AN EXACT SOLUTION FOR THE STEADY STATE PHASE DISTRIBUTION IN AN ARRAY OF OSCILLATORS COUPLED ON A HEXAGONAL LATTICE

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Abstract - When electronic oscillators are coupled to nearest neighbors to form an array on a hexagonal lattice, the planar phase distributions desired for excitation of a phased array antenna are not steady state solutions of the governing non-linear equations describing the system. Thus the steady state phase distribution deviates from planar. It is shown to be possible to obtain an exact solution for the steady state phase distribution and thus determine the deviation from the desired planar distribution as a function of beam steering angle.

INTRODUCTION

Arrays of mutually injection locked oscillators have become of great interest because of their utility in providing properly phased excitations for the elements of a phased array antenna.[1] A number of such schemes have been proposed, fabricated, and tested.[2][3][4][5] These range from linear arrays of three to seven elements to planar arrays up to five by five elements. All of these arrays have been coupled on a rectangular lattice. When feeding phased array elements similarly disposed on a Cartesian lattice, these arrays provide linear phase distributions over the array aperture thus producing a scanned beam. By virtue of the well-known properties of injection locking, phase control is achieved by tuning the resonant (free running) frequencies of the oscillators. If voltage controlled oscillators (VCOs) are used, these frequencies can be controlled via an applied bias voltage. Interestingly, the slope of the linear aperture phase distributions is controlled by adjusting the oscillator free running frequencies on the perimeter of the array only and, further, the needing tuning is constant along each edge of the array and antisymmetric across the array. Thus, in a planar array, only two independent control voltages are required.

Recently, alternative lattices for coupling have been investigated for application in non-Cartesian planar arrays.[6] Of interest because of their isotropy and simplicity are the triangular and hexagonal coupling lattices shown in Figure 1. Note that boundaries of the unit cells of the one are the coupling lines of the other. It has been shown that, in the triangular scheme, planar phase distributions are exact solutions of the non-linear equations describing the steady state system behavior. (In fact, exact solutions of the dynamic equations have also been obtained.) However, in the hexagonal scheme, this is not the case. That is, the desired planar phase distributions are not solutions of the non-linear equations describing the steady state system except at certain discrete azimuth angles about the array normal. They are, however, solutions of the linearized differential equations (the continuum model) approximately describing the system behavior. Thus, we have conjectured that planar phase distributions are approximate solutions in the full nonlinear formulation and have shown numerically that the solutions of the full nonlinear equations are approximately planar. It would be useful to have an estimate of the size of the deviation from planarity exhibited by the exact solution. Derivation of such an estimate is addressed in this paper. Moreover, in obtaining the approximate estimate, it was discovered that an exact solution for the nonplanar phase distribution could be obtained thus providing an exact expression for the deviation from planarity as a function of beam steering angle.

THE STEADY STATE SOLUTION

We begin with the nonlinear differential equation describing the dynamic behavior of one arbitrarily located oscillator located at the point (x, y) in an array coupled on a hexagonal lattice.

$$\frac{\partial \phi_{xy}}{\partial t} = \omega_{tune;xy} - \omega_{ref} - \Delta\omega_{lock}\left[\sin\left(\phi_{xy} - \phi_{x-\delta,y}\right) + \sin\left(\phi_{xy} - \phi_{x+\frac{\delta}{2},y+\frac{\sqrt{3}}{2}\delta}\right) + \sin\left(\phi_{xy} - \phi_{x+\frac{\delta}{2},y-\frac{\sqrt{3}}{2}\delta}\right)\right] \tag{1}$$

Note that there are three sine terms corresponding to the three nearest neighbors to which the oscillator is coupled. (The coupling phase is assumed to be a multiple of 2π.) With the time derivative set equal to zero, this equation relates the oscillator tuning to its phase in steady state. There is one such equation associated with each oscillator in the array resulting in a system of nonlinear equations relating the phase distribution to the tuning distribution over the array. We now assume that the non-perimeter oscillators are all tuned to the same frequency and that the perimeter oscillators are detuned according to the prescription,

$$\begin{aligned}
\omega_{tune}\big|_{x=y\sqrt{3}} - \omega_{ref} &= -\frac{2\pi\, d}{\lambda\sqrt{3}}\sin\theta_0\ \cos\left(\phi_0 - \frac{2\pi}{3}\right)\Delta\omega_{lock} \\
\omega_{tune}\big|_{x=-y\sqrt{3}} - \omega_{ref} &= -\frac{2\pi\, d}{\lambda\sqrt{3}}\sin\theta_0\ \cos\left(\phi_0 + \frac{2\pi}{3}\right)\Delta\omega_{lock} \\
\omega_{tune}\big|_{x=A\sqrt{3}/2} - \omega_{ref} &= -\frac{2\pi\, d}{\lambda\sqrt{3}}\sin\theta_0\ \cos\left(\phi_0\right)\Delta\omega_{lock}
\end{aligned} \tag{2}$$

which has been shown to yield beam steering to angular position (θ_0, ϕ_0) when $\phi_0 = \frac{\pi}{6} \pm n\frac{\pi}{3}$ and n is an integer. $\Delta\omega_{lock}$ is the locking range of the oscillators. We now postulate a solution for the phase distribution, $\phi(x,y)$, in the form of a planar distribution producing a beam pointed in the direction (θ_0, ϕ_0) but, with the additional parameter Δ_{xy} as shown below.

$$\phi(x,y) = -\frac{2\pi\, d}{\lambda}\left[\left(x - \frac{N}{\sqrt{3}}\right)\sin\theta_0\cos\phi_0 + y\sin\theta_0\sin\phi_0\right] + \Delta\phi_{xy} \tag{3}$$

Note that $\Delta\phi_{xy}$ represents the phase deviation of the oscillator at (x, y) from the phase value corresponding to a planar solution. Thus, as $\Delta\phi_{xy}$ approaches zero, the phase distribution becomes planar. The nearest neighboring oscillators are assumed to have the phase (3) with $\Delta\phi_{xy}$ replaced with $-\Delta\phi_{xy}$. This postulated solution (3) is now substituted into the nonlinear equation (1) and, using trigonometric identities, manipulated into the form,

$$\sin\left[2\Delta\phi_{xy} - S\cos\phi_0\right] + \sin\left[2\Delta\phi_{xy} - S\cos\left(\phi_0 - \frac{2\pi}{3}\right)\right] + \sin\left[2\Delta\phi_{xy} - S\cos\left(\phi_0 - \frac{4\pi}{3}\right)\right] = 0 \tag{4}$$

where $S = \frac{2\pi d}{\lambda\sqrt{3}}\sin\theta_0$ and d is the physical spacing of the radiating elements of coupled oscillators. From this the solution for $\Delta\phi_{xy}$ which makes the postulated phase distribution a solution of the nonlinear equation is seen to be,

$$\Delta\phi_{xy} = \frac{1}{2}\tan^{-1}\left\{\frac{\sin\left[S\cos(\phi_0)\right] + \sin\left[S\cos\left(\phi_0 - \frac{2\pi}{3}\right)\right] + \sin\left[S\cos\left(\phi_0 - \frac{4\pi}{3}\right)\right]}{\cos\left[S\cos(\phi_0)\right] + \cos\left[S\cos\left(\phi_0 - \frac{2\pi}{3}\right)\right] + \cos\left[S\cos\left(\phi_0 - \frac{4\pi}{3}\right)\right]}\right\} \tag{5}$$

Substituting this value of $\Delta\phi_{xy}$ into (3) produces the exact solution for the phase distribution over the array. As the steering angle, θ_0, approaches zero, this indicates that the deviation of the aperture phase from planarity approaches zero as the square of θ_0. Note that maintenance of this aperture phase distribution requires a change in the tuning (2) by $4\Delta\phi_{xy}\Delta\omega_{lock}$. This, in turn, implies a change in the ensemble frequency with steering angle. However, it appears that in a rhombic array made of two such triangular arrays this frequency change would be canceled. added

CONCLUSION

Equation (5) provides the desired estimate of the deviation of the exact phase distribution over the hexagonally coupled array from the desired planar distribution and this estimate is, in fact, and exact expression. The corresponding solution for the phase distribution is similarly exact and can be used to determine the radiation pattern modifications due to this phase aberration.

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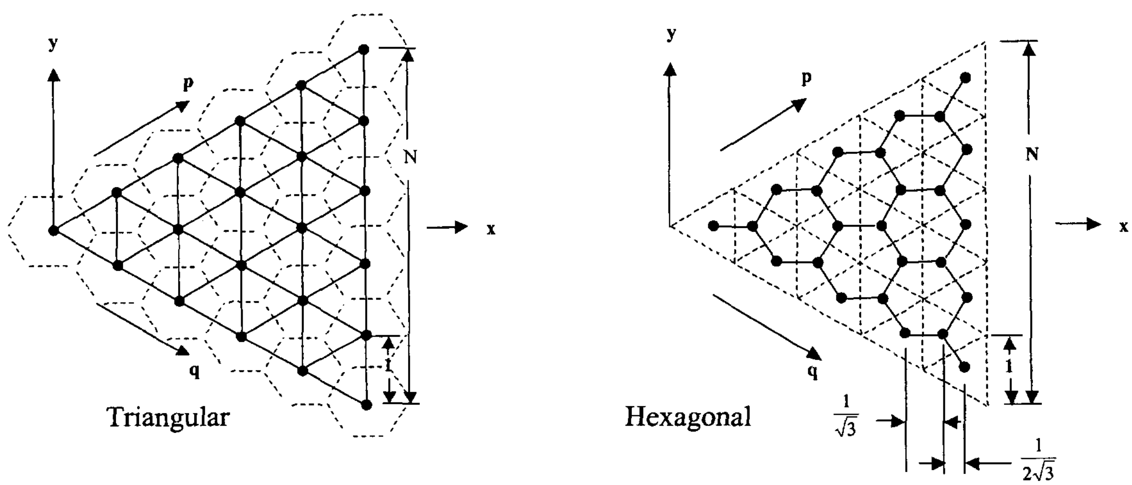

Figure 1. Triangular and hexagonal lattices.