Estimating SBAS Ionospheric Delays Without Grids: The Conical Domain Approach

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ABSTRACT

To ensure the accuracy and integrity of user position estimates based upon Global Positioning System (GPS) measurements, satellite-based augmentation systems (SBAS) require accurate calibration of ionospheric delays. In an SBAS such as the United States' Wide Area Augmentation System (WAAS), slant ionospheric delay error and confidence bounds are derived from interpolated estimates of vertical ionospheric delay modeled on a grid at regularly-spaced intervals of latitude and longitude. Estimation of the vertical delay at each ionospheric grid point relies on the standard thin-shell model of the ionosphere to project measured slant delays to vertical.

This paper presents an alternative model of slant delay measurements that allows direct computation of the user's slant delay estimate without the intervening use of a vertical delay grid. Instead of evaluating fits at points on a grid, each fit is performed in a conical domain with a satellite at the cone vertex. The objective of this approach is to reduce or eliminate sources of error that arise both from the use of the thin-shell model and from the grid interpolation. The effort to develop this approach is motivated, in part, by the need to develop alternatives for modeling slant delays at low latitudes, where complex ionospheric structure, high delay values, and large electron density gradients pose severe challenges for SBAS systems based upon the thin-shell model. We present results comparing the accuracy of the conical domain approach with that achieved using the standard thin-shell model.

INTRODUCTION

Satellite-based augmentation systems (SBAS) for airline navigation are currently under development worldwide. In the United States, the Wide Area Augmentation System (WAAS) was commissioned on July 10, 2003. Other systems at various stages of development and implementation are the European Geostationary Navigation Overlay Service (EGNOS) in Europe and the MTSAT Satellite-Based Augmentation System (MSAS) in Japan, as well as programs in India, China, and Brazil.

The primary goal of SBAS is to enhance the accuracy and integrity of user position estimates based upon Global Positioning System (GPS) measurements. In the absence of selective availability, the ionosphere is the largest source of error for single-frequency users of GPS. Estimating ionospheric delay is an inherently four-dimensional problem, since each GPS slant delay measurement depends upon the position and orientation of its raypath. For example, we can identify the raypath position by two coordinates that locate the ionospheric pierce point (IPP), *i.e.*, the point where the user's raypath intersects a reference ionospheric height, and we can identify the raypath orientation by two angles that specify the direction of the ray at that point.

An SBAS such as the United States' Wide Area Augmentation System (WAAS) derives slant ionospheric delay error and confidence bounds from estimates of vertical ionospheric delay modeled on a grid at regularly-spaced intervals of latitude and longitude. The vertical delay estimate at each ionospheric grid point (IGP) is calculated from a planar fit of neighboring slant delay measurements, projected to vertical using the standard thin-shell model of the ionosphere (see, for example, Mannucci *et al.*, 1999, Birch *et al.*, 2002). Interpolation on the WAAS grid permits estimation of the vertical delay at the IPP of any arbitrary user measurement. The product of the interpolated value and the user's thin-shell obliquity factor provides an estimate of the user's ionospheric slant delay.

Despite its well-known limitations, the thin-shell model of the ionosphere remains popular chiefly due to its simplicity. It reduces a four dimensional problem to two dimensions by relying on the rather crude approximation that the electron density is non-negligible only in the vicinity of a fixed reference height. Two types of error restrict the accuracy of such a model: (1) error due to the implicit assumption that the electron density is azimuthally symmetric near the IPP, and (2) error arising from the slant-to-vertical conversion (*e.g.*, error due to a suboptimal choice of shell height). At mid-latitudes under nominal conditions, the magnitude of error stemming from these sources is small. At low latitudes or at mid-latitudes under disturbed conditions, the accuracy of SBAS systems based upon the thin-shell model suffers due to the presence of complex ionospheric structure, high delay values, and large electron density gradients.

This paper describes an alternative model of slant delay measurements that permits direct computation of the user's slant delay estimate without the intervening use of a vertical delay grid. The objective of this approach is to reduce or eliminate sources of error that arise both from the use of the thin-shell model and from the grid interpolation. The key is to restrict each fit of GPS measurements to a spatial domain encompassing signals from only one satellite. A group of receivers that all look at the same satellite define a cone whose vertex is the position of the satellite; hence, we designate our approach to be a *conical domain* model. An SBAS based on this model replaces fits performed on a grid of IGPs with fits performed within cones that have satellites at the vertices. A user within a given cone evaluates the delay to the satellite directly, using the IPP coordinates of the line-of-sight to the satellite and the broadcast fit parameters associated with the cone.

In the sections that follow, we first review grid-based estimation of ionospheric delay as performed by WAAS. In particular, we review how the thin-shell model is used and the estimation errors that arise as a consequence. We then present the conical domain model and discuss how an SBAS using cone-based estimation might operate. We also examine briefly a hybrid model that combines some of the advantages of both the thin-shell and conical domain models. Finally we report the results of a study comparing the accuracy of the conical domain approach with that achieved using the thin-shell model. We assess accuracy using data sets from both quiet days and days when the ionosphere is disturbed. Also, we examine the dependence of accuracy on latitude by comparing results based upon data sets from networks in Brazil and the United States.

GRID-BASED ESTIMATION OF IONOSPHERIC DELAY

In this section we review how the thin-shell model is currently used in WAAS and the ionospheric modeling errors that arise as a consequence. We emphasize the fact that there is no limiting sense in which fits based on the thin-shell model necessarily converge to the correct solution.

The thin-shell model

GPS signal delay due to the ionosphere is proportional to the number of free electrons along the signal's raypath. The total electron content (TEC) along the raypath may be written as a path integral of electron density along the line-of-sight from the receiver to the satellite:

$$T_{rs} = \int_{\ell_r}^{\ell_s} d\ell \, n_e(\vec{x}(\ell)) \qquad (1)$$

where the subscripts r and s identify, respectively, the receiver and satellite in question, and n_e is the electron density. This integral can be rewritten as an integral over altitude h:

$$T_{rs}(\alpha,\beta,\theta,\phi) = \int_{h_r}^{h_s} dh M(\alpha,h) n_e(h,\psi(h;\alpha,\beta,\theta,\phi)) \quad (2)$$

where

$$M(\alpha,h) \equiv \left[1 - \left(\frac{R_e \cos\alpha}{R_e + h}\right)^2\right]^{-1/2}, \qquad (3)$$

is a geometric factor, R_e is the earth radius, and $\psi(h;\alpha,\beta,\theta,\phi)$ defines the raypath: four parameters are required to specify the raypath uniquely at the ionospheric pierce point, for example:

α --> elevation angle;

β --> azimuthal angle;

θ --> IPP latitude;

ϕ --> IPP longitude.

Thus, solving for slant delay is an inherently four-dimensional problem.

The thin-shell model reduces this four-dimensional problem to two dimensions by introducing the simplifying approximation that the ionosphere is non-negligible only in the neighborhood of a specified reference height h_{iono} (see Fig. 1). Mathematically, this is equivalent to setting h equal to h_{iono} in eq. (3), which allows $M(\alpha, h_{iono})$ to be pulled outside the integral:

$$T_{rs} = M(\alpha, h_{iono}) \cdot V_{rs} \qquad (4)$$

$M(\alpha, h_{iono})$ is designated the thin-shell obliquity factor; it allows an estimate of the vertical delay V_{rs} at the ionospheric pierce point (IPP) to be inferred from a measurement T_{rs} of slant delay.

Slant-delay estimation in WAAS

In the Wide Area Augmentation System (WAAS), slant ionospheric delay error and confidence bounds are derived from estimates of vertical ionospheric delay modeled at points on a grid of latitudes and longitudes. These points, designated ionospheric grid points (IGPs), are spaced uniformly at 5° intervals in the conterminous United States and at 10° intervals in Alaska. To obtain the vertical delay estimate at a given IGP, the thin-shell model is first used to convert a set of slant delay measurements in a given epoch to estimates of the vertical delay at the associated IPPs (see Fig. 2). A planar fit of these values provides the vertical delay estimate at the IGP. All measurements having IPPs that lie within a minimum distance R_{min} of the IGP are included in the fit. If the number of such measurements is less than N_{pts}, the fit radius is expanded until it defines a circle that surrounds N_{pts} points. If the fit radius attains a maximum value of R_{max} and still fails to encompass N_{pts} points, the fit is performed with fewer points, provided that at least N_{min} lie within R_{max}.

Once a vertical delay estimate has been determined at each IGP, the vertical delay at any arbitrary location may be calculated by bi-linear interpolation. The ionospheric slant delay for any WAAS user is estimated by using the thin-shell model a second time, that is, by calculating the product of the user's thin shell obliquity factor and the vertical delay estimate interpolated at the user's ionospheric pierce point (IPP).

Sources of ionospheric modeling error

Under nominal (quiet) ionospheric conditions at mid-latitudes, the WAAS algorithm has been found to provide ionospheric delay estimates of high accuracy. In the presence of significant ionospheric irregularities, however, the thin shell model becomes a major source of estimate error. Such irregularities can arise at mid-latitudes in the presence of disturbed conditions, e.g., in response to the impact of a solar coronal mass ejection upon the earth's magnetosphere. At low latitudes, complex ionospheric structure and sharp gradients in electron density are quotidian features of the ionosphere, due, in part, to the presence of the equatorial (Appleton) anomaly.

Two types of error restrict the accuracy of the thin-shell model. First, the thin-shell approximation implies that the electron density is azimuthally symmetric near the IPP. In other words, the thin-shell obliquity factor will correctly map slant delay to vertical delay only if the slant delay is independent of the azimuthal angle at the IPP. In the thin-shell model, distinct measurements, which share a common IPP, can produce incompatible estimates of vertical delay at the IPP.

There exists a second source of error that arises from the thin-shell model, even when the ionosphere is indeed azimuthally symmetric with respect to the IPP. Since the thin-shell obliquity factor varies monotonically with height, there exists a unique shell height that will permit conversion of a given slant delay measurement to the correct vertical delay at the IPP, and, generally, this height is not known prior to the fit. Thus, delay error can arise due to a suboptimal choice of shell height.

In addition to these inherent limitations of the thin-shell model, there is one other source of delay modeling error in the WAAS algorithm, namely, interpolation error. Even if all vertical delay estimates at the IGPs are correct, interpolation on the WAAS grid will generally introduce error when evaluating the vertical delay at the user's IPP, and this error is converted to slant delay error when the user's vertical delay is mapped to slant delay.

Note that there is no sense in which the WAAS algorithm produces a delay estimate that converges to the correct value. In the limit that a user approaches a reference receiver, the WAAS algorithm does not, in general, provide an estimate of slant delay equal to that measured by the receiver. In the limit that reference receivers are clustered near the user (see Fig. 3), a measurement that passes through an irregularity can corrupt the planar fit, causing the interpolated estimate of the vertical delay at the user's IPP to be significantly in error. A similar problem occurs in the limit that the measurement IPPs converge to the user's IPP. (In fact the problem can be worse in this case: if the receivers are positioned as drawn in Fig. 4, the planar fit will not even correctly determine the direction of the gradient in the vertical delay.) Consequently, reducing the size of the fit radius, introducing a finer IGP mesh or establishing a denser network of receivers will not necessarily mitigate errors arising from the thin shell approximation.

CONE-BASED ESTIMATION OF IONOSPHERIC DELAY

In this section we describe an alternative to the thin-shell model that retains its simplicity but eliminates sources of modeling error. An important feature of this approach is that the slant delay estimate formally converges to the correct value. An SBAS based upon this model will replace fits performed at IGPs with fits performed within conical spatial domains, each having a satellite at the vertex.

The conical domain model

The central idea behind the conical domain approach is that a user looking at a given satellite from an arbitrary location determines the ionospheric delay from a fit of GPS measurements restricted to signals from the *same* satellite. The domain of the fit may be characterized as a measurement cone, where the positions of a pre-selected set of neighboring receivers define the base of the cone, and the vertex is located at the satellite (see Fig. 5). A user, whose line-of-sight to the satellite lies within this cone, can evaluate the slant delay directly according to the position of the raypath. No interpolation on a grid is needed.

The simplest way to parameterize the fit is to solve directly for slant delay and its gradients with respect to a reference raypath. Once the position of the satellite is specified, the raypath of any signal emanating from the satellite is uniquely determined by identifying the location of its IPP. A reference raypath may be defined, for example, as the raypath that passes through the centroid of the IPPs associated with the raypaths to the reference receivers in the cone. In a planar fit model that solves for slant delay directly, each measurement can be approximated as:

$$T_{rs} = T_{rs0} + A_{rs0}(x - x_0) + B_{rs0}(y - y_0), \qquad (5)$$

where T_{rs0} is the slant delay along the reference raypath and A_{rs0} and B_{rs0} are the gradients of the slant delay in the x and y directions, respectively. Here x and y represent two position coordinates that define the distance separating an arbitrary IPP location from the reference IPP. Examples include latitude and longitude, great circle distance coordinates, and Euclidean distance coordinates. In the analysis reported in this paper, we use Euclidean distance coordinates.

An alternative parameterization of the conical domain fit strives to factor out the geometry of the measurement raypaths prior to the fit. A planar fit model that follows this approach approximates each measurement as follows:

$$T_{rs} = M(\alpha, h_{iono}) \cdot \left[V_{rs0} + A_{rs0}(x - x_0) + B_{rs0}(y - y_0)\right] \quad (6)$$

where $M(\alpha, h_{iono})$ is the thin-shell obliquity factor, V_{rs0} is the equivalent vertical delay of the slant delay measurement, and A_{rs0} and B_{rs0} are the gradients of the equivalent vertical delay in the x and y directions as defined above. This formulation retrieves fit parameters by assuming linear deviations of the electron density from local spherical symmetry. (Note that when the ionosphere is spherically symmetric, A_{rs0} and B_{rs0} should vanish.) We use this parameterization of the fit in the analysis discussed below.

In this latter parameterization, the conical domain model bears a superficial resemblance to the thin-shell model. However, an important distinction needs to be made. In the conical domain model, the equivalent vertical delay does not serve as an estimate of the vertical delay of the ionosphere at the measurement IPP. Rather, it should be regarded as the delay that results when all volume elements along the raypath are stacked vertically. Within the pure conical domain model, two different signals that share the same IPP but emanate from different satellites can (and, in general, will) have different equivalent vertical delays. Since neither is treated as an estimate of the vertical delay at the IPP, there is no conflict here, in contrast to the thin-shell model.

The conical domain model also resembles the thin-shell model in that both models reduce an inherently four-dimensional problem to two dimensions. However, the thin-shell model does so by using simplifying assumptions about the ionosphere that introduce error, while the conical domain model makes no assumptions regarding the ionosphere. The conical domain approach attempts to solve a truly two-dimensional problem: once a satellite location has been specified, all signal paths from the satellite may be identified by only two coordinates, *e.g.*, the IPP coordinates. In other words, measurement azimuthal and elevation angles, independent parameters in the thin-shell model, are uniquely determined in the conical domain model by the satellite position and the measurement IPP coordinates. As a consequence, a user's slant delay estimate converges to the correct value (see Fig. 6) in the limit that the receivers converge to the user's location (or, equivalently, in the limit that the measurement IPPs converge to the user's IPP).

In a qualitative sense, the conical domain model represents an approach to addressing the limitations of the thin-shell model that is diametrically opposite to those based on tomography (Blanch *et al.*, 2004, Lejeune *et al.*, 2004). Tomographic approaches generally increase the dimensionality of the problem by moving from slant delay space to electron density space and by solving simultaneously for fit parameters over the entire spatial region of interest. In contrast, the conical domain approach reduces the original four-dimensional problem to a series of local two-dimensional fits, each characterized by a small number of retrieved parameters.

SBAS based upon conical domain approach

An SBAS based upon the conical domain model will differ in several important respects from systems such as WAAS and EGNOS. In particular, such an SBAS would differ significantly from a system that conforms to the standards established in the *Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment* (RTCA, 1996). The most prominent difference is that slant delay error estimates will be derived from fits in a set of conical domains rather than fits at a set of IGPs. This, in turn, will require a redefinition of broadcast message types. Messages that currently transmit to the user the values of parameters associated with fits at each IGP must be modified to provide the values of parameters associated with each conical domain fit.

It should be noted that a grid-based system such as WAAS can provide an estimate of the ionospheric delay along any arbitrary raypath in the atmosphere, that is, it provides solutions to the general four-dimensional problem. This feature can be exploited, for example, to construct maps of vertical delay, an analysis tool that is proving valuable in scientific studies of the ionosphere (see Mannucci *et al.*, 1999). In contrast, a cone-based SBAS will provide delay estimates only for raypaths that emanate from GPS satellites. This is a consequence of reducing the original four-dimensional problem to a set of two-dimensional problems. It is not a serious limitation of cone-based delay estimation, however, since raypaths emanating from GPS satellites are, in general, precisely the raypaths for which delay estimates are sought.

The accuracy and integrity of slant delay estimates in an SBAS based upon the conical domain approach will depend upon the spatial distribution of reference receivers and how they are grouped into cones. The convergence property of the conical domain model argues for employing as many receivers as is practical. In contrast to an SBAS based upon the thin-shell model, more receivers should necessarily lead to higher accuracy. Furthermore, an SBAS based upon the conical domain model can be designed to provide specific regions with enhanced accuracy by increasing the local density of receivers.

Once a network of reference receivers has been established, there will generally exist a tradeoff between accuracy and integrity. Considerations of accuracy favor grouping receivers into many narrow cones, each restricting the spatial extent over which the planar approximation needs to be valid. However, spreading the measurements over many narrow cones will reduce the number of measurements that are used in each fit, and fits based on fewer measurements do not bound the estimate error as reliably, that is, they provide less integrity. Over a region the size of the conterminous United States (CONUS), there would almost certainly need to be multiple cones associated with each satellite. To enhance integrity, cones defined with respect to a single satellite could be designed to overlap.

The hybrid model

This section represents a brief digression from the main thread of our discussion. We have not yet performed a quantitative study of the dependence of delay estimate accuracy on receiver density, either at mid-latitudes or at low latitudes. Such a study might reveal that the density of receivers required to achieve a desired level of accuracy and integrity over a particular region is prohibitively large. In this case, a hybrid model, that combines aspects of both the thin-shell model and the conical domain model, may prove valuable.

The hybrid model augments the conical domain fit with pseudo-measurements based upon delay measurements of signals from neighboring satellites (see Fig. 7). The assumptions of the thin-shell model are retained for purposes of performing slant-to-slant conversion of the measurements associated with the neighboring satellites to the pseudo-measurements associated with the satellite at the cone vertex. The advantage of the hybrid model is that many more observations can be included in the fit. The disadvantage is that these pseudo-measurements are based upon observations that do not lie entirely within the cone and can thereby contaminate the fit if they traverse irregularities that lie outside the cone. Optimal performance may require weighting the pseudo-measurements based upon how closely they are aligned with the axis of the cone. In the hybrid model, as in the thin-shell model, the convergence property is lost.

COMPARISON OF MODELS

In this section we compare the accuracy of the conical domain approach with that achieved by the thin-shell model. Given a fixed set of measurements, we wish to establish whether it is better to determine the user delay by fitting relative few measurements that sample the ionosphere near the user's raypath (the conical domain approach) or by fitting a larger number of measurements that sample a much larger fit domain (the thin-shell approach).

The accuracy of each approach is assessed using a pseudo-user analysis, *i.e.*, each observation in a given data set is treated as a pseudo-user measurement. For every observation. both a thin-shell model fit and a conical domain model fit are calculated, each centered on the pseudo-user IPP in question. When a fit is performed, the pseudo-user measurement is excluded from the fit, and then the value of the measurement is compared to the value predicted by the fit model. The fit parameters used in the study are recorded in Table 1.

	Thin-shell	**Conical domain**
h_{iono}	350 km	350 km
N_{pts}	30	3 - 7
R_{max}	800 – 2100 km	800 – 1500 km
R_{min}	800 km	NA
N_{min}	10	NA

Table 1. Parameters used in the analysis.

We first compare the accuracy of the models using WAAS data sets from days exhibiting different levels of ionospheric disturbance: July 2, 2000, a quiet day, and July 15, 2000, a day on which a major ionospheric storm occurred. Thereafter, we examine data from Brazil to gain some appreciation regarding how the models perform at low-latitudes.

A comparison of model accuracy under quiet and disturbed conditions at mid-latitudes

The data used in the first study consist of WAAS *supertruth* data. Supertruth is based upon data collected by the WAAS network of 25 reference stations spread throughout the continental United States, Alaska, and Hawaii. Supertruth data have been studied extensively in other contexts, and their behavior is well understood. Prior to analysis, these data are post-processed (1) to eliminate interfrequency biases, (2) to remove the effects of cycle slips in carrier phase measurements, (3) to level the carrier phase measurements to the corresponding range measurements, and (4) to filter spurious measurements by taking advantage of the redundancy provided by multiple receivers at each station. The resulting data contain minimal error due to measurement noise and range multipath.

Figure 8 shows the residual error between the pseudo-user estimate and the measured slant delay value for data at 100 second intervals on July 2, 2000. On this day, the ionosphere remained relatively free of disturbance. In each plot, the conical domain residuals are depicted in red, and the thin-shell residuals (nearly invisible) are in blue. Prior to plotting, residuals for both models are converted to equivalent vertical delay using the thin-shell obliquity factor. This serves to remove the geometry and permits the plotting of comparable delay values. Points are plotted only in pairs, *i.e.*, only when both model fit point criteria are satisfied. In other words, if the number of points inside the fit radius for either model is insufficient, no residual is plotted for either model.

Figures 8(a), 8(b), 8(c), and 8(d) show residuals when the fit radius is 800 km and the required number of fit points for the conical domain model is set to 3, 4, 5, and 6, respectively, plotted as a function of geomagnetic local time (in hours). Note that the total number of points plotted in each figure decreases as the number of required fit points rises. This is due to the fact that it becomes progressively more difficult to define a conical domain with the required number of fit points inside 800 km. For each figure we have computed the root mean square (RMS) residual error, σ_{TS} and σ_{CD}, for the thin shell and conical domain models, respectively: (a) σ_{TS} = 17 cm; σ_{CD} = 38 cm; (b) σ_{TS} = 16 cm; σ_{CD} = 16 cm; (c) σ_{TS} = 15 cm; σ_{CD} = 14 cm; and (d) σ_{TS} = 15 cm; σ_{CD} = 14 cm. Note that in nearly all cases the two models provide comparable accuracy (except in (a) where the thin shell model is superior). Comparable results are to be expected since WAAS (and, hence, the thin shell model) has been shown to perform well under quiet conditions.

The situation is dramatically different under disturbed conditions. Figures 9(a), 9(b), 9(c), and 9(d) display an analogous set of residuals for July 15, 2000. On this day a major ionospheric storm occurred that resulted in vertical delay values of unprecedented magnitude in the southern United States. Residuals are plotted for a conical domain fit radius is 1000 km and the required number of fit points set to 4, 5, 6, and 7, respectively. Note that the data set begins at midnight UTC which corresponds to roughly 19:30 hours in geomagnetic local time. Consequently the discontinuity that appears at roughly 19:30 hours is due to a day boundary.

For this data set, the conical domain model clearly out performs the thin shell model. It reduces the overall RMS error by 32- 36 %. This may, in fact, by regarded as an underestimate of the improvement when the ionosphere is highly structured. For much of the day prior to the storm, quiet conditions prevail where, as we have seen above, the conical domain and thin shell models provide similar accuracy. Including this quiet period in the estimation of the overall RMS error serves to average downward the level of improvement that occurs following storm onset.

Figure 9 shows the dependence of RMS error on the fit radius of each model for four different levels of ionospheric disturbance. N_{pts} for the conical domain model is set to 5 in all cases. In each plot, the fit radius is varied from 800 to 1500 km. Figure 9(a) shows that the RMS error for a quiet day (July 2, 2000) has only a weak dependence on fit radius and that the two models provide comparable accuracy. Under moderately disturbed conditions (September 7, 2002), the conical domain demonstrates markedly superior accuracy (Fig. 9(b)). When storm conditions become severe (April 6, 2000 and July 15, 2000), the disparity between the thin shell accuracy and the conical domain accuracy grows strikingly (Figs. 9(c) and 9(d)). It should also be noted that the trend in the conical domain results is toward convergence to zero error at zero fit radius, as expected, whereas the thin shell results, especially Figs. 9(c) and 9(d), do not appear to be converging to zero.

A comparison of model accuracy at low latitudes

The ionosphere at low latitudes is characterized by complicated structure and slant delay values with large magnitudes. The presence of the equatorial (Appleton) anomaly ensures that large gradients in electron density will occur at low latitudes virtually every day. Indeed previous studies of the equatorial ionosphere (*e.g.*, Komjathy *et al.*, 2003) have shown that even under nominally quiet conditions (Komjathy et al. 2003), the ionosphere is typically so complex that the advent of a major magnetic storm does not dramatically increase the level of disturbance sampled by GPS measurements. In this study we will consider data from a nominal day only.

To gain some insight into the comparative performance of the thin shell and conical domain models at low latitudes, we have analyzed data from the Brazilian Network for Continuous Monitoring of GPS (RBMC). Figure 10 displays the locations of dual-frequency GPS receivers in South America including the Brazilian receivers used in this study. Six of the Brazilian stations are treated as pseudo-users to study the impact of the degree of isolation of the user from the receivers used in the fits: PARA and VICO are relatively near other stations in the Brazilian network, BRAZ and UEPP are somewhat more isolated, and IMPZ and CUIB are located far from the other receivers in the network. Again N_{pts} for the conical domain model is set to 5 in all cases.

Figure 11 shows the residuals obtained from the pseudo-measurement analysis of data recorded, February 19, 2002, a date on which the low-latitude ionosphere displayed nominal levels of disturbance. Figure 12 displays the root mean square error for the residuals at each of the six pseudo-user locations. For the highly isolated pseudo-users, the performance of the conical domain model is poor, much worse than the thin shell model. Here it appears that there are simply too few measurements in the vicinity of the user for the conical domain approach to be viable. For the less isolated pseudo-users, the RMS values of the residuals for the conical domain model drop below those of the thin shell model. For the least isolated pseudo-users, this trend continues; the conical domain model has achieved an impressive increase in accuracy.

CONCLUSION

We have presented a new approach to modeling slant delay measurements that addresses limitations of the standard thin shell model of the ionosphere. A satellite-based augmentation system (SBAS) using this approach would permit direct computation of a user's slant delay estimate without the resorting to interpolation on a grid of vertical delay estimates. Instead of evaluating fits at points on a grid, fits would be performed in a conical domain with a satellite at the cone vertex. Unlike the thin-shell model, fits based upon the conical domain model converge to the correct solution in the limit that of sufficiently high reference receiver density.

We have reported the results of an analysis comparing the accuracy of the conical domain approach with that achieved using the standard thin shell model. For a fixed number of measurements in the fit, we find that conical domain model significantly outperforms the thin-shell model provided the fit radius is sufficiently small.

The conical domain approach is a candidate algorithm for modeling slant delays at low latitudes, where complex ionospheric structure, high delay values, and large electron density gradients pose severe challenges for SBAS systems based upon the thin shell model. An important question affecting the practical viability of this approach at low-latitudes concerns the dependence of the desired level of accuracy and the density of receivers required to achieve that accuracy. At present we are making a first attempt at resolving this question, by using simulated data of the equatorial ionosphere and varying the density of reference receivers.

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FIGURE CAPTIONS

Fig.1. Schematic diagram of the thin shell model of the ionosphere.

Fig. 2. Diagram illustrating how WAAS determines user slant delay.

Fig. 3. Diagram illustrating the lack of convergence of the thin shell model as receivers converge to the user's location.

Fig. 4. Diagram illustrating the lack of convergence of the thin shell model as IPPs converge to the user's IPP location.

Fig.5. Schematic diagram of the conical domain model of the ionosphere.

Fig. 6. Diagram illustrating the convergence of the conical domain model as receivers converge to the user's location.

Fig. 7. Schematic diagram of the hybrid model of the ionosphere.

Fig. 8. Pseudo-user slant residual error, projected to equivalent vertical delay, vs geomagnetic local time for WAAS data, July 2, 2000, fit using the thin-shell (blue) and conical domain (red) models: (a) $N_{pts} = 3$; (b) $N_{pts} = 4$; (c) $N_{pts} = 5$; (a) $N_{pts} = 6$.

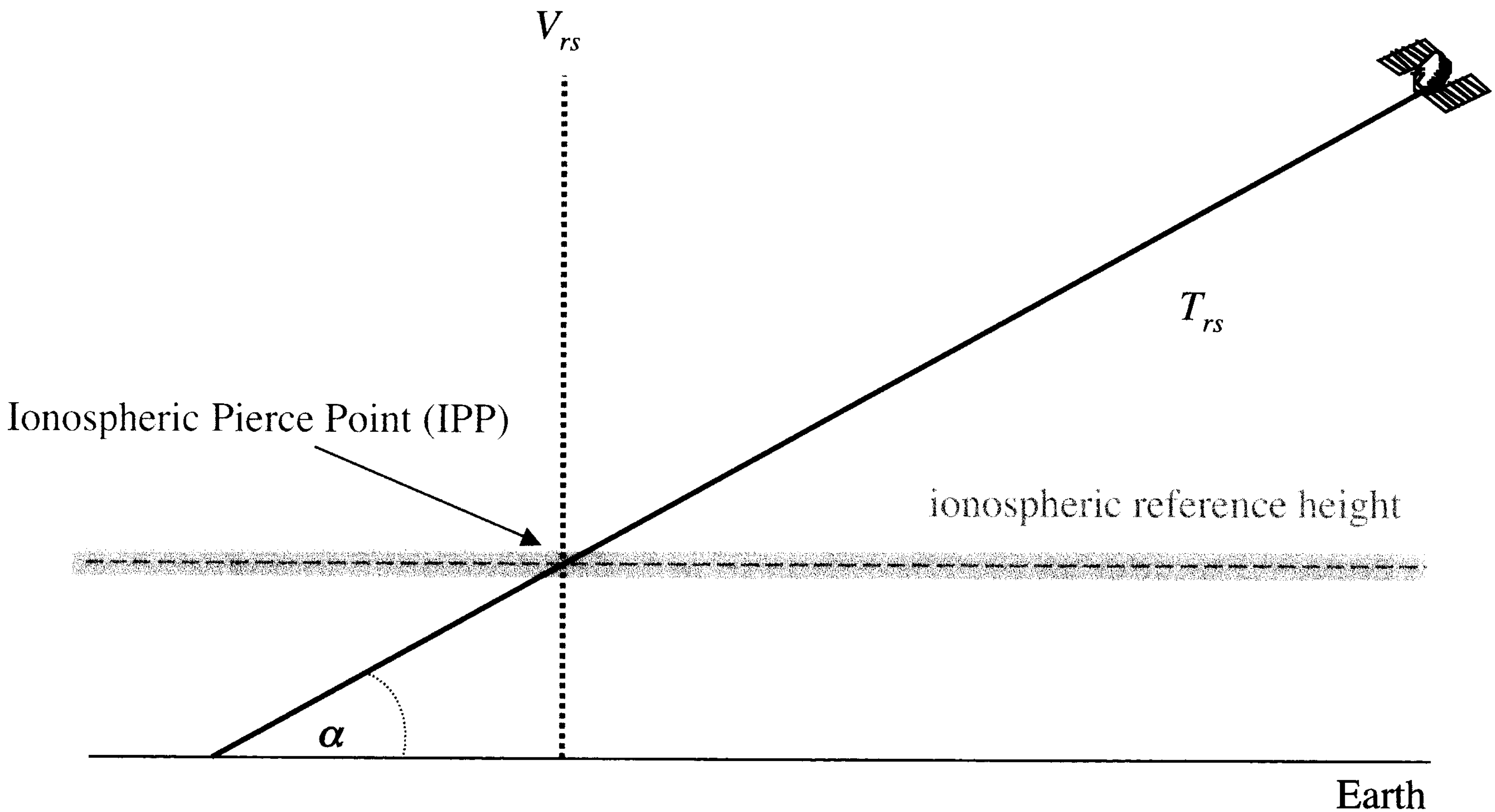
Fig. 9. Pseudo-user slant residual error, projected to equivalent vertical delay, vs geomagnetic local time for WAAS data, July 15, 2000, fit using the thin-shell (blue) and conical domain (red) models: (a) $N_{pts} = 4$; (b) $N_{pts} = 5$; (c) $N_{pts} = 6$; (a) $N_{pts} = 7$.

Fig. 10. Root mean square (RMS) residual for pseudo-user slant residuals, projected to equivalent vertical delay, vs fit radius for WAAS data, fit using the thin-shell (blue) and conical domain (red) models: (a) July 2, 2000; (b) September 7, 2002; (c) April 6, 2000; (d) July 15, 2000.

Fig. 11. Map showing locations of GPS reference receivers in South American network.

Fig. 12. Pseudo-user slant residual error, projected to equivalent vertical delay, vs geomagnetic local time for Brazilian data, July 15, 2000, fit using the thin-shell (blue) and conical domain (red) models, at: (a) IMPZ; (b) CUIB; (c) BRAZ; (d) UEPP; (e) VICO; (f) PARA.

Fig. 13. Root mean square (RMS) residual for pseudo-user slant residuals, projected to equivalent vertical delay, for data at 6 Brazilian locations, fit using the thin-shell (blue) and conical domain (red) models.



V_{rs}
T_{rs}
Ionospheric Pierce Point (IPP)
ionospheric reference height
α
Earth

Fig 1

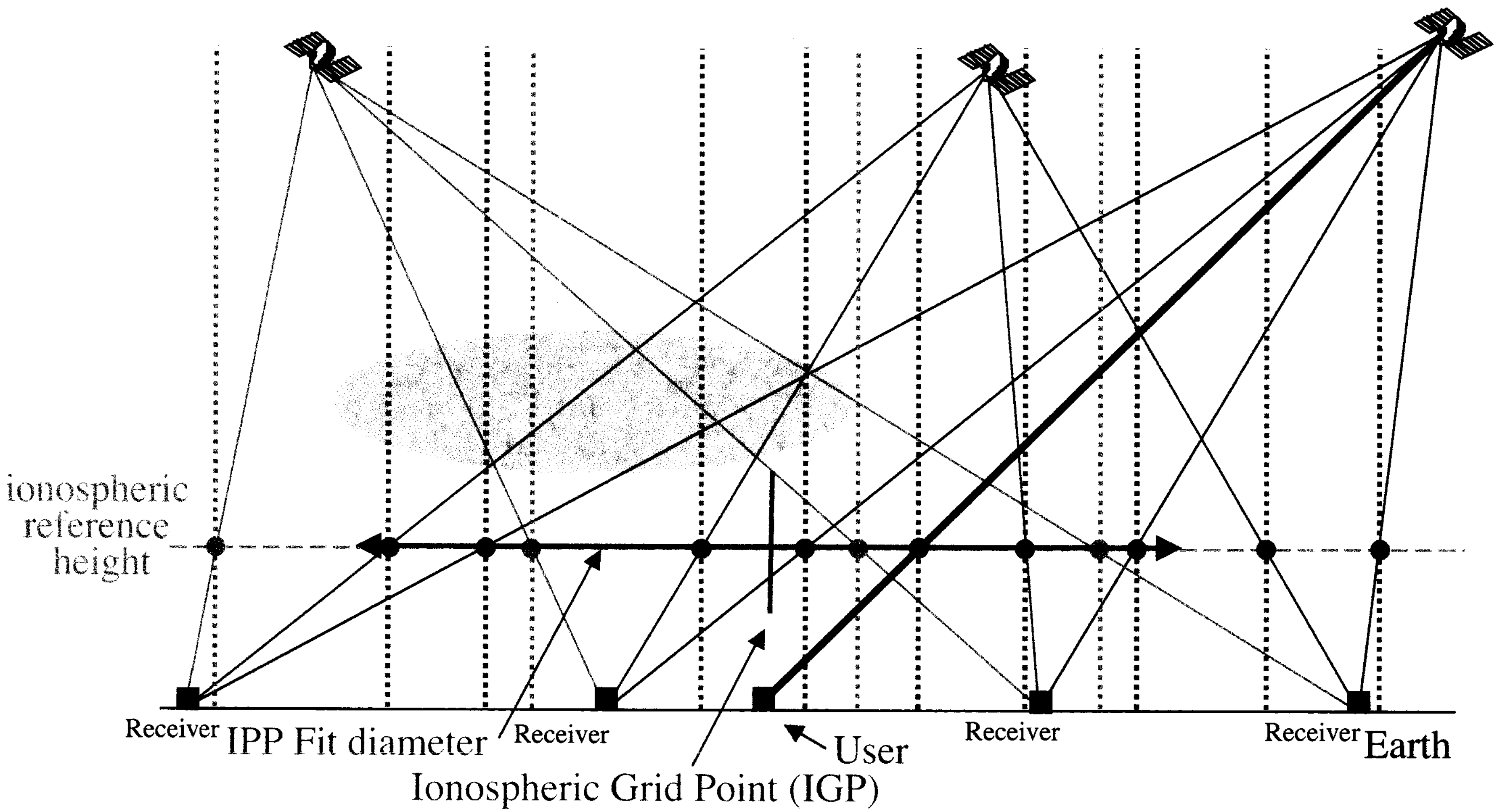

ionospheric
reference
height
Receiver
IPP Fit diameter
Receiver
User
Receiver
Receiver
Earth
Ionospheric Grid Point (IGP)

Fig 2

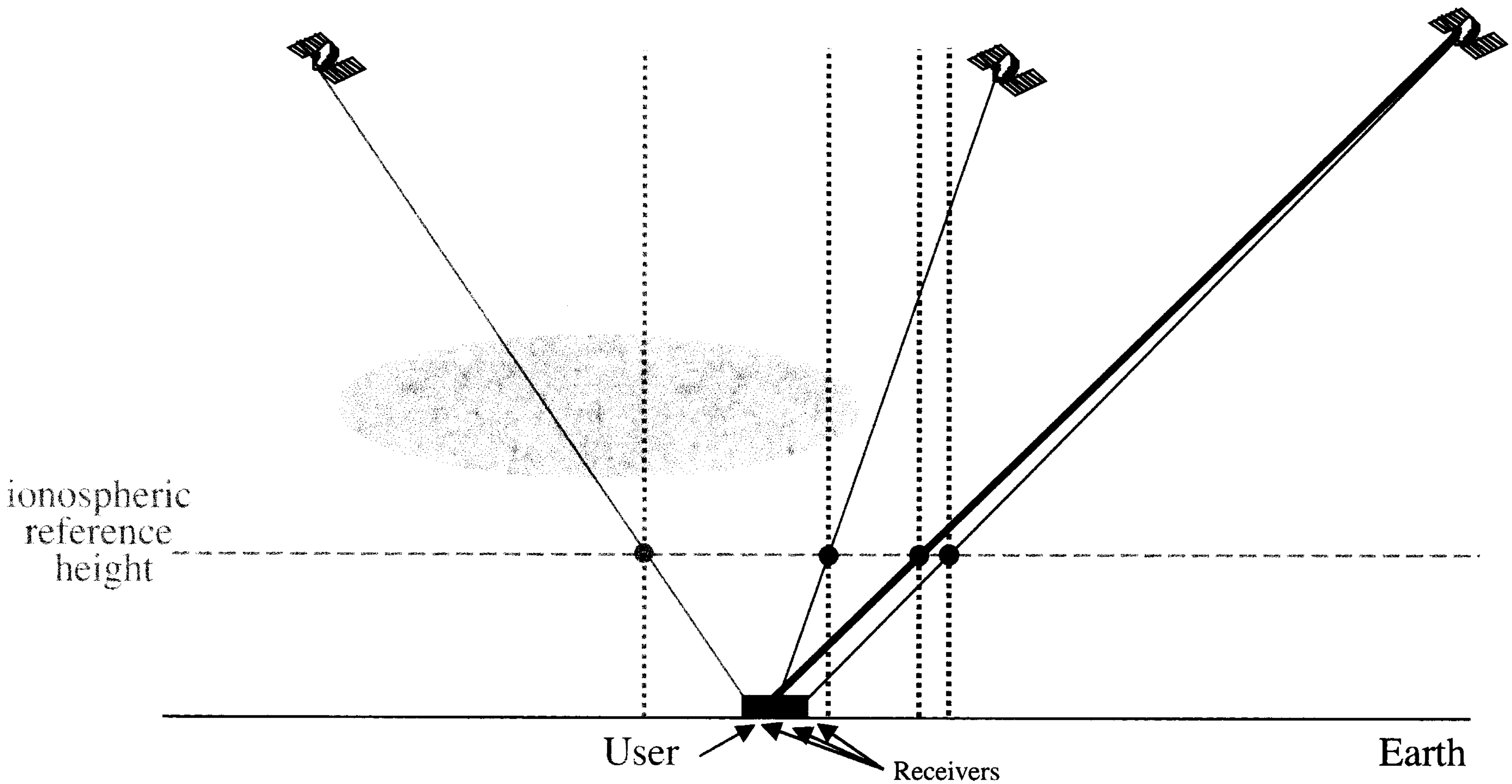


Fig 3

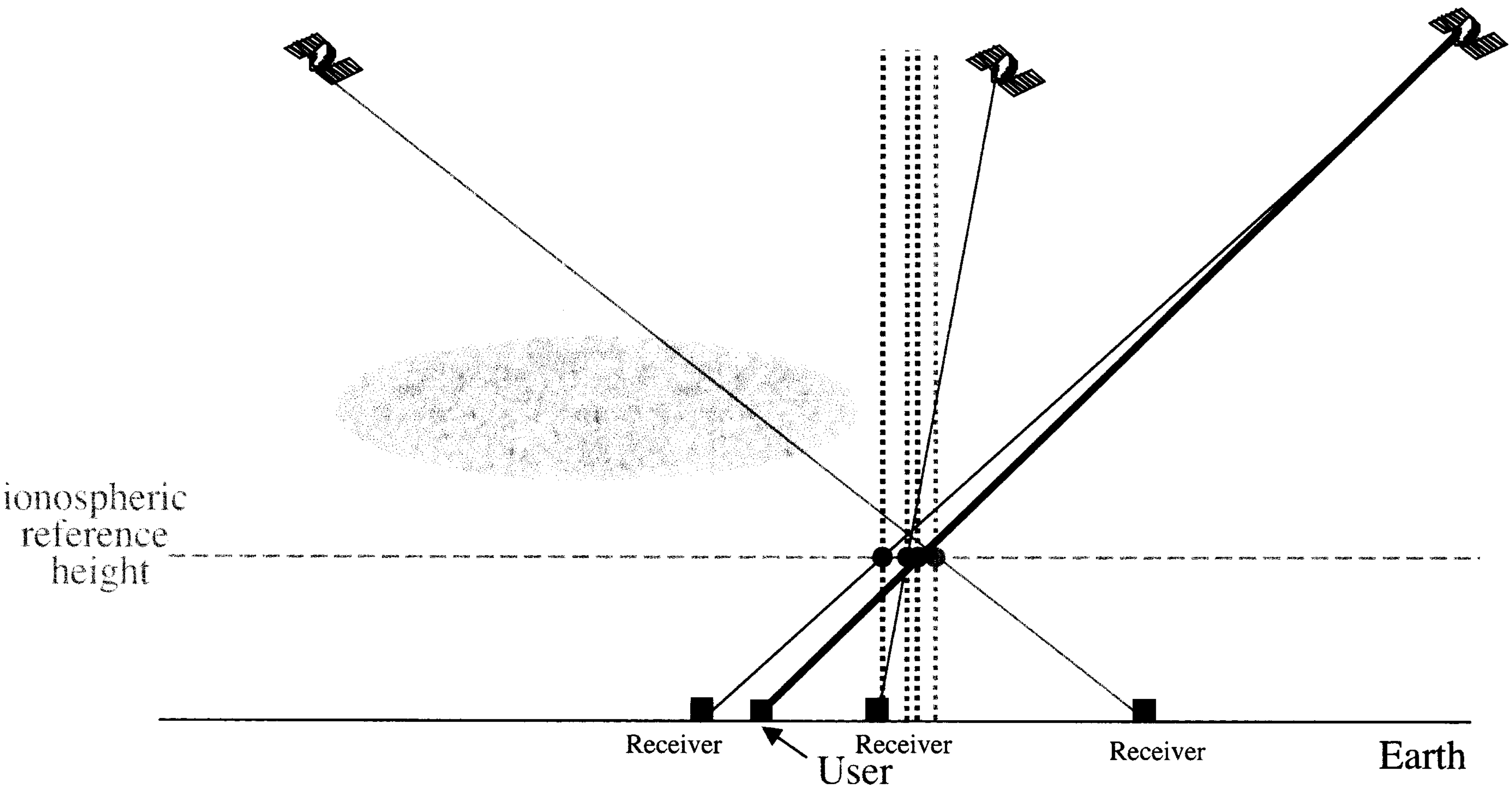

ionospheric
reference
height
Receiver
User
Receiver
Receiver
Earth

Fig 4

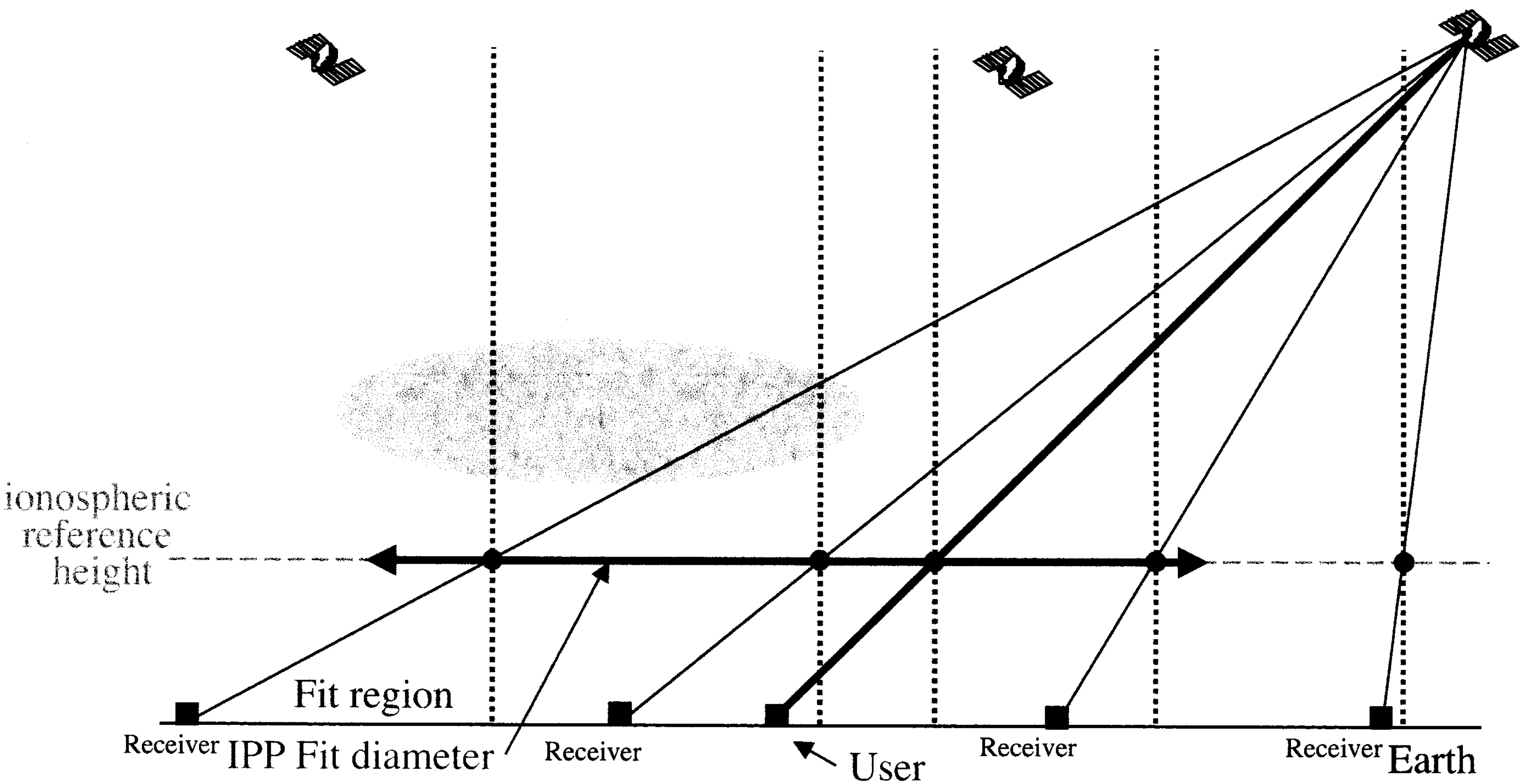

ionospheric
reference
height
Fit region
Receiver
IPP Fit diameter
Receiver
User
Receiver
Receiver
Earth

Fig 5

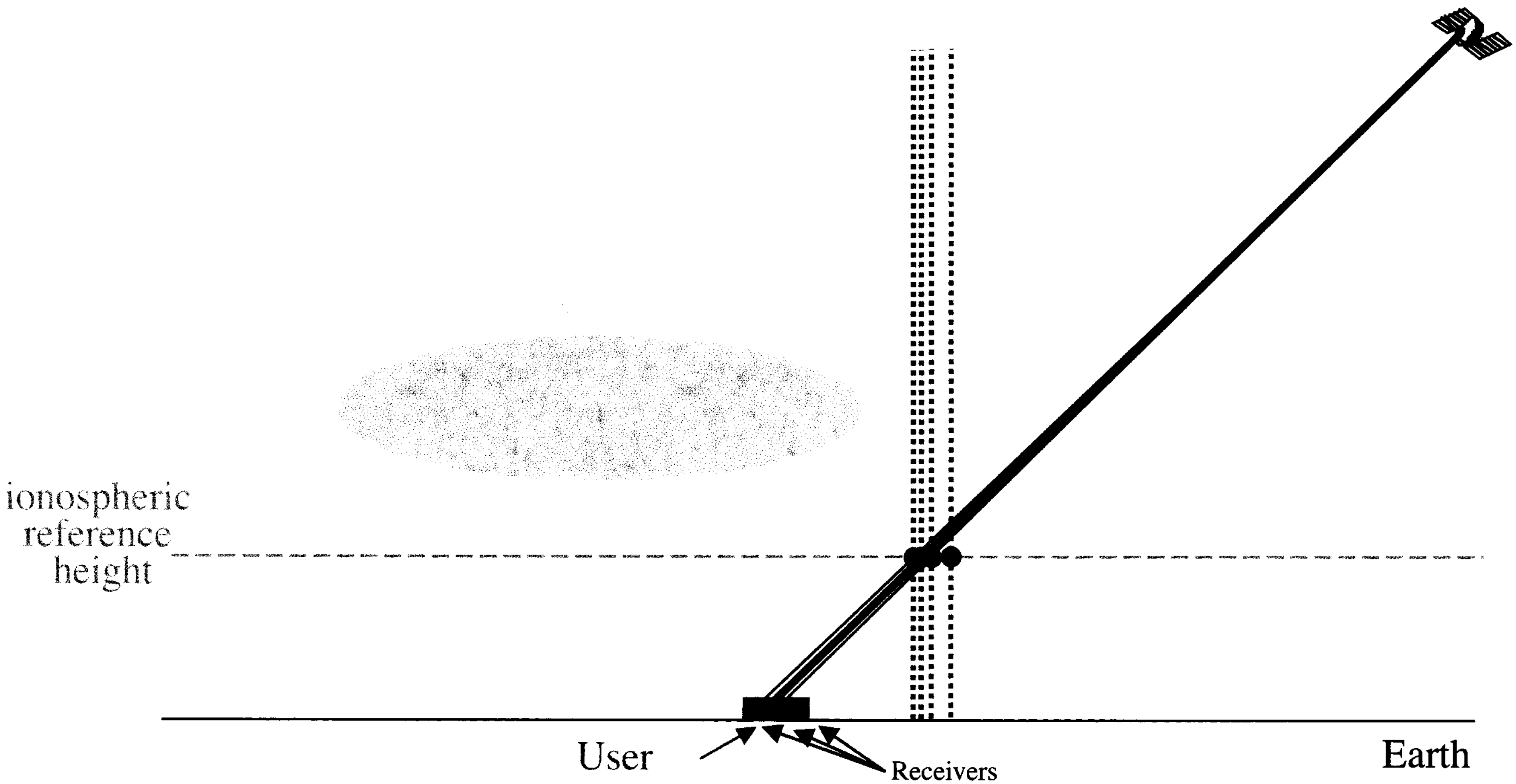


Fig 6

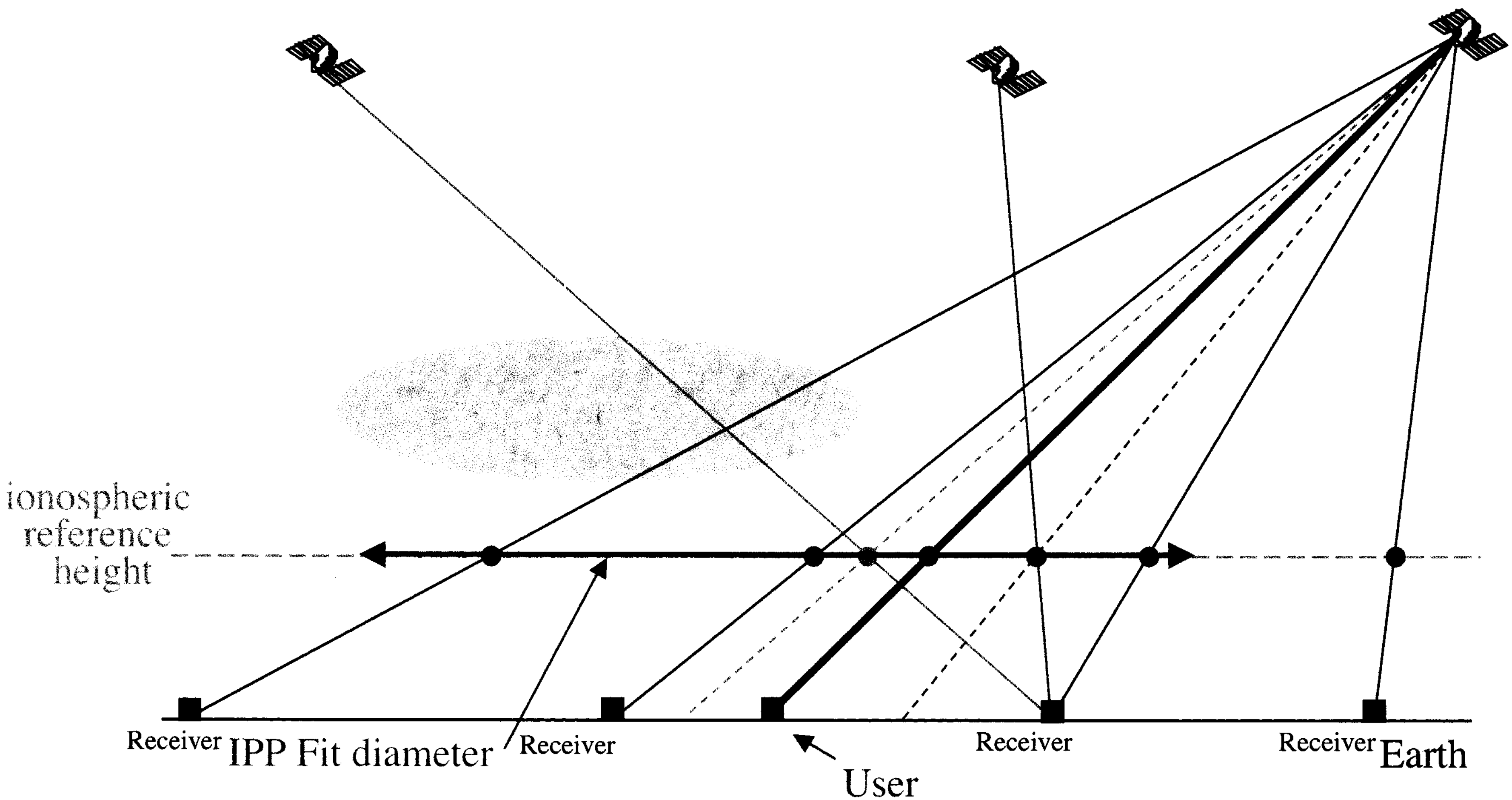

ionospheric
reference
height
Receiver
IPP Fit diameter
Receiver
User
Receiver
Receiver
Earth

Fig. 7

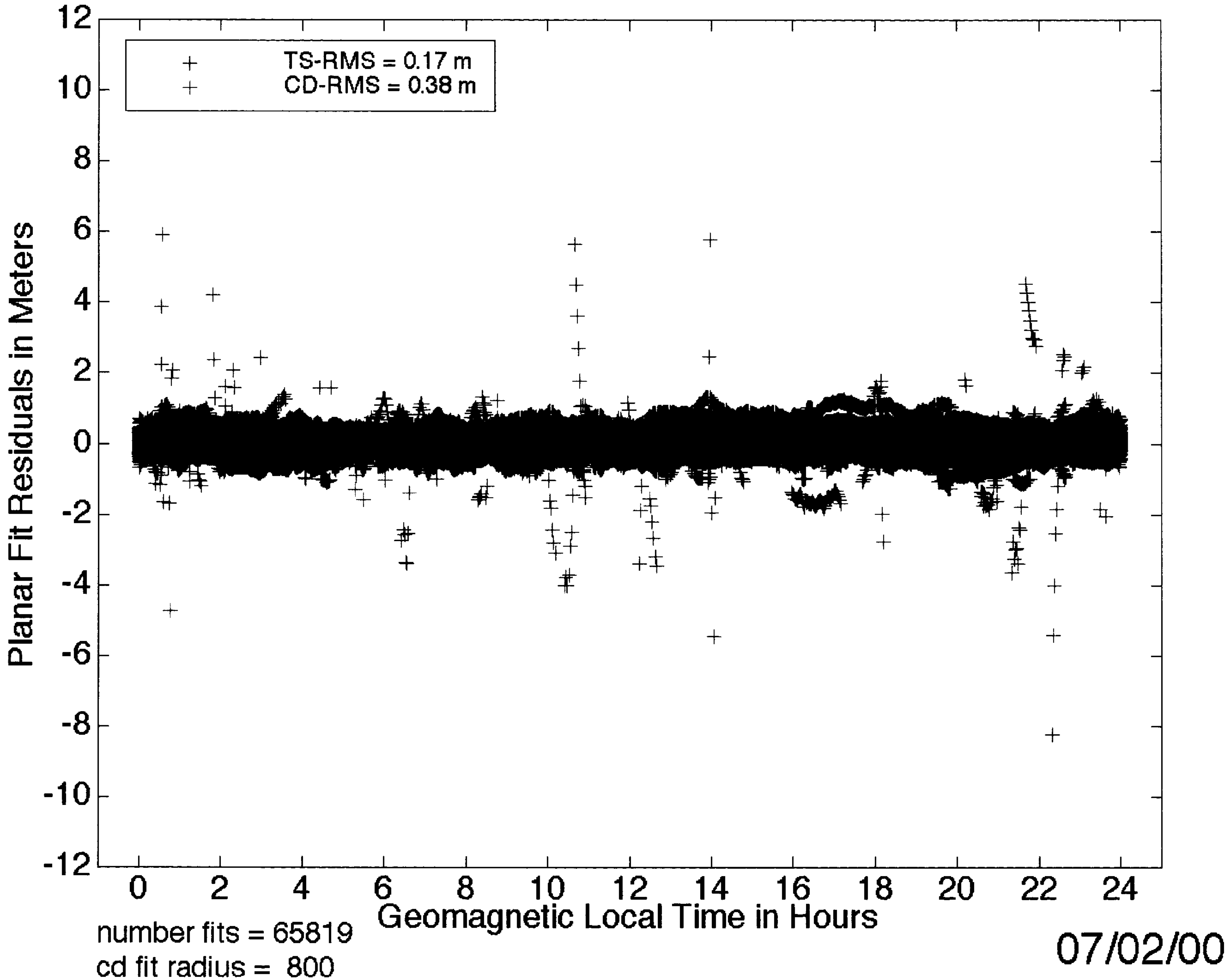
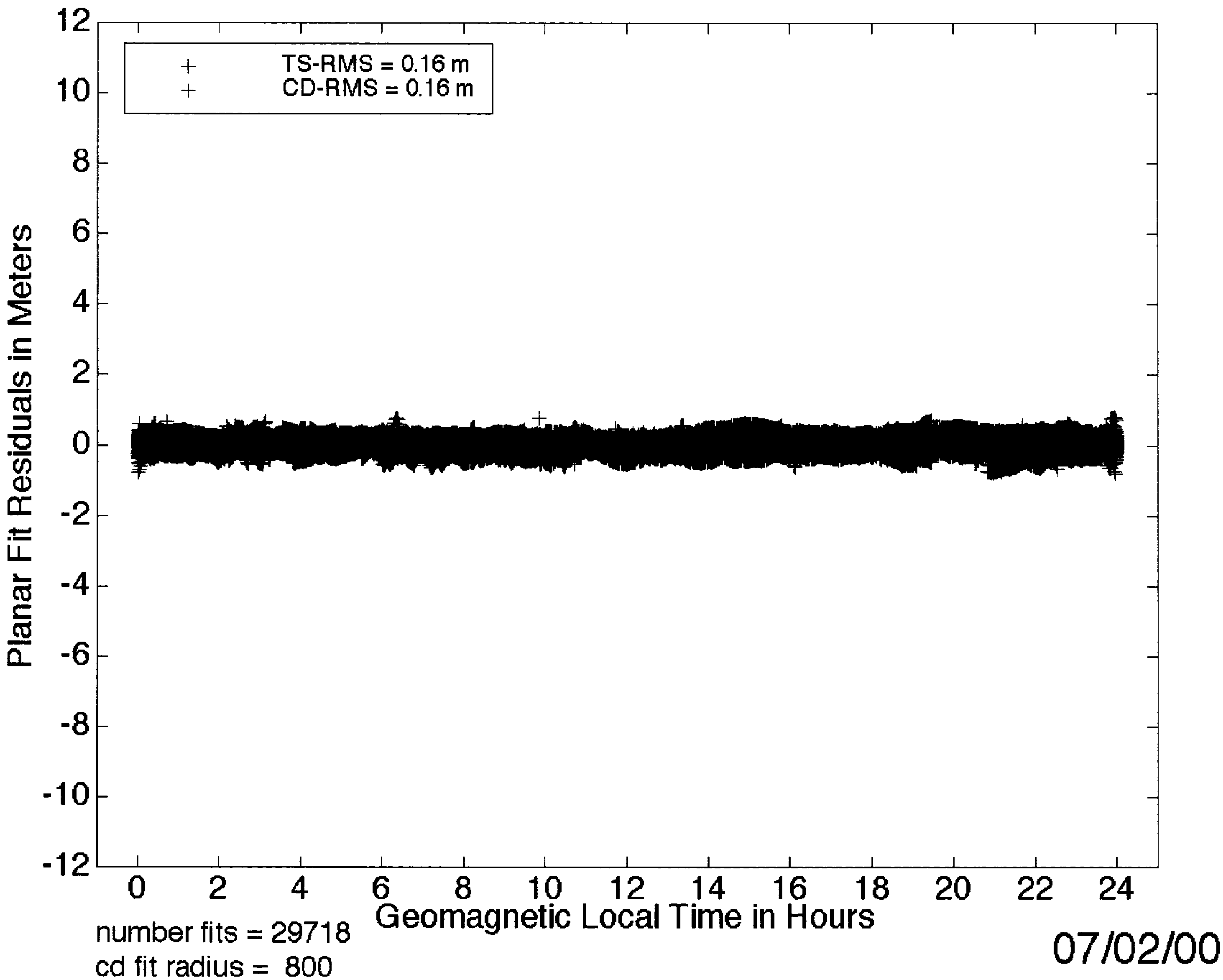
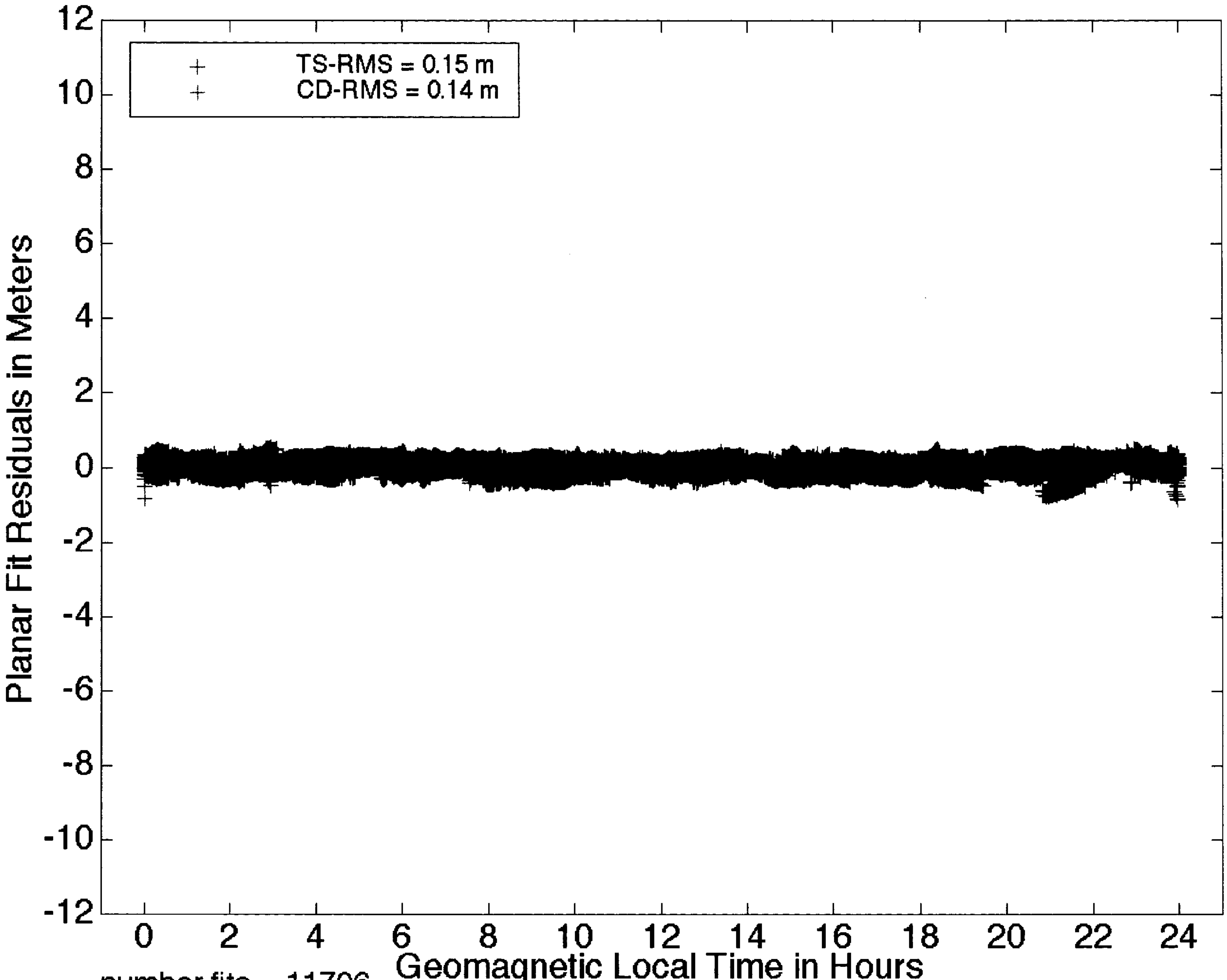

TS-RMS = 0.17 m
CD-RMS = 0.38 m
Planar Fit Residuals in Meters
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
0 2 4 6 8 10 12 14 16 18 20 22 24
Geomagnetic Local Time in Hours
number fits = 65819
cd fit radius = 800
07/02/00

Fig 8(a)



TS-RMS = 0.16 m
CD-RMS = 0.16 m
Planar Fit Residuals in Meters
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
0 2 4 6 8 10 12 14 16 18 20 22 24
Geomagnetic Local Time in Hours
number fits = 29718
cd fit radius = 800
07/02/00

Fig 8(b)



number fits = 11706
cd fit radius = 800

07/02/00

Fig 8(c)

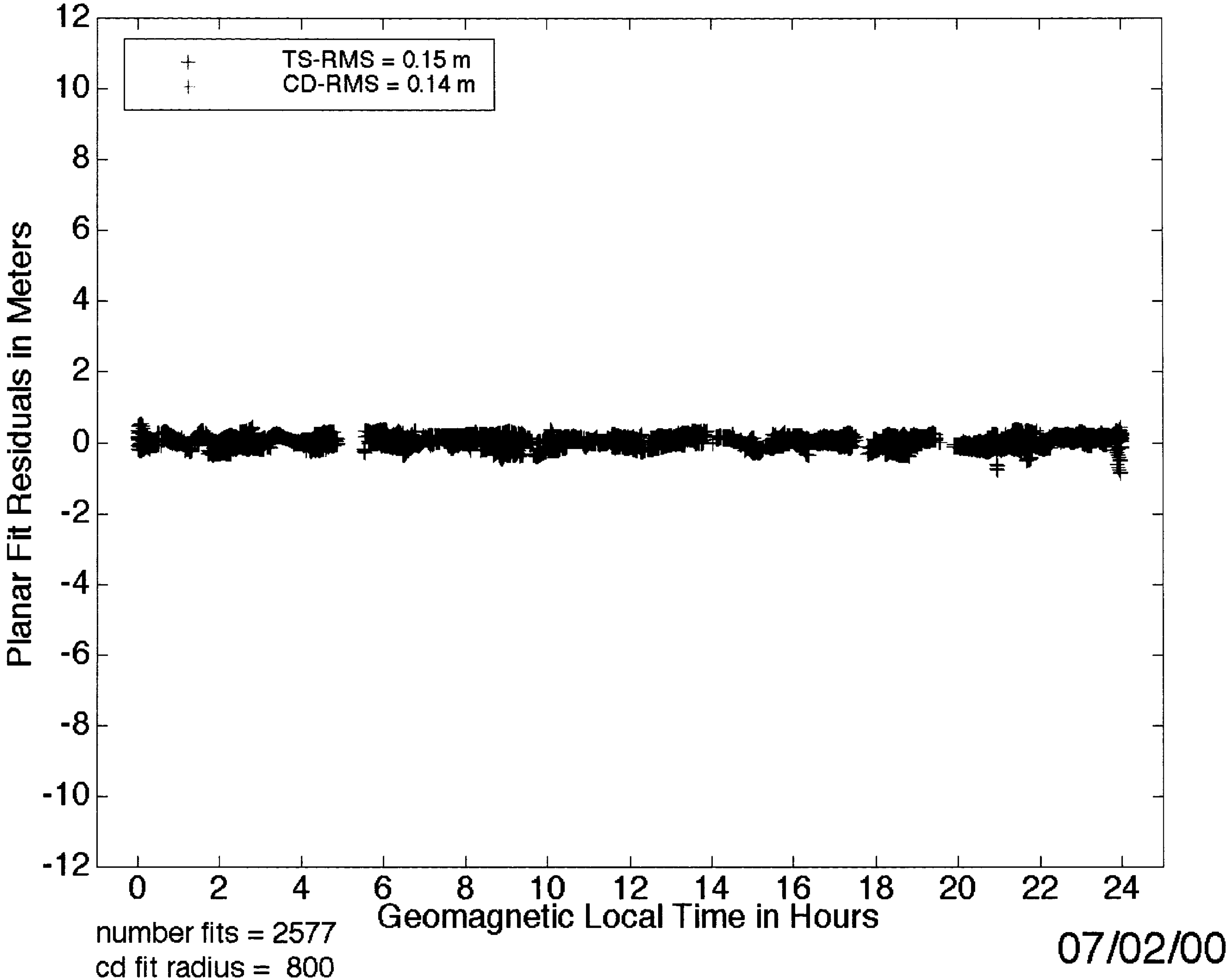
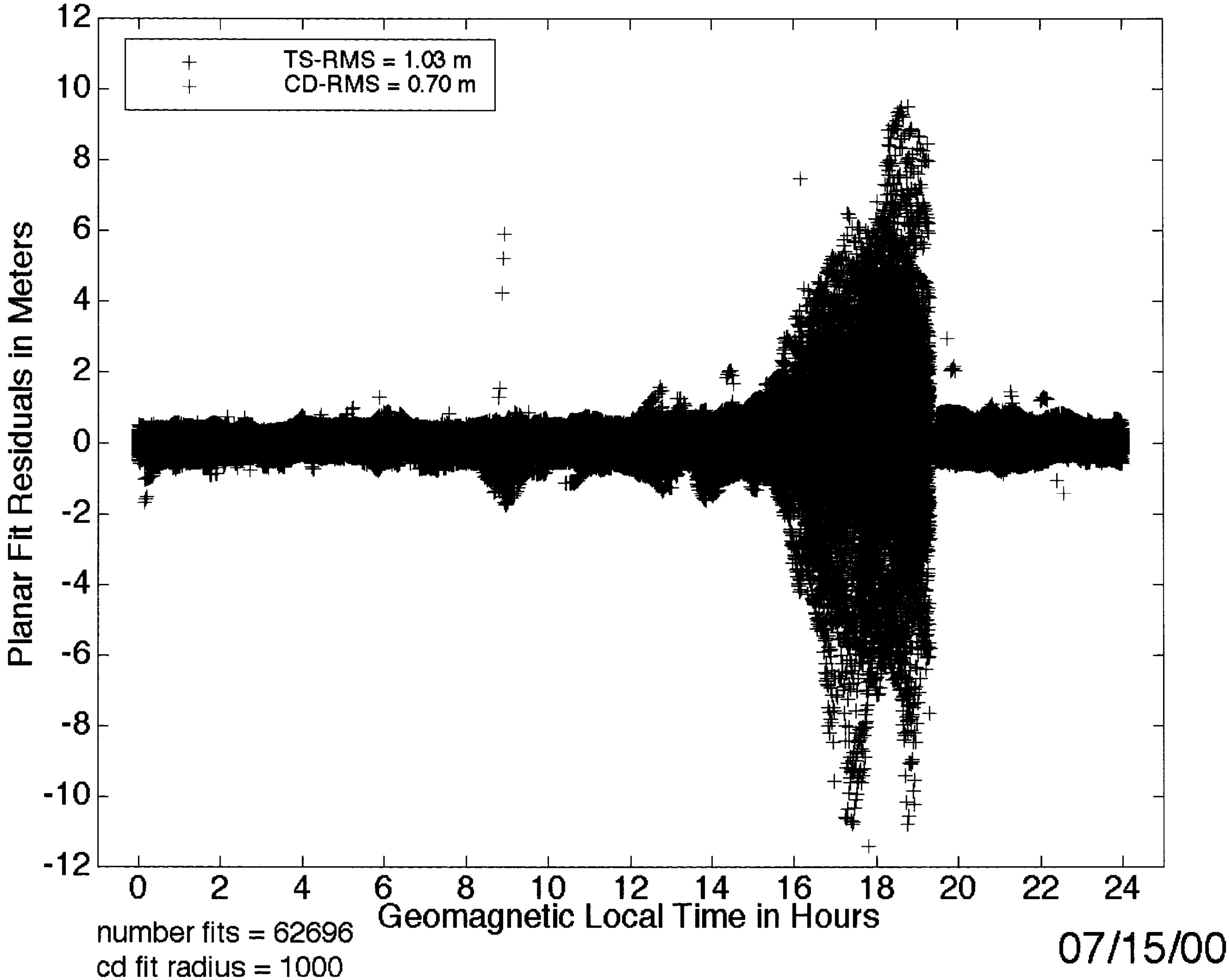

TS-RMS = 0.15 m
CD-RMS = 0.14 m
Planar Fit Residuals in Meters
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
0
2
4
6
8
10
12
14
16
18
20
22
24
Geomagnetic Local Time in Hours
number fits = 2577
cd fit radius = 800
07/02/00

Fig 8(d)



TS-RMS = 1.03 m
CD-RMS = 0.70 m
Planar Fit Residuals in Meters
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
0
2
4
6
8
10
12
14
16
18
20
22
24
Geomagnetic Local Time in Hours
number fits = 62696
cd fit radius = 1000
07/15/00

Fig. 9(a)

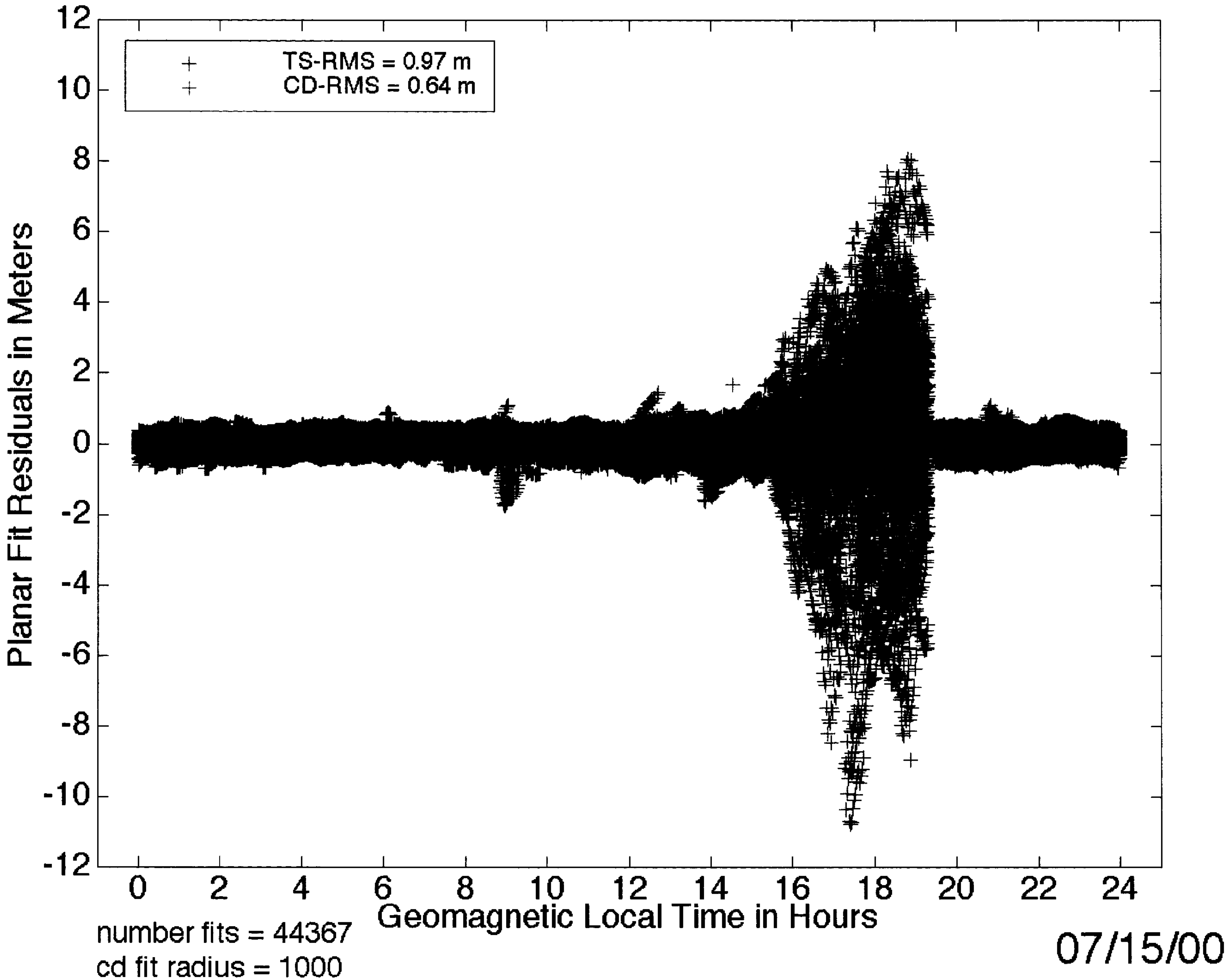

TS-RMS = 0.97 m
CD-RMS = 0.64 m
Planar Fit Residuals in Meters
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
0
2
4
6
8
10
12
14
16
18
20
22
24
Geomagnetic Local Time in Hours
number fits = 44367
cd fit radius = 1000
07/15/00

Fig 9(b)

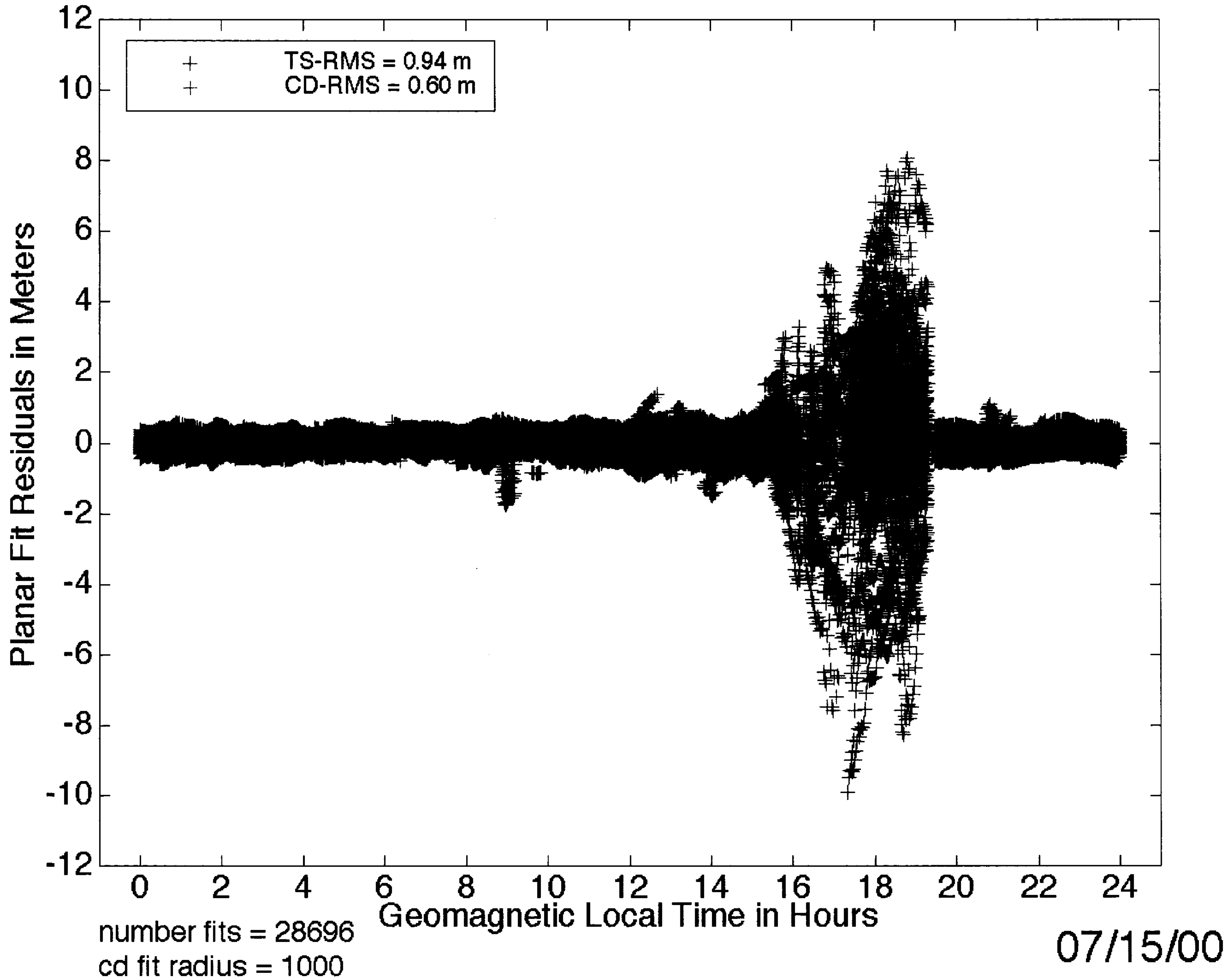
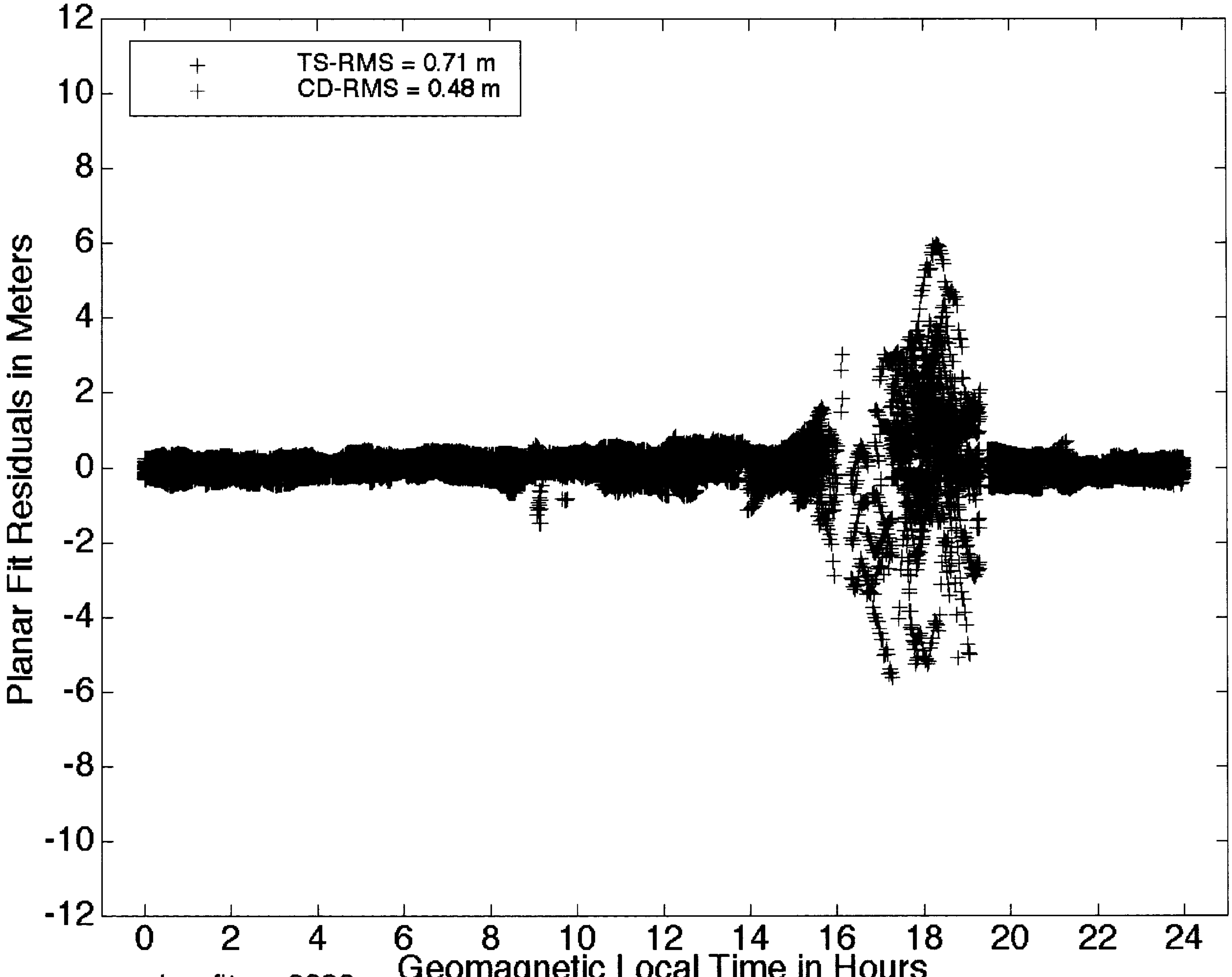

TS-RMS = 0.94 m
CD-RMS = 0.60 m
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
Planar Fit Residuals in Meters
0
2
4
6
8
10
12
14
16
18
20
22
24
Geomagnetic Local Time in Hours
number fits = 28696
cd fit radius = 1000
07/15/00

Fig 9(d)



number fits = 9232
cd fit radius = 1000

07/15/00

Fig 9(d)

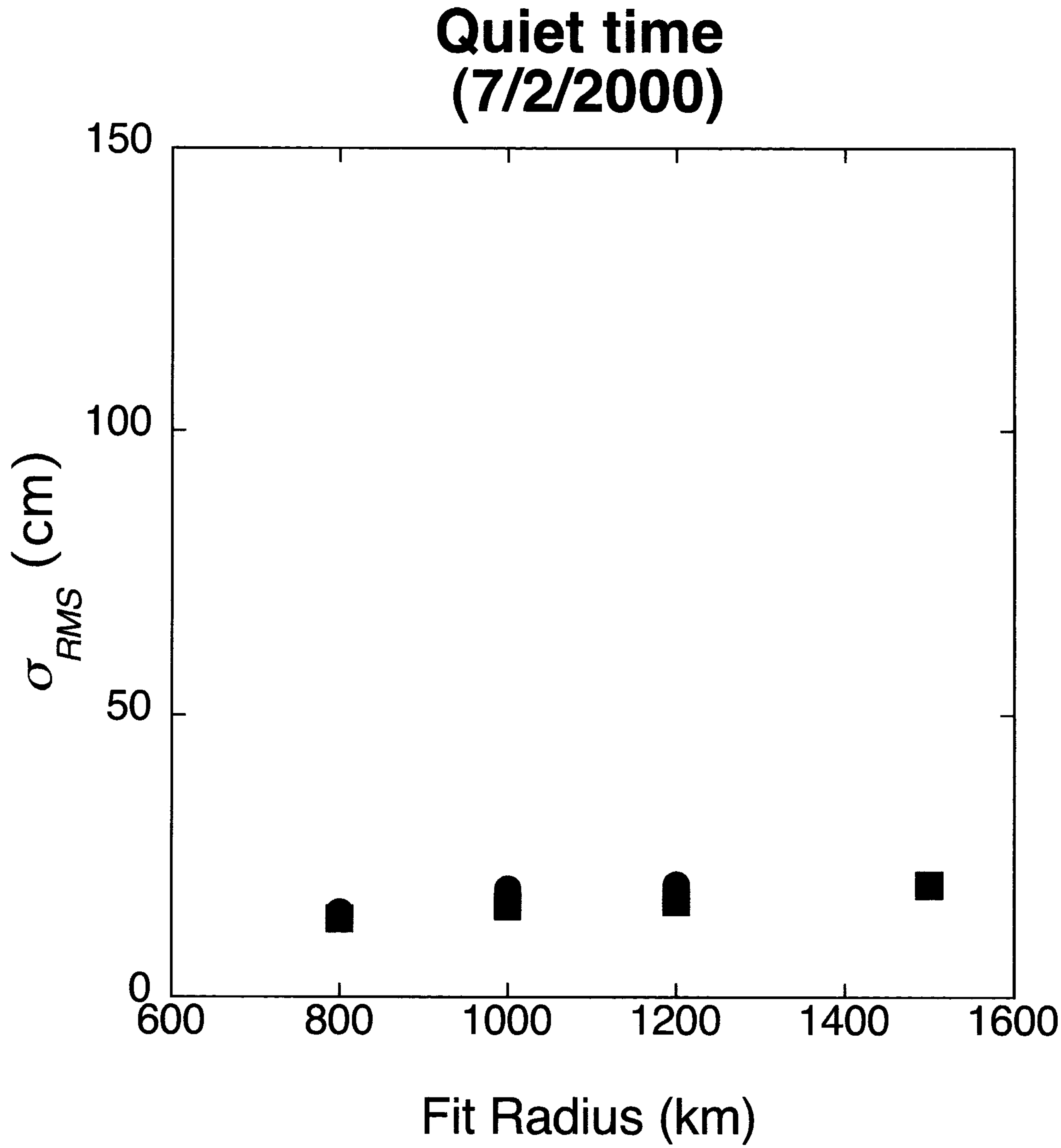

Quiet time
(7/2/2000)
150
100
50
0
σ_RMS (cm)
600
800
1000
1200
1400
1600
Fit Radius (km)

Fig 10(a)

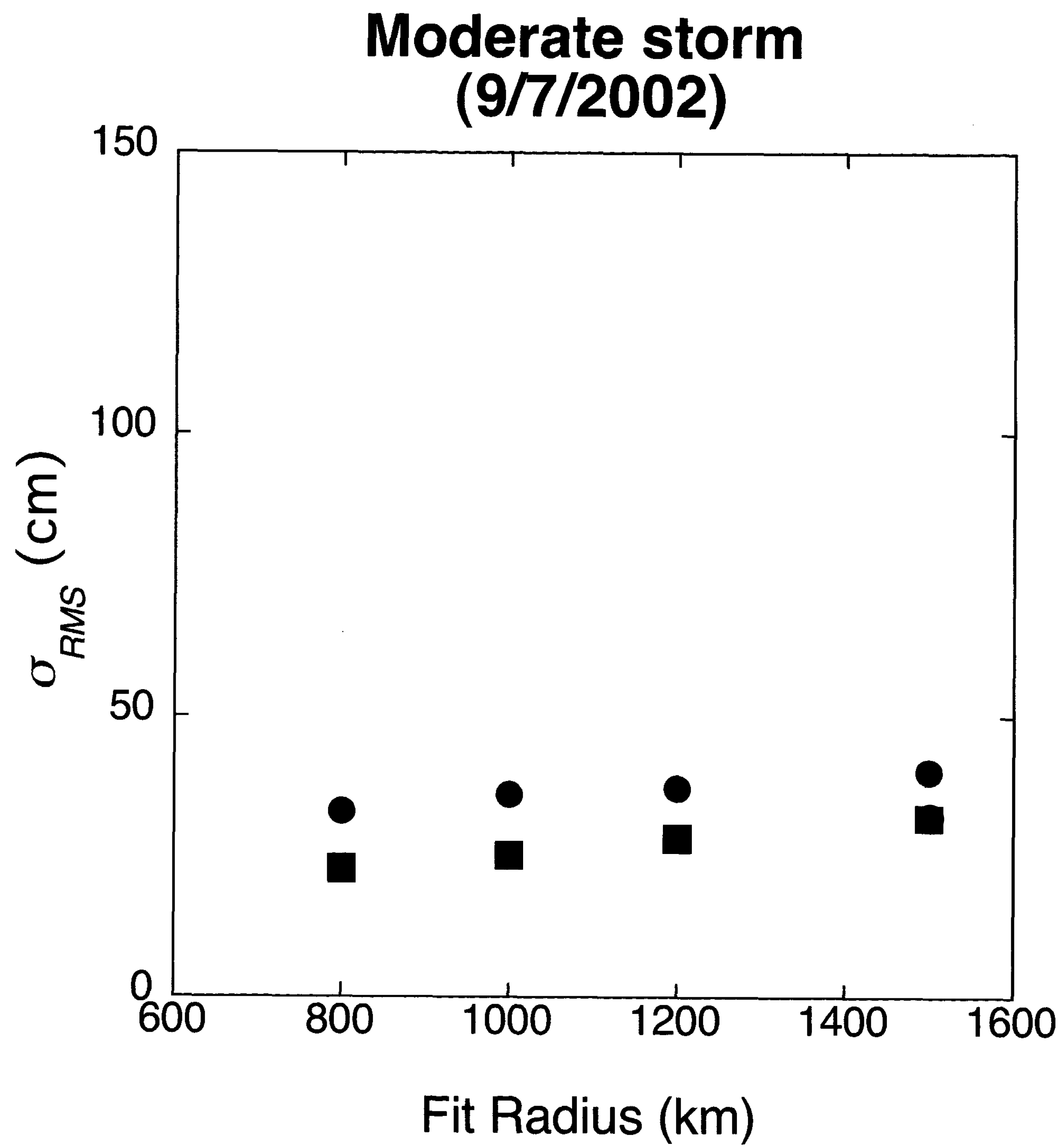
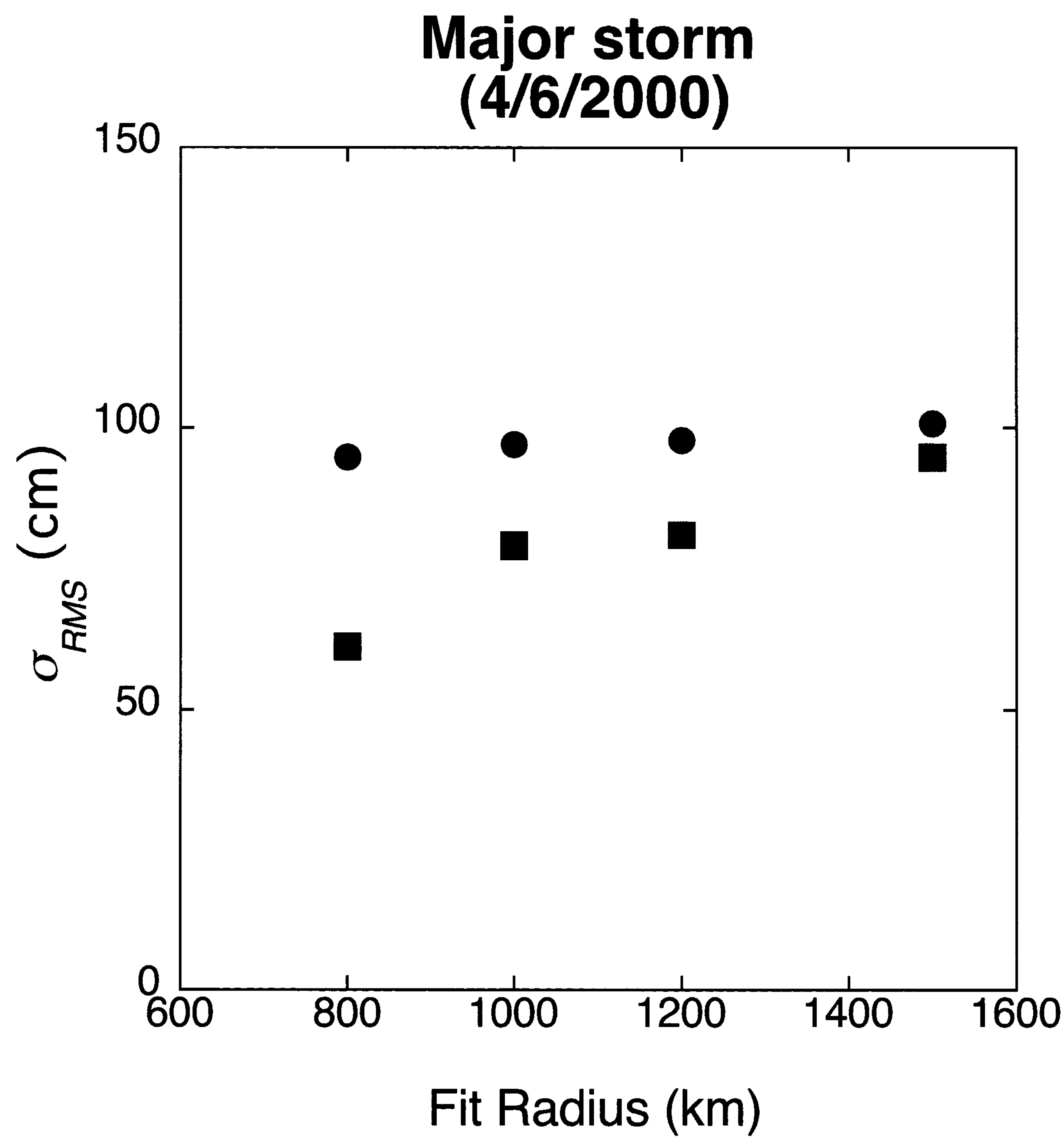

Moderate storm
(9/7/2002)
150
100
50
0
600
800
1000
1200
1400
1600
σ_{RMS} (cm)
Fit Radius (km)

Fig 10(b)


Major storm
(4/6/2000)
150
100
50
0
600
800
1000
1200
1400
1600
σ_{RMS} (cm)
Fit Radius (km)
Fig 10(c)

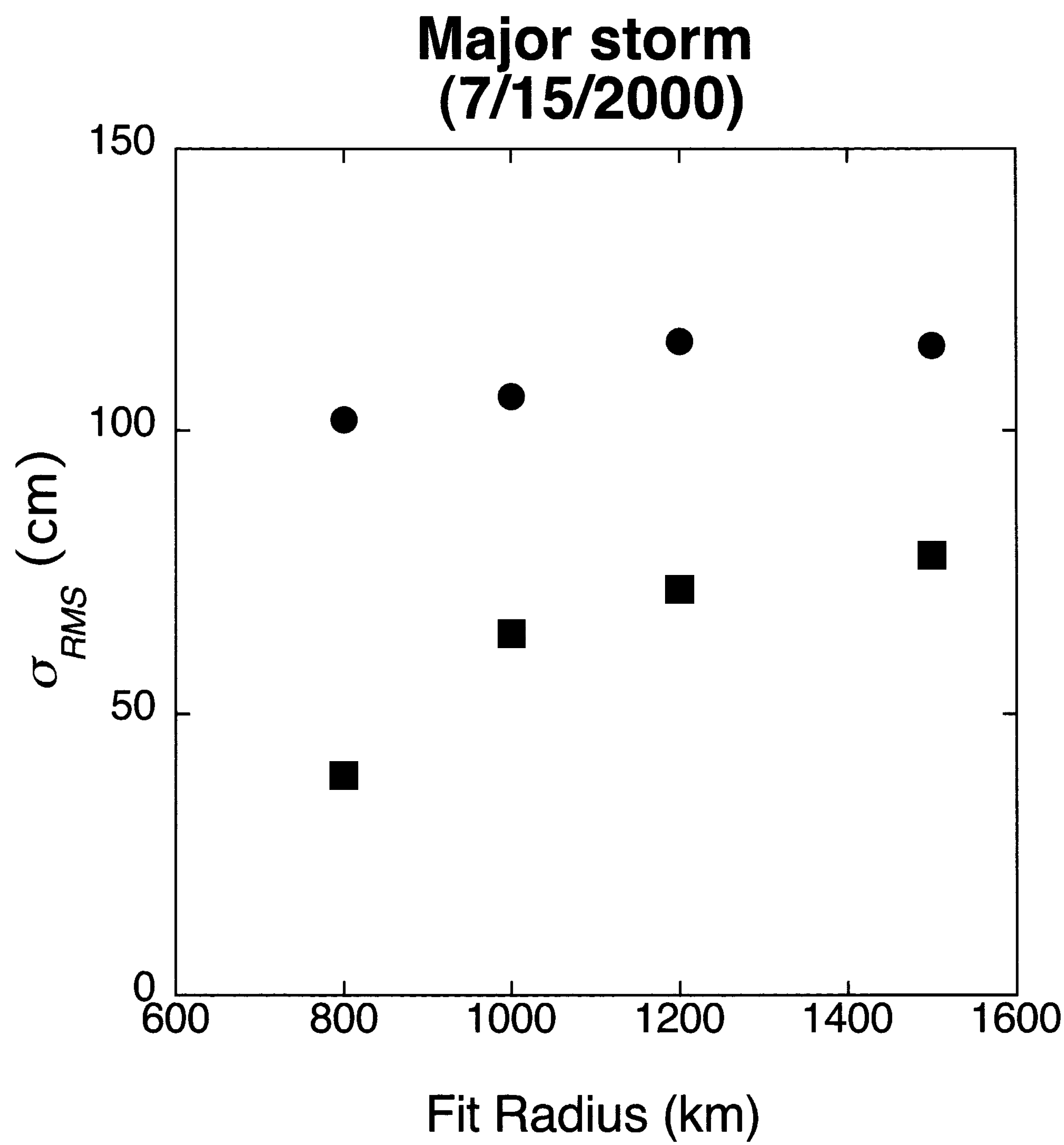

Major storm
(7/15/2000)
150
100
50
0
600
800
1000
1200
1400
1600
σ_{RMS} (cm)
Fit Radius (km)

Fig 10(d)

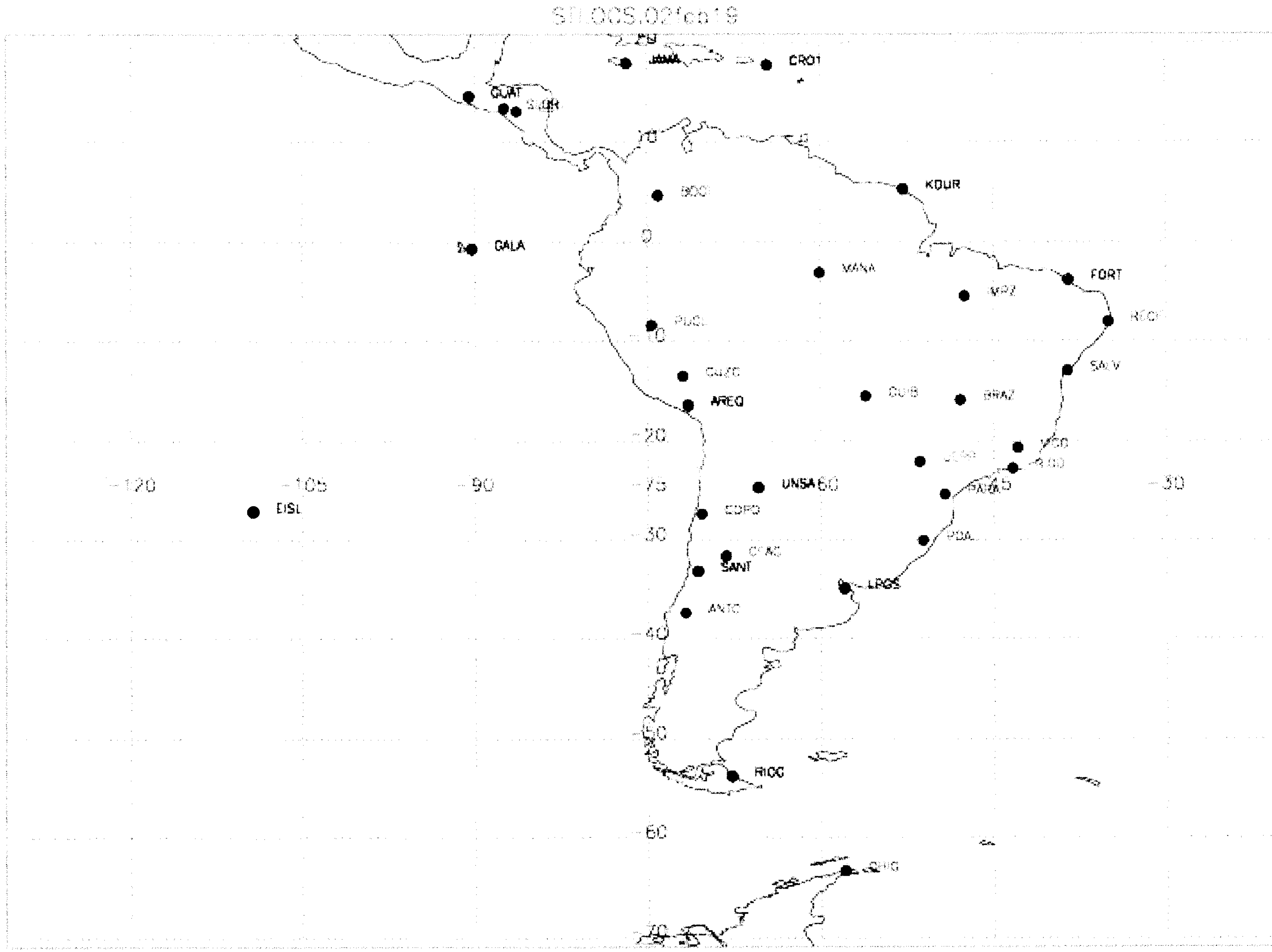
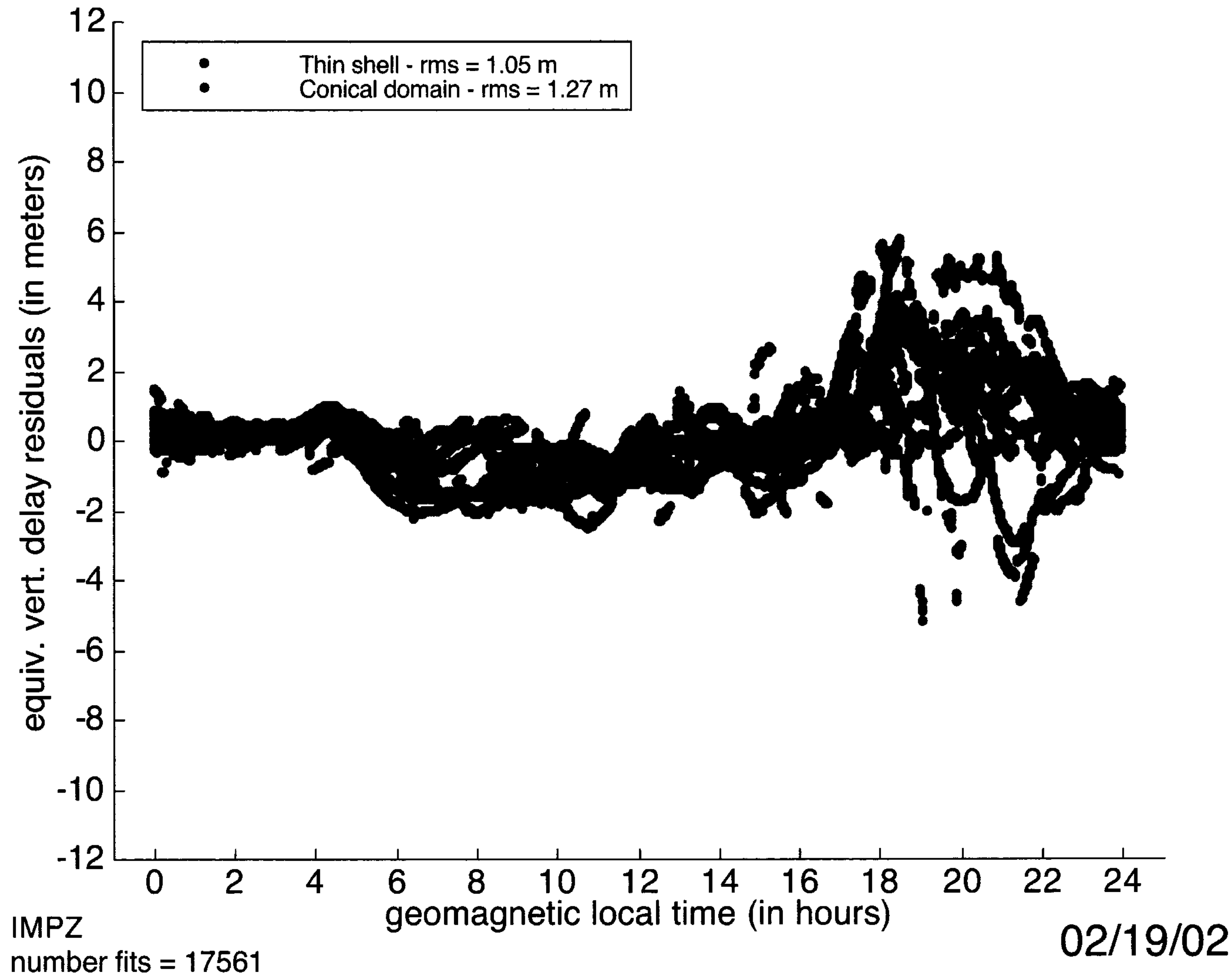
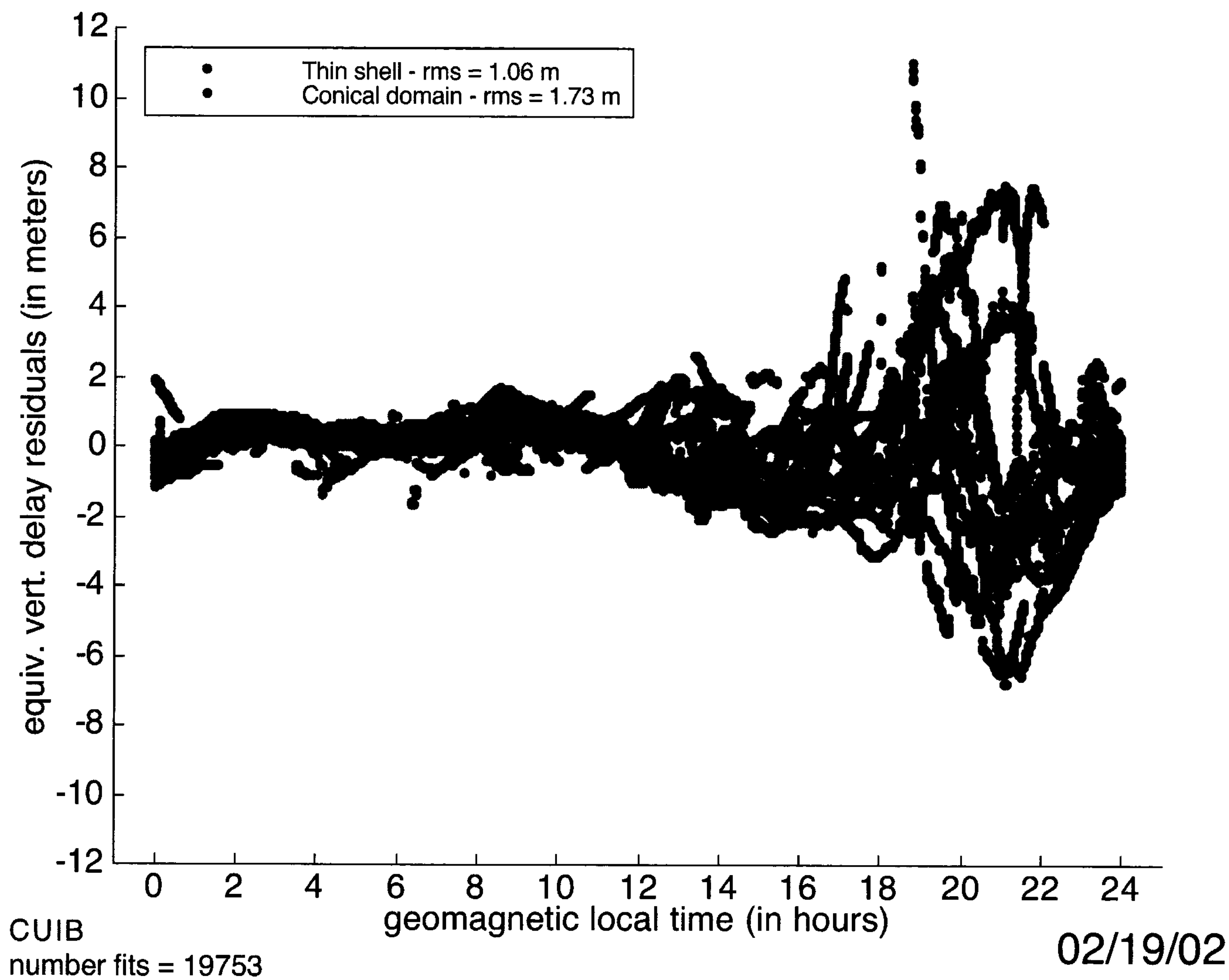

CRO1
GUAT
KOUR
GALA
MANA
FORT
AREQ
UNSA
EISL
SANT
LPGS
RIOG
-120
-105
-90
-75
-60
-45
-30
10
0
-10
-20
-30
-40
-50
-60
-70

Fig 11



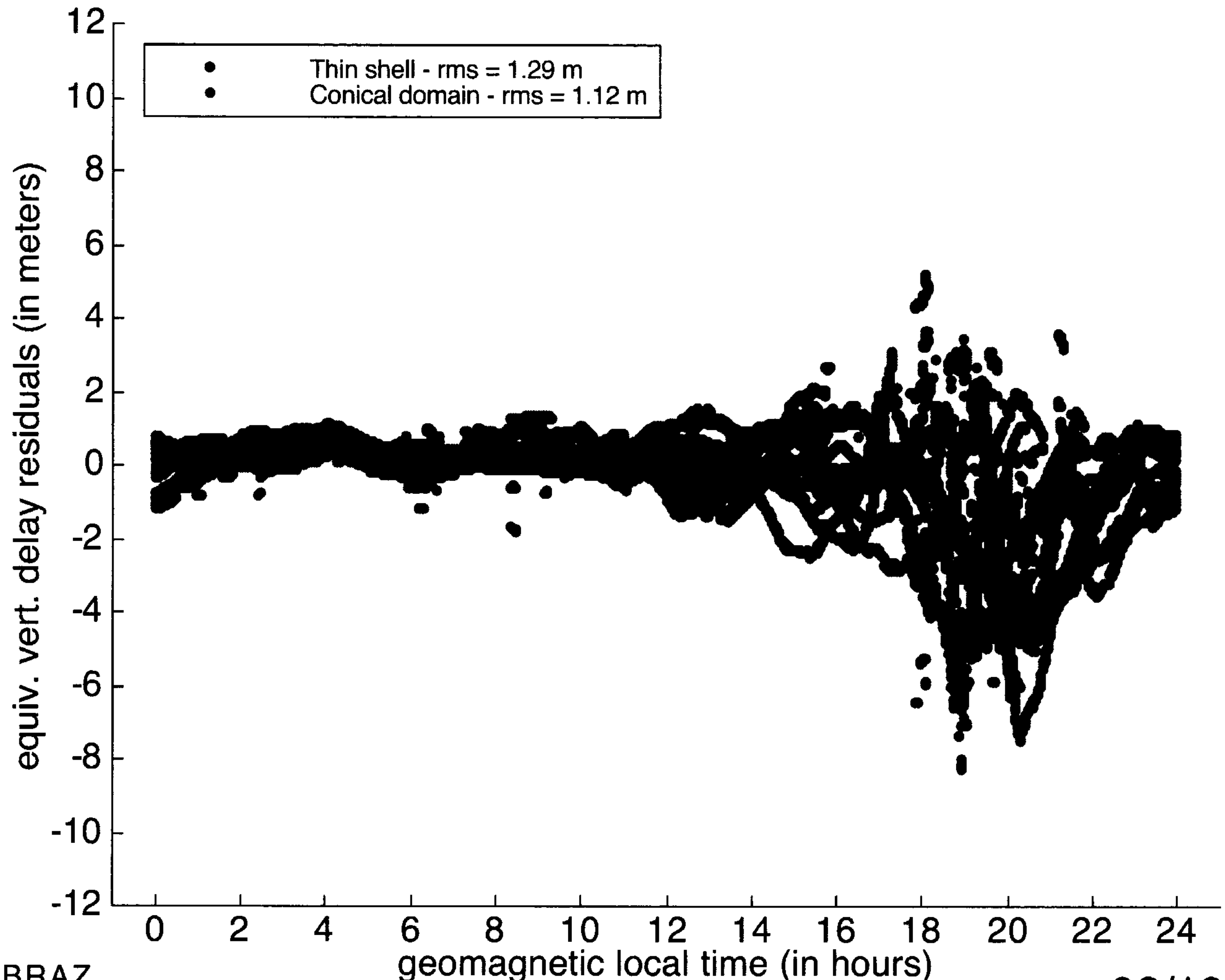
Thin shell - rms = 1.05 m
Conical domain - rms = 1.27 m
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
equiv. vert. delay residuals (in meters)
0 2 4 6 8 10 12 14 16 18 20 22 24
geomagnetic local time (in hours)
IMPZ
number fits = 17561
02/19/02

Fig 12(a)



Thin shell - rms = 1.06 m
Conical domain - rms = 1.73 m
equiv. vert. delay residuals (in meters)
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
0
2
4
6
8
10
12
14
16
18
20
22
24
geomagnetic local time (in hours)
CUIB
number fits = 19753
02/19/02

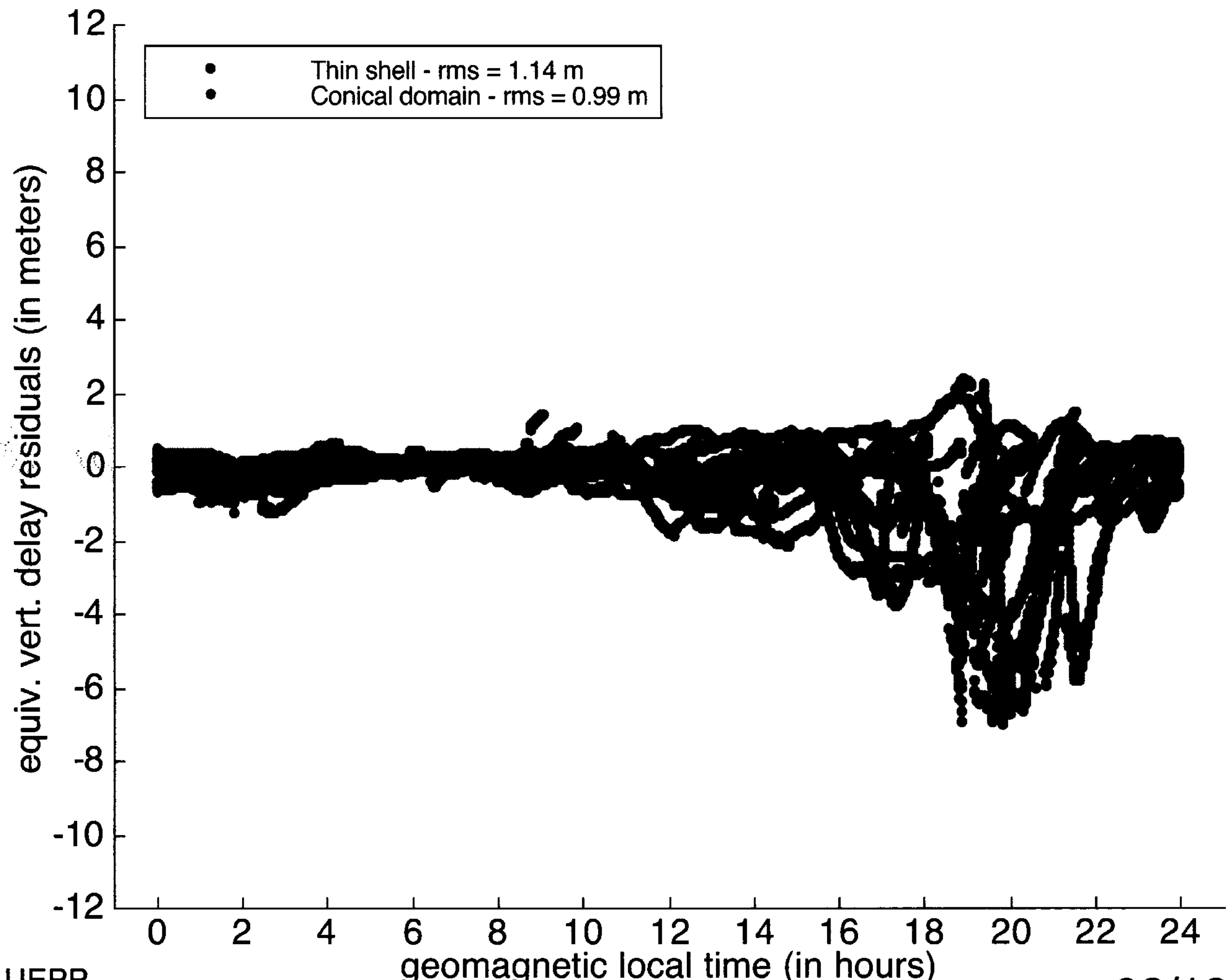
Fig 12(b)



BRAZ
number fits = 20780

02/19/02

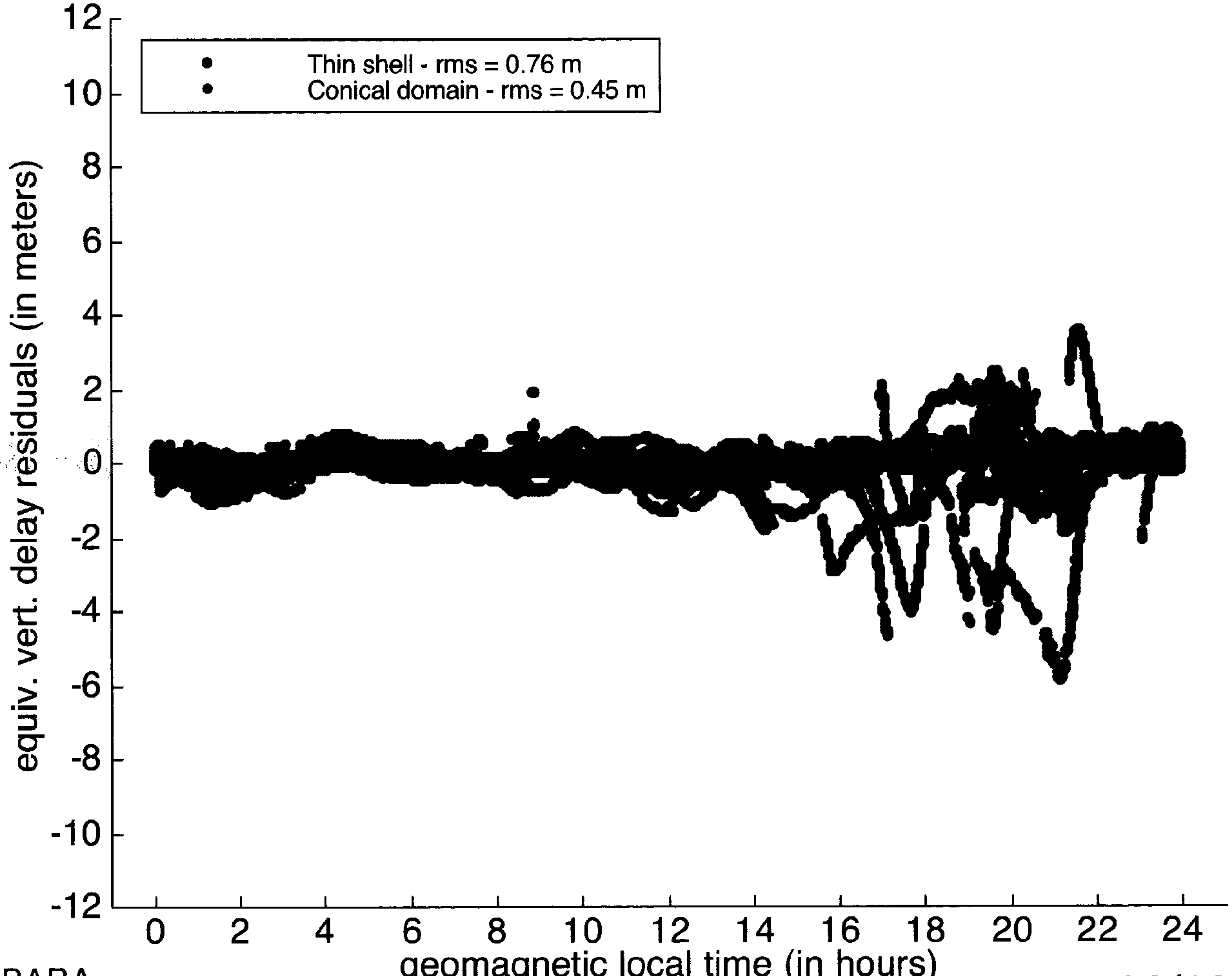
Fig 12(c)



UEPP
number fits = 19627

02/19/02

Fig 12(d)



PARA
number fits = 19301

02/19/02

Fig 12(e)

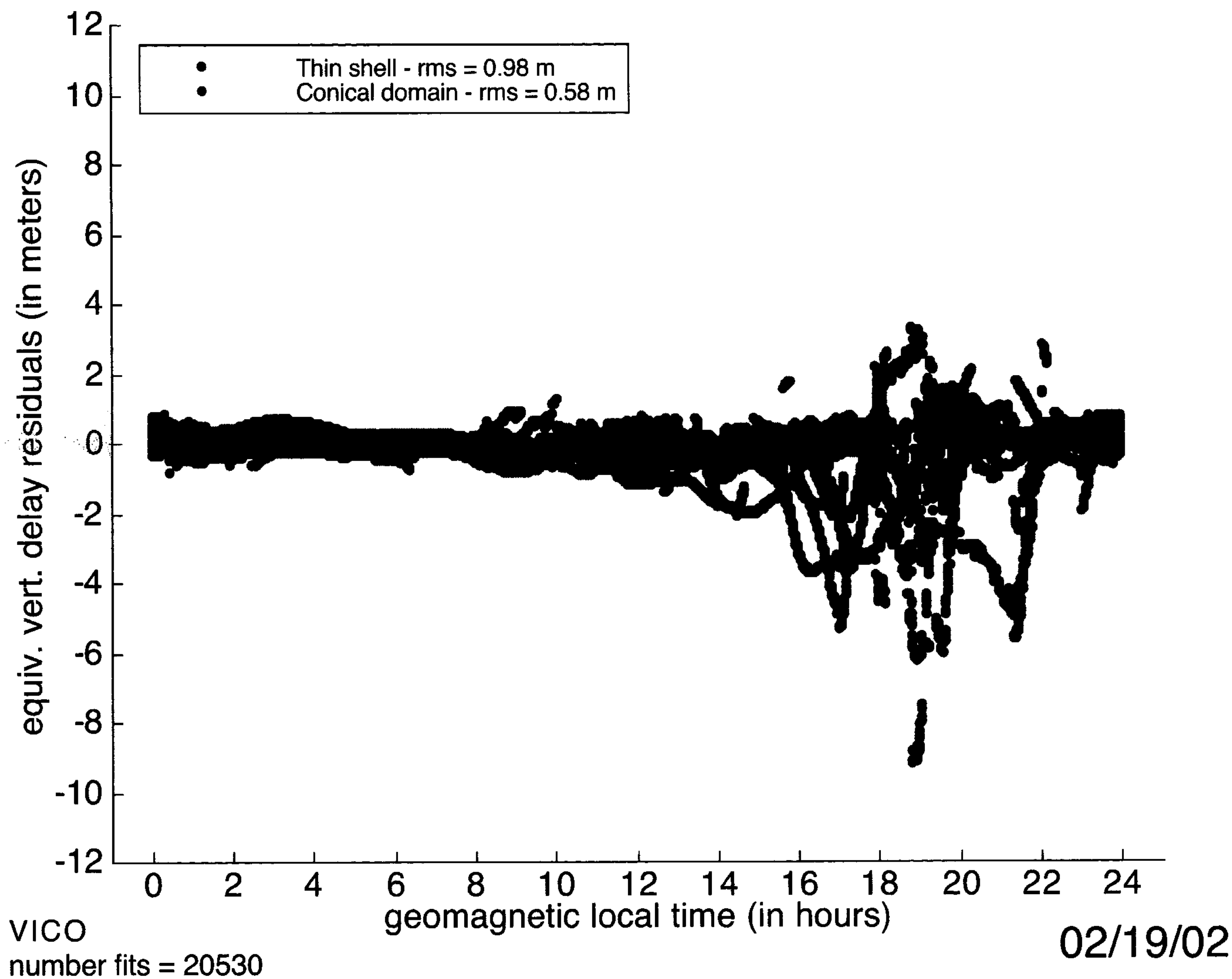
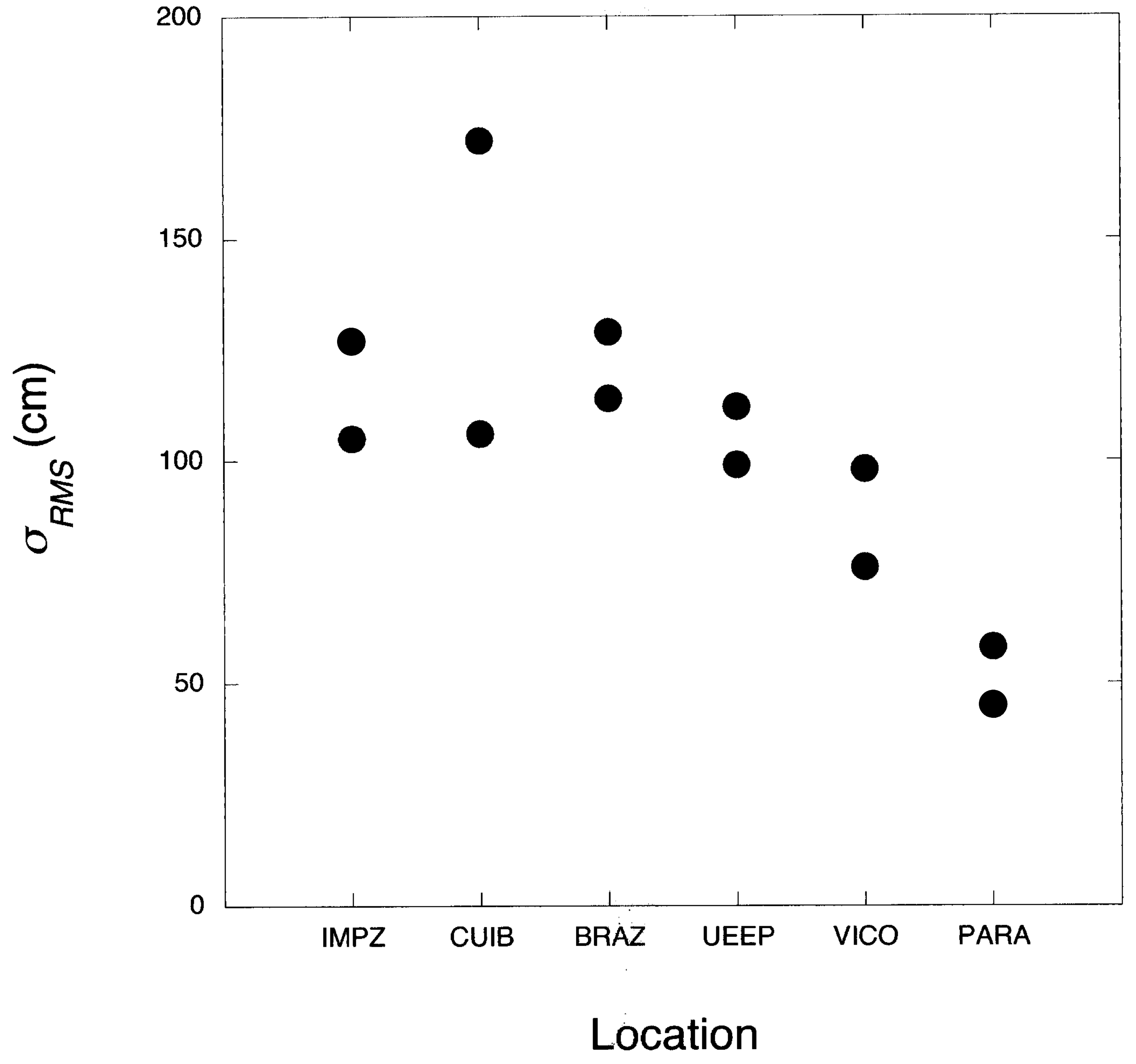

Thin shell - rms = 0.98 m
Conical domain - rms = 0.58 m
equiv. vert. delay residuals (in meters)
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
0 2 4 6 8 10 12 14 16 18 20 22 24
geomagnetic local time (in hours)
VICO
number fits = 20530
02/19/02

Fig 12f



200
150
100
50
0
σ_{RMS} (cm)
IMPZ
CUIB
BRAZ
UEEP
VICO
PARA
Location

Fig 13