A General Stability Condition for the Leader/Follower Formation Flying Control Architecture

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***Abstract*— The Leader/Follower (L/F) architecture is currently being considered as a method for controlling formations consisting of spacecraft, robots, UAVs and other autonomous vehicles. Although ad-hoc stability results exist for the L/F architecture (e.g., stability has been guaranteed under a variety of specific linear and nonlinear control laws), a general stability criterion, *independent of any particular implementation*, has not been previously stated in the open literature. In this paper, a definition of L/F, encompassing previous definitions is given, and a general stability condition for L/F is presented based on hierarchical system theory. The stability condition is applicable to systems with nonlinear, closed-loop dynamics under various structural constraints. Further, the stability condition shows that by using the L/F architecture it is (at least theoretically) possible to reduce formation control design to the more tractable problem of individual vehicle control design.**

I. Introduction

In space applications, NASA and ESA are considering formations of spacecraft for detecting extra-Solar, Earth-sized planets [14], [7] and imaging the event horizon of a black hole [8]. In the atmosphere, formations of UAVs are being studied for reconnaissance, as well as search and rescue missions [1], [31]. Further, multiple underwater vehicles are also being studied for similar tasks, such as searching for mines [11].

We define formation flying as a set of more than one vehicle in which *any of the vehicle dynamic states are coupled through a common control law* [22].[2] Formation guidance and control poses numerous challenges such as avoiding collisions and balancing resources [10]. Various formation control architectures have been developed to address these challenges [15], [21].

For spacecraft formations, the Leader/Follower (L/F) architecture is the most studied control architecture [21].[3] As the name suggests, in L/F some vehicles (called followers) track trajectories with respect to other vehicles (called leaders). An important aspect of L/F is that the control action of each leader must be independent of the motion of each associated follower. In most of the applications considered to date, L/F control designs have focused on (1) a single leader vehicle and a single follower vehicle or (2) a "chain" of vehicles each following the preceding vehicle (e.g. [23], [12], [6], [30]).

In the literature, studies of the L/F architecture (as opposed to L/F controller *design*) have been based on directed graphs [27], [4], [5], [17]. However, these formal definitions are not completely general in the sense that they either (1) limit the scope of the L/F architecture to specific control designs, or (2) make specific assumptions regarding inter-vehicle dependencies to simplify subsequent analyses. The authors have defined L/F in a previous paper [21] using the control dependency directed graph. Although, the control dependency directed graph is similar to directed graphs defined in [4], [5], [18] and [16], it is less restrictive and does not depend on a particular control strategy.

In this paper, we first provide a brief review of directed graphs and hierarchical systems. Next, we present a graph-theoretic definition of formation flying consistent with the definition given above. Moreover, we also present a general, graph-theoretic definition of the Leader/Follower architecture that encompasses previous definitions given in the literature.

We then demonstrate that, under reasonable structural assumptions on vehicle dynamics, our definition of L/F leads to closed-loop error dynamics of an L/F formation that have the form of a hierarchical system. The major contribution of this paper is the application of concepts from hierarchical system theory [25] to the problem of L/F stability. Specifically, we obtain a *general* stability result for the L/F architecture under mild assumptions: our stability result states that if the origin of the tracking error dynamics for each individual vehicle is asymptotically stable, then the origin of the tracking error dynamics for the entire formation is also be asymptotically stable. In short, the formation is stable if all the individual vehicle controllers are stabilizing, regardless of the particular control law or vehicle dynamics. As a result, the Leader/Follower architecture can reduce *formation* control design to the control design of a *single* vehicle.

II. Mathematical Background

In this section, we discuss some important assumptions on formation dynamics, as well as the relevant mathematical background from the theories of directed graphs and hierarchical systems that is needed in the sequel.

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[2]This coupling can be in translational and/or rotational degrees of freedom and in position and/or velocity. For example, a formation may consist of ten spacecraft with synchronized angular velocities.

[3]Note that Leader/Follower has also been referred to as Chief/Deputy [23], Master/Slave [13] and, the traditional terminology from two-spacecraft rendezvous, Target/Chase.

Consider a collection of N vehicles with states z_i, control inputs u_i, desired reference states (i.e., trajectories) d_i, tracking errors $e_i \triangleq d_i - z_i$, and disturbances w_i, where $i = 1, \ldots, N$. For vehicle i, the dynamics may be nonlinear and time-varying, and further, d_i and u_i may depend on the states of any other vehicles. However, we assume that the dynamics, control laws, and reference trajectories are such that the error dynamics can be put in the following form:

$$\dot{e}_i = h_i(t, e_1, \ldots, e_N) \tag{1}$$

where h_i is continuously differentiable, and $h_i(t, 0, \ldots, 0) = 0$, where 0 is the zero vector of appropriate dimension.

A. Directed Graphs

In this section, we first review the definition of a directed graph (or digraph). In addition, four concepts from digraph theory will be used in this paper: vertex degrees, cycles, weak connectedness, and the adjacency matrix. We briefly discuss each concept, and then present two theorems that are used in the sequel. This section summarizes material found in [3].

A directed graph G consists of a set of vertices $\mathcal{V}$ and a set of directed edges $\mathcal{E}$. The vertices are represented by v_i, $i = 1, \ldots, L$, where L denotes the number of vertices. The directed edges, herein referred to as edges, are specified as ordered pairs (v_i, v_j). The edge $(v_i, v_j) \in \mathcal{E}$ means that an arrow is drawn *from* v_i *to* v_j. An edge (v_i, v_i) is referred to as a *self-loop*. One interpretation for an edge (v_i, v_j) is that vertex i (representing vehicle i) is maintaining a desired position with respect to vertex j (representing vehicle j).

The *in-degree* of a vertex is the number of edges directed into the vertex. That is, the in-degree of v_i equals the number of edges of the form $(\cdot, v_i)$. Similarly, the *out-degree* of a vertex is the number of edges directed out of the vertex (i.e., the number of edges of the form $(v_i, \cdot)$).

To define a cycle, the concept of a walk is needed. A *walk* is a sequence of vertices such that each sequential pair forms an edge, and no edge is repeated (i.e., each sequential pair of vertices appears only once in a given order).[4] The length of the walk is the number of vertices in it. For example, if (v_i, v_j) and (v_j, v_k) are edges, then $v_i v_j v_k$ is a walk of length three. A *cycle* is a walk of at least length three with no repeated vertices except that the first vertex must equal the last (e.g. $v_i v_j v_i$ is a cycle, but $v_i v_i$ is not).[5] A digraph without a cycle is called *acyclic*.

A *semiwalk* is a sequence of vertices such that each sequential pair forms an edge without regard to direction (e.g. $v_i v_j$ is a walk if either (v_i, v_j) or (v_j, v_i) is an edge). Note that an "undirected edge" may still only be traversed once in a semiwalk. A digraph is called *weakly connected* if there is a semiwalk between any two vertices.[6]

[4]Since we are not considering digraphs with parallel edges, we have simplified the representation of a walk. Generally, the edges must be indexed, and a walk indicated by an alternating sequence of vertices and edges, with no edge appearing twice.

[5]Cycles are sometimes defined to include self-loops. The exclusion of self-loops is necessary for our formation definitions.

[6]Weak connectedness is usually defined in terms of "undirected paths." However, Theorem 1.2.1 of [9] states that every semiwalk from v_i to v_k also contains an undirected path from v_i to v_k.

For digraphs without *parallel edges*,[7] an $L \times L$ *adjacency matrix* $A = [a_{ij}]$ can be defined as follows:

$$a_{ij} = \begin{cases} 1 & \text{if } (v_i, v_j) \in E \\ 0 & \text{otherwise} \end{cases} .$$

With these definitions, the next two theorems from are used in the sequel.

Theorem 1: Every acyclic digraph has at least one vertex with zero in-degree and at least one vertex with zero out-degree.

Proof: See [3], page 230. □

Theorem 2: A digraph is acyclic if and only if its vertices can be ordered such that its adjacency matrix is upper (or lower) triangular.

Proof: See [3], page 231. □

B. Hierarchical Systems

Hierarchical systems are also used in the stability analysis of the Leader/Follower architecture. We first define hierarchical systems and then present a related stability theorem. The section summarizes material from [25] and [26].

A *hierarchical system* consisting of M subsystems can be written in the following form:

$$\dot{x}_i = f_i(t, x_1, \ldots, x_i) \tag{2}$$

where $i = 1, \ldots, M$, x_i is the state of the ith subsystem, and f_i describes the dynamics of the ith subsystem. Note that f_i can be nonlinear and time-varying.

Let n_i be the dimension of x_i and $\bar{x}_i = [\, x_1^T \cdots x_i^T \,]^T$. The ith *isolated subsystem* is given by

$$\dot{x}_i = f_i(t, 0, \ldots, 0, x_i) \,. \tag{3}$$

Finally, let $B_r \subseteq \mathbb{R}^p$ be a ball about the origin of radius r, where p is the appropriate dimension (e.g., $\bar{x}_i \in B_r$, implies $p = n_1 + \cdots + n_i$).

With these definitions, the next theorem is used subsequently.

Theorem 3: For the hierarchical system given in (2), assume:

(1) f_i is continuously differentiable,
(2) $f_i(t, 0, \ldots, 0) = 0$ for all $t \geq 0$, and
(3) there exist $r > 0$ and $\lambda < \infty$ for $i = 1, \ldots, M$ such that

$$\sup_{t \geq 0} \sup_{\bar{x}_i \in B_r} \left\| \frac{\partial}{\partial \bar{x}_i} f_i(t, \bar{x}_i) \right\| \leq \lambda \,.$$

Then $x = 0$ is a uniformly asymptotically stable equilibrium of the hierarchical system (2) if and only if $x_i = 0$ is a uniformly asymptotically stable equilibrium of the isolated system (3) for $i = 1, \ldots, M$.

[7]Parallel edges are edges that start and end at the same vertex. However, they do not apply to our analysis, and so we do not consider digraphs with parallel edges in this paper.

Proof: See [26], pg. 258. □

Remark: From [25], if condition (3) from Theorem 3 applies when $B_r = \mathbb{R}^p$, then $x = 0$ is the globally exponentially stable equilibrium of the hierarchical system if and only if $x_i = 0$ is the globally exponentially stable equilibrium of each isolated system.

III. Leader/Follower Formation Flying Control Architecture

A. Formation Flying Control

Before addressing the Leader/Follower architecture, we restate our definition of formation flying more compactly using the control dependency directed graph (or simply dependency digraph), which was introduced by the authors in [21].

The vertices of the dependency digraph represent the vehicles in the formation. A directed edge (v_i, v_j) is added to the digraph if the control action of vehicle j is a function of (i.e., depends on) any portion of the state of vehicle i.[8] This dependency can arise in three ways: (1) the control law used to generate u_j depends explicitly on the state z_i, (2) the reference trajectory d_j is a function of z_i, or (3) the control law used to generate u_j depends on the *feedback* control action of vehicle i, which in turn depends on z_i.[9] Note that every vertex will have a self-loop since each vehicle's control action will depend on its own state. The self-loops are omitted from the dependency digraph since they are redundant. Also note that the state z_i can include translational and rotational degrees of freedom of the vehicle. Finally, a digraph similar to the dependency digraph is defined in [25]. However, that digraph also allows state dependencies due to estimation, and it does not assume each subsystem given by z_i is independently actuated.

A corresponding closed-loop error dependency digraph can be defined based on the inter-vehicle dependencies in (1). An edge (v_i, v_j) is added if $h_j(\cdot)$ depends on e_i.

Using the control dependency digraph, formation flying may then be defined as follows. This is definition is identical to the one given in the Introduction.

Definition 1: A collection of N vehicles is *formation flying* if the corresponding control dependency directed graph is weakly connected.

B. Leader/Follower Architecture

Using the control dependency digraph introduced in the previous section, the Leader/Follower formation flying control architecture is simply defined.

Definition 2: A collection of vehicles formation flying is using a *Leader/Follower* architecture if the corresponding error dependency graph is acyclic.

[8]This formulation of an edge may seem anti-intuitive since an arrow from v_i to v_j implies v_j depends on v_i. However, this convention is chosen to match the convention used in the definitions of trees and arboresences in directed graph theory.

[9]In the literature, this third type of dependency has only been used in conjunction with one of the first two. Also, this dependency is highlighted in [16]. Finally, the third type can lead to algebraic loops if the control problem is not well posed.

In this definition, no distinction is made based on the control synthesis technique. Typically, an L/F controller is designed by interconnecting controllers designed for individual vehicles. However, an L/F controller may also result from a constrained multiple-input, multiple-output synthesis technique. Also, [4] and [16] use variations on acyclic digraphs to define more restricted forms of L/F. Definition 2 includes the definitions of L/F in these references.

For an edge (v_i, v_j) in the control dependency digraph, vehicle j is called a *follower*, and vehicle i is called a *leader*. In any L/F algorithm, there is at least one vehicle that does not follow another by Theorem 1 (i.e., at least one vertex will have zero in-degree). This vehicle is referred to as a *formation leader*. Note that a vehicle can have multiple leaders, but care must be taken in this case to ensure that the control problem is well posed.[10] Multiple formation leaders are also possible [28].

IV. General Stability Condition for the Leader/Follower Architecture

The asymptotic stability of an equilibrium of a system implies that *all* the states of the system are stable and approach the equilibrium. However, often only the relative states of a formation are directly meaningful. For example, a deep space multiple spacecraft interferometer may require centimeter level relative spacecraft positioning, but the inertial position of the entire formation may drift arbitrarily. Hence, in the following stability theorem for formations, the error dynamics of the formation leaders may be ignored. However, if this is done, the error states of the formation leaders must be treated as exogenous inputs that only depend on time (i.e., the formation leader error states are effectively a "disturbance").

Let $e = [\, e_1^T, \cdots e_N^T \,]^T$. The main result of this paper is the following theorem.

Theorem 4: For a formation using a Leader/Follower control architecture with error dynamics given by (1), if there exist $r > 0$ and $\lambda < \infty$ such that

$$\sup_{t \geq 0} \sup_{e \in B_r} \|\nabla_e h_i(t, e_1, \ldots, e_N)\| \leq \lambda,$$

and the origin of isolated error dynamics $\dot{e}_i = h_i(t, 0_1, \ldots, 0_{i-1}, e_i, 0_{i+1}, \ldots, 0_N)$ is asymptotically stable, then origin of the formation error dynamics is asymptotically stable.

Proof: By definition, the Leader/Follower closed-loop error dependency digraph is acyclic. By Theorem 2, the vehicles (vertices) can be renumbered so that the adjacency matrix of the closed-loop error dependency digraph is upper triangular. This in turn implies that the error dynamics can be represented as a hierarchical system. Finally, since the requisite

[10]The L/F algorithm discussed in [12] considers a chain of vehicles maintaining constant offsets. Except for the fleet leader, each vehicle uses the velocity of the vehicle immediately in front of it, and the acceleration and velocity of the single formation leader. In this case, each vehicle has two leaders.

assumptions are met, Theorem 3 guarantees the asymptotically stability of the origin of the error dynamics. □

Theorem 4 can be used to show the stability of the Leader/Follower control designs in [28] and [16], though it does not apply during the leader switching of [16]. This switching results in a reference trajectory that is not continuously differentiable. Theorem 4 also extends the stability results of [27] to include other than "tree" digraphs. This theorem also provides a theoretical framework for extending the stability of a specific control design between a single leader and a single follower spacecraft, such as in [2], [19] or [29], to an entire Leader/Follower formation.

V. Conclusions

By introducing the control dependency directed graph, both formation flying itself and the Leader/Follower formation control architecture were defined in a straightforward manner. The key aspect of a formation is that vehicle states are coupled through control laws. This coupling is such that for *any* vehicle either its control actions are dependent upon the state of another formation member, or a change in its state affects the control actions of another formation member.

The key aspect of the Leader/Follower control architecture is that the control dependency digraph is acyclic. As a result, an L/F formation is a hierarchical system: the control actions of leaders are independent of the followers. Further, the general stability theorem presented in this paper theoretically justifies the extension of single leader/single follower control laws to an entire formation. That is, L/F formation control design can be reduced to the design of a controller for a single follower, which is then copied as many times as necessary.

We have only used Theorem 2 to show that, through a renumbering of subsystem indices, an acyclic system is a hierarchical system. Specific algorithms for renumbering can be found in [25] and [24]. Also note that there are other forms of Leader/Follower-based stability such as string and mesh stability [20]. These types of stability apply further constraints (e.g. tracking errors do not grow as a disturbance propagates through a hierarchy).

The full version of this paper will include specific constraints on the structure of the vehicle dynamics, control laws and state reference trajectories so that the error dynamics have the form given in (1).

Acknowledgment

This research was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

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