Interplanetary Optical Navigation 101

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Overview

- Context of optical navigation
- Brief history of opnav
- Geometry modeling
- Camera modeling
- Image modeling
- Orbit determination sensitivity to opnav
- Onboard autonomous opnav

INTRODUCTION

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Putting opnav in context

- JPL's Navigation and Mission Design Section has two related tasks:
  - Mission Design:
    - Calculate a nominal trajectory which meets the diverse needs of the scientists on a particular mission
    - Locate *deterministic* maneuvers required to fly the nominal trajectory
    - Locate *statistical* maneuvers which correct errors in the actual trajectory
  - Navigation:
    - Estimate the flight path of a spacecraft (orbit determination) using three fundamental data sets:
      - Ground-based astrometric data
      - Radio metric tracking data (doppler, range, ΔDOR)
      - **Onboard optical navigation data**
    - Calculate trajectory corrections (maneuver determination)

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Ground-based astrometry

- Operational spacecraft navigation relies heavily on *a priori* estimates of the ephemerides of the planetary systems being encountered
  - Planetary ephemerides (Myles Standish, Skip Newhall)
  - Satellite ephemerides (Bob Jacobson, Jay Lieske, George Null)
  - Asteroid and comet ephemerides (Don Yeomans, Paul Chodas, Steve Chesley; Brian Marsden, Gareth Williams; Andrea Milani; others)

- All ephemerides depend on reliable astrometry: 400-year-old eclipse observations of Galilean satellites, 90 years of transit circle measurements, photographic and CCD astrometry, Hubble data, ...

- These ephemerides are updated by the flight projects

- Covariance matrix embodies the information content of the observations

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Radio data

- Doppler: the workhorse
  - Direct measure of radial velocity
  - Very sensitive to radial acceleration (gravity determination)
  - Rotation of earth gives geocentric (α, δ) to a few μrad

- Range: measure of round-trip light time
  - Subject to systematic error from propagation effects, clock errors

- ΔDOR: measure of offsets $(\Delta\alpha, \Delta\delta)$ from quasars
  - First used for Voyager
  - Accurate to nanoradians, but not trivial to acquire

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Orbit determination

- Orbit determination is not just a 6-dimensional problem:

$$\mathbf{r}(T), \dot{\mathbf{r}}(T) = f\big(\mathbf{r}(T_0), \dot{\mathbf{r}}(T_0), \text{maneuvers, ephemerides, other accels}, \ldots\big)$$

- Ground-based astrometry gives the heliocentric position and velocity of the target

- Radio tracking gives the geocentric position and velocity of the spacecraft

- Onboard imaging gives two dimensions of the target-centered position of the spacecraft

- It's up to the magic of linearized least squares to put the three together and produce a best estimate of the parameters that can affect the trajectory

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Maneuvers

- To zeroth order, a spacecraft maneuver is an impulse—an instantaneous change in velocity without changing the position
  - Calculate difference between desired aimpoint and estimated one
  - Calculate partials of these with respect to spacecraft velocity at maneuver time
  - Invert 3×3 system of equations
- In practice, a maneuver requires finite time, plus maybe additional fuel (and ΔV) required to orient the spacecraft correctly and reorient afterwards
  - Use spacecraft-specific models for calculating net force due to turning
  - Use finite differences along integrated trajectory to get partials
  - Iterate!
- Placement of maneuvers is a trade-off between fuel consumption and OD accuracy
- Sometimes the encounter time is allowed to vary, which provides a degree of freedom and therefore saves fuel

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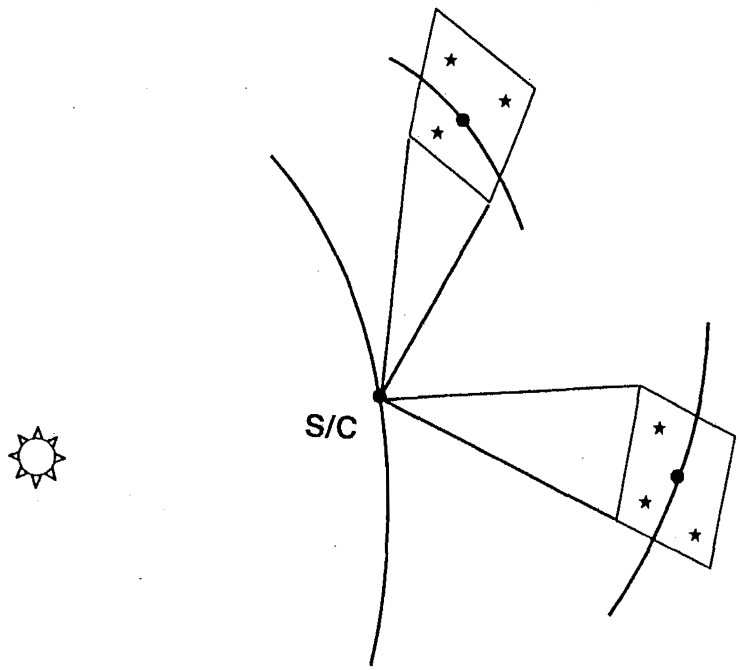
Basics of opnav

- Similar to classical photographic astrometry, but using spacecraft pictures of foreground objects against background stars



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Basics of opnav

- Measure image coordinates (x, y) of stars and objects, infer (α, δ) of objects
- *Very* useful for determining $\mathbf{B} \cdot \mathbf{R}$ and $\mathbf{B} \cdot \mathbf{T}$
- Also good for getting orbits of satellites (both old and new)
- Requires a decent star catalog (now, Tycho-2; in 15 years, who knows?)
- Requires adequate calibration of the spacecraft camera for focal length, distortion, pixel shape; we hope the hardware is stable!
- No atmosphere to contend with, just cosmic rays, CCD defects, and a point-spread function that's well matched to the pixel size
- Small field of view yields few reference stars (sometimes only one) $\rightarrow$ scheduling observations is a task unto itself

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A brief history of opnav

- 1967: Mariners 6 and 7: First demo of opnav

- 1971: Mariner 9: Nineteen frames of Phobos and Deimos nailed $\mathbf{B} \cdot \mathbf{T}$ and $\mathbf{B} \cdot \mathbf{R}$ to a few km, TOF to 1 second, *without radio*

- 1973: Mariner Venus/Mercury: We discovered light-time and aberration effects the hard way (21-pixel residuals)

- 1976: Vikings 1 and 2: Routine use of opnav

- 1979–89: Voyagers 1 and 2: centerfinding accuracy improves; specialized star catalogs introduced; system moves from Modcomp and Univac to VAX

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A brief history of opnav

- 1991–93: Galileo: Gaspra and Ida encounters, "single-frame mosaic" and "jail-bar" techniques for getting the most bang for the buck
- 1994: system moves from VAX to Unix machines
- 1998–2000: NEAR: first operational use of landmark tracking
- 1999–2001: Deep Space 1: first operational use of autonomous onboard opnav
- 2003–08: Cassini
- Future JPL missions: Deep Impact, Stardust, DAWN
- Future APL missions: MESSENGER, New Horizons

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Typography

- Scalars are *italic*
- Vectors are **boldface**
  - Considered to be column vectors
  - Leading superscript to denote the coordinate system
- Matrices are **boldface sans-serif**
  - Often considered to be operators, premultiplying vectors or other matrices
  - Trailing subscript to denote the "from" coordinate system
  - Leading superscript to denote the "to" coordinate system
- Example: ${}^{b}\mathbf{r} = {}^{b}\mathsf{T}_{a}\,{}^{a}\mathbf{r}$

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Coordinate systems

- **Inertial**, or Earth Mean Equator of 2000.0:
  - — z-axis toward North Celestial Pole
  - — x-axis toward "First Point of Aries" (Υ)
  - — y-axis completes right-handed triad ($\hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{i}}$)
- **Camera** coordinates (M, N, L):
  - — L-axis along boresight, from camera to target
  - — M- and N-axes horizontal and vertical in focal plane
- **Body-fixed** coordinates:
  - — z-axis toward body's angular momentum vector (north pole)
  - — x-z plane contains body's prime meridian
  - — y-axis completes right-handed triad
- **Mount** or "az/el" coordinates:
  - — Natural system in which the camera moves
  - — Two rotations from this system to specify camera pointing vector
  - — Camera twist is therefore determined

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Coordinate transformations

- Rotation matrices do not change the direction of a vector! They change the components of a vector. They *do* change the coordinate axes.

- Elementary rotation $\mathbf{R}_i(\theta)$ rotates the coordinate axes by angle θ about axis i:

$$\mathbf{R}_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}, \qquad \mathbf{R}_2(\theta) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix},$$

$$\mathbf{R}_3(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Any arbitrary rotation can be written as a sequence of elementary rotations. This sequence is not unique and is therefore defined by convention.

- The inverse of a rotation matrix is its transpose:

$$^{a}\mathbf{T}_{b} = {}^{b}\mathbf{T}_{a}^{-1} = {}^{b}\mathbf{T}_{a}^{\mathrm{T}}$$

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Position vectors

- We deal with the *apparent* position of some *target* as seen from the *camera*—the direction from which light appears to come.

- A target may be the center of a Solar System body, some feature on a body, a star, or even another spacecraft. Denote its position and velocity at time T by $\mathbf{t}(T)$ and $\dot{\mathbf{t}}(T)$.

- The camera may be on a spacecraft or on a body. Denote its position and velocity at time T by $\mathbf{r}(T)$ and $\dot{\mathbf{r}}(T)$.

- Easiest to do everything in inertial coordinates, referred to the barycenter of the Solar System

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Position vectors

- Example 1: Stickney (the largest crater on Phobos)
  - Crater relative to center of Phobos (body-fixed): ${}^{b}\mathbf{s}$
  - Orientation of Phobos: ${}^{b}\mathbf{T}_{i}(T) = \mathbf{R}_3(W)\,\mathbf{R}_1(90^\circ - \delta_p)\,\mathbf{R}_3(90^\circ + \alpha_p)$ where (α_p, δ_p, W) are functions of T
  - Transform to inertial: ${}^{i}\mathbf{s} = {}^{b}\mathbf{T}_{i}^{\mathrm{T}}(T)\,{}^{b}\mathbf{s}$
  - Add vector from Mars barycenter to center of Phobos (satellite ephemeris)
  - Add vector from Solar System barycenter to Mars barycenter (planetary ephemeris)

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Position vectors

- Example 2: Camera at Palomar Observatory
  - Start with the geodetic latitude φ, longitude λ, and elevation h
  - Convert to rectangular coordinates
  - Calculate transformation from inertial to body-fixed:

$$ {}^{i}\mathbf{T}_b = \mathbf{R}_2(-x)\,\mathbf{R}_1(-y)\,\mathbf{R}_3(\theta_G)\,\mathbf{R}_1\big(-(\varepsilon+\Delta\varepsilon)\big)\,\mathbf{R}3(-\Delta\psi)\,\mathbf{R}1(\varepsilon)\,\mathbf{R}_3(-z_A)\,\mathbf{R}_2(\theta_A)\,\mathbf{R}_3(-\zeta_A) $$

  - (ζ_A, θ_A, z_A) for precession
  - $(\Delta\psi, \Delta\varepsilon)$ for nutation
  - θ_G for the Earth's rotation (sidereal time)
  - (x, y) for polar motion

  - Transform to inertial: ${}^{i}\mathbf{s} = {}^{b}\mathbf{T}_i^{\mathrm{T}}\,{}^{b}\mathbf{s}$
  - Add vector from Solar System barycenter to Earth (planetary ephemeris)

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Position vectors

- Example 3: a star
  - Catalogs provide position (α_0, δ_0) at some epoch T_0, proper motion (μ_α, μ_δ), and maybe parallax π
  - Catalogued positions represent the *true* position of the star as seen from the barycenter of the Solar System
  - Calculate $\alpha(T) = \alpha_0 + \mu_\alpha(T - T_0)$ and $\delta(T) = \delta_0 + \mu_\delta(T - T_0)$
  - Calculate the distance $r = 1\ \mathrm{pc}/\pi$
  - Convert from spherical to rectangular coordinates

- The above method assumes constant angular rates. A better assumption is constant velocity, but one needs to know the radial velocity and the distance. Besides, foreshortening effects make a difference for only a handful of nearby stars.

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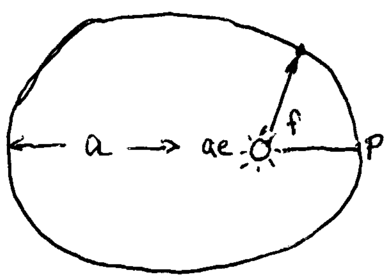
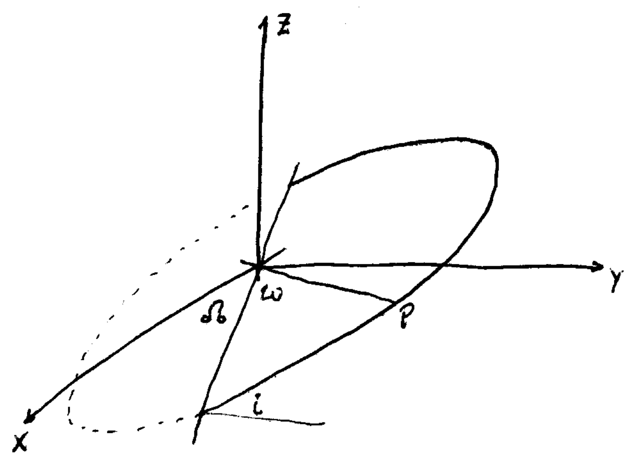
Orbit geometry

- In classical celestial mechanics, the trajectory of a massless particle relative to a point mass (the "one-body problem") is defined by the initial position $\mathbf{r}$ and velocity $\dot{\mathbf{r}}$ of the particle and by the mass M of the central body

- A bound orbit (elliptical) is characterized by 6 "Keplerian" orbital elements:
  - — Semimajor axis a
  - — Eccentricity e
  - — Inclination i
  - — Longitude (or R.A.) of ascending node ☊ (or Ω)
  - — Argument of pericenter ω (or longitude of pericenter $\varpi \equiv$ ☊ $+ \omega$)
  - — Mean anomaly M (or true anomaly f or time of pericenter T)





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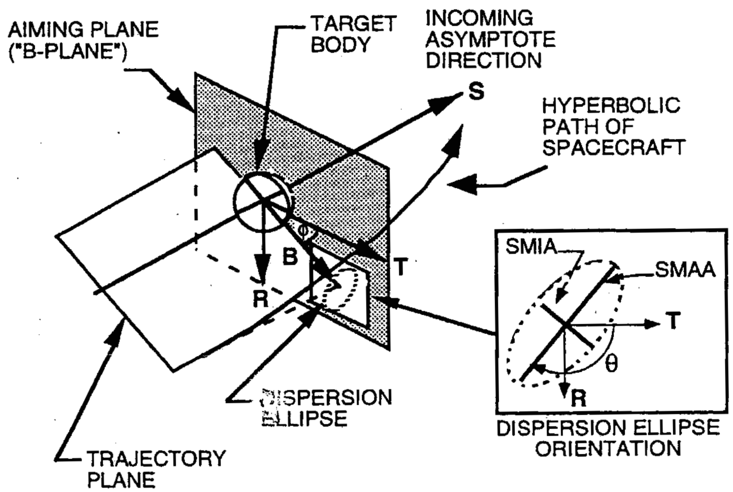
Orbit geometry

- A flyby trajectory (hyperbolic) may characterized by these 6 parameters:
  - **B · R**
  - **B · T**
  - Time of pericenter (often "linearized")
  - Right ascension of incoming asymptote
  - Declination of incoming asymptote
  - Hyperbolic excess velocity V_∞



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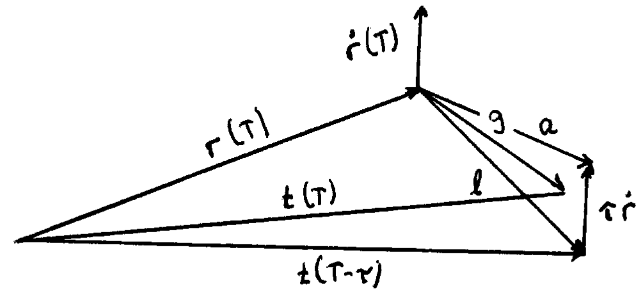
Position vectors



- *Geometric* position of the target:

$$\mathbf{g}(T) = \mathbf{t}(T) - \mathbf{r}(T)$$

- *True* position, accounting for light time:

$$\mathbf{l}(T) = \mathbf{t}(T - \tau) - \mathbf{r}(T)$$

where $\tau \equiv |\mathbf{l}(T)|/c$ is the light time

- *Apparent* position, accounting for stellar aberration:

$$\mathbf{a}(T) = \mathbf{l}(T) + \tau\dot{\mathbf{r}}(T)$$

(this is classical formula; special relativity is different)

- Note that these calculations are all done in inertial J2000.0 coordinates.

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Camera orientation

- Brute force: ${}^{c}\mathbf{T}_i = \mathbf{R}_3(\phi)\,\mathbf{R}_2(90° - \delta)\,\mathbf{R}_3(\alpha)$
  - — Useful for 3-axis stabilized spacecraft
  - — Requires all three angles to be known

- Via mount coordinates: ${}^{c}\mathbf{T}_i = \mathbf{R}_2(\mathrm{El})\,\mathbf{R}_3(\mathrm{Az})\,{}^{m}\mathbf{T}_i$
  - — Useful for spacecraft or telescope mounts with 2 degrees of freedom
  - — Assumes that camera twist is known in the mount system
  - — Determine Az and El by transforming the camera pointing vector (expressed
  - — in mount coordinates) from rectangular to spherical coordinates:

$$
\begin{aligned}
{}^{m}\hat{\mathbf{L}} &= {}^{m}\mathbf{T}_i\,{}^{i}\hat{\mathbf{L}} \\
\mathrm{Az} &= \operatorname{atan2}\left({}^{m}L_2, {}^{m}L_1\right) \\
\mathrm{El} &= \cos^{-1}\,{}^{m}L_3 = \operatorname{atan2}\left(\sqrt{{}^{m}L_1^2 + {}^{m}L_2^2}, {}^{m}L_3\right)
\end{aligned}
$$

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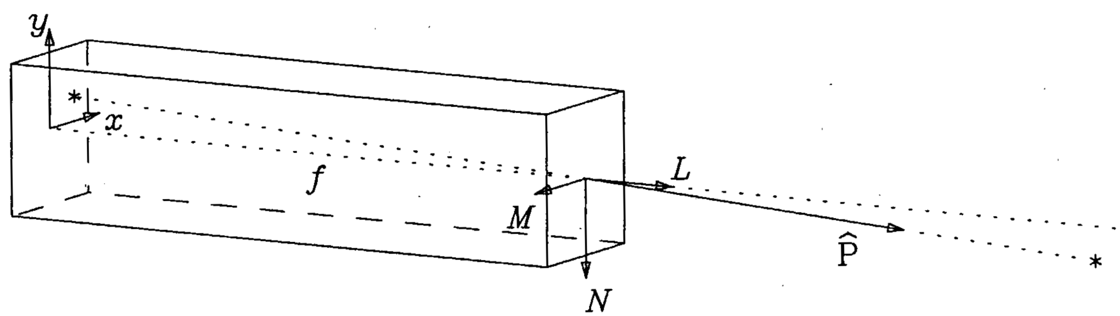
Camera geometry

- To first order, any camera behaves like a pinhole (*gnomonic projection*)
- Rays from the rest of the universe travel through the pinhole and form a real image on the focal plane, located at distance f from the pinhole
- The image is upside down, so we define (x, y) coordinates in the image plane such that x and y are antiparallel to M and N



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Camera geometry

- Given ${}^i\mathbf{a}$, the *apparent* vector from the camera to the target:
  - Rotate into camera coordinates:

$$ {}^c\mathbf{a} \equiv (M, N, L)^{\mathrm{T}} = {}^c\mathbf{T}_i \, {}^i\mathbf{a} $$

  - Project into the focal plane:

$$ \begin{pmatrix} x \\ y \end{pmatrix} = f \begin{pmatrix} M/L \\ N/L \end{pmatrix} $$

- Note that f, x, and y have units of length (conventionally mm)

- The resulting (x, y) are *ideal* camera coordinates

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Camera aberrations

- Real optics can suffer from five classical third-order aberrations:
  - Spherical aberration: light does not come to a perfect focus, but the image location is unaffected
  - Coma: light is spread out into a V shape; affects brighter images more
  - Distortion: the image is displaced radially by an amount proportional to the cube of the field angle
  - Astigmatism: out-of-focus images are elliptical rather than circular
  - Curvature of field: the focal surface is not planar

- Also, the detector (CCD, film, Vidicon, whatever) may not be mounted exactly perpendicular to the optical axis:
  - "Tip-tilt" or "decentering" error
  - A square image is turned into a trapezoid

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Camera aberrations

- Distortion and tip-tilt are modeled as corrections to the ideal coordinates in the image plane:

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} x(x^2+y^2) & xy & x^2 \\ y(x^2+y^2) & y^2 & xy \end{pmatrix} \begin{pmatrix} \epsilon_r \\ \epsilon_M \\ \epsilon_N \end{pmatrix}$$

  where the ϵ's are coefficients giving the magnitude of each effect

- The distorted positions are then:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x + \Delta x \\ y + \Delta y \end{pmatrix}$$

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Image coordinates

- We do not measure (x', y') directly.
- An image is a 2-dimensional array of measured brightness levels (DN, for "data number"; a.k.a. ADU, for "analog-to-digital units") as recorded by a detector inside the camera
- The *sample* or column coordinate s runs horizontally, left to right
- The *line* or row coordinate l runs vertically, top to bottom
- Linear mapping from (x', y') to (s, l):

$$\begin{pmatrix} s \\ l \end{pmatrix} = \begin{pmatrix} s_0 \\ l_0 \end{pmatrix} + \begin{pmatrix} K_x & K_{xy} \\ K_{yx} & K_y \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

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Camera geometry summary

$$^{i}\mathbf{l}(T) = {}^{i}\mathbf{t}(T - \tau) - {}^{i}\mathbf{r}(T)$$

$$^{i}\mathbf{a}(T) = {}^{i}\mathbf{l}(T) + \tau\, {}^{i}\dot{\mathbf{r}}(T)$$

$$^{c}\mathbf{a}(T) = {}^{c}\mathbf{T}_{i}\, {}^{i}\mathbf{a}(T) = (M, N, L)^{\mathrm{T}}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = f \begin{pmatrix} M/L \\ N/L \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x(x^2 + y^2) & xy & x^2 \\ y(x^2 + y^2) & y^2 & xy \end{pmatrix} \begin{pmatrix} \epsilon_r \\ \epsilon_M \\ \epsilon_N \end{pmatrix}$$

$$\begin{pmatrix} s \\ l \end{pmatrix} = \begin{pmatrix} s_0 \\ l_0 \end{pmatrix} + \begin{pmatrix} K_x & K_{xy} \\ K_{yx} & K_y \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

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Camera geometry summary

- Therefore the predicted s and l are a function of:
  - All the parameters which affect $\mathbf{r}$ and $\dot{\mathbf{r}}$
  - All the parameters which affect $\mathbf{t}$ and $\dot{\mathbf{t}}$
  - Camera pointing angles (however defined)
  - Camera focal length f
  - Camera distortion parameters $(\epsilon_r, \epsilon_M, \epsilon_N)$
  - Pixel zero-point offsets (s_0, l_0)
  - Pixel mapping matrix $(K_x, K_{xy}, K_{yx}, K_y)$

- Parameters which affect $\mathbf{r}$, $\dot{\mathbf{r}}$, $\mathbf{t}$, and $\dot{\mathbf{t}}$ are *dynamic parameters*

- The others are *non-dynamic* or *optical parameters*

FILTERING

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Fundamentals of filtering

- Linearized multidimensional least-squares

- The set of M observed quantities is a column vector $\mathbf{x}_o$

- The set of "correct" values for these quantities (*i.e.*, without any observation error) is a column vector $\mathbf{x}$, each of whose elements is a function of N parameters $\mathbf{q}$: $\mathbf{x} = \mathbf{x}(\mathbf{q})$

- The actual value of the parameters is their nominal value plus some unknown correction to them: $\mathbf{q} = \mathbf{q}_0 + \Delta\mathbf{q}$

- The set of computed values (based on the nominal values of the parameters) is $\mathbf{x}_c = \mathbf{x}(\mathbf{q}_0)$

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Fundamentals of filtering

- Expand $\mathbf{x}$ in a Taylor series:

$$\mathbf{x} = \mathbf{x}_c + \left.\frac{\partial \mathbf{x}}{\partial \mathbf{q}}\right|_{\mathbf{x}=\mathbf{x}_c} \Delta \mathbf{q}$$

  where the partials matrix has N rows and M columns

- Rewrite as

$$\frac{\partial \mathbf{x}}{\partial \mathbf{q}} \Delta \mathbf{q} = \mathbf{x} - \mathbf{x}_c$$

- The right-hand side still contains $\mathbf{x}$, the ideal values of the observations. Replace by $\mathbf{x}_o$, the actual observations, so that the RHS now contains the residual vector $\mathbf{b} = \mathbf{x}_o - \mathbf{x}_c$

- The M *condition equations* are therefore the vector equation

$$\mathbf{A} \Delta \mathbf{q} = \mathbf{b}$$

  where $\mathbf{A}$ is the partials matrix.

- If $M > N$ the problem is overdetermined

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Fundamentals of filtering

- The solution is found by minimizing $\chi^2 = ||\mathbf{x}_o - \mathbf{x}||^2$

- "Square up" by multiplying both sides by $\mathbf{A}^{\mathrm{T}}$:

$$\mathbf{A}^{\mathrm{T}}\mathbf{A}\Delta\mathbf{q} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

  which is now a set of N *normal equations*

- Invert to solve for $\Delta\mathbf{q}$:

$$\Delta\mathbf{q} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

- Mathematically correct, but numerically unstable!

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Covariance matrix

- Divide each of the condition equations by the measurement uncertainty:

$$\mathbf{SA}\Delta\mathbf{q} = \mathbf{Sb}$$

where $\mathbf{S} = \mathrm{diag}\,(1/\sigma_1, \ldots, 1/\sigma_\mathrm{M})$. This makes the condition equations dimensionless.

- Form the normal equations:

$$(\mathbf{SA})^\mathrm{T}\,(\mathbf{SA})\Delta\mathbf{q} = (\mathbf{SA})^\mathrm{T}\,\mathbf{Sb}$$

or

$$(\mathbf{A}^\mathrm{T}\mathbf{WA})\Delta\mathbf{q} = \mathbf{A}^\mathrm{T}\mathbf{Wb}$$

where $\mathbf{W} = \mathbf{S}^\mathrm{T}\mathbf{S} = \mathrm{diag}\,(1/\sigma_1^2, \ldots, 1/\sigma_\mathrm{M}^2)$ is the *weighting matrix*

- The *information matrix* is $\mathbf{A}^\mathrm{T}\mathbf{WA}$

- The covariance matrix is $\mathbf{P} = (\mathbf{A}^\mathrm{T}\mathbf{WA})^{-1}$
  - — Note that $\mathbf{P}$ is independent of the residuals
  - — One can perform a covariance analysis before measurements are made

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A priori constraints

- One is often unwilling to let a parameter change too much

- *A priori* covariances are used to restrict $\Delta\mathbf{q}$

- They are also helpful for systems that are otherwise underdetermined

- These are really additional condition equations, $\Delta\mathbf{q} = \mathbf{0} \pm$ some appropriate covariance

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Factorization methods

- "Squaring up" the condition equations and inverting the resulting normal equations is mathematically unstable

- Loss of precision in forming σ^2 can produce near-singular $\mathbf{A}^{\mathrm{T}}\mathbf{W}\mathbf{A}$

- Standard Kalman filter technique *subtracts* a symmetric matrix from the current covariance, possibly producing a negative element on the diagonal

- Much better to work with a square root of either the covariance matrix or the information matrix
  - — No loss of precision
  - — No possibility of negative eigenvalues

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U-D filter

- Write $\mathbf{P} = \mathbf{U}\,\mathbf{D}\,\mathbf{U}^{\mathrm{T}}$ where

$$\mathbf{U} = \begin{pmatrix} 1 & U_{12} & U_{13} & \dots & U_{1N} \\ 0 & 1 & U_{23} & \dots & U_{2N} \\ 0 & 0 & 1 & \dots & U_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{D} = \mathrm{diag}\,(\mathrm{D}_1, \mathrm{D}_2, \mathrm{D}_3, \dots, \mathrm{D}_\mathrm{N})$$

- Our implementation works with a one-dimensional “UD array” arranged

$$D_1, U_{12}, D_2, U_{13}, U_{23}, D_3, \dots, D_N$$

- As each condition equation is processed, the D_i are multiplied by a factor which is ≤ 1 by construction; no subtractions

- Great for processing one measurement at a time

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SRIF filter

- "Square Root Information Filter" uses $\mathbf{P}^{-1} = \mathbf{R}\,\mathbf{R}^{\mathrm{T}}$ where $\mathbf{R}$ is triangular

- Data points (condition equations) are incorporated using Householder transformations to retriangularize $\mathbf{R}$

- Great for processing a bunch of measurements at once

- Also numerically stable

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Stochastic parameters

- One often deals with families of related parameters:
  - Stochastic accelerations on the spacecraft (daily?)
  - Pointing angles for each opnav picture
  - Right ascension and declination for each star

- The total number of these parameters may not be known in advance

- Each condition equation has partials with respect to only one parameter in each family

- Use one parameter per family when filtering
  - Process all measurements depending on one parameter together
  - Save the current $\Delta\mathbf{q}$ and appropriate rows of P
  - Zero out Δq_i and reinitialize ith row and column of P

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Smoothing

- The final $\Delta\mathbf{q}$ and $\mathbf{P}$ contain the solution and covariance for only the last member of each family of stochastic parameters

- Smoothing works backwards, applying subsequent data to improve the estimate of earlier values of the stochastic parameters
  - Read in the saved values of $\Delta\mathbf{q}$ and $\mathbf{P}$
  - Compare the saved $\Delta\mathbf{q}$ to the final solution
  - Get ∂(stochastic parameter)/$\partial\mathbf{q}$ from the correlations in $\mathbf{P}$
  - Apply this to the difference in $\Delta\mathbf{q}$, add to the saved value of the stochastic parameter
  - Sigmas for the stochastics may likewise be obtained

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Filtering summary

- Solve for corrections $\Delta \mathbf{q}$ to the nominal values of the parameters
- Linearize; $\mathbf{A}$ is the partial derivatives of the observations with respect to the observations
- Use factorized methods to obtain the solution and its covariance
- Use stochastic filtering and smoothing
- Iterate if necessary

IMAGE PROCESSING

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Image basics

- An image is a 2-dimensional array of measured brightness levels (DN, for "data number"; a.k.a. ADU, for "analog-to-digital units") as recorded by a detector inside the camera

- The *sample* or column coordinate s runs horizontally, left to right

- The *line* or row coordinate l runs vertically, top to bottom

- The top left pixel is either $(1, 1)$ or $(0, 0)$, depending on whether you speak Fortran or C

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Image basics

- In a CCD, the DN represents the DC voltage level measured at an amplifier on the chip, then converted from analog to digital.
- The measured voltage is the sum of a constant bias plus a component proportional to the number of free electrons accumulated in that pixel.
- The *gain* of the chip, g, is the number of electrons per DN.
- Free electrons can be generated in several ways:
  - — Thermal noise
  - — Electrons liberated by cosmic rays
  - — Photoelectrons from background or scattered light
  - — Photoelectrons from light emanating from the target(s) of interest
- The DN value reported for a pixel is the bias plus the free electrons (divided by g), integrated over the surface area of the pixel and over the time of the exposure

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Image noise

- All free electrons are generated in random processes which obey Poisson statistics
  - The measurement error for a given number of electrons is

$$\sigma_{e^-} = \sqrt{e^-}$$

  - The measurement error *measured in DN* is

$$\sigma_{\mathrm{DN}} = \sqrt{g\,\mathrm{DN}_{e^-}}$$

    where DN_{e^-} is that portion of the measured DN attributable to electrons (*i.e.*, after the bias is subtracted)

- The on-chip electronics has its own read noise, usually measured in e^-

- The A/D converter produces an integer value for its output DN. The *quantization noise* is 1/12 DN.

- The bias level itself may vary with time, producing additional noise

- The total σ_{DN} is the RSS of all of these contributions

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Point spread function

- For a variety of reasons (diffraction, errors in fabrication, design decisions), a point source produces a *point spread function* or PSF which is not a delta function.
  - Geometric optics: light rays map onto points in the focal plane (ray tracing). A bundle of incoming parallel rays from a light source at infinity need not converge to one point on the detector.
  - Physical optics: diffraction effects. An entrance aperture of diameter d produces an *Airy pattern* of width approximately λ/d radians or $f\lambda/d$ mm in the focal plane.

- A well-designed camera has a pixel size well matched to the *full width at half maximum* (FWHM) of the PSF:
  - Pixels too big $\rightarrow$ you lose information that is present in the focal plane
  - Pixels too small $\rightarrow$ the information content per pixel decreases

- We need the PSF to "light up" more than one pixel in order to obtain the *centroid* of an image to subpixel accuracy

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Centroiding: stars and small objects

- For best results, need $S/N > 7$, FWHM ~ 1 pixel, low smear

- Two techniques for centroiding star images:
  - "Box filter": convolve the actual image with a pixellated PSF, then interpolate the convolution results to find the maximum response
  - "DN filter": fit a PSF, represented as an analytic funtion, to the actual image, solving for brightness, background, etc., as well as (x, y)

- Accuracy varies with star brightness:
  - Overexposed stars are often unusable
  - Well-exposed stars are best (about 0.05–0.20 pixel)
  - Faint stars with $S/N < 3$ also have poor centroids (about 0.5 pixel)

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Centroiding: stars and small objects

- It doesn't matter much what PSF is used, as long as it's circularly symmetric and is monotonically decreasing with increasing radius
  - Gaussian, $\exp(-r^2/2\sigma^2)$; $\mathrm{FWHM} = 2\sqrt{\ln 4}\sigma$
  - $1/(1 + (r/r_0)^2)$; $\mathrm{FWHM} = 2\mathrm{r}_0$

- Even if the actual PSF varies across the field, fitting with a constant PSF is OK. Systematic differences in the centroid caused by a PSF mismatch can usually be absorbed in the camera calibration constants.

- If the camera is rotating during the exposure, the image will be trailed.
  - Convolve the PSF with a line segment, obtaining a smeared PSF, and use that to fit the images.
  - Solve for the line segment length and direction, which are constant for all images in the picture.
  - This is a nonlinear problem, so iteration is necessary.

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Centroiding: resolved objects

- Three techniques for centroiding extended bodies:
  - "Limb scans"
  - "Jailbars"
  - DN fitting

- Centroiding typically good to about 0.25 to 0.5 pixel

- Accuracy can degrade as the image gets bigger; there is usually a minimum uncertainty *in kilometers* due to topography

- Systematic errors can arise from mismodeling of surface characteristics

- Surface model determination is often done off-line from operations

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Centroiding: limb scans

- Interpolate DN array along a radial "scan line" from the assumed center. This yields a one-dimension array of interpolated DN values.

- Model the expected brightness using some appropriate phase law convolved with-the PSF

- Slide the modeled brightness along the actual DN array to locate the limb or terminator along the scan line

- Fit a projected triaxial ellipsoid to the set of limb and terminator points thus determined

- Iterate until converged

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Centroiding: jailbars

- Some spacecraft (Galileo, MESSENGER) return only pieces of an opnav picture in order to reduce data volume (bits to ground)

- Onboard image analysis:
  — locates the image of the desired target
  — extracts every nth row or column within the image
  — transmits only the extracts

- Opnav processing is like limb scans, but the scan lines are always either horizontal or vertical; no interpolation is required

- Good at determining the target coordinate in the direction of the scans

- Not so good at determining the target coordinate perpendicular to the scans

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Centroiding: DN fitting

- Model the shape as a sum of spherical harmonics:

$$r(\lambda, \varphi) = \sum_{n=0}^{N} \sum_{m=0}^{n} (A_{nm} \cos m\lambda + B_{nm} \sin m\lambda)\, P_n^m(\sin \varphi)$$

- Model the albedo similarly

- Model the reflectance law and phase function (Hapke, or Minnaert, or Lambert, or ...)

- Compute the expected DN in each pixel

- Compute $\partial \mathrm{DN} / \partial (s, l, \text{model parameters})$

- Perform a least-squares fit and iterate

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Centroiding: landmarks

- A *landmark* is in essence the tip of a vector which is constant in body-fixed coordinates—a fiducial point

- In operations, a landmark should be easy to identify, impossible to misidentify, and its observed image coordinates should not depend on the viewing geometry

- For NEAR:
  - Drag the mouse (freehand!) around a crater rim
  - Software fits an ellipse to these points
  - The center of the ellipse is the direction to the landmark
  - The landmark lies on the crater's axis, in the plane of the rim

- For future missions:
  - Construct a model of the surface (or portion thereof) using photoclinometric techniques
  - Construct the predicted brightness in each pixel
  - Box filter the actual picture to get $(\Delta s, \Delta l)$

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Camera calibration

- Take a set of pictures of a dense star field (an open cluster about the size of the camera's field of view is best)

- Mosaic with lots of overlap:
  - Minimum of 5 pictures in an $\times$ pattern
  - 3×3 mosaic with $1/4$ FOV between centers is better

- Overall scale (f) from comparing pixel separation to angular separation

- Higher-order terms from relative image movement from one picture to the next

- Solution parameters, in increasing level of complexity:
  - f
  - $\epsilon_r, \epsilon_M, \epsilon_N$
  - K_{yx}, K_y
  - s_0, l_0

- Always solve for camera pointing (three angles, stochastic)

- Do not solve for K_x or K_{xy}

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