide low latency (15 minute or better) position fixes on a global coverage sense. This constellation also represents a scalable solution since a second 6/2/1 constellation can be added to the first to create a 12/4/1 without reconfiguring the first.

The ability for a kinematic solution obtained by the Polar 8/2/1 in two-way global coverage would be useful in an emergency situation where the astronauts would need to have immediate navigation information. It adds significant robustness because an 8/2/1 constellation can easily be reconfigured to a Polar 6/2/1 configuration in the event of a failure of one satellite or two satellites, if the failures occur in separate planes. This leads to the Polar 8/2/1 being a desirable constellation.

CONCLUSIONS

Generalized DoP allows the effects of multiple radiometric measurements to be assessed in the same manner that standard measures of DoP are used to assess the information relevant to position fixing that is associated with a single set of radiometric mearsurements. In the current case the effect of integrating multiple radiometric measurements in time are assessed in order to allow the performance of sparse constellations around the moon to be compared with fully populated constellations that provide only kinematic solutions. With this innovation, the basis of comparison can be changed to a domain that is more closely aligned with user requirements, namely the latency associated with achieving a particular level of precision in the state estimate.

Restriction to the use of kinematic solutions, as is done with analysis based on static DoP, biases the selection of a constellation to those with more satellites. The use of dynamic solutions allows for integrating radiometric signals over a period of time to improve the system availability and thus allow for the consideration constellations with fewer satellites. Application of generalized DoP to the evaluation of inherent navigation capability for users on the lunar surface brought by constellations of orbiting spacecraft around the moon has thereby eliminated this bias. The analysis method described here has thus provided for a set of recommendations for the build-up of a moon-orbiting sparse constellation of spacecraft.

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