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The Blocking Probability of Geiger-Mode Avalanche Photo-diodes

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Abstract

When a photon is detected by a Geiger-mode avalanche photo-diode (GMAPD), the detector is rendered inactive, or blocked, for a certain period of time. In this paper we derive the blocking probability for a GMAPD whose input is either an unmodulated, Bernoulli modulated or pulse-position-modulated (PPM) Poisson process. We demonstrate how the PPM and Bernoulli cases differ, illustrating that the PPM blocking probability is larger than the Bernoulli. The blocking rates may be decreased by focusing the incident light on an array of detectors. We show that the binomial output statistics of an array of GMAPDs may be modeled as Poisson and measure the error in this approximation via the relative entropies of the two distributions.

1 Introduction

An optical communications channel essentially consists of a modulated optical transmitter, the optical channel, and an optical receiver. The receiver focuses the incident optical field onto a detector and processes the detector output to extract the transmitted information. In an idealized system, the arrival of photons at the detector may be modeled as a Poisson point process. An ideal detector for this signal accurately records each incident photon, reproducing the Poisson process exactly at its output. A Geiger-mode avalanche photo-diode (GMAPD), e.g. [1], presents one method to efficiently count photons. When a photon is absorbed by a GMAPD, the detector produces a large pulse at its output, which, being easily distinguished from thermal background noise, may be detected and mapped to a photon count. However, after a photon is detected the GMAPD is rendered inactive, or blocked, for a certain period of time. This recovery time is required to flush the electrical charge from the avalanche structure. The detector produces no response to incident photons during the blocking period, reducing the efficiency of the detection process[1].

In this paper we derive the blocking probability for a GMAPD for various modulations of the source. We begin in Section 2.1 with the simplest case, that of an unmodulated Poisson input to the detector. In Section 2.2, we consider a Bernoulli modulated source, and in Section 2.3, a pulse-position modulated (PPM) source. We show in Section 3 that for the same duty cycle the Bernoulli and PPM blocking probabilities agree at low signal power but diverge as signal power increases, with the PPM blocking rates being higher than the Bernoulli.

The deleterious effect of blocking can be mitigated by focusing the incident signal on an array of GMAPDs. Assuming the same signal power is now spread over an array, the probability that a photon is incident on a blocked detector is reduced, increasing the effective detection efficiency. In Section 4 we look at the output statistics of the sum of the array elements. We show that the array output statistics are binomial and may be approximated as a Poisson process whose mean is the total mean photon flux incident on the array, scaled by the probability that a detector is not blocked.

Concurrent analysis observed the appropriateness of the Poisson approximation[2] and showed a low rate approximation to the unmodulated blocking probability[3]. In this paper exact expressions for the blocking probability

[1] D. Boroson, "The communications penalty of refresh times in Geiger-mode detectors", Internal memorandum, Lincoln Laboratory, April, 2004.

[2] V. Vilnrotter, "Approximate probability density of the output of a GMAPD array", Internal memorandum, Jet Propulsion Laboratory, January, 2004.

[3] V. Vilnrotter, "The effects of non-zero recovery time on GMAPD detector arrays," Internal memorandum, Jet Propulsion Laboratory, January, 2004.

are derived for unmodulated, memoryless modulated, and PPM modulated sources. We illustrate the accuracy of the Poisson approximation via the relative entropy and show the rate of convergence of the approximation for the unmodulated case.

2 Blocking probability for a single detector

To simplify analysis, we divide time into discrete slots of some fixed duration and assume the detector is either blocked or not blocked for the duration of each slot. We assume that all incident photons are absorbed, i.e., a 100% quantum efficiency, and that only incident photons may trigger a detector output, i.e., no dark noise events.

Let $X(n)$ denote the number of photons incident on a GMAPD detector during the nth time slot, L the number of slots the detector is blocked (inactive) after a slot in which a photon is absorbed, and $S(n)$ the state of the detector at the beginning of slot n.

$S(n)$ denotes the remaining time, in slots, that the detector will be blocked. Hence $S(n)$ takes values in $\{0, 1, \ldots, L\}$ with $S(n) = 0$ indicating that the detector is not blocked and $S(n) = i > 0$ indicating that that detector is blocked at the beginning of the nth slot. When $S(n) = i > 0$, the detector remains blocked throughout the duration of slots $n, n+1, \ldots, n+i-1$, becoming unblocked at the beginning of slot $n+i$. We may model this evolution of the detector state via the Markov model illustrated in Fig. 1, where $q_1 \stackrel{\text{def}}{=} P[S(n) = L|S(n-1) = 0]$ and $q_0 \stackrel{\text{def}}{=} P[S(n = 0)|S(n-1) = 0]$.

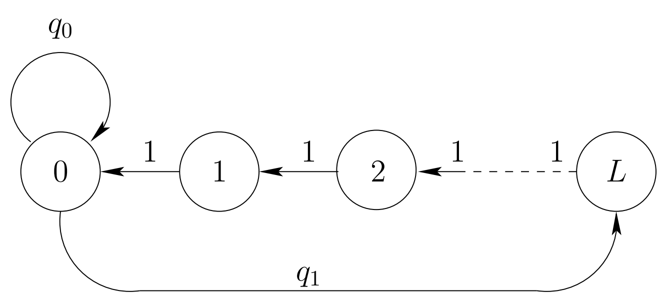

Figure 1: Markov model of detector state

In the following subsections we determine $\mu_j(n) \stackrel{\text{def}}{=} P[S(n) = j]$ under various models for $X(n)$. Of particular interest is $\mu_B(n) \stackrel{\text{def}}{=} 1 - \mu_0(n)$, the blocking probability for the detector. We assume throughout that $X(n)$ is Poisson.

2.1 Unmodulated source

We first consider the simplest case, where $X(n)$ is an unmodulated source. Let $X(n)$ be a Poisson random variable with mean λ, where λ is constant for all n and $X(n)$ and $X(m)$ are independent for $m \neq n$. Since $S(n)$ is a function of $X(n-i), i \geq 1$, $X(n)$ and $S(n)$ are independent–i.e., the number of incident photons during any time interval is independent of the detector state.

Here, $q_0 = P[X(n) = 0] = e^{-\lambda}$ and

$$q_1 = P[X(n) > 0] = 1 - q_0 = 1 - e^{-\lambda}.$$

For $0 < q_0 < 1$, the Markov chain of Figure 1 is aperiodic and irreducible, hence the detector state probabilities converge to steady state values $\mu_j = \lim_{n\to\infty} \mu_j(n)$. The μ_j satisfy

$$\begin{aligned}
\mu_0 &= q_0\mu_o + \mu_1 \\
\mu_1 &= \mu_2 = \cdots = \mu_L \\
\sum_{i=0}^{L} \mu_i &= 1
\end{aligned}$$

Solving for μ_0, μ_B yields

$$\mu_0 = \frac{1}{1+Lq_1} \tag{1}$$

$$\mu_B = \frac{Lq_1}{1+Lq_1} \tag{2}$$

As $\lambda \to \infty$, we have $q_1 \to 1$ and $\mu_0 \to \frac{1}{L+1}$, i.e., the detector cycles through all of the states continuously and is in the unblocked state in one out of every $L+1$ time slots.

2.2 Memoryless modulated source: Bernoulli

We now consider the case where $X(n)$ is modulated by a memoryless source. Let $a(n) \in \{0,1\}$ be a Bernoulli process that modulates the source. A signal is present in the nth slot if $a(n) = 1$, otherwise no signal is present. The mean photon count $\lambda(n) \stackrel{\text{def}}{=} E[X(n)]$ is given by

$$\lambda(n) = \begin{cases} \lambda_s + \lambda_b & \text{if } a(n) = 1 \\ \lambda_b & \text{if } a(n) = 0. \end{cases}$$

where λ_s and λ_b represent signal and background means, respectively.

We assume $a(n), a(m), n \neq m$ are independent and identically distributed with $P[a(n) = 1] = 1/M$. It follows that $X(n)$ and $X(m)$ are independent and, hence, $X(n)$ and $S(n)$ are independent. Hence the analysis follows that of Section 2.1, with a simple change to the transition probabilities.

In this case the transition probabilities q_0 and q_1 in Figure 1 are given by

$$\begin{aligned}
q_0 &= P[X(n) = 0|S(n) = 0] \\
&= P[X(n) = 0] \\
&= P[X(n) = 0|a(n) = 0]P[a(n) = 0] + P[X(n) = 0|a(n) = 1]P[a(n) = 1] \\
&= e^{-\lambda_b}\frac{M-1}{M} + e^{-(\lambda_s+\lambda_b)}\frac{1}{M}
\end{aligned} \tag{3}$$

and

$$\begin{aligned}
q_1 &= 1 - q_0 \\
&= \left(1 - e^{-\lambda_b}\right)\frac{M-1}{M} + \left(1 - e^{-(\lambda_s+\lambda_b)}\right)\frac{1}{M}.
\end{aligned} \tag{4}$$

The probabilities μ_0, μ_B are given by (1), (2), with q_1 given by (4).

2.3 Modulation with memory: PPM

We now extend the analysis to the case where $X(n)$ is an M-ary PPM source. Each PPM symbol consists of M slots, one of which contains a pulse. We assume throughout that the blocking duration is larger than the PPM order, i.e, $L > M$. Let $a_k(n)$ be the sequence of binary slot-values at the output of an M-ary PPM mapping, with $a_k(n) \in \{0,1\}$, $n \in \{1,\ldots,M\}$, denoting the transmitted value in the nth slot of the kth PPM symbol. We assume the PPM symbols are drawn independently and with equal likelihood.

In an analogous manner, let $S_k(n)$ denote the state of the detector and $X_k(n)$ the number of photons incident on the detector in the nth slot of the kth symbol. Thus, $X_k(n)$ is Poisson with mean

$$\lambda_k(n) = \begin{cases} \lambda_s + \lambda_b & \text{if } a_k(n) = 1 \\ \lambda_b & \text{if } a_k(n) = 0. \end{cases}$$

Since the $a_k(n)$ are correlated, $X_k(m)$ and $X_k(n)$ are correlated. Since $S_k(n)$ is a function of the prior photon arrivals, $S_k(n)$ and $X_k(n)$ are correlated, which will be the case for any modulation that contains memory. This correlation complicates the analysis.

The blocking probabilities may be obtained by observing the state of the detector periodically with period M. I.e., we track the state of the detector during the ith slot, stepping in time increments of M slots.

To that end, let $\mu_{k,i}(n) = P[S_k(n) = i]$, the probability that the detector is in state i at the beginning of the nth slot of the kth symbol. We will see that the state probabilities for any slot may be derived from those for another single slot. We will treat the case $n = 1$, the first slot of the PPM symbol.

Let $Q(i,j) = P[S_k(1) = j | S_{k-1}(1) = i]$ be the transition probability matrix for the first slot. Let

$$\begin{aligned} p_{00} &= P[X_k(n) = 0 | a_k(n) = 0] = e^{-\lambda_b} \\ p_{10} &= P[X_k(n) > 0 | a_k(n) = 0] = 1 - e^{-\lambda_b} \\ p_{01} &= P[X_k(n) = 0 | a_k(n) = 1] = e^{-(\lambda_b + \lambda_s)} \\ p_{11} &= P[X_k(n) > 0 | a_k(n) = 1] = 1 - e^{-(\lambda_b + \lambda_s)}, \end{aligned}$$

the conditional probabilities of photons being incident on a detector in slot n.

Consider transitions from the unblocked state. If photons are first observed in the jth slot of the PPM symbol, there will be a transition to state $L - M + j$. The probability of this transition is

$$Q(0, L-M+j) = \begin{cases} p_{00}^{M-1} p_{01} & j = M - L \\ \frac{j-1}{M} p_{01} p_{00}^{j-2} p_{10} + \frac{1}{M} p_{00}^{j-1} p_{11} + \frac{M-j}{M} p_{00}^{j-1} p_{10} & 1 \le j \le M \\ 0 & \text{otherwise} \end{cases}$$

where we use the case $j = M - L$ to denote that no photons are observed during the symbol, in which case we remain in the unblocked state.

Consider transitions from the ith blocked state. If $i \ge M$ the detector remains blocked for the duration of the symbol, hence

$$Q(i,j) = \begin{cases} 1 & i \ge M, j = i - M \\ 0 & i \ge M, j \ne i - M \end{cases}$$

For $i < M$, the detector is blocked for the first i slots, returning to the unblocked state in the $(i+1)$th slot. The case then follows in a similar manner as just considered for the unblocked state with PPM order $M' = M - i$. If photons are first observed in the jth slot after reaching the unblocked state, there will be a transition to state $L - M' + j$. The probability of this transition is

$$Q(i, L-M'+j) = \begin{cases} \frac{M-M'}{M} p_{00}^{M'} + \frac{M'}{M} p_{00}^{M'-1} p_{01} & i < M, j = M' - L \\ \frac{j-1}{M} p_{01} p_{00}^{j-2} p_{10} + \frac{1}{M} p_{00}^{j-1} p_{11} + \frac{M-j}{M} p_{00}^{j-1} p_{10} & i < M, 1 \le j \le M' \\ 0 & \text{otherwise,} \end{cases}$$

where the case $j = M' - L$ corresponds to observing no photons after arriving in the unblocked state. The transition probability to the unblocked state now has a contribution from the all zeros symbol which appears with probability $(M - M')/M$.

Let $\boldsymbol{\mu}_k(1) = [\mu_{k,0}(1), \ldots, \mu_{k,L}(1)]$, the vector of state probabilities for the first slot of the kth symbol. On the graph characterized by Q, there is a path from the unblocked state to any other state and visa-versa, hence the graph is irreducible. Since there is a self-loop, it is also aperiodic. Hence for large k the state probabilities $\boldsymbol{\mu}_k(1)$ approach steady state values which we denote by $\boldsymbol{\mu}(1) = [\mu_0(1), \ldots, \mu_L(1)]$. $\boldsymbol{\mu}(1)$ is the unique left-eigenvector of Q with eigenvalue 1 that sums to 1:

$$\boldsymbol{\mu}(1)Q = \boldsymbol{\mu}(1), \tag{5}$$
$$\sum_{i=0}^{L} \mu_i(1) = 1.$$

Given $\boldsymbol{\mu}(1)$, we may obtain the detector state probabilities for slots $n = 2, \ldots, M$ as follows. Note that

$$\mu_i(n) = \mu_{i+1}(n-1) \text{ for } 1 \leq i \leq L-1,$$

hence only $\mu_L(n)$ and $\mu_0(n)$ remain to be determined. Either is sufficient, since we may obtain the other from the constraint $\sum_{i=0}^{L} \mu_i(n) = 1$. Since $\mu_i(n)$ is periodic in i with period M for $i > 0$, i.e., $\mu_i(n) = \mu_{i+M}(n)$ for $0 < i \leq L - M$, we have $\mu_L(n) = \mu_{L-M}(n)$.

In the general case the blocking probability is a function of the slot position in the PPM symbol. However, it follows from the periodicity of $\mu_i(n)$ that if L is a multiple of M then $\mu_L(n) = \mu_1(n-1)$ and

$$\sum_{i=1}^{L} \mu_i(n) = \sum_{i=1}^{L} \mu_i(n-1)$$

Hence the blocking probabilities μ_B, which are given by the above sums, agree for all slot positions.

3 Comparison of PPM and Bernoulli source blocking probabilities

As we have shown, computing the blocking probability for a PPM modulated source is more involved than for a Bernoulli source. We may, however, approximate the PPM blocking probability with the Bernoulli, keeping the duty cycle constant. In this section we show the accuracy of this approximation, and argue that the PPM blocking probability is always larger than the Bernoulli.

Figure 2 illustrates μ_B as a function of λ_s for $M = 64$, $L = 100$, $\lambda_b = 0.01$. The blocking probability for a Bernoulli source is given by (2), where q_1 is given by (4). The PPM blocking probabilities are given by solving (5) and the comments that follow. Since L is not a multiple of M, the blocking probability is a function of the slot position $n \in \{1, 2, \ldots, 64\}$

For small λ_s, the blocking probability is dominated by the noise slots and the Bernoulli and PPM cases converge (as would any modulation scheme) to

$$\lim_{\lambda_s \to 0} \mu_B = \frac{Lp_{10}}{1 + Lp_{10}}$$
$$\approx \frac{L\lambda_b}{1 + L\lambda_b}$$

where the approximation holds for small λ_b. As λ_s increases, the Bernoulli case approaches the asymptotic value

$$\lim_{\lambda_s \to \infty} \mu_B = \frac{L(p_{10}(M-1)+1)}{M + L(p_{10}(M-1)+1)}$$

The PPM blocking probability grows more rapidly in λ_s than the Bernoulli, achieving a larger asymptotic value. Although we don't have a closed form expression for the PPM case, this behavior can be seen as follows. Suppose the detector has just entered the unblocked state. As the signal slot mean increases, the probability that the arrival of a signal slot will block the detector increases. For a Bernoulli source, the mean time until a signal slot appears is

$$\sum_{i=1}^{\infty} \frac{i}{M} \left(\frac{M-1}{M}\right)^{i-1} = M$$

Whereas with PPM the mean waiting time given the detector has become unblocked in the nth slot of the symbol, $n \in \{1, 2, \ldots, M\}$ is

$$\begin{aligned} \sum_{i=n}^{M} \frac{i-n+1}{M} + \sum_{i=1}^{M} \frac{(M-n+1+i)(n-1)}{M^2} \\ = \frac{M^2-(n-1)^2+Mn}{2M} \\ < M \end{aligned} \tag{6}$$

Hence we expect the blocking probability to grow more rapidly with PPM. Since (6) also gives the mean time since the last pulse, it roughly predicts which slot position n is most likely to be blocked, second-most likely to be blocked, etc., based on the relative values of (6)–a smaller mean time correlating with a larger blocking probability. This slot position dependence of the blocking probability is illustrated in Figure 2. The exact order of the asymptotes is also a function of L and higher order moments. Figure 3 illustrates the largest asymptotic values from Figure 2.

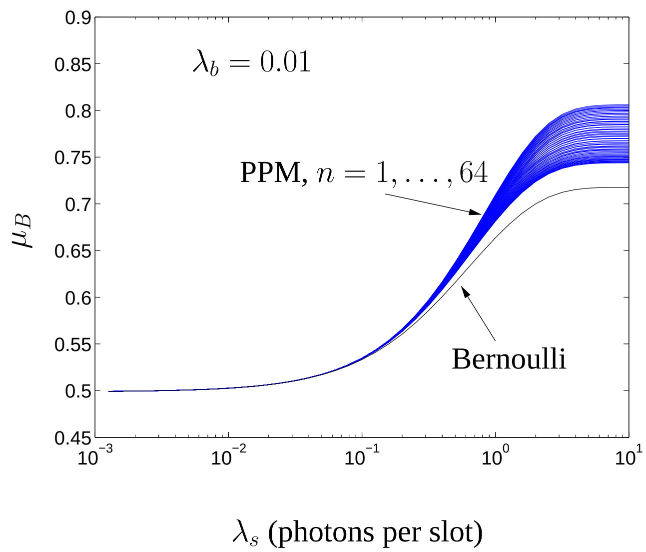

Figure 2: Bernoulli and PPM blocking probabilities, $M = 64, L = 100, n \in \{1, \ldots, 64\}$, Poisson channel, $\lambda_b = 0.01$

Figure 4 illustrates the blocking probability in linear and log-domain for the Bernoulli and PPM cases with $M = 64$, $L = 128$ and a range of λ_b. Here we've chosen L to be a multiple of M so that the PPM slot blocking probabilities agree in order to show the behavior as a function of λ_b. Over a range of incident background rates, PPM orders and blocking durations we begin to see the PPM and Bernoulli cases deviate for $\lambda_s > 0.1$.

4 Output statistics for an array of detectors

One method to combat blocking is to focus the same incident light power on an array of detectors. This lowers the incident power per detector, decreasing the blocking probability, which is monotonic in the incident power. The sum of the detector outputs will approximate the input process. In this section, we look at the output statistics from an array with N detectors. To simplify analysis, we assume the source is unmodulated.

Let $S^{(i)}$ denote the state, $X^{(i)}$ the input and $Y^{(i)}$ the output of the ith detector during the nth slot. Let λ denote the mean incident photons per slot for the entire array. We assume that for all i, $X^{(i)}$ is Poisson with mean $\lambda_i = \lambda/N$.

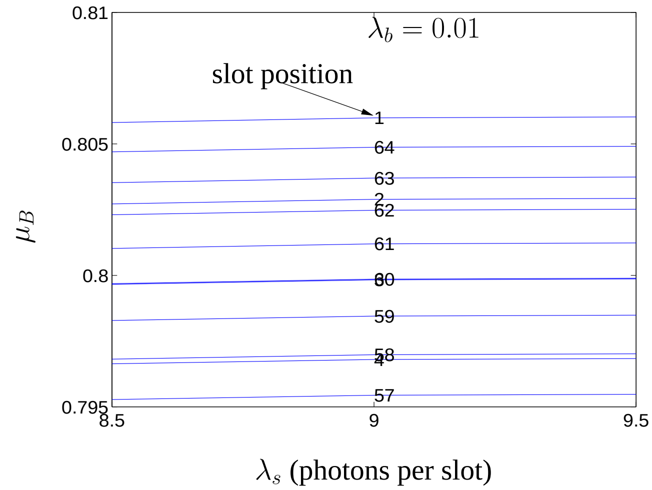

Figure 3: PPM blocking probability asymptotes, $M = 64, L = 100$, Poisson channel, $\lambda_b = 0.01$

The output of a detector is one if the detector is unblocked and photons are incident; otherwise the output is zero. Hence we have

$$Y^{(i)} = \begin{cases} 1 & \text{if } S^{(i)} = 0 \text{ and } X^{(i)} > 0 \\ 0 & \text{otherwise} \end{cases}$$

and $Y^{(i)}$ is Bernoulli distributed with

$$\begin{aligned} p &= P[Y^{(i)} = 1] \\ &= P[X^{(i)} > 0, S^{(i)} = 0] \\ &= \mu_0 P[X^{(i)} > 0] \\ &= \frac{q_1}{1 + Lq_1} && (7) \\ &= \frac{1 - e^{-\lambda_i}}{1 + L(1 - e^{-\lambda_i})} && (8) \end{aligned}$$

We assume the incident photon counts $X^{(i)}$ and $X^{(j)}$, $i \neq j$, are sums of events from non-overlapping increments of a Poisson point process and, hence, independent. Since $S^{(i)}$ and $S^{(j)}$ are functions of independent processes, they are also independent, as are $Y^{(i)}$ and $Y^{(j)}$. Hence, the output of the N-element array,

$$Z_N = \sum_{i=1}^{N} Y^{(i)},$$

is a binomial random variable with

$$P_{Z_N}(k) = P[Z_N = k] = \binom{N}{k} p^k (1-p)^{N-k}$$

It is well known, e.g., [2], that when N is large the binomial distribution is closely approximated by a Poisson with the same mean. Hence we may approximate Z_N as Poisson with mean Np. Here we quantify the accuracy of that approximation.

Let P_λ be the Poisson distribution with mean λ. What is an appropriate measure of the closeness of P_{Z_N} to P_λ? We will measure the similarity via the relative entropy

$$D(P_{Z_N} || P_\lambda) = \sum_{k=0}^{\infty} P_{Z_N}(k) \log \frac{P_{Z_N}(k)}{P_\lambda(k)} = \sum_{k=0}^{N} P_{Z_N}(k) \log \frac{P_{Z_N}(k)}{P_\lambda(k)}$$

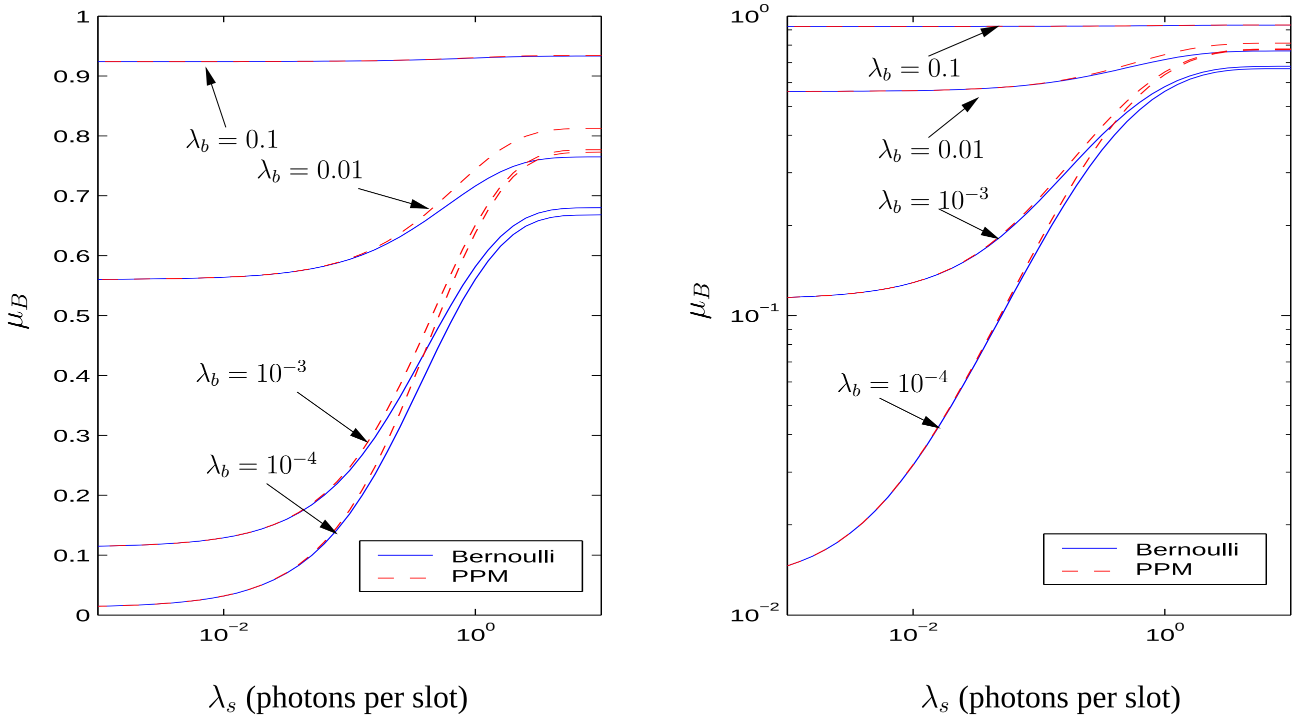

Figure 4: Bernoulli and PPM blocking probabilities, linear and log-domain, $M = 64, L = 128$, Poisson channel, $\lambda_b \in \{0.1, 0.01, 0.001, 0.0001\}$

where we truncate the summation since $P_{Z_N}(k) = 0$ for $k > N$, using the convention $0 \log 0 = 0$ ($\lim_{x\to 0} x \log x = 0$). The relative entropy is a natural measure of the distance between the distributions. For example, the mean log-likelihood-ratio, an optimum decision criterion for statistical tests, may be expressed in terms of the divergence, e.g., [3]. Moreover, a bound on the relative entropy yields a bound on the L_1-distance via Pinsker's inequality

$$D(P_{Z_N}||P_\lambda) \geq \frac{1}{2}||P_{Z_N} - P_\lambda||^2 = \frac{1}{2}\left(\sum_{k=0}^{\infty} |P_{Z_N}(k) - P_\lambda(k)|\right)^2$$

which assigns a more intuitive meaning to the magnitude of the dissimilarity. Figure 5 illustrates $D(P_{Z_N}||P_{Np})$ for the unmodulated source as a function of λ with $L = 128$.

In [2] the authors show

$$D(P_{Z_N}||P_{Np}) \leq \frac{p^2}{1-p}$$

With constant λ, $p \to 0$ as $N \to \infty$, hence the distribution of Z_N approaches P_{Np}. In the case of an unmodulated source we may approximate (7) by

$$p \approx \frac{1}{L + N/\lambda}$$

for small λ/N. Hence, for large N, we have

$$\begin{aligned} D(P_{Z_N}||P_{Np}) &\leq \frac{1}{(L+N/\lambda)(L+N/\lambda-1)} \\ &\leq \left(\frac{\lambda}{N}\right)^2 \end{aligned}$$

and the divergence goes as $(\lambda/N)^2$.

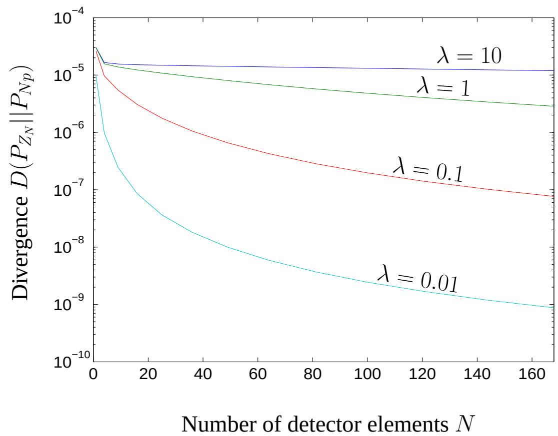

Figure 5: Divergence $D(P_{Z_N}||P_{Np})$, $L = 128$

4.1 Scaled mean approximation

For constant λ, as $N \to \infty$, we have $p \to 0$ and $Np = \lambda$, and the output of the array equals the input. For finite, but sufficiently large, N, we've seen Z_N is well-approximated by a Poisson distribution with mean Np. Using $N(1 - e^{-\lambda/N}) \approx \lambda$, the mean may be approximated by $Np \approx \lambda\mu_0$, which is a simple, intuitive, scaling of the photon arrival rate by the nonblocking probability of a single element. Figure 6 illustrates how close $\mu_0\lambda$ is to Np as a percentage of Np and Figure 7 illustrates the distance between the corresponding relative entropies.

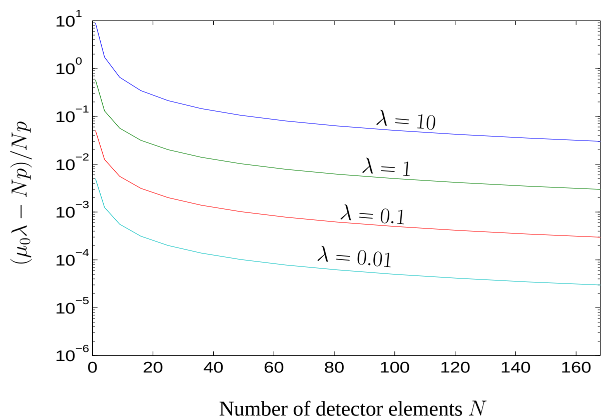

Figure 6: Closeness of mean approximation: $(\mu_0\lambda - Np)/Np$

4.2 Correlated detectors: modulation, turbulence

Several mechanisms may cause the outputs of the detectors in the array to be correlated. For example, if the source is modulated, the means of the incident rates will be given by an identical random process. Another source of correlation arises from atmospheric turbulence. Turbulence induces correlated 'hot spots' where the signal is focused on a subsection of the array.

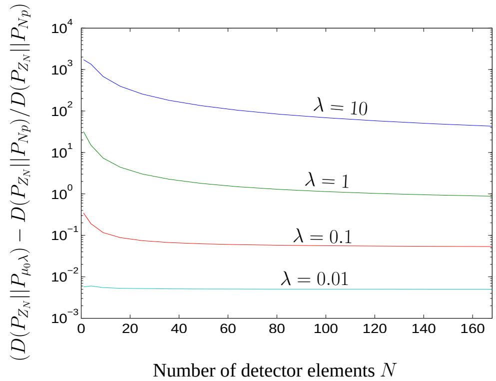

Figure 7: Closeness of relative entropy approximation: $(D(P_{Z_N}||P_{\mu_0\lambda}) - D(P_{Z_N}||P_{Np}))/D(P_{Z_N}||P_{Np})$

As shown in [2], the sum of the detector outputs, even when correlated and not identically distributed, may be approximated as Poisson so long as no one detector dominates the sum and the outputs are not strongly dependent. Hence, under these assumptions, the array output may be modeled as Poisson with mean $\sum_{i=1}^{N} E[Y^(i)]$, the sum of the means of the correlated Bernoulli random variables corresponding to the outputs of the detector elements.

5 Conclusions

We have illustrated how to compute the blocking probabilities for a GMAPD with either an unmodulated or modulated source. Extending the PPM modulated results to other modulations would be straightforward. We quantifed the accuracy of approximating the output of an array of GMAPDs, with an unmodulated source, as Poisson, showing this approximation is accurate even for a small number of detectors.

Although the blocking probabilities for the PPM and Bernoulli modulated sources diverge, that divergence only occurs when the blocking probability is large. In practical applications, a GMAPD array should utilize a number of detectors sufficiently large as to make the blocking probability small. In this case the simpler Bernoulli approximation is accurate, although care should be taken since the Bernoulli approximation is a lower bound to the PPM blocking probability.

Any real-world optical communications channel would be modulated and experience fading such that the outputs of the array would be correlated. However, it remains reasonable to approximate the array output as Poisson. The error in this approximation may be quantified applying the results on [2].

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