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Carl G. Sauer, Jr
Jet Propulsion Laboratory
Pasadena, CA

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Carl G. Sauer, Jr‡

ABSTRACT

The L1 Diamond is a configuration of four solar sail spacecraft traveling in approximately a diamond arrangement and situated on the Sun side of the Sun-Earth L1 Lagrange point. The purpose of this diamond arrangement will be to determine the large scale three-dimensional structure of solar disturbances propagated toward the Earth. These spacecraft will have a wide separation of 100 to 500 Earth radii and solar sails will maintain this separation in non-Keplarian orbits. In this paper equations are derived to calculate the solar sail characteristic acceleration as a function of the position in the L1 Diamond formation. An L1 Diamond formation with a separation of 250 Earth radii is used as an example.

INTRODUCTION

Over the last several years there has been increased interest in the use of solar sails[1] to support a number of near Earth *Space Physics* missions. In a previous paper[2] the author presented results of a study examining the use of solar sails for two high energy *Sun Earth Connection* (*SEC*) missions, a Solar Polar Imaging mission and a Heliopause mission. Another class of *SEC* missions, more suitable for near term solar sail technology, involve Earth synchronous missions in which the sail is placed along or adjacent to the Earth-Sun line, maintaining that position for some specified period of time. An example of one such proposed mission is *Geostorm Warning*, which would be stationed between the Earth and the Sun near L1 to monitor and detect disturbances in the solar wind.[3]

The research described in this paper was performed at the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California under contract with the National Aeronautics and Space Administration.

‡ Principal Member of the Engineering Staff, Outer Planets Mission Analysis Group, Navigation and Mission Design Section, Jet Propulsion Laboratory, California Institute of Technology, Senior Member AAS, Senior Member AIAA.

This paper considers a more extensive application of the *Geostorm Warning* concept in which a group of four solar sails would be placed in a diamond arrangement adjacent to the L1 to achieve a multi-point measurement of the solar environment. As presented in the 2003 SEC roadmap, the science objectives for this mission would be to discover the properties of the solar-wind turbulence as a function of separation and time, to measure possible spatial symmetries of the turbulence, to discover associations of the turbulence with supra-thermal and energetic particles, and to measure the spatial variation in convected and propagating waves, shocks and other disturbances in the solar wind. As presently conceived, this mission would be launched on a Delta launch vehicle with a ballistic transfer to a *Halo* orbit in the vicinity of L1. At this point the individual solar sail spacecraft would be released and independently travel to their assigned locations. A conceptual diagram of one L1 Diamond concept is shown in fig. 1 where the origin of the L1 Diamond is offset from L1 by a small amount, d, towards the Sun. In the concept used in this paper and shown in fig. 1, there is a monitoring position radially inward from the origin of the L1 Diamond by the reference separation, there are two side points in the ecliptic separated from the L1 Diamond origin by half the separation distance, and there is a monitoring position at the separation distance directly above the L1 Diamond origin.

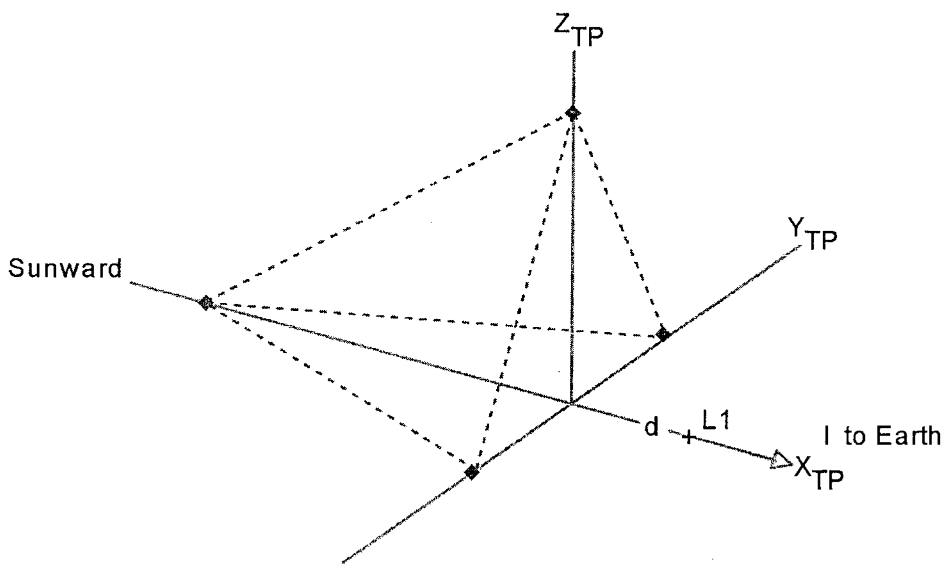

Figure 1 The L1 Diamond

A near term solar sail is eminently suited for such a monitoring mission since the required sail acceleration is significantly less than that required for one of the high energy *SEC* missions examined in Ref. 1. An advantage a solar sail has over other propulsion concepts is that no fuel is consumed and the total time on station is only constrained by other considerations than fuel consumption. There are several ways to configure the L1 Diamond formation. Probably the best way would be to have the diamond positions located such that all of the solar sails would have the same characteristic acceleration. The problem with having the same accelerations for all four diamond positions is that there may be a large difference in the separation distances between the pair of side positions and the other two positions. A second way to configure the L1 Diamond, that used in this paper, is to have the separations the same in the L1 Diamond formation but with different sail characteristic accelerations.

Although a solar sail transfer from the Earth to a L1 Diamond monitoring point is feasible,[4] the present idea is to deliver the four solar sails into a *Halo* orbit around L1, similar to that used in *Genesis*, with a single launch to a ballistic trajectory before releasing the individual solar sails. When the sail acceleration is relatively low, such as that considered for this mission, the L1 Diamond positions are near L1 and are only possible inside L1 towards the Sun. The influence of the Earth and Moon must thus be considered for this L1 Diamond mission which monitors the solar environment.

In this paper, equations are presented enabling the sail characteristic acceleration and sail normal angle to be found as a function of a position in the L1 Diamond formation. A solar sail with real physical properties such as non-unity reflectivity for the sail material is employed for the mission studies for this L1 Diamond mission and the sail characteristic acceleration and sail normal orientation with respect to the sun-line are found as a function of separation distance for each of the monitoring positions. The rationale of using characteristic acceleration, defined as the acceleration of an ideal sail at 1 AU and normal to the sun line, to define sail performance is to enable the results to be independent of payload mass and sail loading.

SAIL FORCE MODEL

The general form for the forces normal to the sail surface, F_N, and tangent to the sail surface, F_T, for a flat surface is given by,

$$\begin{aligned} F_N &= S_0 r_s^{-2} f_N(\alpha) \\ F_T &= S_0 r_s^{-2} f_T(\alpha) \end{aligned} \tag{1}$$

where S_0 is the solar radiation force per unit area on a perfectly reflective flat surface at 1 AU equal to approximately 9.026 x 10^{-6} Nm^{-2}. In the above equations r_S is the heliocentric distance from the Sun in AU, α is the angle between the solar illumination and sail normal, and $f_N(\alpha)$ and $f_T(\alpha)$ are functions relating the dependence of the normal and tangential forces on the angle of incidence of the sunlight. For a perfectly reflecting flat surface there is no tangential component of force and the thrust force and sail normal are coincident so that,

$$\begin{aligned} f_N(\alpha) &= \cos^2\alpha \\ f_T(\alpha) &= 0 \end{aligned} \tag{2}$$

The force in eq. 2 varies as the square of the cosine of the incidence angle, α, since both the force acting on the sail and also the effective area of the sail presented to the Sun vary as the cosine of the incidence angle.

When a more realistic model of a flat sail is considered that is characterized by less than unity reflectance, specular reflections, and radiated energy from the sail, a more complicated expression is needed to define the forces experienced by the sail. The derivation of the resultant forces depend upon a detailed knowledge of the physical properties of the sail material such as the solar reflectance, the fraction of specular reflected light, the emissivity of the front and back surfaces of the sail and an integration of the non-Lambertian re-radiated energy from the sail.

Based upon the above physical properties for a real flat sail model, the normal and tangential force dependencies are given by

$$\begin{aligned} f_N(\alpha) &= c_1 \cos^2\alpha + c_2 \cos\alpha \\ f_T(\alpha) &= c_3 \sin\alpha \, \cos\alpha \end{aligned} \quad (3)$$

where the constants c_1, c_2 and c_3 in the above equations are functions of the physical properties of the sail.[5] One set of coefficients used in the Solar Sail Halley Rendezvous Study[6] in 1976 were,

$$\begin{aligned} c_1 &= 0.9136 \\ c_2 &= -0.0054 \\ c_3 &= 0.0864 \end{aligned} \quad (4)$$

and are nearly the same as the coefficients shown below that are being used to calculate solar sail performance in this L1 Diamond study,

$$\begin{aligned} c_1 &= 0.9150 \\ c_2 &= 0.0001 \\ c_3 &= 0.0850 \end{aligned} \quad (5)$$

The normal and tangential force dependencies from eq. 3 are resolved into radial, f_R, and transverse or circumferential, f_S, components to give,

$$\begin{aligned} f_R(\alpha) &= (c_1 - c_3)\cos^3\alpha + c_2 \cos^2\alpha + c_3 \cos\alpha \\ f_S(\alpha) &= (c_1 - c_3)\sin\alpha \cos^2\alpha + c_2 \sin\alpha \cos\alpha \end{aligned} \quad (6)$$

A comparison of the force vector dependencies for a real set of sail parameters[6] indicates that the sail acceleration goes to zero when the solar incidence angle is approximately 72 degrees and the sail force vector angle is around 55 degrees from the radial direction. For an ideal set of sail parameters the above two angles are the same and the sail acceleration does not go to zero until the angles are at 90 degrees. Trajectory simulations of solar sails with a real set of physical properties must take into consideration these angle constraints such that they are not exceeded and result in an erroneous generation of the sail trajectory. The set of real sail parameters in eq. 5 are used in this study rather than an ideal set since the side positions in the L1 Diamond may require large values of the force vector angle which approach the angle constraints above. The above equations do not consider contributions due to a deviation of the sail from flatness and a more complicated function may be required to adequately model the response of the sail as a function of the solar incidence angle when the shape function of the sail is known.

SAIL FORCE EQUATIONS

An ideal model of the Earth-Sun geometry is assumed to derive the equations defining the required sail characteristic acceleration. In this model a circular orbit of the Earth-Moon barycenter is assumed at exactly 1 AU and the combined gravitational attraction of the Earth and Moon are used. This simplification allows the required sail acceleration to be calculated as a function of the position in the L1 Diamond formation. In the current scenario the relative positions of the monitoring points are assumed to be kept constant with respect to the origin of the L1 Diamond.

As shown in fig. 1, a rotating coordinate system is defined with the origin at the Earth-Moon barycenter and the negative x axis directed toward the Sun with the z axis normal to the Ecliptic and the y axis completing a right-handed coordinate system. Denoting the distance from the Earth of L1 by l and the origin of the L1 diamond configuration by a displacement d from L1, the position, $\boldsymbol{x}$ of a monitoring point in the L1 Diamond with respect to the Earth is,

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_{TP} + l + d \\ y_{TP} \\ z_{TP} \end{bmatrix} \qquad (7)$$

where x_{TP}, y_{TP} and z_{TP} represent the displacement from the origin of a position in the L1 Diamond formation, x_{TP}, l and d being generally negative. In the same coordinate frame but with the Sun now at the center and with the positive x axis towards the Earth, the position $\boldsymbol{x}_S$ of a position in the L1 Diamond formation is given by,

$$\boldsymbol{x}_s = \begin{bmatrix} x + r_E \\ y \\ z \end{bmatrix} = r_s \begin{bmatrix} \cos\theta\cos\varphi \\ \sin\theta\cos\varphi \\ \sin\varphi \end{bmatrix} \qquad (8)$$

where r_s is the distance of the L1 Diamond position from the Sun, and θ and φ are the ecliptic longitude and latitude respectively of the displacement of the position of the sail from the Sun-Earth line. In eq. 8 r_e is the distance from the Sun to the Earth equal to 1 AU.

A Sun centered spherical-polar coordinate system is now defined with one axis toward the particular L1 Diamond position, a second axis normal and in the longitudinal direction and the third axis in the latitudinal direction. Defining unit vectors u_R, u_θ and u_φ respectively in these directions, the position R_S of a point in the L1 Diamond formation is then given by,

$$R_S = r_s \mathbf{u}_R \tag{9}$$

and the position R_E of the Earth in this coordinate frame is,

$$R_E = r_e \mathbf{u}_e \tag{10}$$

where $\mathbf{u}_e$ is a unit vector in the direction of the Earth given by,

$$\mathbf{u}_e = \cos\theta \cos\varphi \, \mathbf{u}_R - \sin\theta \, \mathbf{u}_\theta - \cos\theta \sin\varphi \, \mathbf{u}_\varphi \tag{11}$$

In order for the solar sail to maintain a constant position in the diamond formation, the radial velocity v_R and latitudinal velocity v_φ at this position must be zero and in order to have the same angular rate as the Earth-Moon, the longitudinal velocity v_θ must be equal to,

$$v_\theta^2 = \mu_s r_s^2 r_e^{-3} \cos^2\varphi \tag{12}$$

The equations of motion in this spherical-polar coordinate system are given by,

$$\begin{aligned}
\dot{v}_R &= (v_\theta^2 + v_\varphi^2) r_s^{-1} - \mu_s r_s^{-2} - \mathbf{u}_R^T \mathbf{G} + a_R \\
\dot{v}_\theta &= (v_\theta v_\varphi \tan\varphi - v_R v_\theta) r_s^{-1} - \mathbf{u}_\theta^T \mathbf{G} + a_\theta \\
\dot{v}_\varphi &= -(v_\theta^2 \tan\varphi + v_R v_\varphi) r_s^{-1} - \mathbf{u}_\varphi^T \mathbf{G} + a_\varphi
\end{aligned} \tag{13}$$

where a_R, a_θ and a_φ are the components of sail acceleration in the three directions. The vector $\mathbf{G}$ in the above equations represents the attraction of the Earth-Moon system and the effects of the indirect terms that result because the origin of the coordinate system is the Sun rather than solar system barycenter. The vector $\mathbf{G}$ in eq. 13 is equal to,

$$\mathbf{G} = \mu_e r_e^{-2} \mathbf{u}_r + \mu_e r^{-3} (r_s \mathbf{u}_r - r_e \mathbf{u}_e) \tag{14}$$

and can be rewritten as,

$$\mathbf{G} = \mu_e r_e^{-2} \{ r_e^2 r_s r^{-3} \mathbf{u}_R + (1 - r_e^3 r^{-3}) \mathbf{u}_e \} \tag{15}$$

In the above equations, μ_s is the gravitational constant of the Sun and μ_e is the gravitational constant of the Earth-Moon system and r is the distance from the Earth to the L1 Diamond position and is equal to the magnitude of the position vector $\mathbf{x}$ appearing in eq. 7.

Setting the radial and latitudinal velocities to zero and also the time derivatives of the velocities in eq.13 to zero since each component of velocity is constant with respect to time, eq.13 becomes,

$$\begin{aligned}
\dot{v}_R &= \mu_s r_s^{-2}(r_s^3 r_e^{-3}\cos^2\varphi - 1) - \mu_e r_e^{-2}\{r_s r_e^2 r^{-3} + (1 - r_e^3 r^{-3})\boldsymbol{u}_e \bullet \boldsymbol{u}_R\} + a_r = 0 \\
\dot{v}_\theta &= -\mu_e r_e^{-2}(1 - r_e^3 r^{-3})\boldsymbol{u}_e \bullet \boldsymbol{u}_\theta + a_\theta = 0 \\
\dot{v}_\varphi &= -\mu_s r_s^{-2}(r_s^3 r_e^{-3}\sin\varphi\cos\varphi) - \mu_e r_e^{-2}(1 - r_e^3 r^{-3})\boldsymbol{u}_e \bullet \boldsymbol{u}_\varphi + a_\varphi = 0
\end{aligned} \tag{16}$$

It is convenient to set the gravitational attractions in eq. 16 to,

$$\begin{aligned}
\mu_s r_s^{-2} &= g_s = 5.9300835\, mm/s^2 \\
u_e r_e^{-2} &= g_{em} = g_s / 328900.
\end{aligned} \tag{17}$$

and, from eq. 16 and eq. 17, the three components of acceleration required to keep the solar sail spacecraft in the desired position are,

$$\boldsymbol{a} = \begin{bmatrix} a_R \\ a_\theta \\ a_\varphi \end{bmatrix} = g_s \begin{bmatrix} 1 - r_s^3 r_e^{-3}\cos^2\varphi \\ 0 \\ r_s^3 r_e^{-3}\sin\varphi\cos\varphi \end{bmatrix} + g_{em} \begin{bmatrix} r_s r_e^2 r^{-3} + H\cos\theta\cos\varphi \\ -H\sin\theta \\ -H\cos\theta\sin\varphi \end{bmatrix} \tag{18}$$

where the components of the sail acceleration in eq. 18 are in the radial, longitudinal and latitudinal directions and the variable H is defined as,

$$H = (1 - r_e^3 r^{-3}) \tag{19}$$

In order to calculate the sail normal angle and, from it, the sail characteristic acceleration, the acceleration vector $\boldsymbol{a}$ is expressed as a function of the magnitude a of the acceleration adjusted to a 1 AU distance, the sail force vector cone angle γ and the out-of-plane angle β as,

$$\boldsymbol{a} = \begin{bmatrix} a_R \\ a_\theta \\ a_\varphi \end{bmatrix} = a\, r_s^{-2} \begin{bmatrix} \cos\gamma \\ \sin\gamma\cos\beta \\ \sin\gamma\sin\beta \end{bmatrix} \tag{20}$$

Since the sail normal angle α and force vector angle γ are the same for an ideal sail, the characteristic acceleration for an ideal sail is found by dividing the acceleration magnitude, a, by the square of the cosine of the sail force vector angle, γ. For a real set of sail parameters, where the actual physical characteristics of the sail material are considered, the cone angle γ of the force vector and sail normal angle α are not in the same direction and the sail normal angle and sail characteristic acceleration are not easily solved in a closed form. In this case a one parameter search is used to determine the sail normal angle and from that, the sail characteristic acceleration.

L1 DIAMOND FORMATION FOR 250 R_E SEPARATION

To illustrate the above analysis, the sail acceleration and sail normal angle is calculated for each of the monitoring points in a L1 Diamond configuration with a 250 R_E separation. In this case the four locations are identified as follows, first at 250 R_E from the origin of the L1 Diamond towards the Sun with coordinates (-250:0:0), next a separation of 125 R_E from the origin on both the right and left side positions with coordinates (0:125:0) and (0:-125:0) and last a point 250 R_E above the diamond origin with coordinates (0:0:250). An offset between L1 and the origin of the L1 Diamond, shown in the previous equations, is included to reduce the magnitude of the required characteristic acceleration of the solar sail necessary to maintain the desired orientation. The effect of this offset is shown in fig.2 where the required characteristic acceleration is shown for each of the L1 Diamond positions and as a function of the radial offset of the L1 Diamond origin towards the sun.

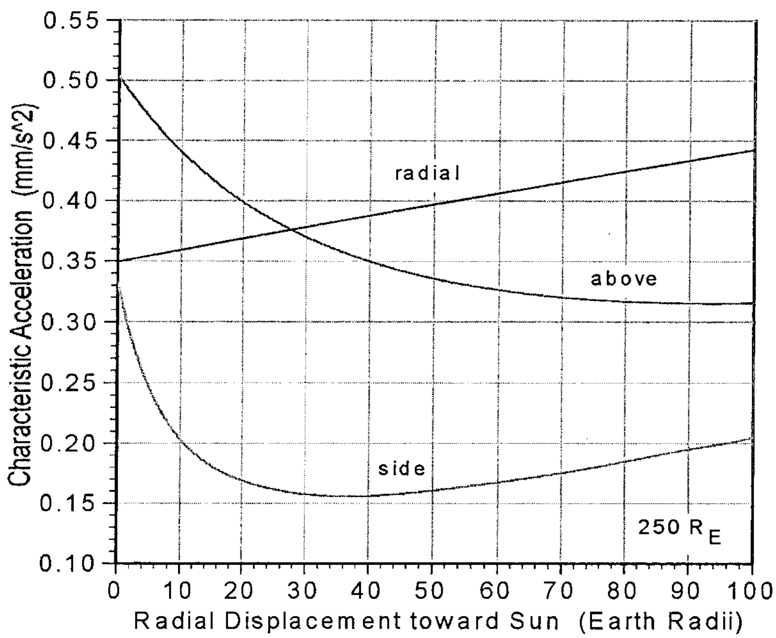

Figure 2. Sail Characteristic Acceleration

Only a single side location is shown since both the right and left positions have the same characteristic acceleration and sail normal angle α although opposite values of the out-of-plane angle β. The out-of-plane angle is not defined for the radial location since the sail normal angle is zero for this position.

If an offset of the origin of the L1 Diamond were not included, the characteristic sail acceleration would be large since the sail normal angles for both the side and out of ecliptic positions are high as seen in fig. 3 at a zero offset of the origin. Including an offset such that the origin of the L1 Diamond is further from L1 results in a reduction in the sail normal angle and a consequent reduction in the required sail characteristic acceleration. An offset of 30 R_E is used in the examples presented in this paper and was chosen such that the sail accelerations for the two side positions were minimized and the sail accelerations for the radial and out of ecliptic positions were approximately equal.

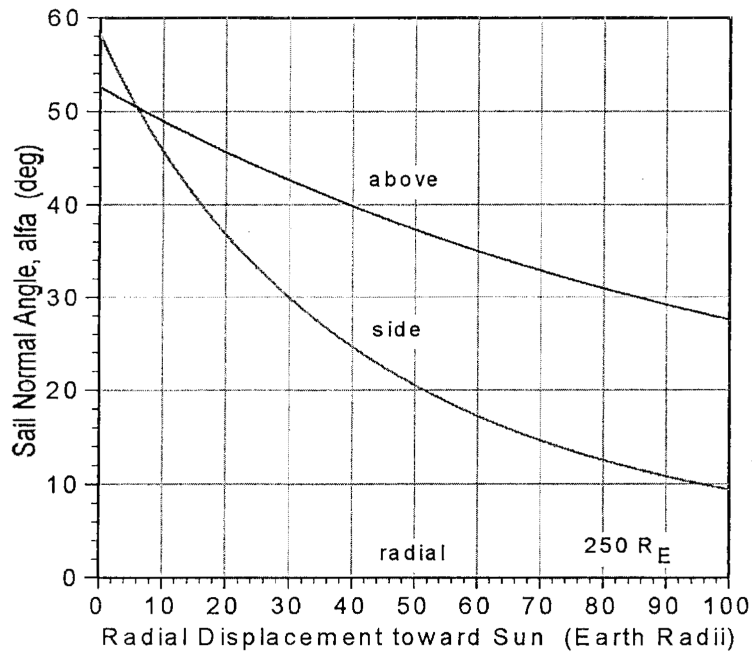

Figure 3. Sail Normal Angle

The values of sail characteristic acceleration and sail normal angle for the 30 R_E offset of the L1 Diamond origin shown in figs 1 and 2 were verified by a simulation of the trajectory for each of the L1 Diamond positions. The Earth-Moon barycenter and Sun were used as attracting bodies and the Earth-Moon barycenter was assumed in a circular orbit around the Sun. This simulation resulted in less than a 1 km variation from the desired location after a period of one year thus verifying that the equations for the acceleration and orientation presented previously were correct.

SAIL TRANSFER TO L1 DIAMOND LOCATION

The previous sections were characterized by an ideal model for the Sun-Earth-Moon system where there were no perturbations from bodies other than the Earth-Moon. If a more realistic model is assumed for the Earth-Moon system where actual positions of the Earth and Moon are used, it is not expected that the L1 Diamond formation would maintain its shape over the period of a year. As mentioned before the present mission scenario considers a ballistic launch to a *Halo* orbit similar to that used for the *Genesis* mission. This *Halo* orbit is a quasi-stationary orbit around the L1 location and requires relatively small corrective maneuvers to maintain the orbit over extended periods of time. The solar sail transfer from the *Halo* orbit to each of the desired locations also used the more exact physical model of the sail parameters. Although the L1 Diamond mission is not planned until the middle of the next decade, an example of a launch in 2008 into a *Halo* orbit was available from the *Geostorm Warning* study[3] and the examples in this paper used this *Halo* orbit to illustrate the transfer from that orbit to each of the L1 Diamond positions.

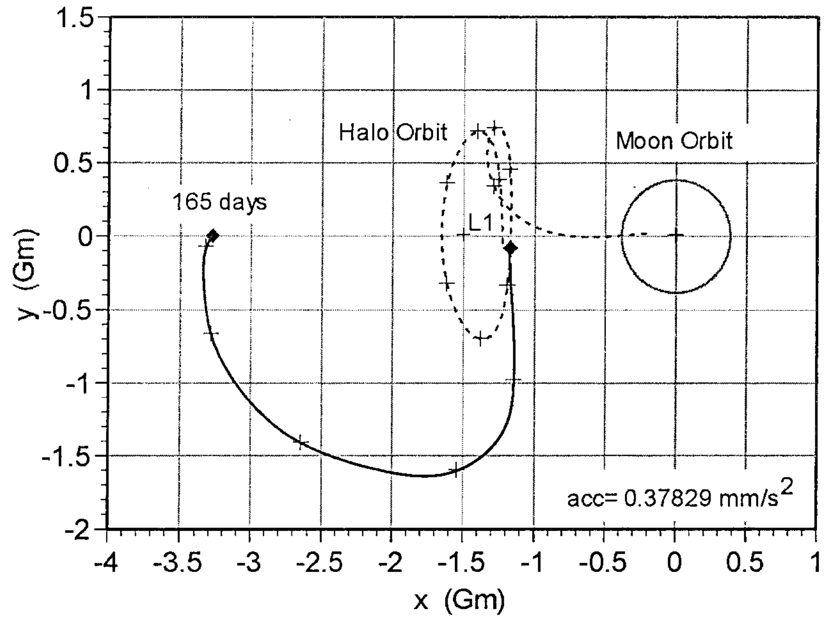

Figure 4. Sail Transfer from Halo Orbit to -30:-250:0:0

Fig. 4 is an ecliptic view of the *Halo* orbit (the dashed line) and the sail trajectory to the position (-30:-250:0:0) in the radial L1 Diamond position where the offset of the diamond origin in Earth radii is indicated by the -30 in the description of the sail position. In this and the following figures the trajectories are shown in an Earth centered, rotating coordinate frame with the negative *X* axis directed toward the Sun with tic marks at 30 day intervals. The trajectory simulation for these transfers uses the Earth, Moon and Venus as perturbing bodies. The units in these plots of transfer trajectories are Giga-meters or millions of kilometers. The orbit of the Moon is shown to give an illustration of the scale of the trajectories and the sail characteristic acceleration and the location of L1 are also shown. In these figures the departure point on the Halo orbit is optimized to minimize the transfer time to the position in the L1 Diamond formation. In each case the sail characteristic acceleration given in fig.2 was used for each of the transfers with the sail steering direction optimally directed.

Figs. 5 and 6 show the sail transfer trajectories from the *Halo* orbit to the right and left side L1 Diamond positions. Although one would expect both sail transfers to appear similar, there is some difference since the *Halo* orbit is not quite symmetric around the L1 position and furthermore is inclined somewhat to the ecliptic. This inclination is not apparent in the above figures and results in some asymmetry in the departure conditions and may be the cause of the difference in flight time for the transfer to these two side locations.

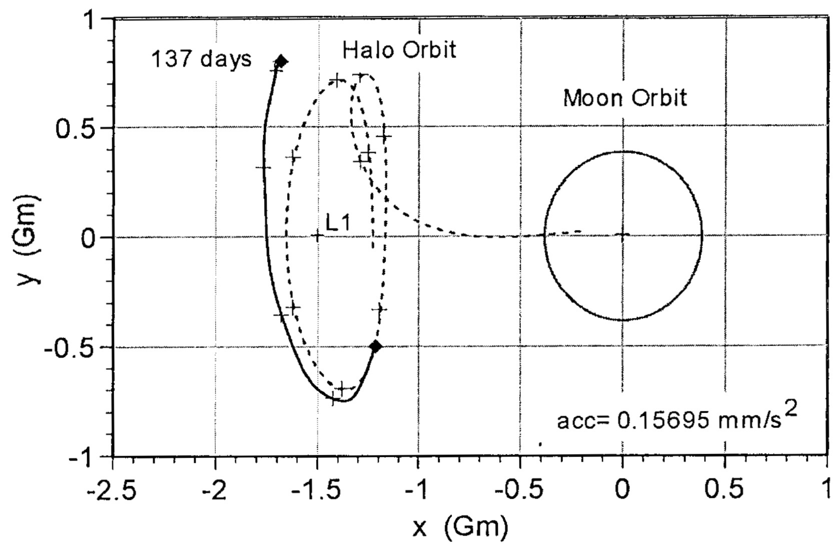

Figure 5. Sail Transfer from Halo Orbit to -30:0:125:0

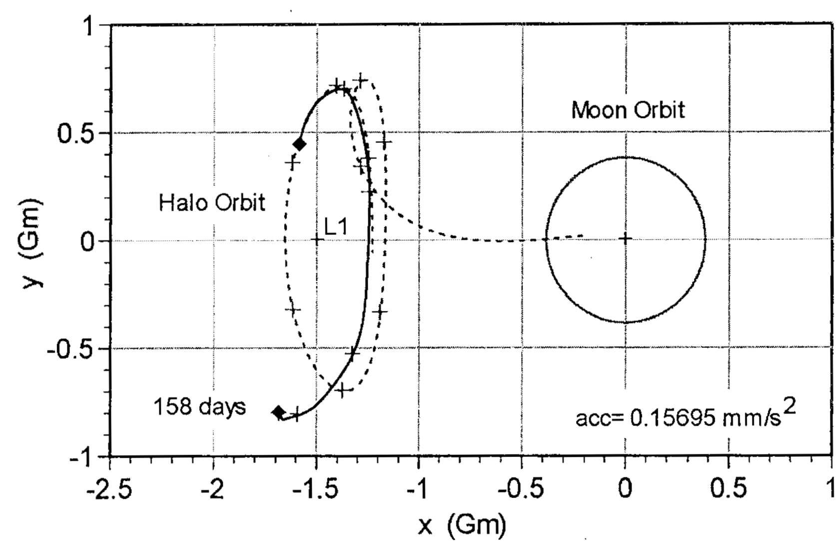

Figure 6. Sail Transer from Halo Orbit to -30:0:-125:0

Figs. 7 and 8 show the sail transfer trajectory to the L1 Diamond position that lies above the ecliptic. Fig.7, like the previous examples, shows a projection of the *Halo* and sail trajectories on the ecliptic. The optimum departure point on the *Halo* orbit for this transfer appears close to that of two of the previous three examples. In an actual mission it would be more practical to depart on subsequent orbits on this *Halo* orbit in order to allow time for the release and checkout of each of the sail spacecraft. This was not done for these trajectories since the integrated trajectory file for the *Halo* orbit that was used for these examples was not extended in time sufficiently to allow the departure point to be optimized on subsequent orbits.

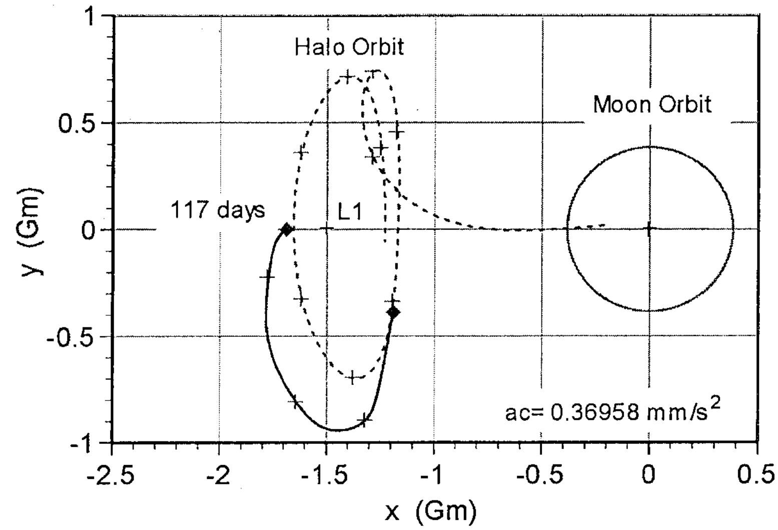

Figure 7. Sail Transer from Halo Orbit to -30:0:0:250

Since this last L1 Diamond position extends above the ecliptic, it is interesting to examine the out-of-plane component of the transfer trajectory. A projection of the trajectory on the X-Z plane normal to the Y axis is shown in fig. 8 and shows not only the out-of-ecliptic part of the trajectory but also the extent of the *Halo* orbit that is out of the ecliptic plane. It is uncertain to what extent the L1 *Halo* orbits differ from one another and to the one used here when launched at different times of the year since the geometry with respect to the ecliptic plane should vary to some extent. As such the preceding solar sail trajectories should be considered more as examples to illustrate the sail transfer from a *Halo* orbit rather than

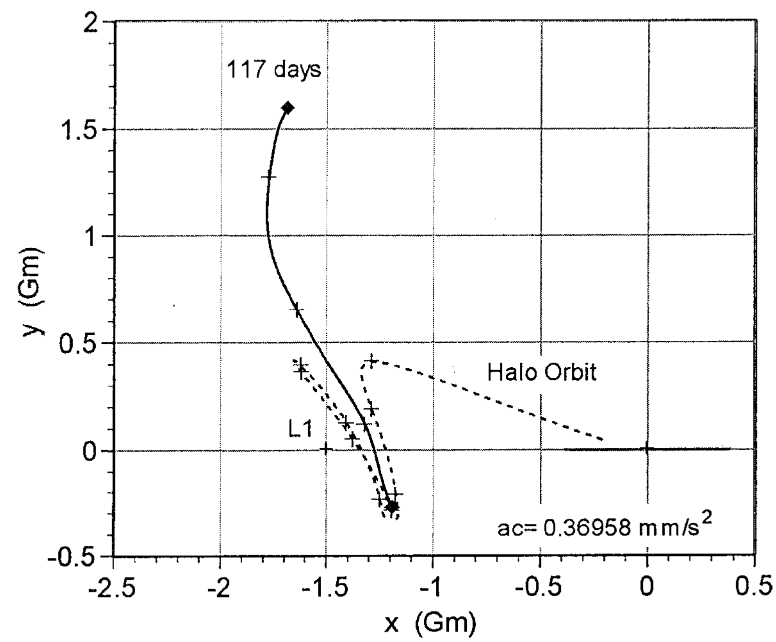

Figure 8. Sail Transer from Halo Orbit to -30:0:0:250

actual design points. It is conceivable that there is an Earth departure date into a *Halo* orbit that will result in the best combination of sail transfer trajectories from the *Halo* orbit to the desired L1 Diamond positions.

ORBIT SUSTENANCE

In a sense L1 is mainly a mathematical abstraction and in the real world, with all the various perturbations together with an elliptical Earth orbit around the Sun, L1, if it does exist, varies in position to some extent. Although the L1 Diamond formation can be kept together in an ideal Earth-Moon system as was assumed in generating the sail force equations, it is doubtful if the same can be realized when the actual positions of the Earth, Moon and other bodies are present. In this case it is desirable to try to keep the Diamond arrangement regulated as close as possible by continuously re-targeting the sail spacecraft. Various strategies to do this re-targeting have been examined by Yen[3] in investigating station keeping for the *Geostorm Warning* mission and the similar strategies are equally applicable to this mission.

However only a cursory examination was made in regard to station keeping during the observation phase of this L1 Diamond mission. The strategy that was examined was to keep the sail characteristic acceleration the same as defined previously but to change the nominal values of the fixed sail angles α and β slightly from the values defined previously in order to return to the original position after some period of time. In order to return to this position a third variable is needed, namely the time. Since there are only three variables available to re-target the spacecraft, it is not possible to null the velocity errors. A fixed sail steering direction was chosen since it was the easiest to simulate in the present solar sail trajectory software, however other steering strategies are probably better since this strategy leaves residual velocity errors that tend to increase on subsequent trajectory phases. The final result being that the sail will eventually be unable to correct and re-target to the desired location.

The path traced out for the first L1 Diamond position, in the radial direction towards the Sun from L1, is shown in fig. 9. In this and following figures, the deviation from the specified L1 Diamond position is shown in units of Earth radii. For this radial L1 Diamond position, the solar sail was able to re-target to the original position in 382 days with a maximum excursion of approximately 55 Earth radii in the negative Y direction and a residual velocity error of close to 40 m/s upon returning to the starting position.

Upon re-targeting, the sail normal angle α increased from 0 degrees to 0.4 degrees and the out-of plane angle β, previously undefined, now became 190.1 degrees. If the sail had not been re-targeted to the original position but instead had kept the original values of α and β, the position error after 382 days would have been nearly 1760 Earth radii. This position error illustrates to some extent the sensitivity to sail steering that will be present for this type of solar sail mission in the vicinity of L1.

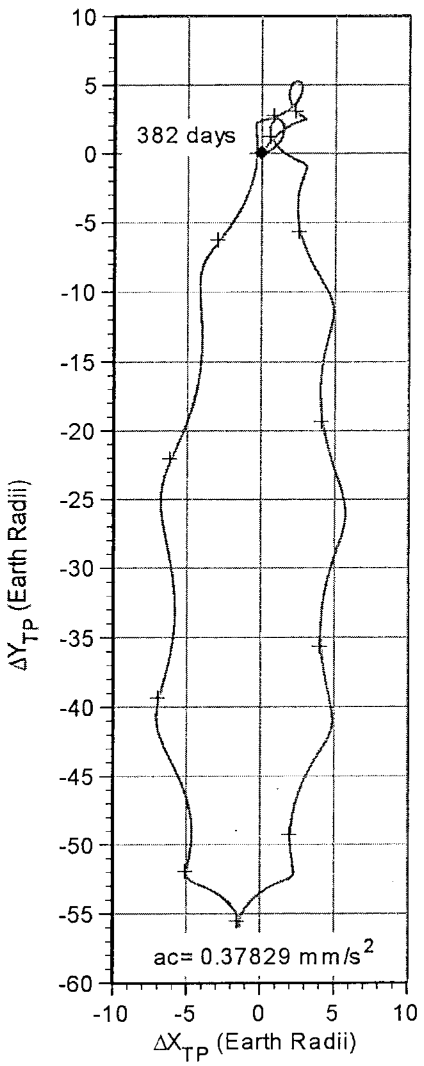

Figure 9. Varition in -30:-250:0:0

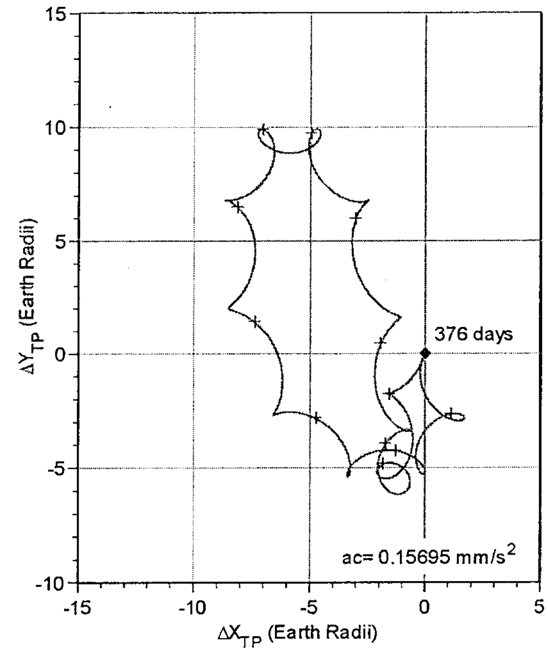

Figure 10. Variation in -30:0:125:0

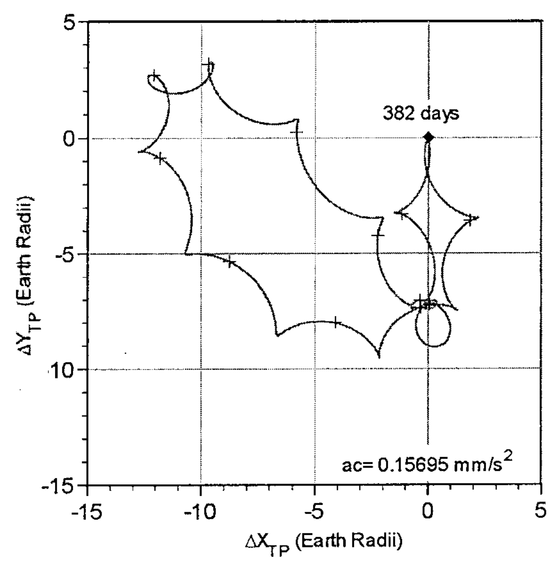

Figure 11. Varition in -30:0:-125:0

The deviation from the station location for each of the L1 Diamond side positions is shown in figs. 10 and 11. As expected the maximum deviation is less than that for the radial L1 Diamond position while the time to re-target is about the same. An interesting aspect shown in these two figures is the effect of the Moon on the sail trajectory. The tick marks on the figures are spaced at 30 day increments and are close to the separation time of the cusps in these figures. The effect of the Moon can also be seen in fig. 9 for the radial position offset although it is a little more subtle because of the greater distance from the Earth-Moon system. As a consequence of the re-targeting, the fixed sail normal angle α decreased by 1.6 degrees for the position in the positive Y direction and decreased by 2.7 degrees for the position in the negative Y direction. The out-of-plane angle β changed by 0.1 degrees for both side positions and the velocity error at the re-targeted positions amounted to 24 m/s and 19 m/s respectively for the positive and negative side positions.

The deviation of the sail trajectory from the out-of-ecliptic L1 Diamond position is shown in figs.12 and 13. Fig.12 shows a projection of the sail trajectory on the ecliptic where the effects of the perturbation of the Moon are readily apparent. A projection of the trajectory normal to the X-Z plane, shown in fig.13, indicates a more subtle influence of the Moon in the plane normal to the ecliptic.

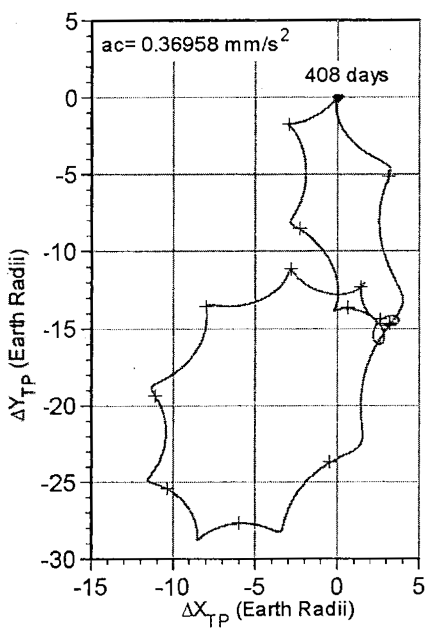

Figure 12. Variation in -30:0:0:250

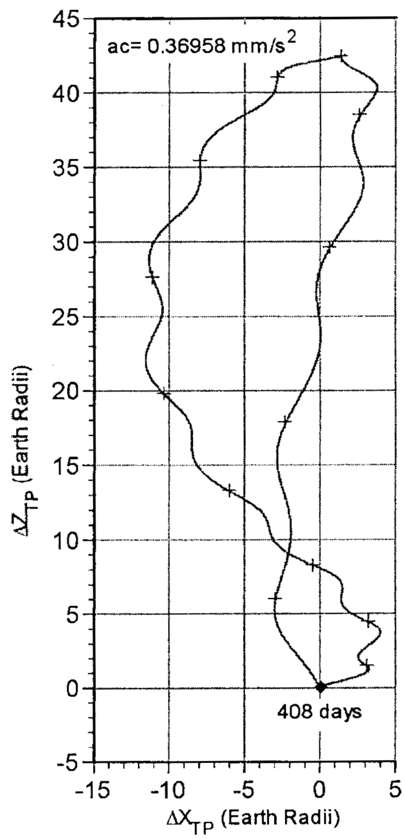

Figure 13. Variation in -30:0:0:250

A time of 408 days is required to re-target to the L1 Diamond position located above the ecliptic plane, this time being longer than that required for the other L1 Diamond positions. The deviations from the nominal position during the re-targeting phase are larger than those for the side positions but not quite as large as the deviation for the radial offset position. The horizontal deviation of the sail trajectory from the nominal point is around 30 Earth radii while the vertical deviation is slightly greater at 42 Earth radii. In order to re-target, the sail normal angle decreased by 1.3 degree from the nominal value and the out-of-plane angle β increased from 90 degrees to 91.1 degrees. The residual velocity at the re-targeted position above the ecliptic plane is approximately 30 m/s, a little less than that for the radial offset position.

SUMMARY AND CONCLUSIONS

This paper has defined the solar sail design parameters necessary for a set of solar monitoring stations around L1. This long term "L1 Diamond" mission requires only modest values of sail characteristic acceleration but is likely to present difficulties in maintaining the diamond formation over extended periods of time. The ultimate goal of this mission is, not only to keep the sail spacecraft spacing in the formation relatively constant, but also to be able to vary the spacing to get a different spatial resolution of the solar environment. The ability to change the spacing between the sail spacecraft will affect the entire configuration of the L1 Diamond and will be the subject of additional studies in the future as will examinations of the L1 Diamond configurations with equal values of sail characteristic acceleration for each of the diamond positions.

The maintenance of the Diamond formation is also of great importance and will be a strong function of the particular sail steering strategy that is adopted. This concern must also be addressed in the future and will require changes, not only in the solar sail trajectory simulation software, but also in navigation software that will be required to actually fly and navigate the sail spacecraft. The examples presented in this paper are intended more to provide a general description of the L1 Diamond mission and to familiarize the reader with some the problems inherent in this type of solar sail mission.

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REFERENCES

1. Mulligan, Patricia, *Solar Sails for Operational Space Community*, Space Times, Magazine of the AAS, Vol 38, No 1, Jan-feb, 1999

2. Sauer, Carl G., Jr., *Solar-Sail Trajectories for Solar Polar and Interstellar Probe Missions*, AAS Paper 99-336, AAS/AIAA Astrodynamics Specialist Conference, Girdwood, Alaska, Aug 16-19, 1999

3. Yen, Chen-Wan L, *Solar Sail Geostorm Warning Mission Design*, AAS Paper 04-107, 2004 Spacflight Mechanics Conference, Maui, Hawaii, Feb 8-12, 2004.

4. Powers, R.B., and Coverstone-Carrol, V., *Optimal Solar Sail Orbit Transfers to Synchronous Orbits*, AAS Paper 99-334, AAS/AIAA Astrodynamics Specialist Conference, Girdwood, Alaska, Aug 16-19, 1999

5. McInnes, Colin R., *Solar Sailing, Technology, Dynamics and Mission Applications*, Springer-Praxis Series in Space Science and Technology, Praxis Publishing Ltd, Chichester. UK, 1999

6. Sauer, Carl G. Jr., *A Comparison of Solar Sail and Ion Drive Trajectories for a Halley's Comet Rendezvous Mission*, AAS Paper 77-4, AAS/AIAA Astrodynamics Conference, Jackson. Wyoming, Sep 7-9, 1977