Nonlinear Optics and Crystalline Whispering Gallery Mode resonators

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ABSTRACT

We report on our recent results concerning fabrication of high-Q whispering gallery mode (WGM) crystalline resonators, and discuss some possible applications of lithium niobate WGM resonators in nonlinear optics and photonics. In particular, we demonstrate experimentally a tunable third-order optical filter fabricated from the three metalized resonators; and report observation of parametric frequency doubling in a WGM resonator made of periodically poled lithium niobate (PPLN).

Keywords: Whispering Gallery Modes, Parametric Frequency Conversion, Tunable Optical Filter, High-Order Optical Filter, Periodically Poled Lithium Niobate

1. INTRODUCTION

High-Q whispering gallery modes (WGMs) (morphology-dependent resonances)[1–4] were first observed in liquid droplets,[5,6] as well as in solidified droplets of fused amorphous materials, such as fused silica.[7–11] Although those materials are characterized by small optical attenuation, the highest quality factor of WGMs has remained limited by Rayleigh scattering of residual surface roughness.[12] This is generally the case, even though a resonator formed by surface tension forces has nearly a defect-free surface characterized by molecular-scale inhomogeneities.

Liquids and amorphous materials form only a small part of high quality optical materials suitable for fabrication of WGM resonators. For instance, some crystals are transparent enough to sustain high-Q WGMs, on one hand; and are nonlinear enough, to allow us to manipulate continuously by the WGMs' characteristics, on the other.

Artificial crystals generally have high purity, high index of refraction, and the high stability of structure, and thus are ideal for fabrication of WG resonators. Ultimate Q-factors of WG resonators made of fused silica degrade in the atmosphere due to diffusion of atmospheric water into the material.[9] Water molecules, however, cannot diffuse into some crystals. Typically, crystals also have very low intrinsic absorption of light. For example, absorption of sapphire determined by light scattering due to imperfection of the crystalline structure is less than $\alpha = 1.3 \times 10^{-5}$ cm^{-1} at $\lambda = 1$ μm[13] which corresponds to $Q \simeq 8 \times 10^9$. Light absorption for the crystalline quartz may also be estimated using absorption in fused silica, which is extensively studied for fiber optic applications, with $\alpha \leq 5 \times 10^{-6}$ cm^{-1} at $\lambda = 1.55$ μm[14]; this corresponds to $Q \geq 1.2 \times 10^{10}$ for quartz.

The problem is how to produce WGM resonators with such materials. Melting is obviously not suitable for materials with crystalline structure, because it destroys the initial crystal purity and stoichiometry. Moreover, during solidification, the original spherical droplet of the melt turns into a rough body with multiple facets and crystal growth steps. Mechanical preparation looks unsuitable as well because any residual roughness would result in serious limitation on the achievable Q-factors. Hence, it is commonly believed that high-Q optical WGM resonators cannot be fabricated with transparent crystals.

Surprisingly, it was shown recently, that mechanical preparation results in fabrication of high-Q lithium niobate WGM resonators. Resonant interaction of light and microwaves was achieved using the resonators. A new kind of electro-optic modulator and photonic receiver based on this interaction was also suggested[15,16] and realized.[17–19]

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The efficiency of nonlinear optical processes increases with the increasing Q-factor of the modes of WGM resonators. The maximum quality factor of WG modes in lithium niobate resonator reported in[19] was less than 5×10^7 at $\lambda = 1.55\ \mu m$, which approximately corresponded to values expected from the intrinsic absorption of congruent lithium niobate cited by crystal producers ($\alpha \leq 5 \times 10^{-3}\ \mathrm{cm}^{-1}$). On the other hand, $Q \approx 2 \times 10^8$ at $\lambda = 2.014\ \mu m$ ($\alpha \leq 5 \times 10^{-4}\ \mathrm{cm}^{-1}$) was reported earlier for a MOTIRR – multiple total-internal-reflection resonator, analogous to a WGM resonator, used in optical parametric oscillators pumped at 1064 nm.[20] This suggests that the Q-factors achieved were limited by the fabrication process, and that the above values for the absorption of congruent lithium niobate may be inaccurate.

We found experimentally that it is possible to obtain crystalline WGM optical resonators with very high Q-factors ($Q > 10^9$), similar to that of surface-tension-formed resonators, by adopting simple polishing techniques. With this approach, the original crystal structure and composition is preserved, and the unique linear and nonlinear crystal properties are enhanced with the small volume of the high-Q resonator.[1–3,5–7,21] Total internal reflection at the walls of the WG resonators provides the effect of an ultra-broad band mirror, allowing very high Q-factors across the whole material transparency range. This property makes WG crystal resonators a unique tool for optical material studies. With our fabrication process,we have achieved Q-factor limited in value only by the absorption of the material.

We succeeded in fabricating toroidal lithium niobate WG mode resonators with quality-factors as high as $Q = 2 \times 10^8$ at $\lambda = 1.31\ \mu$m, using diamond polishing of the rim of flat disk preforms. The disks typically have radius of 0.75−7.5 mm and thickness of 0.05−1 mm. Our measurement lowers the experimentally achieved optical absorption floor of congruent as well as stoichiometric $LiNbO_3$ up to $\alpha \leq 4 \times 10^{-4}\ \mathrm{cm}^{-1}$ at that wavelength. We have also obtained $Q = 5 \times 10^7$ at $\lambda = 0.78\ \mu$m.

Preliminary experiments show that our polishing technique is applicable not only to the lithium niobate, but also to a variety of other crystalline materials, ranging from crystalline quartz to sapphire. For instance, we reached $Q = 2 \times 10^9$ at $\lambda = 1.31\ \mu$m in a sapphire resonator. Qs of the same order were obtained for other transparent crystals, and the details will be discussed in the follow-up publications.[22] This makes a crystalline WGM resonator a more general tool for measurement of small optical losses and other properties of crystals, as well as a versatile building block for novel experiments in quantum and nonlinear optics.

To illustrate possible applications of high-Q crystalline and, in particular, lithium niobate resonators, in nonlinear optics and photonics, we performed two experiments, results of which are reported in this paper. We fabricated a third-order tunable optical filter using three metal-coated $LiNbO_3$ disc WGM resonators. The filter have 29 MHz bandwidth and can be tuned in range of ±12 GHz by applying DC voltage in range of ±150 V to the metal coating. Because free spectral range of the resonators is approximately 13.3 GHz, the filter may be tuned practically at any optical frequency in the transparency range of lithium niobate. Large optical tuning was realized due to small thickness (50 μm) of the discs.

In our second experiment we demonstrated parametric frequency upconversion with periodically poled lithium niobate disc WGM resonator. Due to high Q-factor ($Q > 10^7$ for unloaded, and $Q > 5 \times 10^6$ for loaded resonator), we realized frequency doubling at 1550 nm with almost 50% efficiency at 25 mW pump power. The efficiency was restricted due to not optimal structure of the periodical poling of the resonator material, though the method allows to obtain much higher efficiency with proper poling. The follow-up studies of the parametric processes in WGM PPLN resonators are also important because it has been predicted that an optical parametric oscillator (OPO) based on a WGM PPLN resonator might have power threshold below a microWatt[23] – orders of magnitude less than that of the state-of-the-art OPOs, typically at 0.5 mW level.[24]

2. THIRD ORDER TUNABLE FILTER

One of obvious applications of optical resonators is fabrication of optical filters. Standing alone and coupled optical fiber resonators are widely used as such filters.[25,26] Newly developed WGM resonators are also promising for achieving that goal because of their small size, low losses, and integrability into optical networks.[27–34]

In particular, high-order optical filters with specific pass-band frequencies, high selectivity, large free spectral range, and wide tunability are desired. While the first three requirements could be fulfilled with small WGM resonators in Chebyshev or Butterworth designs and low insertion losses[27,35–37]; the tunability is quite difficult

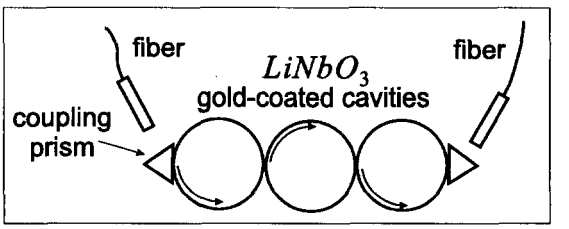

Figure 1. A scheme of a third order tunable optical filter made of gold-coated lithium niobate discs.

to achieve. For example, this conclusion is valid for fused silica microspheres, since the usual method for their fabrication is based on melting a fiber tip, and cannot produce a precise, predetermined geometry and, therefore, a precise resonance frequency. Mechanical trimming of WGMs with applied strain[38–40] and temperature[41] tuning have been previously used for the controlled tuning of the resonance frequency in WGM microresonators. All-optical tunable filter design based on discontinuity-assisted ring resonators was proposed theoretically,[42] but no experimental implementation of the configuration was reported so far, up to our knowledge.

Recently, a technique for WGM resonance trimming utilizing a photosensitive coating was used with microring resonators. In that study, glass microrings were dipped in a polymer coating material and were exposed to UV light. This method produced resonators with relatively small Q (about 800) because of the polymer-induced absorption; but it still allowed large tunability of the optical resonance of the microring, enough for wavelength selective applications.[43]

Another approach for trimming the frequency of microresonators exploits the photosensitivity of germanate silica glass.[44–46] When exposed to UV light, this material undergoes a small permanent change in structure that alters its index of refraction.[44] This property is used in writing fiber Bragg gratings.[47] In the case of a WGM resonator, the spatially uniform change in the index of refraction results in a uniform translation of the resonant frequencies of a WGM microresonator. Such a tunable resonator as well as a second-order optical filter based on two coupled resonators, one of which was tunable, was experimentally realized for high-Q (10^8) WGMs.[48–50]

We here report on the realization of a miniature resonant electro-optically tunable Butterworth third-order filter. Our filter is based on three gold-coated disc WGM cavities fabricated from a commercially available lithium niobate wafer (See Figs. 1 and 2). The filter, operating at the 1550 nm wavelength, is further development of the tunable filter design based on single lithium niobate resonator.[50,51]

While tunable single-resonator filters are characterized by their finesse which is equal to the ratio of the filter free spectral range and the filter bandwidth, our three-resonator filter has much more rare spectrum compared with a standing alone WGM resonator, similarly to the coupled fiber-ring resonators.[25,26] The tuning speed of the filter is approximately 10 ns, while the real spectrum shifting time is determined by filter's bandwidth and does not exceed 30 μs. We observed at least -40 dB suppression of the channel cross-talking rate for 50 MHz channel spacing. For comparison, a conventional Fabry-Perot tunable filter may have a finesse of 100, 125 GHz bandwidth, and tuning speed in a millisecond range. Fabry-Perot filters also meet -20 dB channel-to-channel isolation condition for 50 GHz channel spacing.[52,53]

The differences in the size of the cavities is rather important for the device fabrication. Our aim was to produce spectral lines of all the resonators of a similar width to allow the realization of a complex spectral line structure. If resonances of the interacting cavities have differing widths, then as they are made to approach each other, the height of the narrower resonance will simply track the shape of the wider ones, which is of no use for the filter application. The size of a cavity affects the quality of its resonance since cavities of similar size have similar quality factors. Our experiment proves that we are able o fabricate similar resonators approximately with the same parameters.

Fig.3 depicts spectrum obtained in the experiment with three gold-coated lithium niobate resonators. To highlight the filter performance we plotted also the theoretical third-order Butterworth fit of the curve. Obviously, the three-cavity filter has much faster rolloff compared with the Lorentz line of the same full width at the half

Figure 2. A picture of the filter. Light is introduced into and extracted from the gold-coated WGM resonators with diamond prism couplers. Optical fibers (not shown in the picture) are connected to optical collimators to minimize insertion losses.

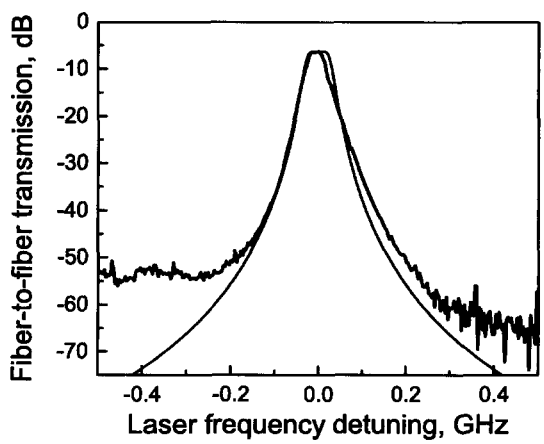

Figure 3. Transmission curve of the filter and its fit with Butterworth profile function $\tilde{\gamma}^6/[(\nu - \tilde{\nu}_0)^6 + \tilde{\gamma}^6]$, where $\tilde{\gamma} = 29$ MHz, $\tilde{\nu}_0$ determines the center of the filter function and primarily depends on the resonators' geometrical dimensions. Voltages applied to the resonators vary near zero in 10 V range to properly adjust frequencies of each individual resonator and construct the collective filter function as shown.

maximum. On the other hand, the filter function does not look exactly like third order one because of small differences between the cavity Q-factors and dimensions.

Transmission and reflection of a monochromatic electromagnetic wave of frequency ν by a WGM optical lossless resonator may be characterized by coefficients

$$T = \frac{\gamma}{\gamma + i(\nu - \nu_0)}, \quad R = \frac{i(\nu - \nu_0)}{\gamma + i(\nu - \nu_0)}, \tag{1}$$

where T and R describe the amplitude transmission and reflection respectively, γ and ν_0 are the linewidth and

resonance frequency of a mode of the resonator (we assume that $|\nu - \nu_0|$ is much less than the cavity free spectral range). The power transmission $|T|^2$ through the resonator is Lorentzian.

When three cavities are placed in series, the light amplitude transmission coefficient is

$$T = \frac{T_1 T_2 T_3}{(1 - R_1 R_2 \exp[i\psi_{12}])(1 - R_2 R_3 \exp[i\psi_{23}]) - R_1 R_3 |T_2|^2 \exp[i(\psi_{12} + \psi_{23})]}, \tag{2}$$

where T_j and R_j ($j = 1, 2$) are the transmission and reflection coefficients of the resonators, and ψ_{jk} is the phase shift introduced by the coupling of resonators j and k.

Let us consider the case of resonators with slightly different resonance frequencies and the values of linewidth, and assume that $\exp(i\psi_{jk})$ are properly adjusted. Under optimum conditions[27] power transmission through the system is

$$|T|^2 \simeq \frac{\tilde{\gamma}^6}{\tilde{\gamma}^6 + (\nu - \tilde{\nu}_0)^6}, \tag{3}$$

where $\tilde{\gamma}$ is the bandwidth of the filter, and $\tilde{\nu}_0$ is the central frequency of the filter. The transmission through the resonator system is small for any frequency when the resonant frequencies of the modes are far from each other ($|\nu_j - \tilde{\nu}_0|^2 \gg \tilde{\gamma}^2$). The transmission becomes close to unity when the mode frequencies are close to each other compared with the modes' width γ. Finally, the transmission for the off resonant tuning is inversely proportional to ν^6, not to ν^2, as for a single resonator, Lorentzian, filter. Those are the properties of the third order filters; they much our experimental observations.

A schematic diagram of the tunable filter configuration is shown in Fig.1. A Z-cut disk resonators have in 3.3 mm diameter and 50 μm in thickness.[50] The resonator perimeter edge was polished in the toroidal shape with 40 μm curvature radius. We studied several nearly identical disks. The repeatable value of the loaded quality factor of the main sequence of the resonator modes we used was $Q = 5 \times 10^6$ (the observed maximum was $Q = 2 \times 10^8$), which corresponds to 30 MHz bandwidth of the mode.

Light was sent into, and retrieved out of, the resonator via coupling diamond prisms. The repeatable value of fiber-to-fiber insertion loss was approximately 6 dB. The maximum transmission was achieved when light was resonant with the resonators' modes. Tuning of the filter was realized by applying voltage to the top and bottom disks' surfaces coated with gold. The coating is absent on the central part of the resonator perimeter edge where WGMs are localized.

The maximum frequency shift of the TE and TM mode may be found from[54]

$$\Delta\nu_{TE} = \nu_0 \frac{n_e^2}{2} r_{33} E_Z, \quad \Delta\nu_{TM} = \nu_0 \frac{n_o^2}{2} r_{13} E_Z, \tag{4}$$

where $\nu_0 = 2 \times 10^{14}$ Hz is the carrier frequency of the laser, $r_{33} = 31$ pm/V and $r_{13} = 10$ pm/V are the electro-optic constants, $n_e = 2.28$ and $n_o = 2.2$ are the refractive indexes of $LiNbO_3$, E_Z is the amplitude of the electric field applied along the cavity axis. We worked with TM modes because they have better quality factors than TE modes. If the quality factor is not very important, it is better to work with TE modes because their electro-optic shifts are three times as much as those of TM modes for the same values of applied voltage.

Experimentally measured electro-optic tuning of the filter spectral response and tuning of center wavelength with applied voltage is shown in Fig.4. The filter exhibits linear voltage dependence in ± 150 V tuning range, i.e. the total tuning span exceeds the FSR of the resonator. Changing the tuning voltage from zero to 10 V shifted the spectrum of the filter by $1.3 - 0.8$ GHz for TM polarization, in agreement with theoretical value. Though, theoretically, this value does not depend on the resonator properties and is related to the fundamental limitations of optical resonator based high speed electro-optical modulators,[55] the different results for different resonators measured in our experiment occur due to imperfectness of the cavity metal coating as well as due to partial destruction of the coating during polishing procedure.

The insertion loss in our setup occurs primary due to inefficient coupling to the WGM. We believe that antireflection coating of the coupling prisms or use of special gratings placed on high-index fibres may reduce the losses significantly.

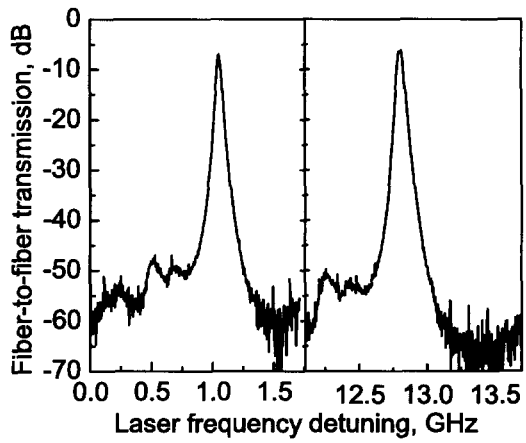

Figure 4. Transmission curve of the filter and the same curve reconstracted at, approximately, 12 GHz frequency shift from the initial transmission curve center ν_0. Optimum voltages are different for all the resonators and are scaled from 100 V to 150 V were applied to the discs.

3. FREQUENCY DOUBLING WITH WGM PPLN RESONATOR

It is known that WGM resonators are unique because of their ability to store light in microscopic spatial volumes for long periods of time, resulting in strong enhancement of nonlinear interactions of various kinds.[1–3, 5–7, 21] As an example of the potential of the WGM crystalline resonators for nonlinear optics applications, we demonstrated parametric frequency doubling in a WGM resonator made of periodically poled lithium niobate.

We achieved efficient frequency doubling at $\lambda = 1.55$ μm and $\lambda = 1.319$ μm using the same WGM resonator made of periodically poled $LiNbO_3$ (PPLN).[56] We found that the maximum Q-factors are the same for both the PPLN resonators and our original congruent lithium niobate resonators, $Q = 2 \times 10^8$ at $\lambda = 1.319$ μm, – proving that the periodical poling does not change linear absorption of the material.

Frequency conversion may be realized when energy and momentum conservation (phase matching condition) for photons are fulfilled during the nonlinear interaction. For example, the energy conservation law gives a trivial condition for frequency doubling

$$\lambda_p = 2\lambda_s, \tag{5}$$

where λ_p and λ_s are the pump and signal wavelengths in vacuum. On the other hand, momentum conservation is not generally satisfied in crystals: $2k_p - k_s \neq 0$, where k_p and k_s are wave vectors of the pump and signal waves. This problem can be handled using a quasi-phase matching technique,[56] which is based on producing a modulation of the nonlinear susceptibility of the material with a period Λ such that

$$k_s - 2k_p = \frac{2\pi}{\Lambda} \rightarrow \Lambda = \frac{\lambda_s}{n_s - n_p}, \tag{6}$$

where n_s and n_p are indices of refraction for the signal and pump light. Spatial modulation of nonlinear susceptibility may be obtained by local flipping of the sign of the spontaneous polarization, i.e. by flipping the spins in ferroelectric domains. From the point of view of optical nonlinearity, oppositely polarized domains exhibit opposite signs of second-order susceptibility.[56]

To achieve quasi-phase matching for parametric frequency doubling in a WGM resonator one should take into account not only the frequency dependent dispersion of the host material of the dielectric resonator, but also the dispersion introduced by the internal geometrical mode structure. Modes inside a WGM resonator are localized close to the resonator rim. The shorter the wavelength, the closer is the mode to the resonator surface, and

Figure 5. A picture of a whispering gallery mode resonator made of a periodically poled lithium niobate wafer.

the longer is the mode path. This geometrical property of WG modes significantly changes the phase matching condition compared with bulk material.

The frequency of the main sequence of high order TE WG modes may be estimated from

$$\frac{2\pi R}{\lambda}n(\lambda) + \sqrt{\frac{n^2(\lambda)}{n^2(\lambda)-1}} \simeq \nu + \alpha_q \left(\frac{\nu}{2}\right)^{1/3} + \frac{3\alpha_q^2}{20}\left(\frac{2}{\nu}\right)^{1/3}, \tag{7}$$

where λ is the wavelength in vacuum, ν is the mode order, $n(\lambda)$ is the wavelength dependent index of refraction, R is the radius of the resonator, and α_q is the qth root of the Airy function, $Ai(-z)$, which is equal to 2.338, 4.088, and 5.521 for $q = 1, 2, 3$, respectively.[57] The phase matching condition (6) should be rewritten here as

$$\nu_s - 2\nu_p = \nu_\Lambda, \tag{8}$$

where ν_s, ν_p, and ν_Λ are numbers of signal, pump, and the nonlinearity modulation modes.

We used a WGM resonator fabricated from a commercial periodically poled flat Z-cut $LiNbO_3$ substrate, with TE modes corresponding to the extraordinary waves in the material (Fig.5). The resonator has a radius of $R = 1.5$ mm and thickness of $d = 0.5$ mm. The curvature of the rim is approximately 1.2 mm. The lithium niobate substrate is periodically poled with $\Lambda = 14$ μm period. The poling is made in stripes, as schematically shown in the left part of Fig.6.

WG modes encounter many periods of nonlinearity modulation, not just a single period, because they are localized close to the surface of the resonator. This provides us with the opportunity to achieve frequency doubling at a wide range of frequencies including those that do not correspond to the original planar poling period. Indeed, for frequency doubling at $\lambda = 1.55$ μm we have $n_s = 2.179$, $n_p = 2.138$, $\Lambda_{bulk} = 19$ μm, $\Lambda_{resonator} = 17.9$ μm (i.e. $\nu_{1.55} = 526$), and for doubling $\lambda = 1.319$ μm we find $n_s = 2.145$, $n_p = 2.197$, $\Lambda_{bulk} = 12.8$ μm, $\Lambda_{resonator} = 12.24$ μm (i.e. $\nu_{1.319} = 770$). We found that, for the resonator that we are using in the experiment, the amplitude of the corresponding harmonics of the nonlinearity of the poled crystal are $\chi^{(2)}_{1.55} \simeq 1.6 \times 10^{-3}\chi^{(2)}\cos(\nu_{1.55}\phi)$ and $\chi^{(2)}_{1.319} \simeq 8 \times 10^{-5}\chi^{(2)}\cos(\nu_{1.319}\phi)$, where $\chi^{(2)}$ is the nonlinearity of the material. We observed doubling for both of these frequencies.

The experimental setup is shown in Fig.7. Light from a pigtailed pump laser is sent into a WGM PPLN resonator through a specially designed collimator and a diamond prism coupler. The coupling efficiency exceeds 50%, i.e. more than a half of the pump power enters the cavity. The cavity output is sent through a lens

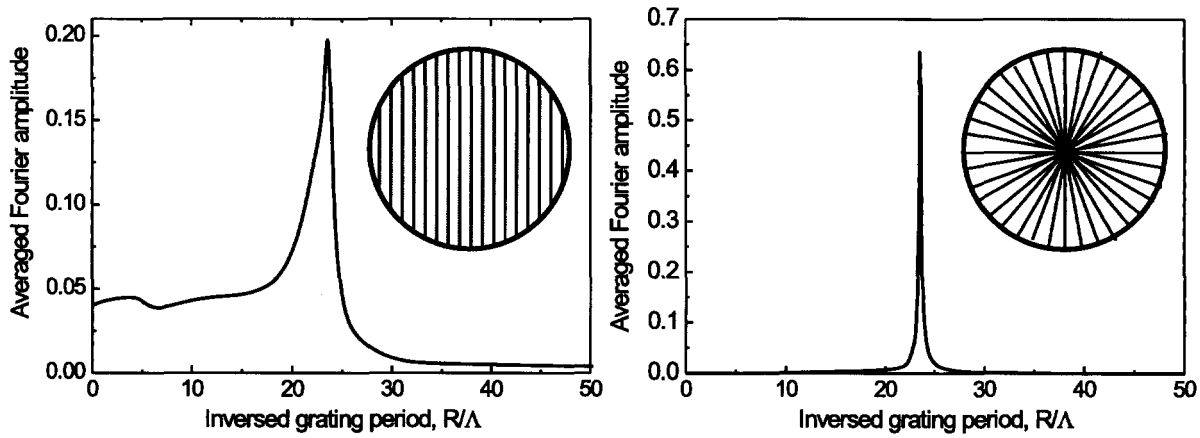

Figure 6. Scheme of two possible periodical poling structures ("striped" and "flower-like") for a lithium niobate WGM resonator and the corresponding spatial harmonics of the nonlinearity of the poled crystal.

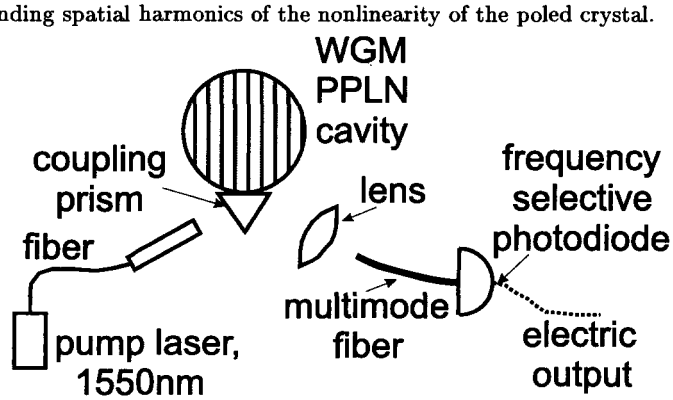

Figure 7. Scheme of the experimental setup.

to a multimode optical fiber connected to a selective photodiode, which cuts off the leftovers of the pump radiation, entering the multimode fiber along with the generated signal. We calibrated the measurement data and subtracted the losses due to unperfect optical links in the system.

The absorption spectrum of the resonator measured by frequency scanning of the pump laser at 1.55 μm and the emission spectrum for second harmonic at 775 nm, taken simultaneously, are shown in Fig. 8.

It is interesting to note that phase matching conditions are not periodic with the resonator free spectral range (FSR). The resonator FSR (13.6 GHz) is easily identifiable through periodic pattern of the absorption peaks in Fig. 8. Only one mode for the second harmonic is efficiently coupled to the pump, within more than 80 GHz scan of pump laser - roughly six times the free spectral range. The bandwidth of the second harmonic nonlinear resonance, in units of pump laser frequency tuning, is approximately 5 MHz.

The efficiency of the frequency doubling at $\lambda = 1.55$ μm if shown in Fig. 9. The maximum signal power observed in the experiment was approximately 12.3 mW. The pump power at this point was 25 mW at the input port of the resonator. For the frequency doubling at $\lambda = 1.319$ μm the maximum efficiency was 2×10^{-2} for the pump power available (30 mW). This relatively small efficiency at 1.319 μm may be explained by the small effective value of nonlinearity, or in other words, the poor efficiency of the chosen grating poling period.

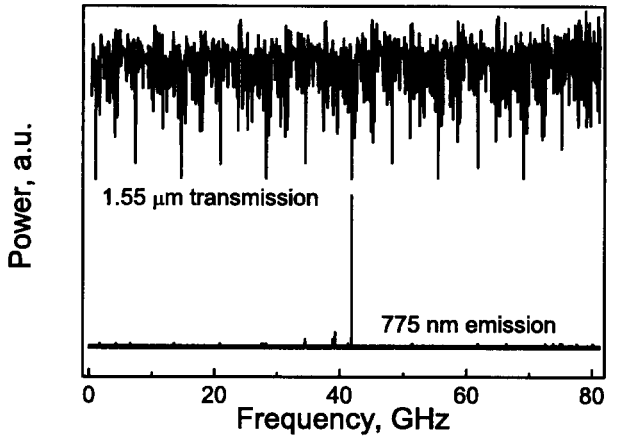

Figure 8. Transmission and emission spectra of the resonator.

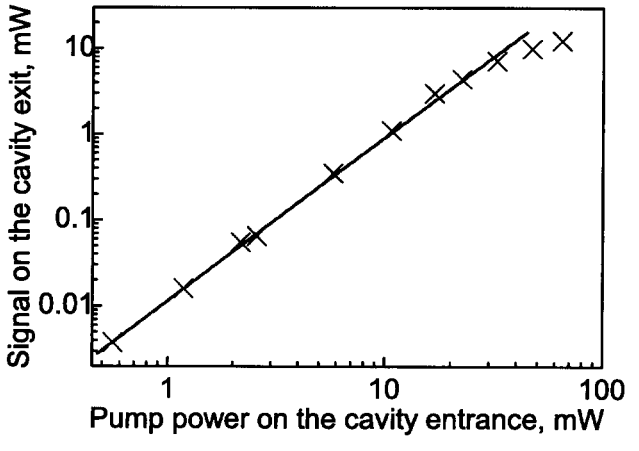

Figure 9. Conversion efficiency of the frequency doubling process as function of normalized input pump power.

The conversion efficiency of the frequency doubling may be found theoretically and compared with the experiment. The interaction Hamiltonian in slowly varying amplitude and phase approximation is

$$H = \hbar g((a_p^\dagger)^2 a_s + a_s^\dagger a_p^2), \tag{9}$$

where the coupling constant is

$$g = 2\pi\omega_p \frac{\chi_{1.55}{}^{(2)}}{\epsilon_p} \frac{V_{spp}}{V_p} \sqrt{\frac{2\pi\hbar\omega_s}{\epsilon_s V_s}}, \tag{10}$$

$V_{spp} = \int_V \Psi_s \Psi_p^2 dV < (V_p, V_s)$ is the mode overlap integral, a_p ($a_p^\dagger$) and a_s ($a_s^\dagger$) are mode annihilation (creation) operators for the signal and pump electromagnetic field in the resonator modes.

Using this Hamiltonian we derive equations of motion

$$\dot{a_s} = -\gamma_s a_s - i g a_p^2 + F_s, \tag{11}$$

$$\dot{a_p} = -\gamma_p a_p - 2 i g a_p^\dagger a_s + F_p, \tag{12}$$

where F_p, F_s are the Langevin forces, γ_p and γ_s are pump and signal decay rates respectively. The expectation value $\langle F_p \rangle$ describes pumping from outside of the system. We can write the expression $|F_p|^2/\gamma_p^2 = 4W_p Q_p/(\hbar\omega_p^2)$, where $Q_p = \omega_p/(2\gamma_p)$ is the mode quality factor, W_p is the power of the pump radiation (in vacuum).[23]

Solving Eqs. (11) and (12) in steady state and neglecting quantum fluctuations we find the photon number for the second harmonic generated in the system and derive the output power of the signal with respect to the pump power

$$\frac{W_{s\ out}}{W_{p\ in}} = \frac{6}{\mathcal{S}} \left[(1 + \mathcal{S} + \sqrt{\mathcal{S}(1+\mathcal{S})})^{1/3} + (1 + \mathcal{S} - \sqrt{\mathcal{S}(1+\mathcal{S})})^{1/3} - 2 \right]^2 \tag{13}$$

where $\mathcal{S} = 54 W_{p\ in}/W_0$ is the saturation parameter, and

$$W_0 = \frac{n_p^4 n_s^2}{64\pi^3 \left[\chi_{1.55}^{(2)}\right]^2} \left(\frac{V_p}{V_{pps}}\right)^2 \frac{\omega_p V_s}{Q_p^2 Q_s} \tag{14}$$

is the saturation power.

For our experimental parameters ($V_{pps}/V_p = 0.3$, $V_s \approx 2\pi R \times 2R(2\pi/\nu_s)^{1/2} \times (R/\nu_s^{2/3}) = 10^{-6}$ cm^3, $n_p = 2.138$, $n_s = 2.179$, $\chi_{1.55}^{(2)} = 7 \times 10^{-10}$ CGS, $\omega_p = 2 \times 10^{15}$ s^{-1}, loaded Q-factors: $Q_s \simeq 8 \times 10^6$ and $Q_p \simeq 1.2 \times 10^7$) calculated value of the saturation power is 190 mW. The 300 mW value for the saturation power was derived from our experimental data. It can easily be seen that increasing the Q-factors of the modes and producing exact periodical poling for the appropriate wavelengths will result in orders of magnitude reduction of the required pump power, while keeping the high conversion efficiency intact.

The conversion efficiency in our experiment saturates faster than the theoretical prediction. We believe this effect is due to temperature fluctuations. We were unable to keep the temperature stable for the applied high laser power. Even a small amount of heat in the nonlinear resonator moves the modes and shifts the resonator to a regime where phase matching is not fulfilled.

Our experiments clearly show that we can fabricate high-Q optical WG resonators from crystals using relatively simple mechanical polishing techniques. This manner of resonator preparation does not destroy the crystalline structure, nor does it affect the domain pattern that is important for quasi-phase matching of the fundamental and second harmonics supported by WG modes in the resonator. Furthermore, domain boundaries do not induce any additional optical loss in periodically poled material, allowing a quality-factor $Q = 2 \times 10^8$ to be achieved, both in PPLN and in congruent lithium niobate resonators.

4. CONCLUSION

In this paper we show that it is possible to produce crystalline whispering gallery mode resonators with Q-factors of the same order as the maximal Q-factors of fused silica resonators ($Q > 10^9$). This opens opportunities for both fundamental science and engineering.

For instance, we have demonstrated a coupled system of three metalized $LiNbO_3$ resonators. Such a system is basically a tunable third order filter with a sharp roll-off. The technique may also be used to produce other tunable complex filter functions with any desired line shapes. The filter may be utilized for high-density telecommunication networks, and in RF photonics applications.

As an example of application of the cavities in nonlinear optics, we realized frequency doubling experiment with a PPLN WGM resonator. Our experiment proves that the nonlinear optics with high-Q WG resonators is important and interesting because it results in interaction of light with tens and hundreds of meters of a material. It is practically impossible to fabricate a high quality stoichiometric crystal of this size. With WG mode resonators, however, surface reflection does the trick: for light circulating inside the millimeter-size resonator, the crystal appears to be very long.

Our experimental results are in an agreement with theory. The third-order tunable optical filter and high efficiency nonlinear parametric frequency converter discussed above are only the beginning of application of crystalline WGM cavities in optics.

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